Discussion 5

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Interval Estimation

- An interval estimator for θ is defined by two random variables $[\hat{\theta}_L, \hat{\theta}_U]$, i.e.

$$\mathbb{P}(\hat{\theta}_L \leq \theta \leq \hat{\theta}_U) = 1 - \alpha$$

  where α is a called the significance level.

- **Pivotal method**
  1. A pivot Q is a function of the sample measurements and θ.
  2. The pdf of Q does not depend on the parameter θ.

- The idea:

$$\mathbb{P}(a \leq Q \leq b) = 1 - \alpha$$
$$\Rightarrow \mathbb{P}(\hat{\theta}_L \leq \theta \leq \hat{\theta}_U) = 1 - \alpha$$

  via some algebraic transformation.

Large-sample confidence intervals

- Pivotal method based on CLT.
- If the target parameter θ is μ, p, $\mu_1 - \mu_2$, or $p_1 - p_2$, then for large samples,
$$Z = \frac{\hat{\theta} - \theta}{\sigma_{\hat{\theta}}} \approx N(0, 1).$$
- In this case we have that the (approximated) endpoints for a $100(1 - \alpha)\%$ confidence interval for θ are given by
$$\hat{\theta}_L = \hat{\theta} - z_{\alpha/2}\sigma_{\hat{\theta}}, \ \hat{\theta}_U = \hat{\theta} + z_{\alpha/2}\sigma_{\hat{\theta}}$$

Sample Size Calculation

- Set up

$$W > \text{Margin of Error (Error of Estimation)} \Longrightarrow n \geq ?$$

A sample size should be a whole number.

- Example
  - Let $X_1, \ldots, X_n \sim i.i.d. N(\mu, \sigma^2)$ The two-sided $1-\alpha$ confidence interval for mean, μ, is

$$\left[\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right].$$

  - $\frac{\sigma}{\sqrt{n}} z_{\alpha/2}$ is called a margin of error.
  - If we want mean μ to be within w standard units away from the estimate $\bar{X}$, we can set

$$w \geq z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \Longrightarrow n \geq \frac{\sigma^2}{w^2} z_{\alpha/2}^2.$$

Example

- Let $X_1, \ldots, X_n \sim i.i.d. Unif(0, \theta)$. Find a two sided confidence interval for the parameter θ.