

# Analysis-suitable parameterization for isogeometric analysis

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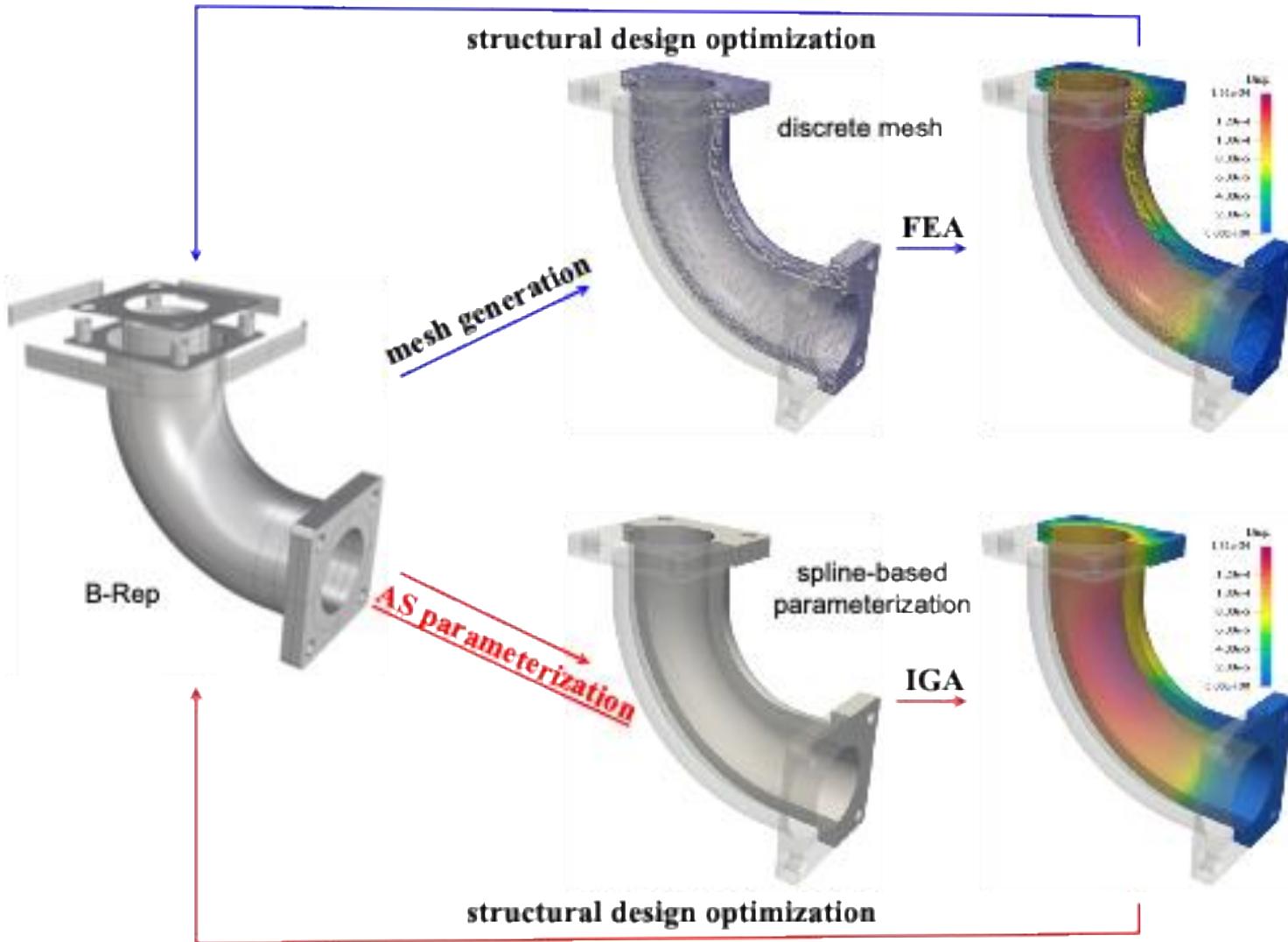
导师: 朱春钢教授(大连理工大学)、Matthias Möller (TU Delft)

# Outline

- 1. Background and motivation**
- 2. Overview of the algorithms**
- 3. Applications**
- 4. Conclusions and outlook**

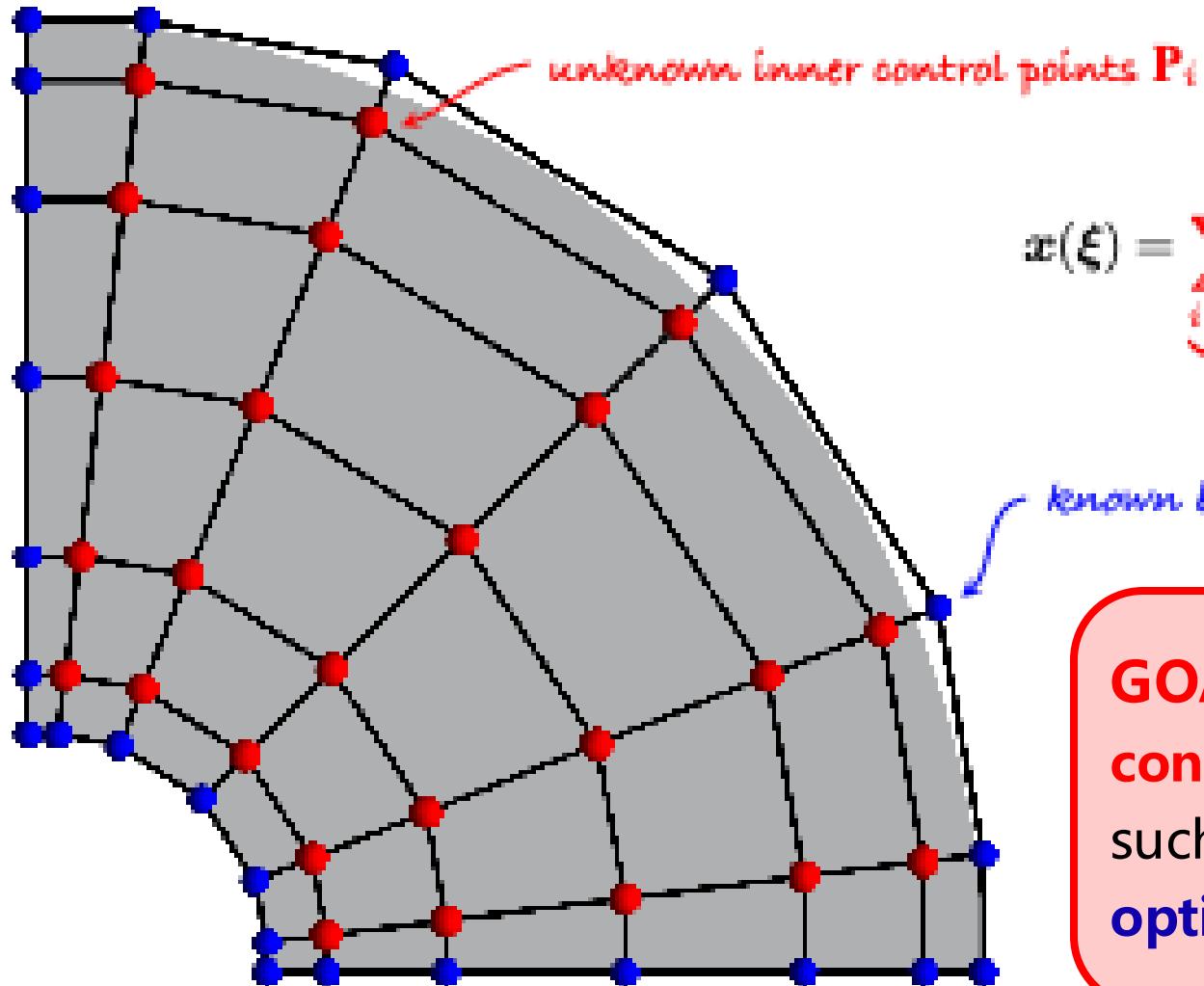
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Design-Through-Analysis product development workflow

- Introduced by T.J.R. Hughes et al., 2005.
- **Key Idea:** Approximate physical fields with **the same basis functions** used in CAD model representation.
- Benefits:
  - **Unified design and analysis;**
  - **Precise, efficient geometry;**
  - **No data type conversion;**
  - **Simplified mesh refinement;**
  - **Continuous high-order fields;**
  - **Superior approximation.**
- Broad applications: shell analysis, fluid-structure interaction, shape and topology optimization, etc.



$$x(\xi) = \underbrace{\sum_{i \in I_r} P_i R_i(\xi)}_{\text{unknown}} + \underbrace{\sum_{j \in I_B} P_j R_j(\xi)}_{\text{known}}$$

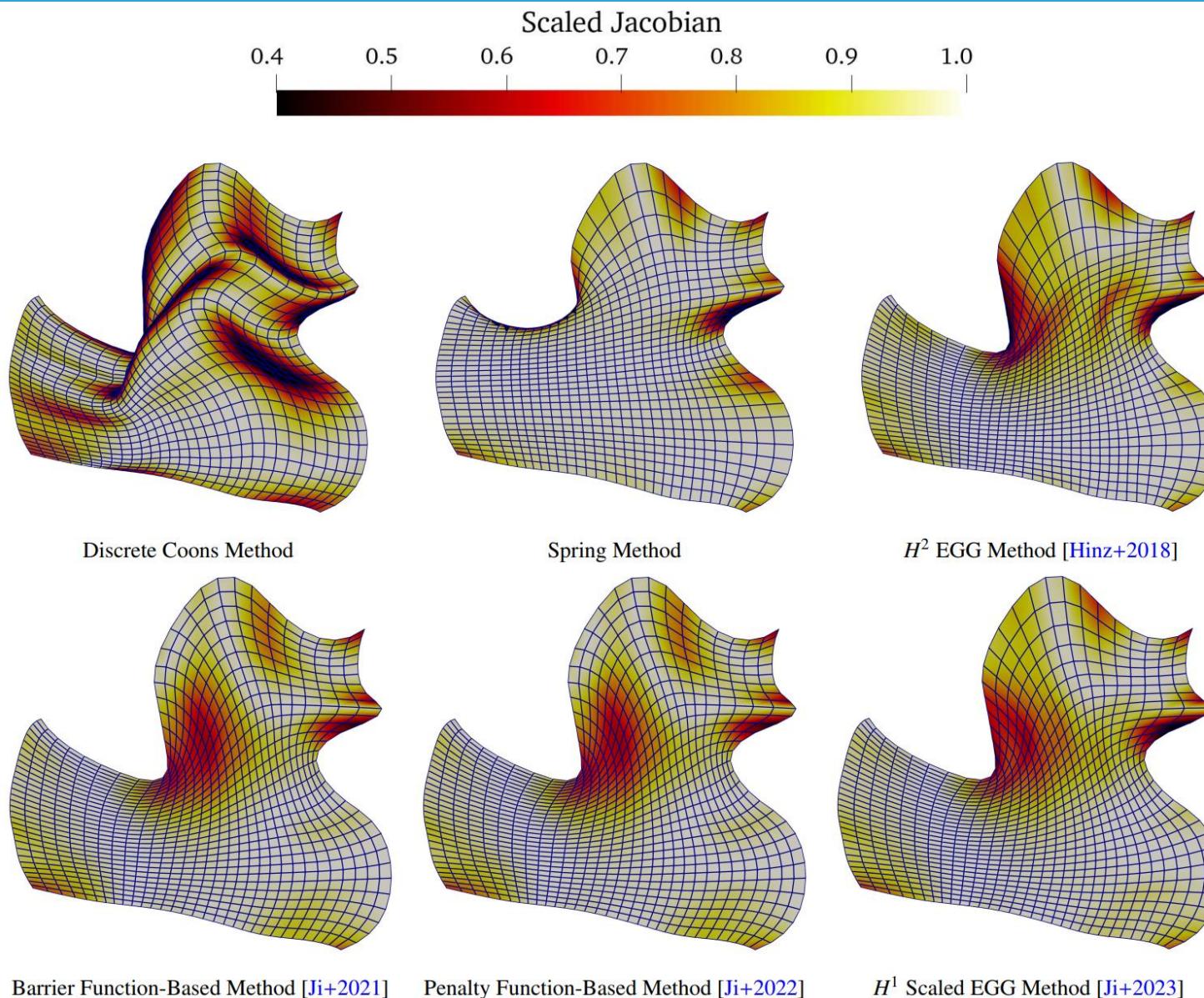
known boundary control points  $P_j$

**GOAL:** to construct **unknown inner control points  $P_i$**  (or basis functions  $R_i$ ) such that  $x$  ensures **bijectivity** and exhibits **optimal orthogonality** and **uniformity**.

➤ Parameterization quality significantly affects downstream analysis!

