- The random variables $Y_1, Y_2, \cdots, Y_n$ are called a **random sample** from the population distribution $f_Y(y)$ if
    1. $Y_1, Y_2, \cdots, Y_n$ are **mutually independent**.
    2. The marginal pdf (pmf) of each Y_i is the same function $f_Y(y)$.

- Suppose $Y_1, \cdots, Y_n$ from $f(y;\theta)$ is a random sample. Then, the quantity

$$T = T(Y_1, \ldots, Y_n)$$

  is called a **statistic** if it depends only on the random sample. i.e., T cannot depend on the unknown parameter θ.

- The probability distribution of T is called its **sampling distribution**.

$$T = T(Y_1, \cdots, Y_n) \sim f_T(t).$$

Moment Generating Function (MGF)

Moment

- rth moment: $E(X^r),\ r = 1, 2, \cdots$
- rth central moment: $[E(X - \mu)^r],\ r = 1, 2, \cdots$

Moment Generating Function (MGF)

$$m_X(t) = E(e^{tX})$$

provided that the expectation is finite for $|t| < a$ with some $a > 0$.

- All moments can be generated from MGF if MGF is finite.
- If a random variable X has a finite MGF m_X^t, then the rth Moment is obtained

$$E(X^r) = m_X^{(r)}(0)$$

Gamma Distribution $X \sim Gam(\alpha, \beta)$

$$f(x \mid \alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}, \text{ for } x > 0, \alpha > 0, \beta > 0$$

- $E(X) = \alpha\beta$
- $V(X) = \alpha\beta^2$
- $m_X(t) = (1 - \beta t)^{-\alpha}$ for $t < \frac{1}{\beta}$
- $\text{Exp}(\beta) = Gam(1, \beta)$
- $\chi^2_n = Gam(\frac{n}{2}, 2)$