Discussion 6

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Properties of Point Estimators and Methods of Estimation

- Properties
  1. Unbiasedness - MSE, Relative Efficiency
  2. Consistency
  3. Sufficiency

- Methods
  1. Methods of Moment (Mom) Estimator
  2. Maximum Likelihood (ML) Estimator
  3. MVUE : Rao-Blackwell Thm

Unbiased Estimator

- Let $\hat{\theta}$ be an estimator for a parameter θ. Then $\hat{\theta}$ is an *unbiased estimator* if
$$\mathbb{E}[\hat{\theta}] = \theta.$$

- Bias
$$\mathbb{B}(\hat{\theta}) = \mathbb{E}[\hat{\theta}] - \theta.$$

- How do we choose the best estimator among unbiased estimators? $\Rightarrow$ *MSE*
$$\mathrm{MSE}_\theta(\hat{\theta}) = \mathbb{E}[(\hat{\theta} - \theta)^2] = \mathbb{V}[\hat{\theta}] + [\mathbb{B}(\hat{\theta})]^2.$$

Example

Let $X_1, X_2, \ldots, X_n$ denote a random sample from a uniform distribution on the interval $(0, \theta)$.

1. Find an unbiased estimator of θ based on a sample mean.
2. Find an unbiased estimator of θ based on a $X_{(n)}$.
3. Compare the two estimators for θ.