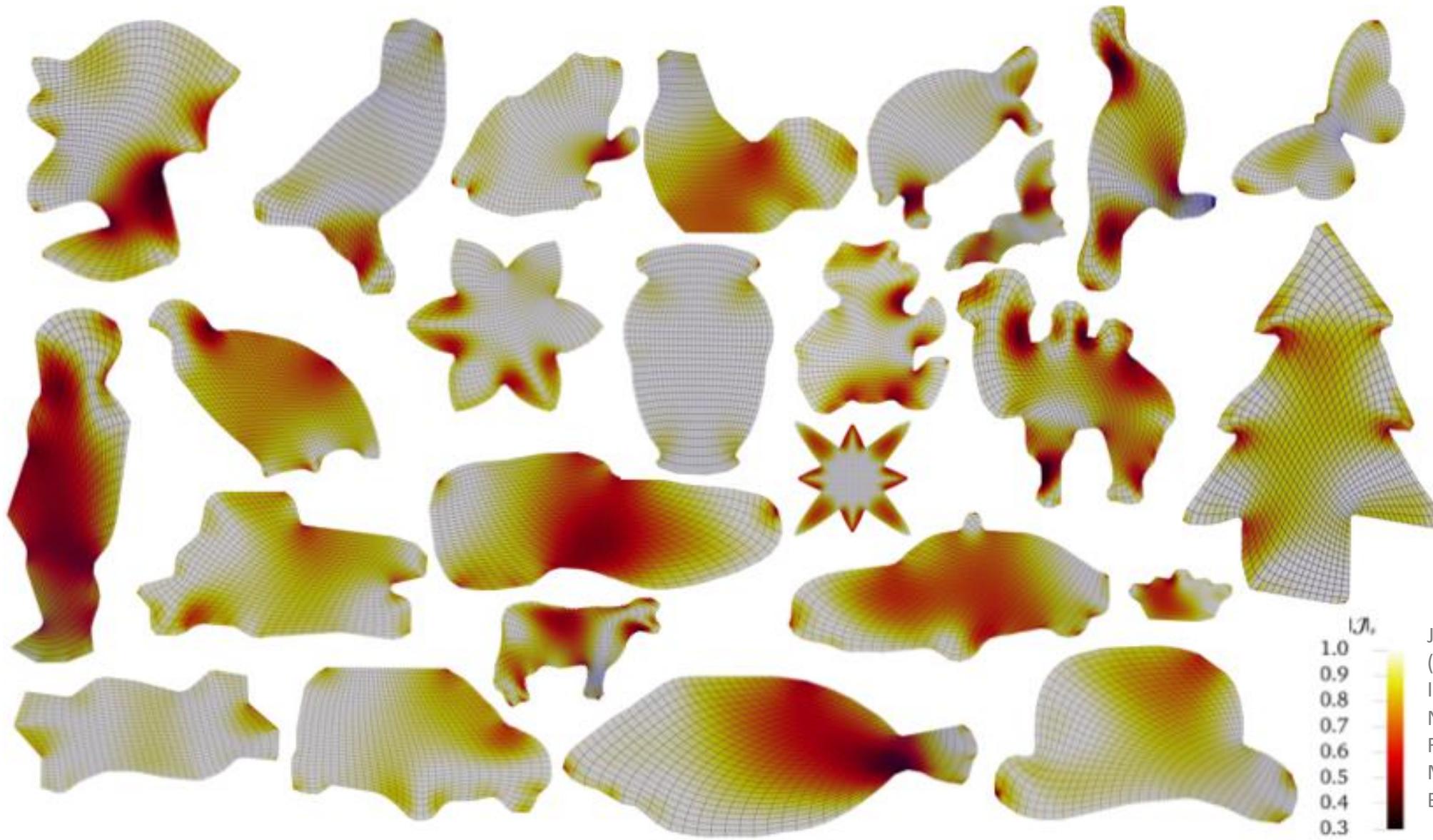
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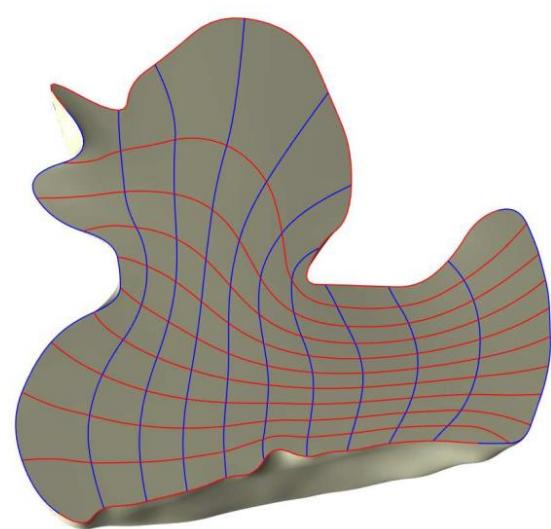
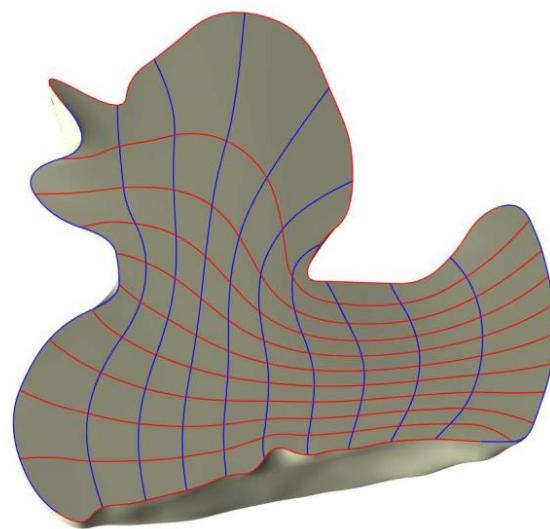
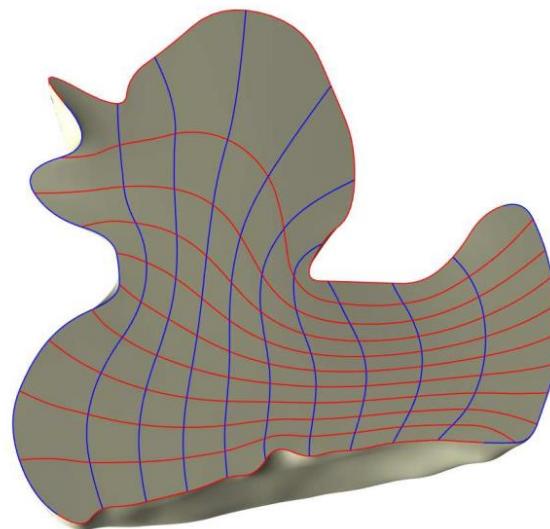
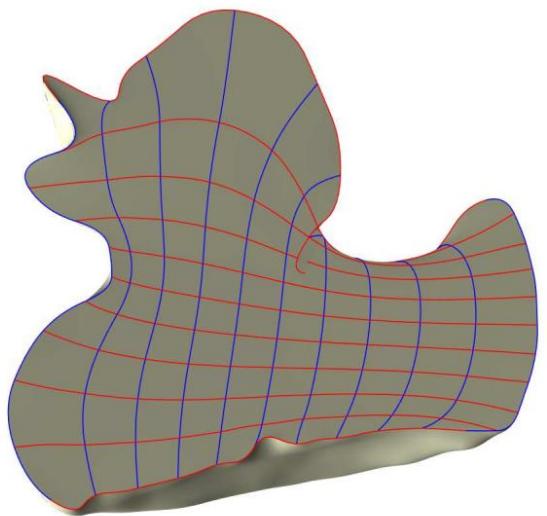
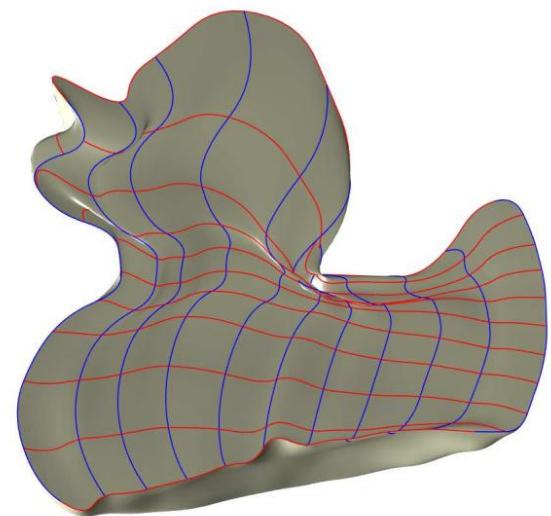
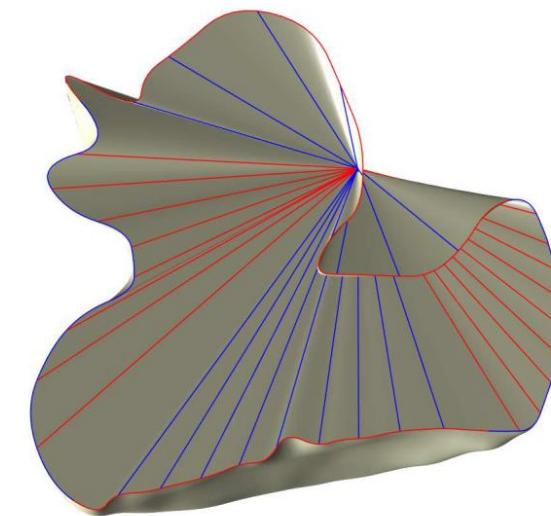


- Algebraic methods:
  - Coons patch
  - Spring patch
- Optimization-based methods:
  - Barrier-function-based <sup>1</sup>
  - Penalty-function-based <sup>2</sup>
- PDE-based methods:
  - Elliptic grid generation <sup>3</sup>
  - Improved EGG <sup>4</sup>

- 
1. Ji, Y. et al. (2021). *JCAM*, 396, 113615.
  2. Ji, Y. et al. (2022). *CAGD*, 94, 102081.
  3. Hinz, J. et al. (2018). *CAGD*, 65, 48-75.
  4. Ji, Y. et al. (2023). *CAGD*, 102, 102191.



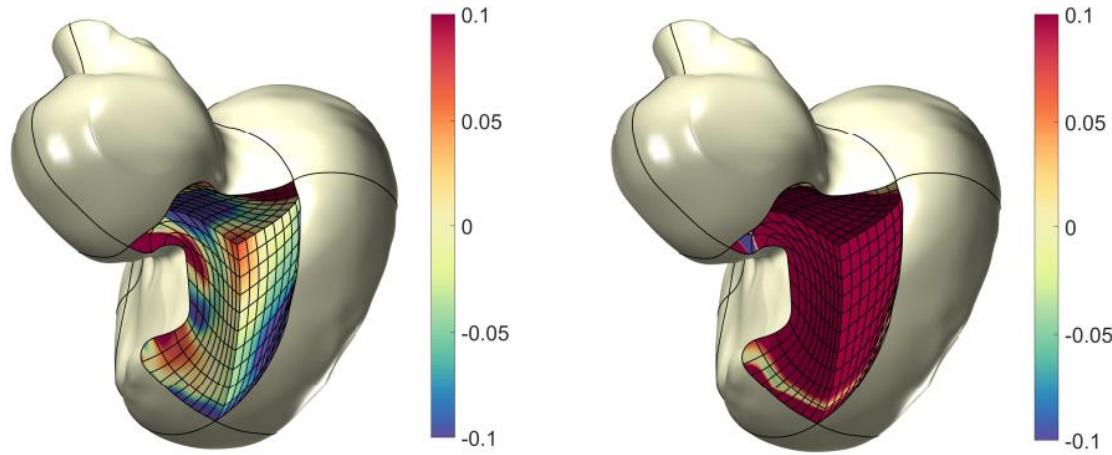
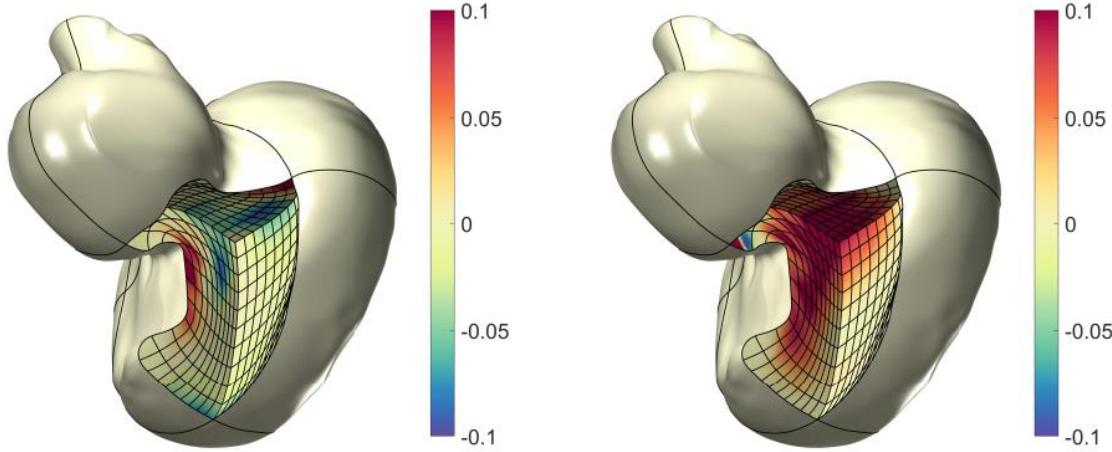
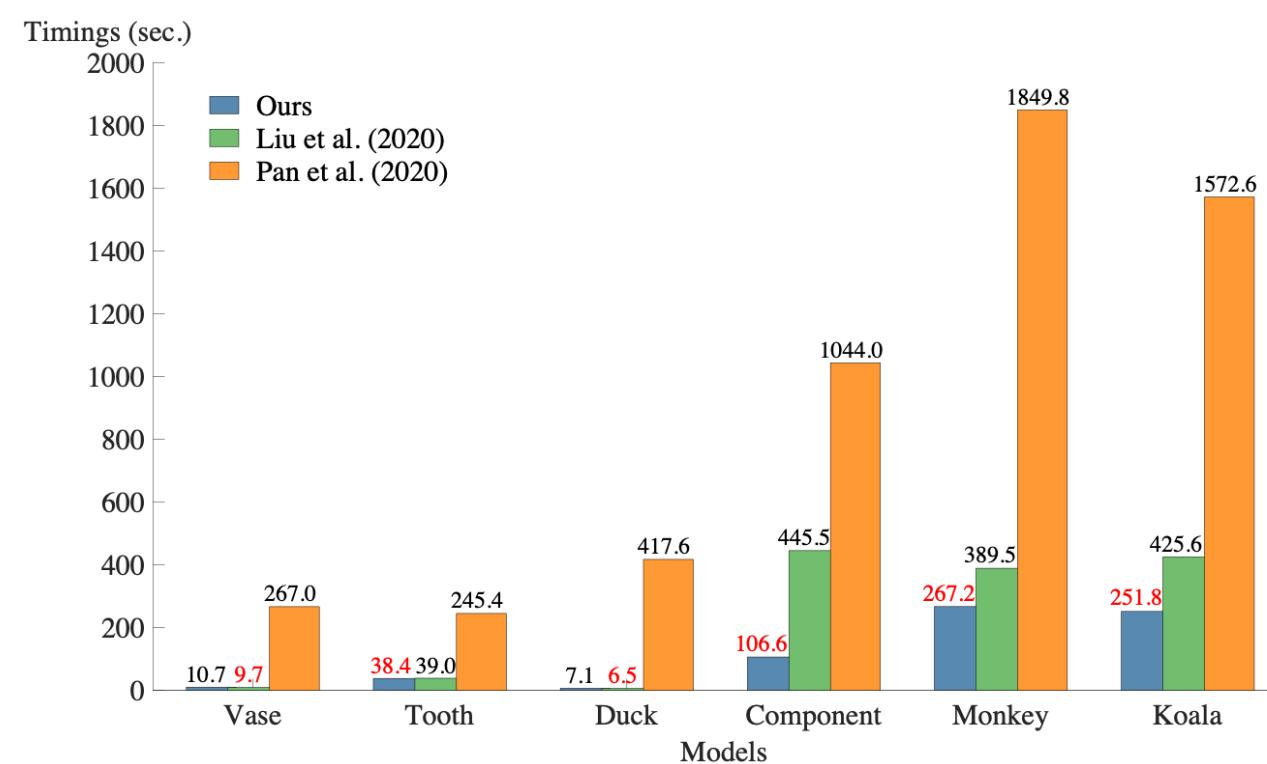
Ji, Y., Möller, M., Verhelst, H.M. (2024). Design Through Analysis. In: Bodnár, T., Galdi, G.P., Nečasová, Š. (eds) Fluids Under Control. Advances in Mathematical Fluid Mechanics. Birkhäuser, Cham.



Same point

Coons Patch

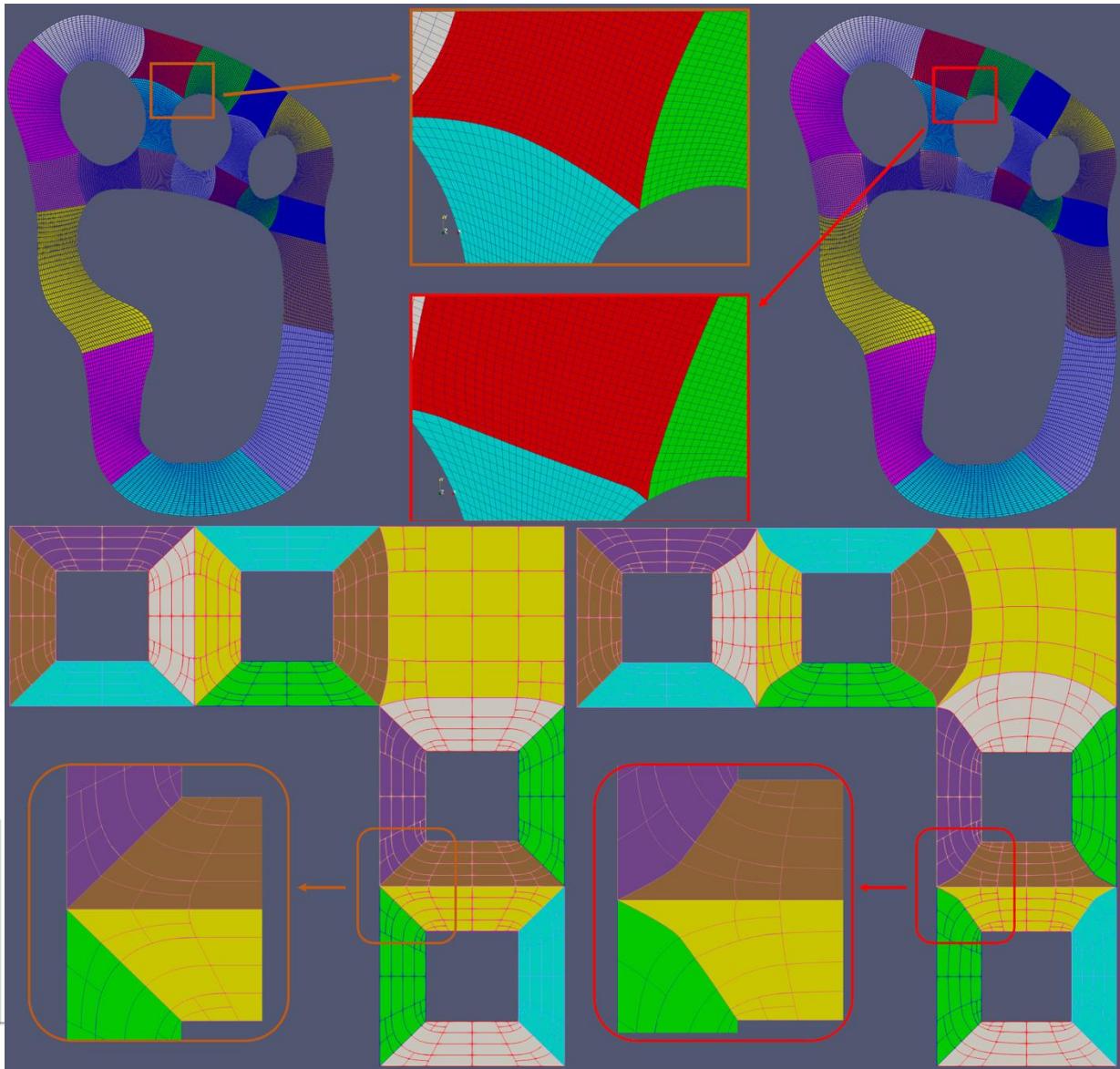
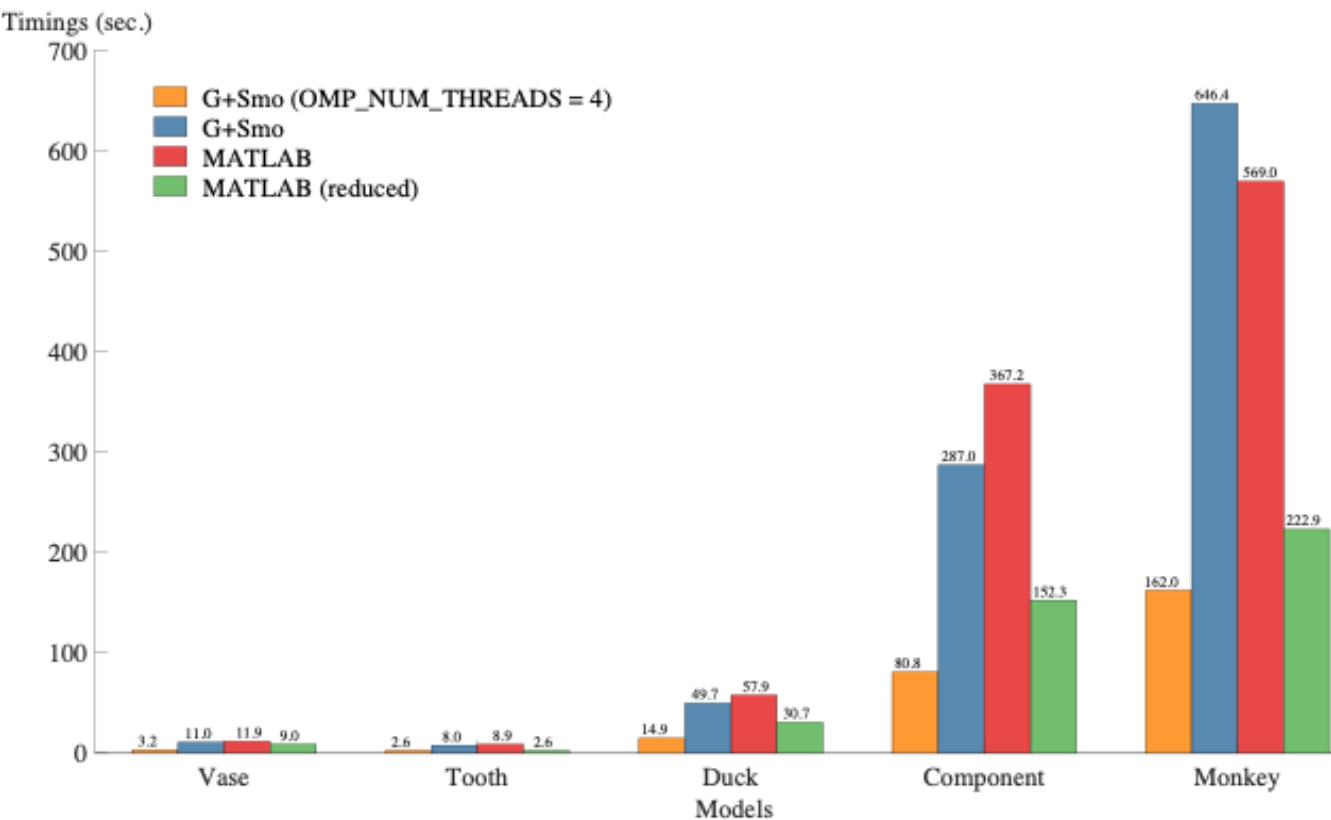
Smoothness energy

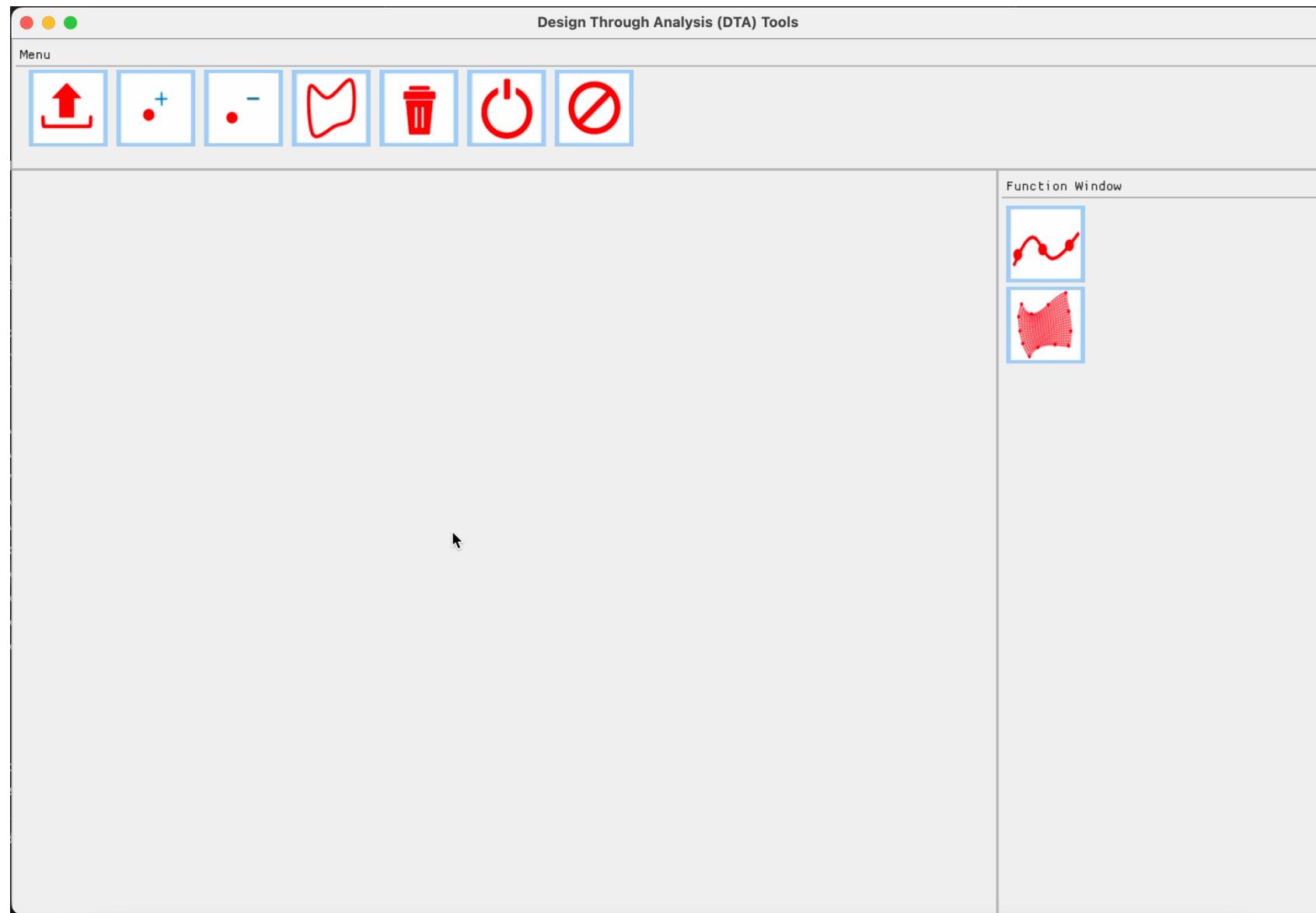

 $m_{SJ}^{Ours} - m_{SJ}^{Pan}$ 
 $m_{unif.}^{Pan} - m_{unif.}^{Ours}$ 

 $m_{SJ}^{Ours} - m_{SJ}^{Liu}$ 
 $m_{unif.}^{Liu} - m_{unif.}^{Ours}$ 


- **Positive values** indicate our method performs better;
- Efficiency comparison (MATLAB vs. C++ (Pan+ & Liu+)):
  - Significantly **faster** than Pan et al. (2020);
  - Large-Scale Models: Outperforms Liu et al. (2020).

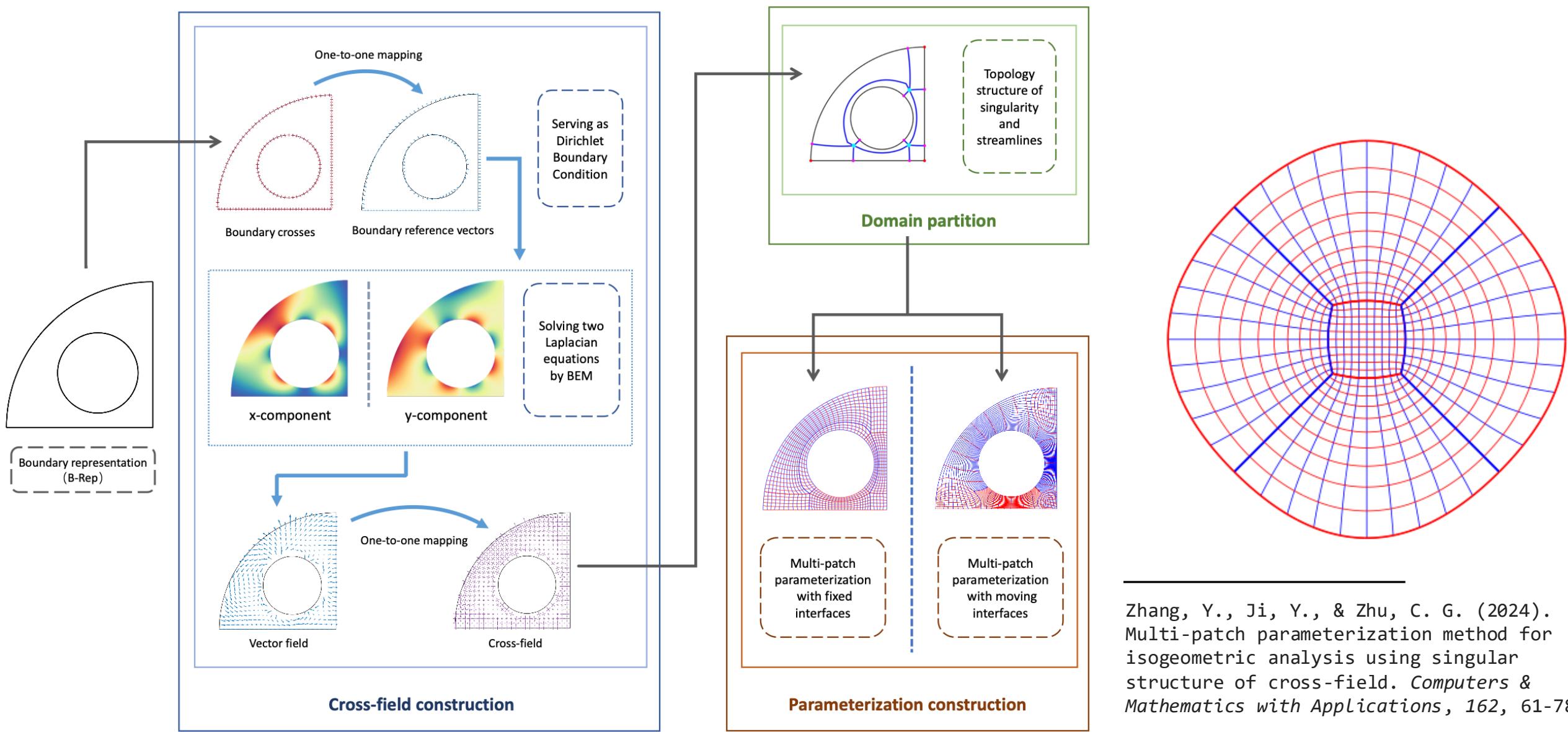
1. Pan, M., Chen, F., & Tong, W. (2020). CMAME, 359, 112769.
2. Liu, H., Yang, Y., Liu, Y., & Fu, X. M. (2020). CAGD, 79, 101853.

- In our released G+Smo implementation,  
**3-4x speed-up**;
- Suitable for **multi-patch** and **THB-spline parameterization**;

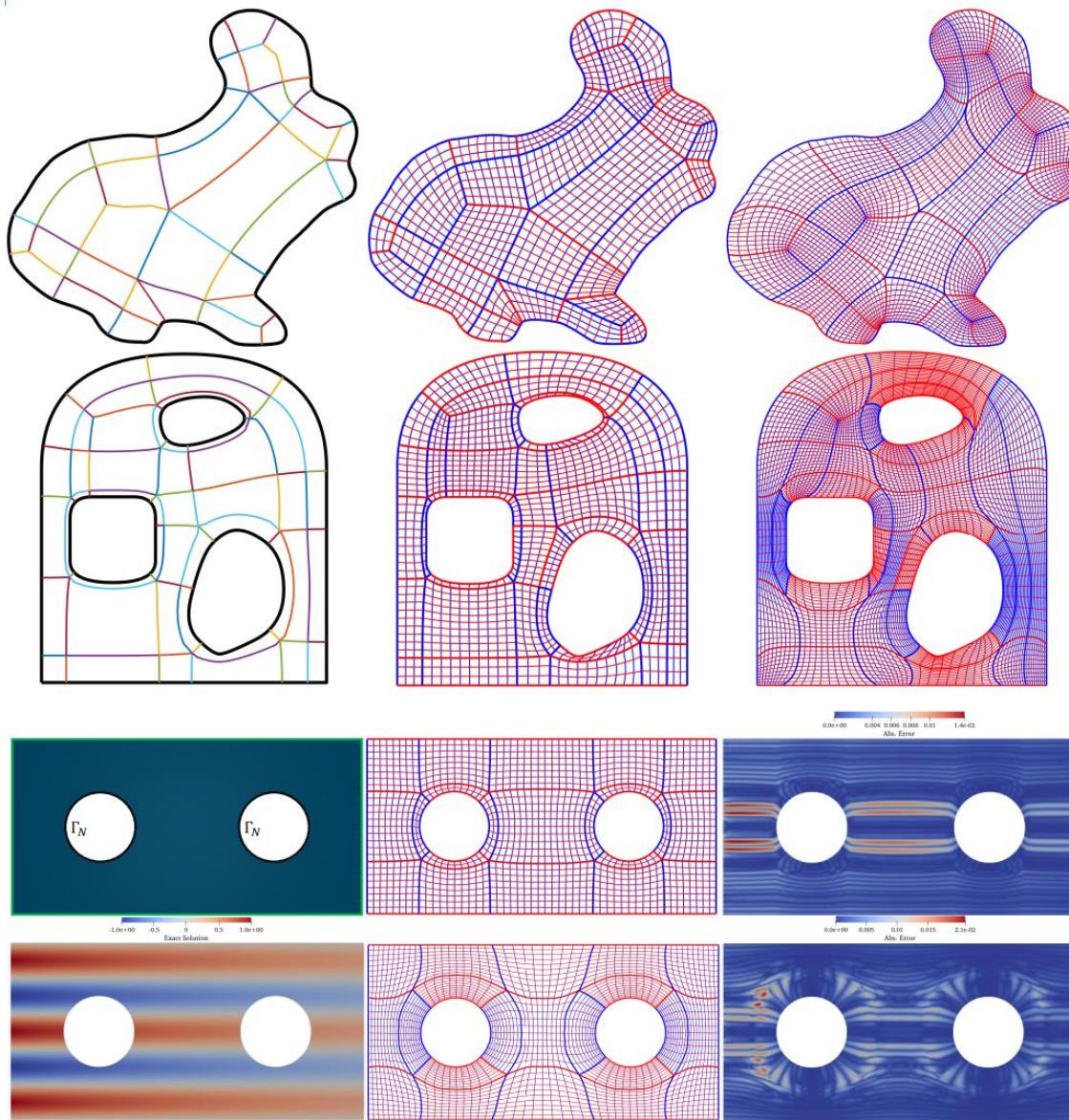




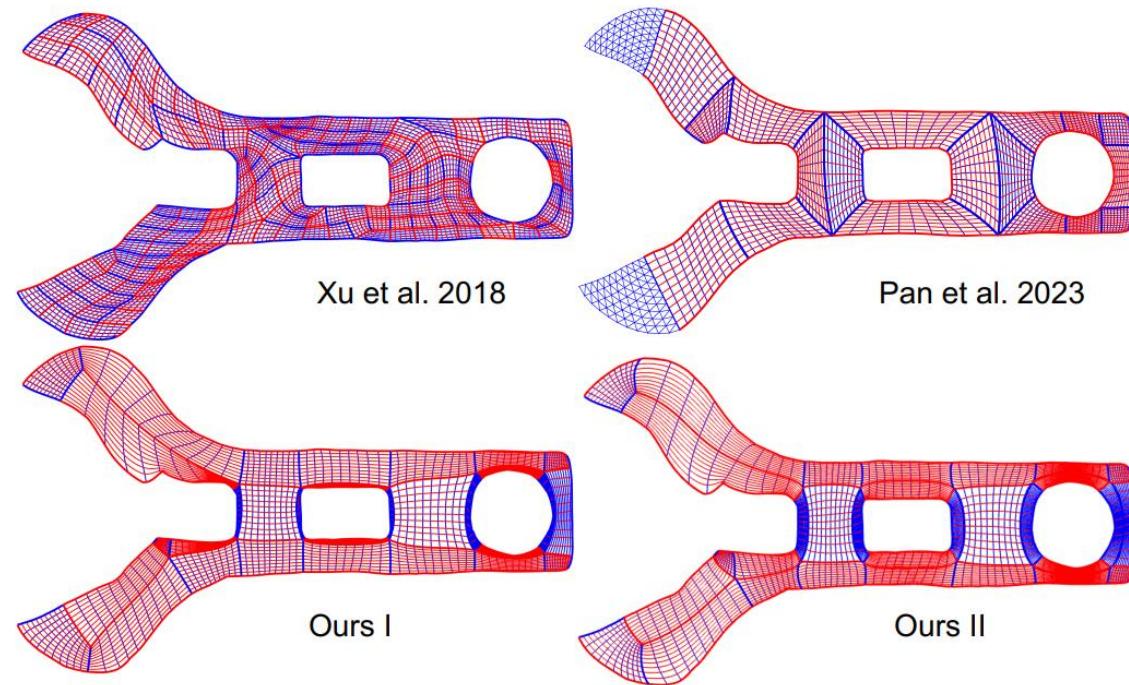
# Multi-patch parameterization using cross-field

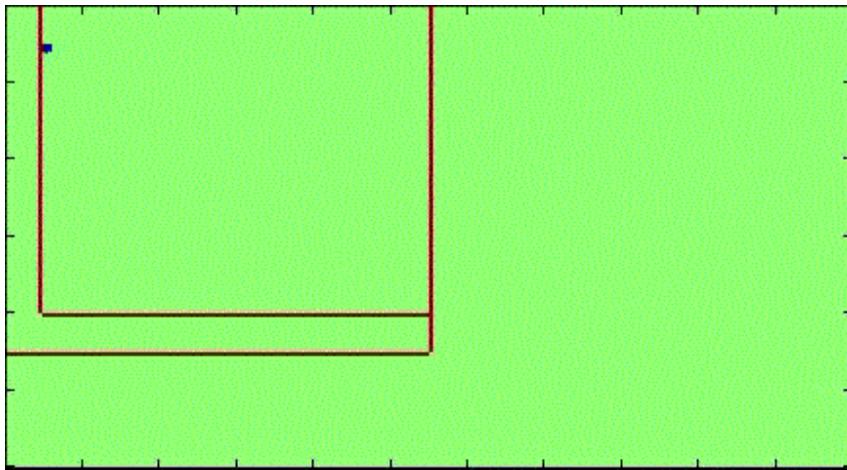


Zhang, Y., Ji, Y., & Zhu, C. G. (2024). Multi-patch parameterization method for isogeometric analysis using singular structure of cross-field. *Computers & Mathematics with Applications*, 162, 61-78.



Model	#Patch	Method	$ \mathcal{J} _s$			unif.		
			min.	avg.	min.	avg.	max.	
rabbit	33	Coons	-0.8593	0.9628	0.7030	<b>0.9410</b>	1.0524	
		fixed-I	<b>0.2204</b>	<b>0.9504</b>	0.6103	0.9544	<b>0.9982</b>	
		moving-I	0.02918	0.9283	<b>0.0000</b>	0.9550	1.0000	
3 holes	46	Coons	-0.5492	0.9710	0.8008	0.9573	1.0958	
		fixed-I	<b>0.1545</b>	<b>0.9716</b>	0.8007	0.9573	0.9978	
		moving-I	0.1461	0.9361	<b>0.6791</b>	<b>0.9571</b>	<b>0.9968</b>	

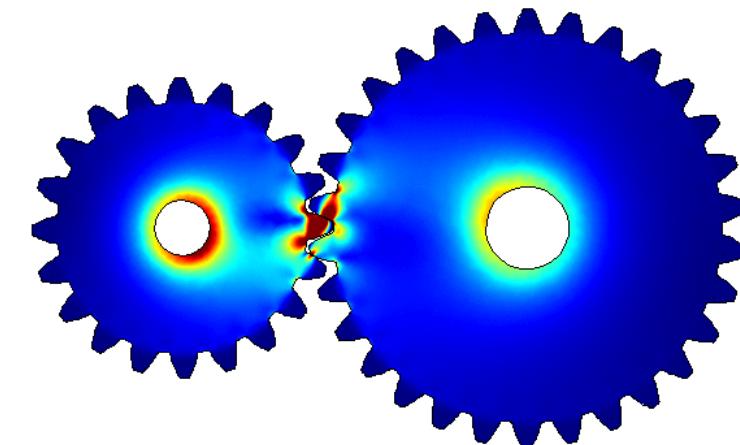




Wave propagation



Laser printing

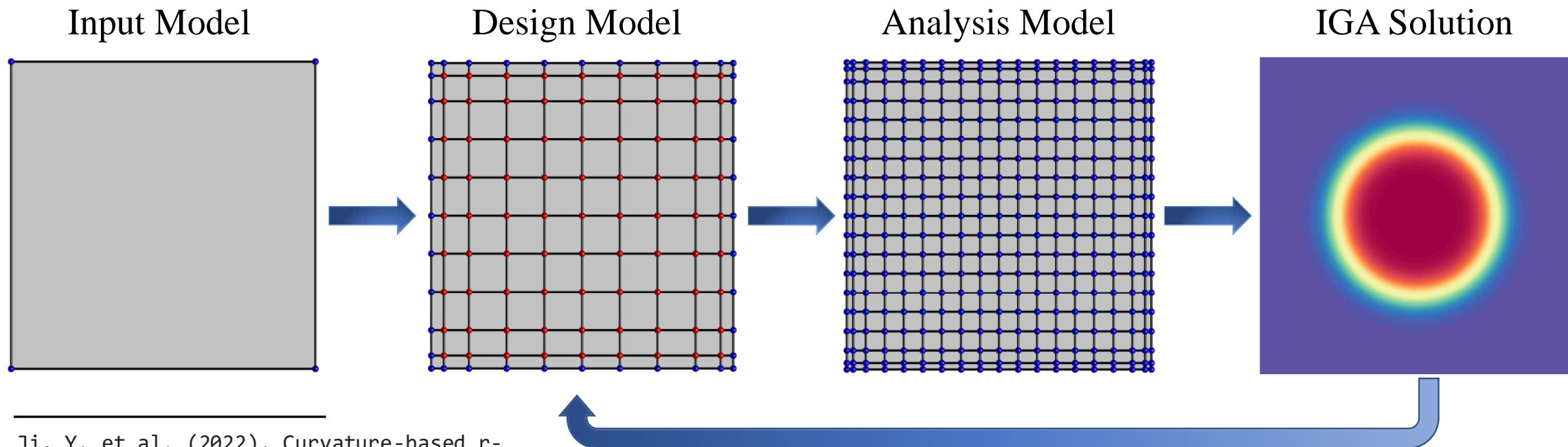


Stress distribution

- **Localized and anisotropic features** extensively exist in physical phenomena;
- **Isotropic parameterizations are not efficient** for such problems;
- Anisotropic parameterizations (r-adaptivity):
  - Enhance per-DOF accuracy while keeping constant total DOFs.
  - Keep the sparse pattern of mass matrix and stiffness matrix.

## Anisotropic parameterizations are often solution-dependent:

- Need **good numerical solution accuracy** to drive parameterization;
- Adjust as few control points as possible **for high efficiency**;
- **Bi-level strategy:** a coarse level (design model) to update the parameterization for efficiency and a fine level (analysis model) to perform analysis for accuracy.

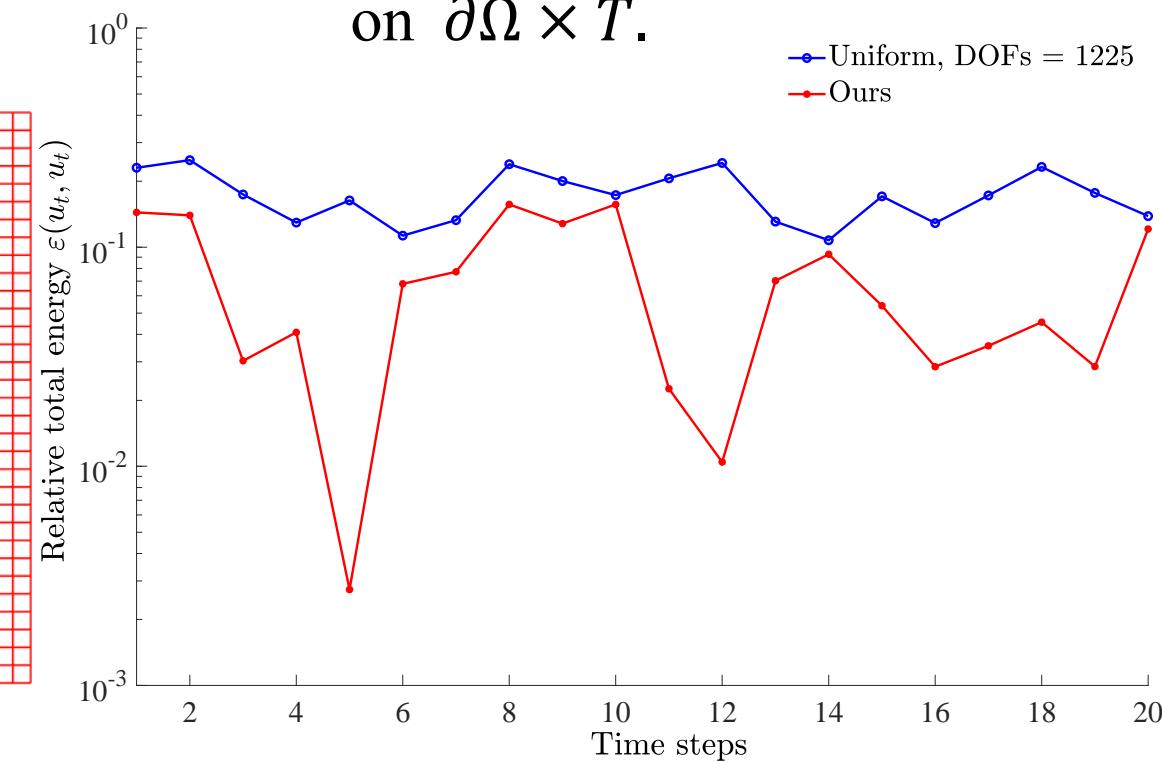
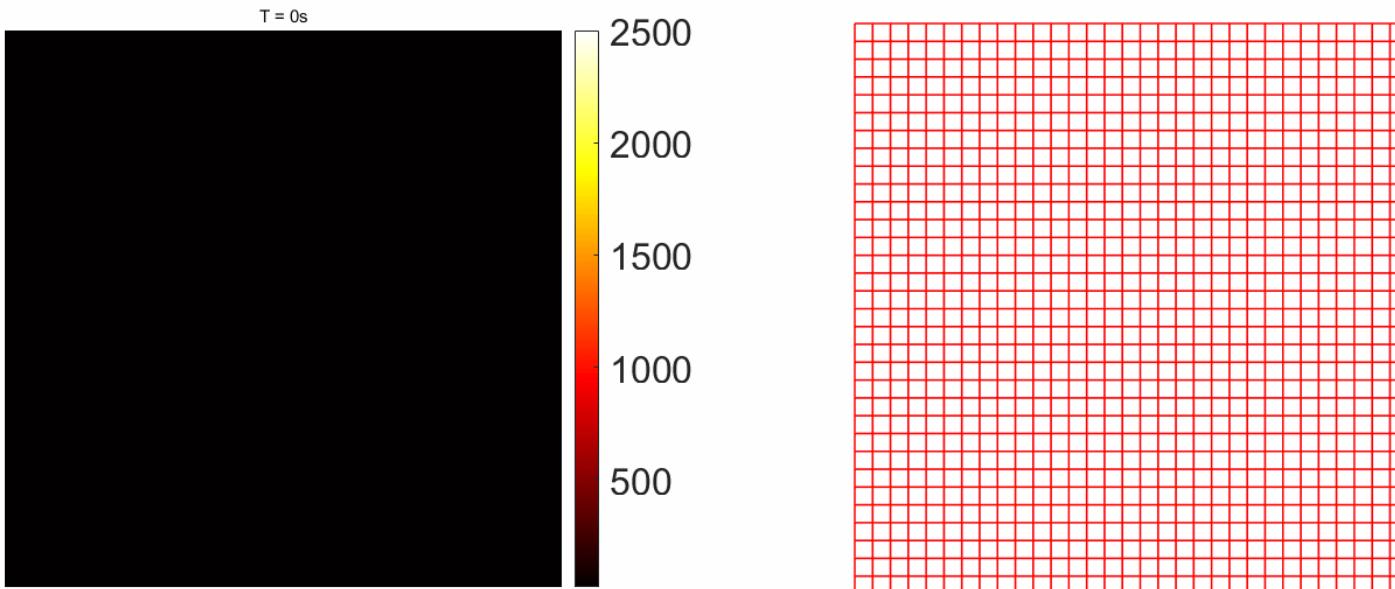


Ji, Y. et al. (2022). Curvature-based r-adaptive planar NURBS parameterization method for isogeometric analysis using bi-level approach. *Computer-Aided Design*, 150, 103305.

Objective function and sensitivity analysis

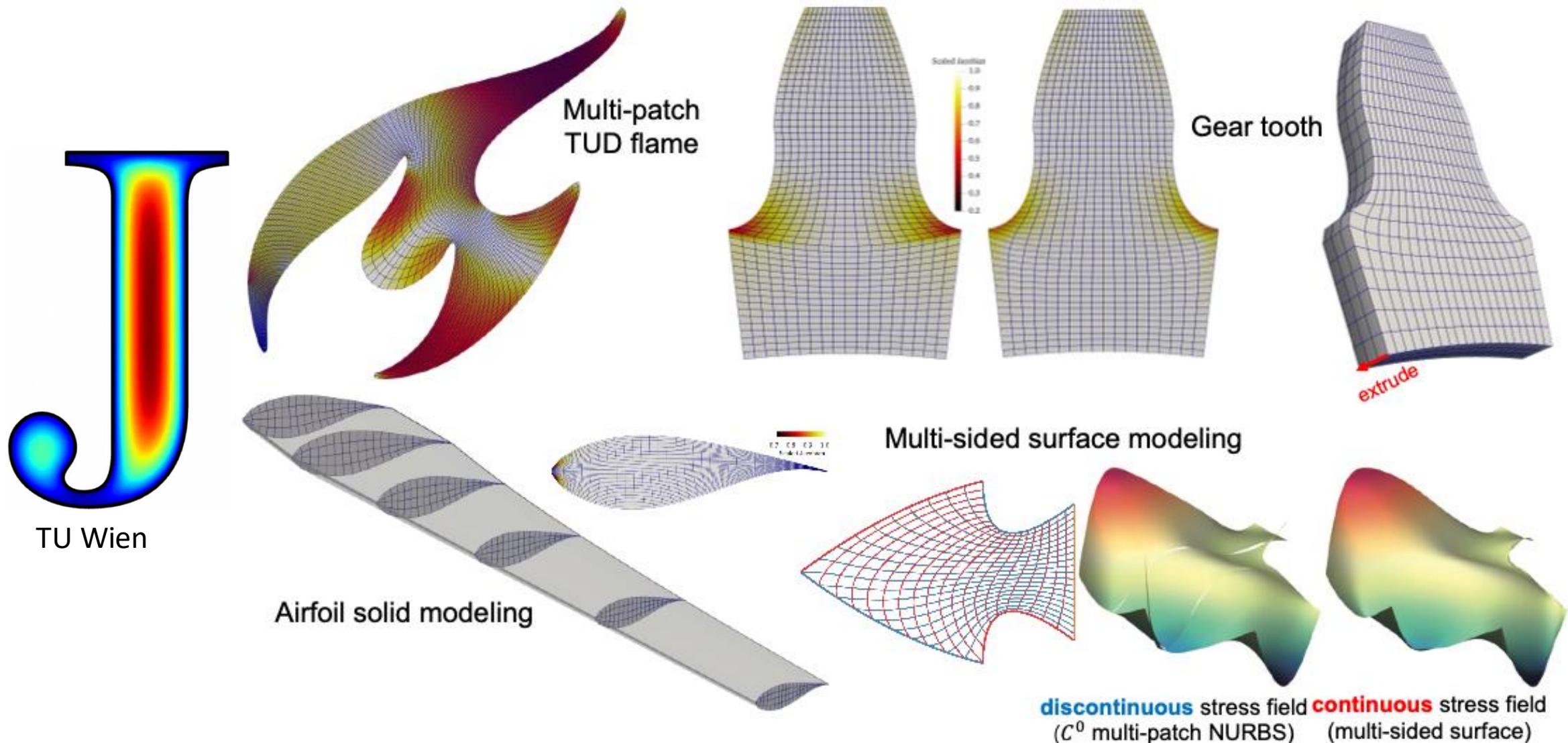
- Consider a two-dimensional linear heat transfer problem with a moving Gaussian heat source:

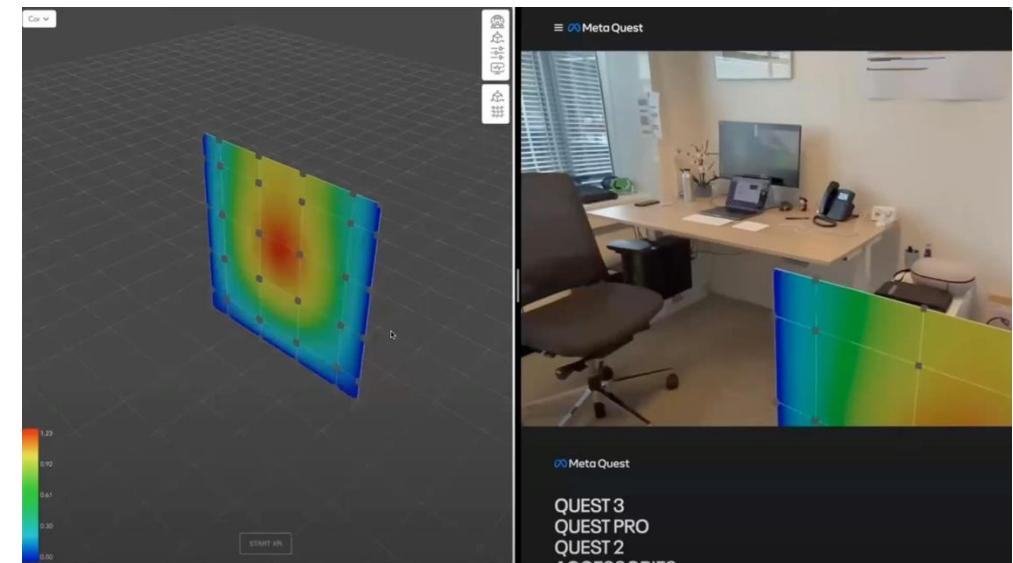
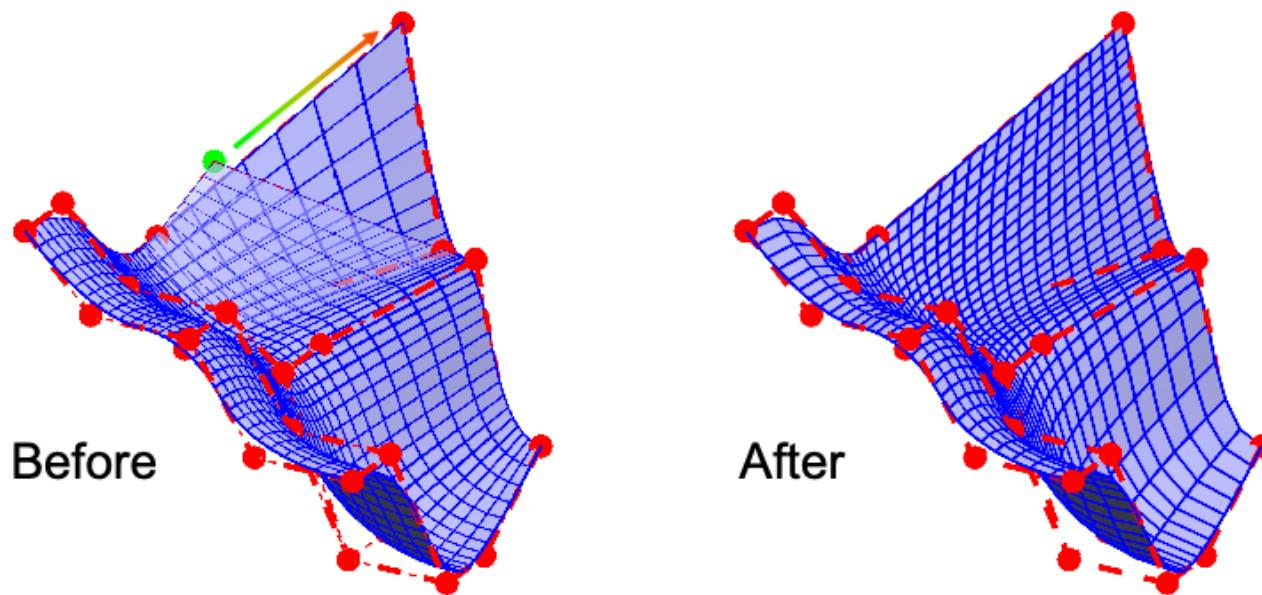
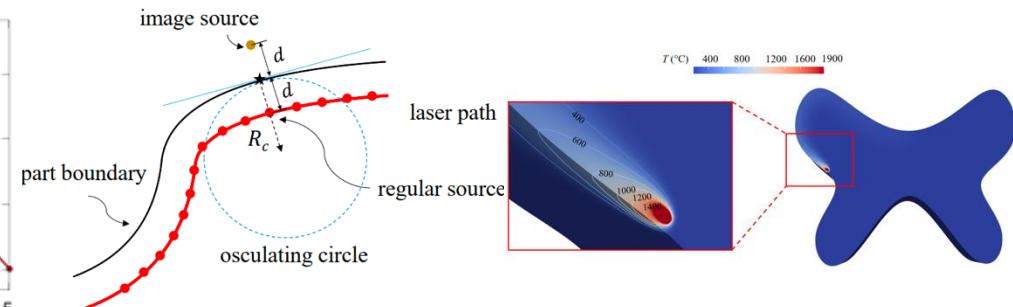
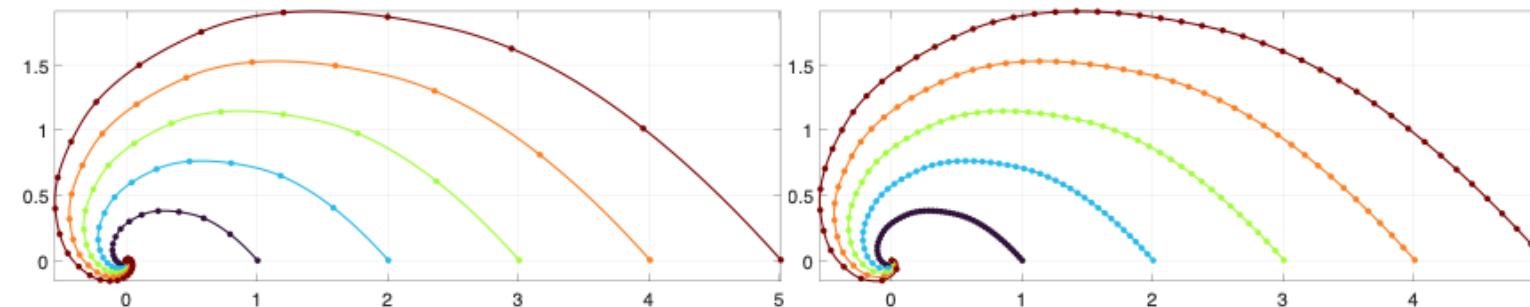
$$\begin{cases} C_p \rho \frac{\partial u(\mathbf{x}, t)}{\partial t} - \nabla \cdot (\kappa \nabla u(\mathbf{x}, t)) = f(\mathbf{x}, t), & \text{in } \Omega \times T, \\ u(\mathbf{x}, t) = u_0, & \text{in } \Omega, \\ \kappa \nabla u(\mathbf{x}, t) = 0, & \text{on } \partial\Omega \times T. \end{cases}$$



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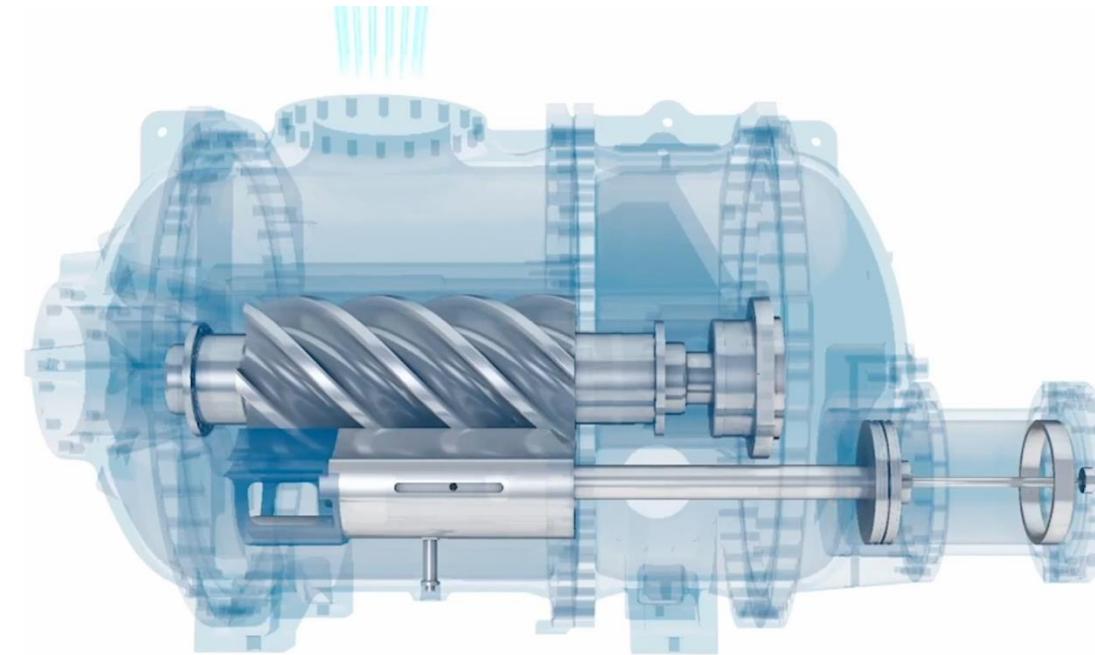
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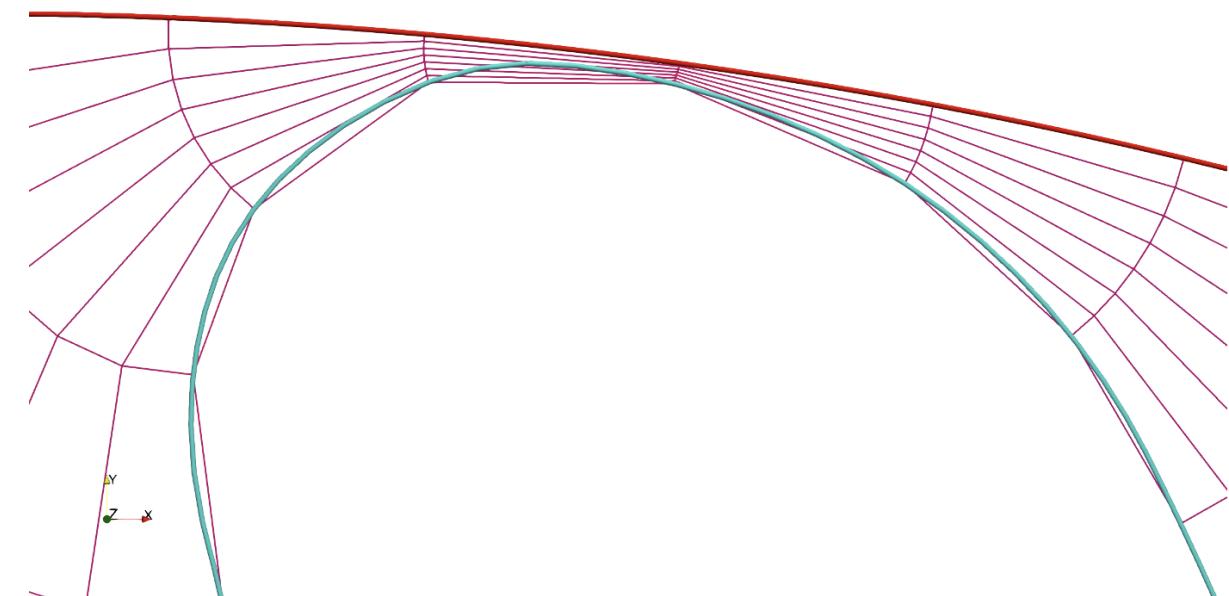
- Curve/surface reparameterization while keeping the geometry;
  - Curve – metal additive manufacturing
  - Surface – VR/AR

Yang, Y., Ji, Y. Möller M, Ayas C. Computational Efficient Process Simulation of Geometrically Complex Parts in Metal Additive Manufacturing, Submitted.



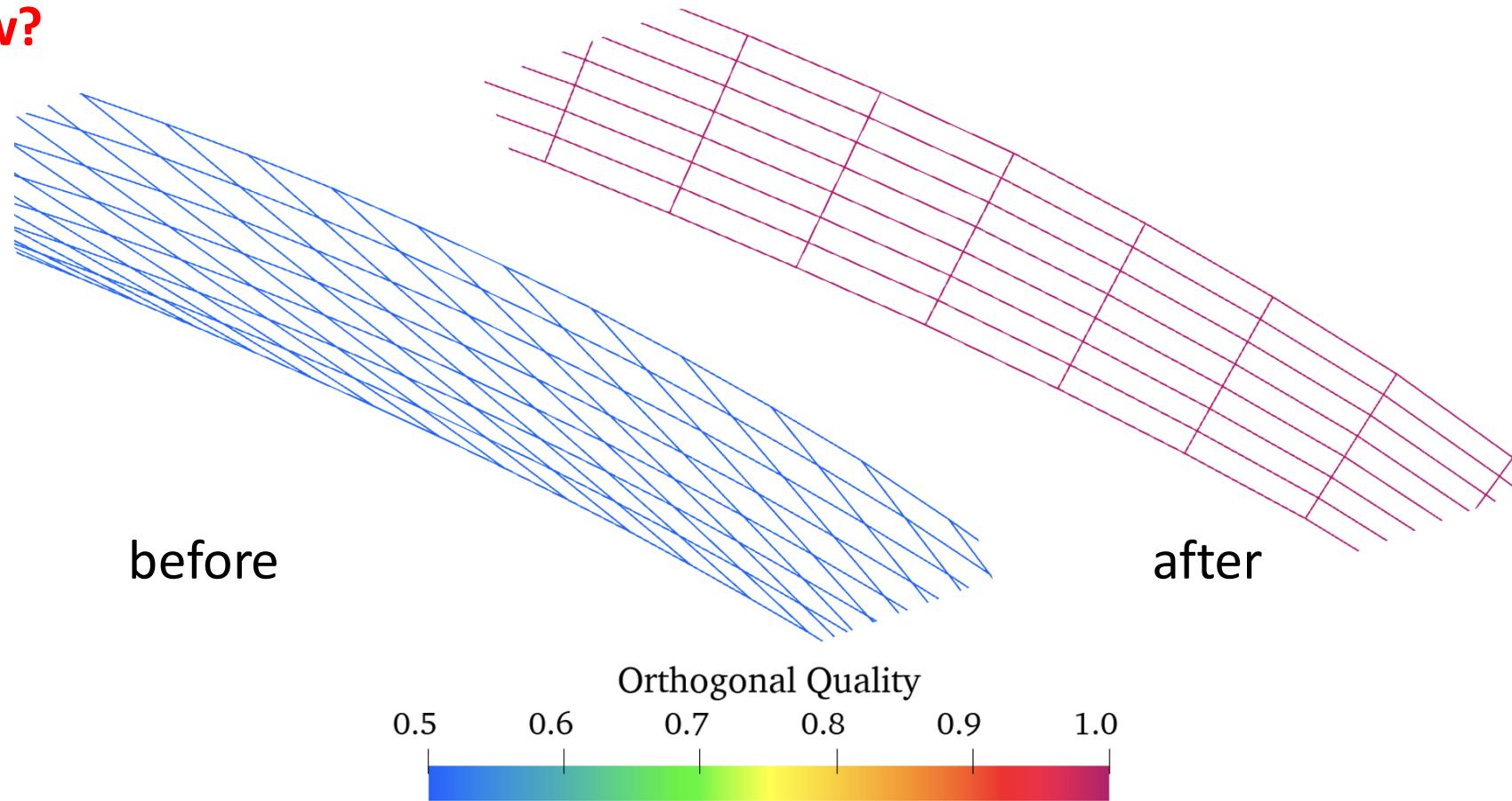
Rotary twin-screw compressor

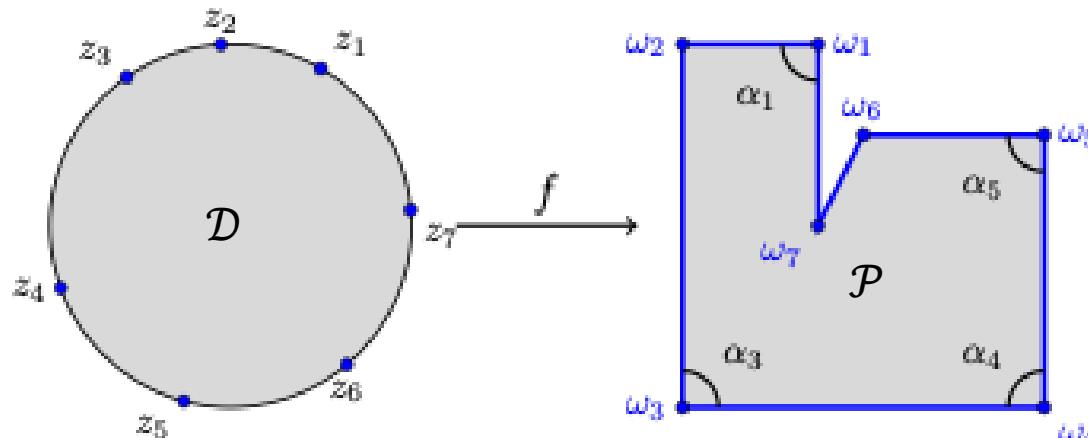
- **Structured mesh generation** is a crucial preprocessing step in the simulation-based analysis of twin-screw machines.
- However, the existing mesh generators typically produce only **linear meshes with straight-sided cells**;
- **Analysis-suitable, high-order NURBS parameterizations.**



Source: [https://www.gascompressors.co.uk/  
technologies/oil-floodedscrew-compressor/](https://www.gascompressors.co.uk/technologies/oil-floodedscrew-compressor/)

- **Parameter speed** of the boundary curves significantly affects the mesh quality;
- Mesh quality is greatly improved by using the **boundary reparameterization technique**.
- **So, how?**

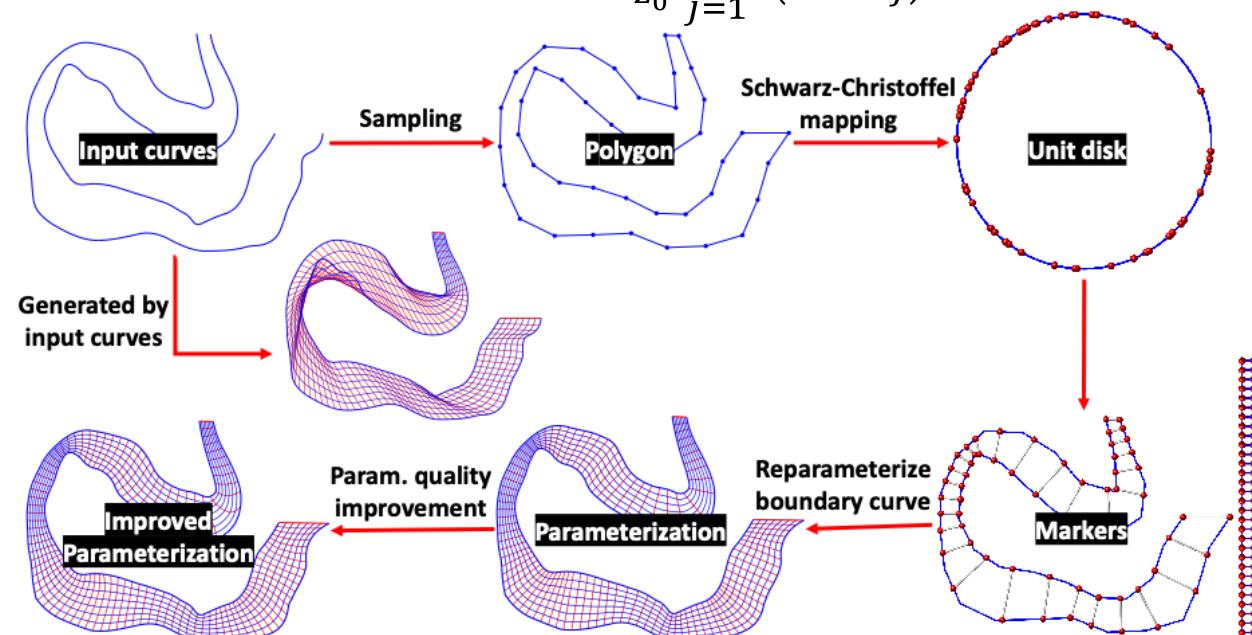




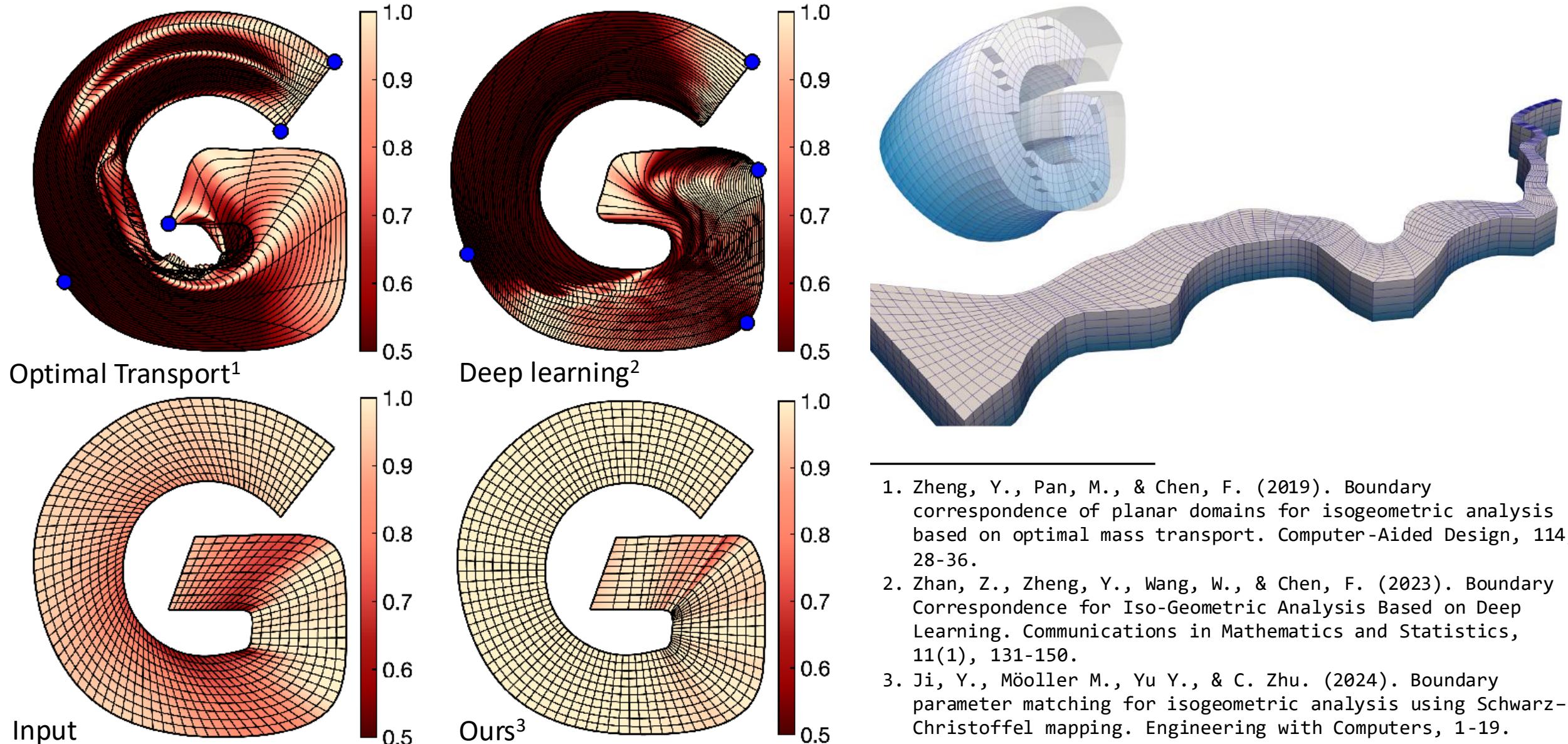
- Solving the Schwarz-Christoffel parameter problem for  $\{z_j\}$  numerically allows us to compute sets of markers on the two opposite curves that can be used to reparametrize one curve w.r.t. the other.
- Solving the parameter problem is far from easy, **the CRDT algorithm<sup>[1]</sup>** is adopted and implemented.

- **Riemann mapping theorem:**  $\exists$  analytic function  $f$  with non-zero derivative such that  $f(\mathcal{D}) = \mathcal{P}$ .
- **Schwarz-Christoffel formula**

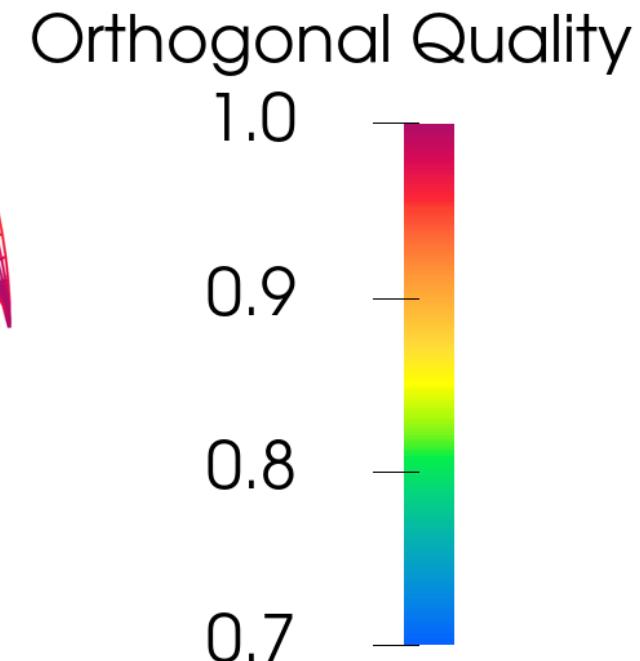
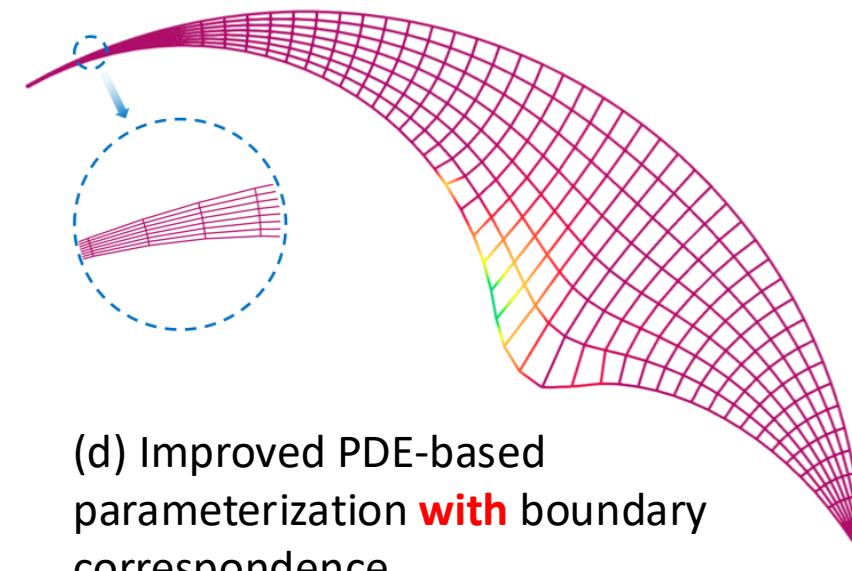
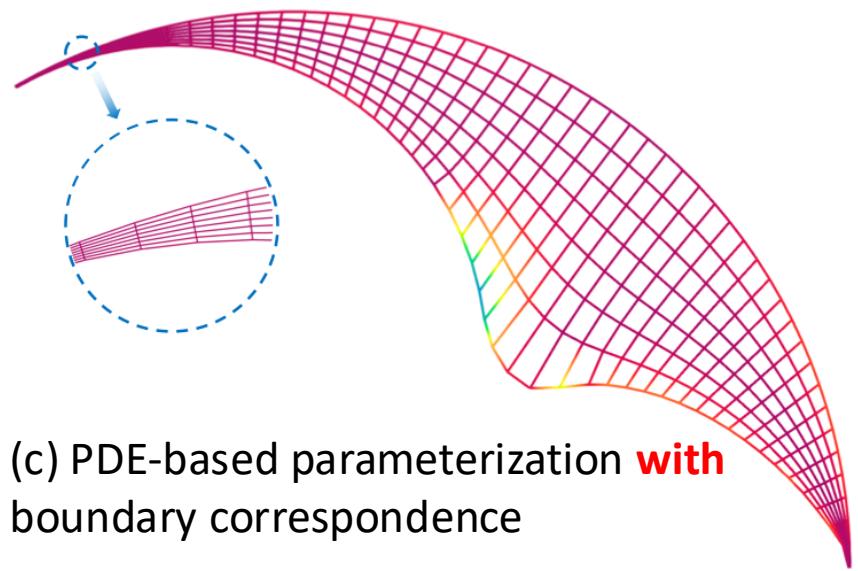
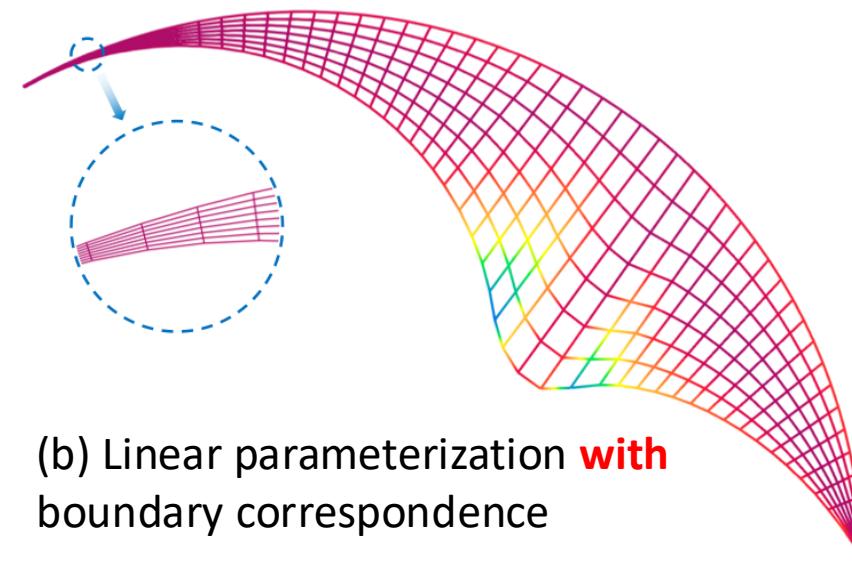
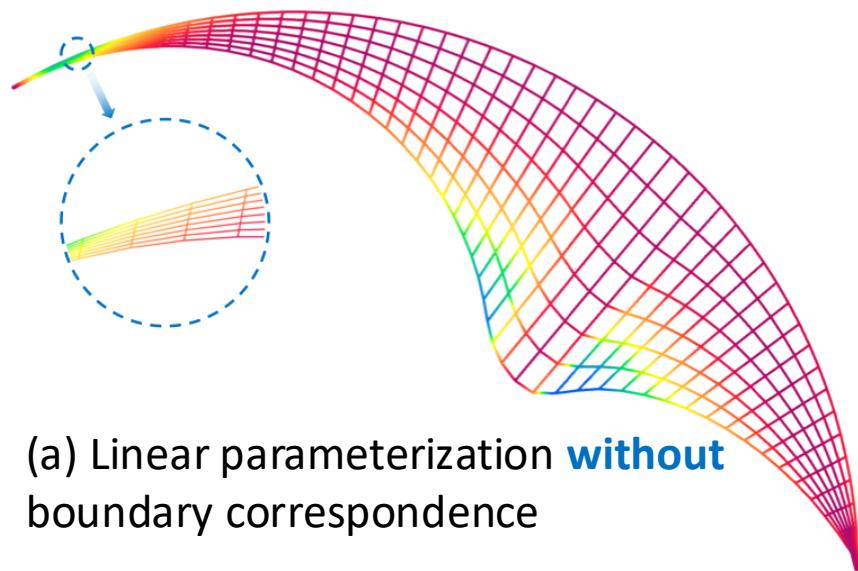
$$f(z) = f(z_0) + C \int_{z_0}^z \prod_{j=1}^n \left(1 - \frac{\zeta}{z_j}\right)^{\beta_j} d\zeta$$

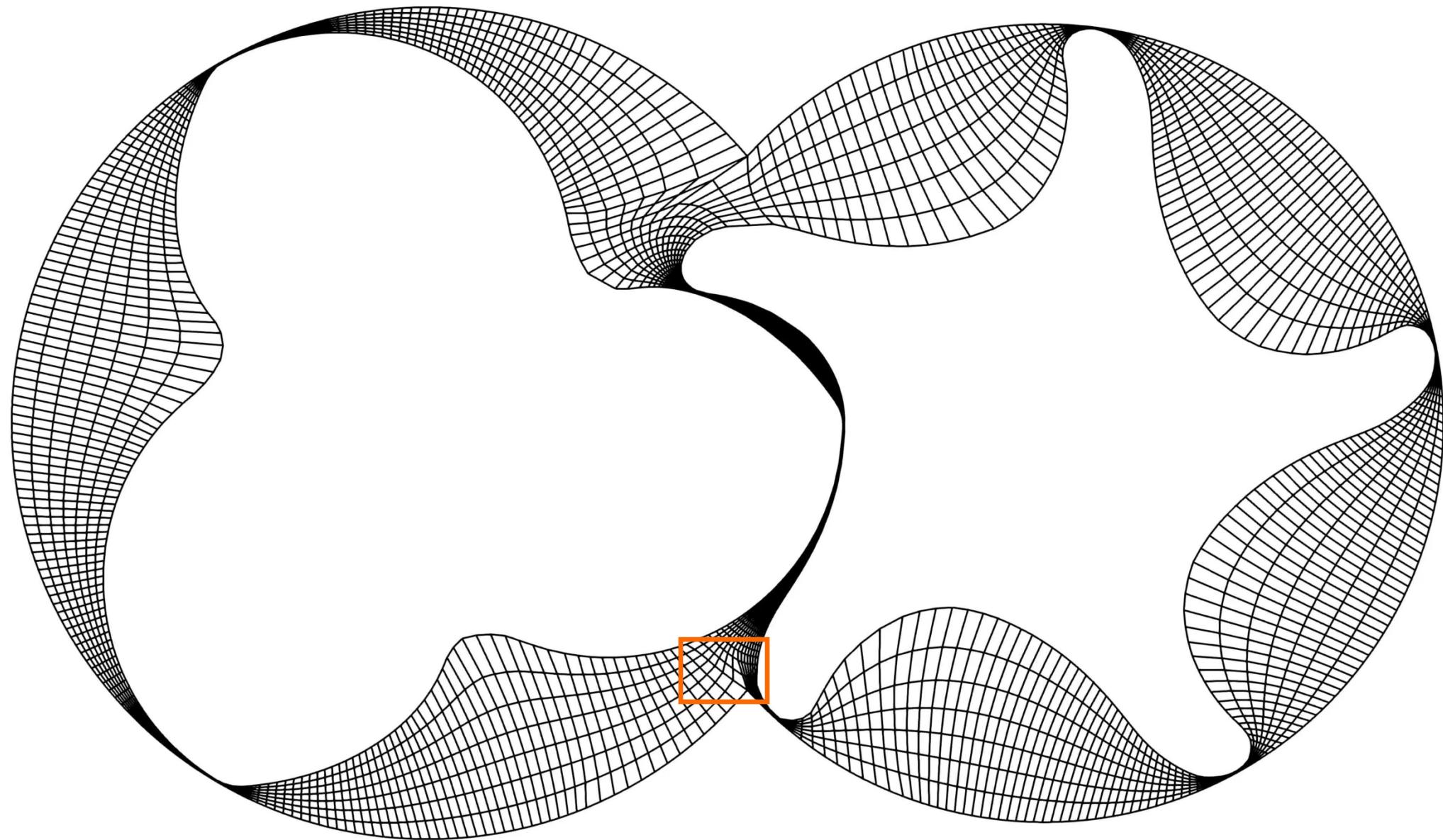


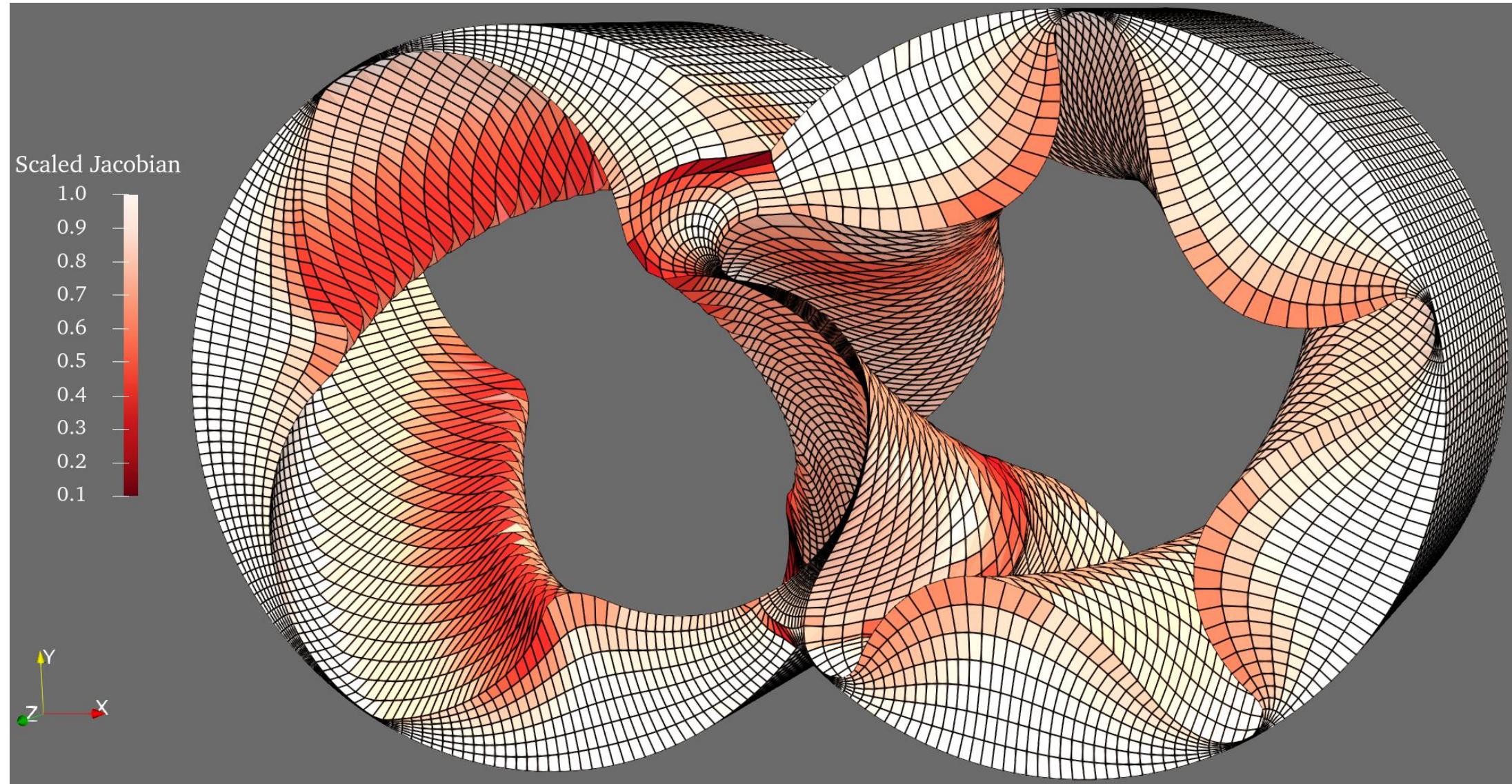
1. Driscoll, T. A., & Vavasis, S. A. (1998). Numerical conformal mapping using cross-ratios and Delaunay triangulation. SIAM Journal on Scientific Computing, 19(6), 1783-1803.



# Comparison of different parameterization methods



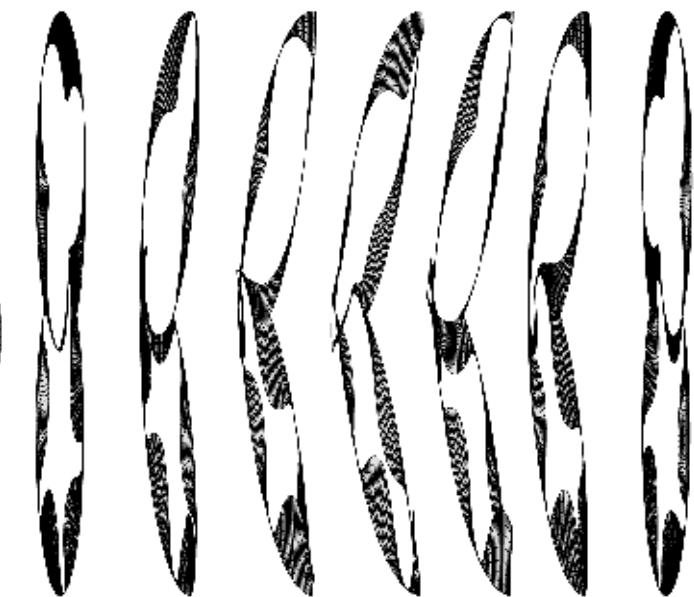
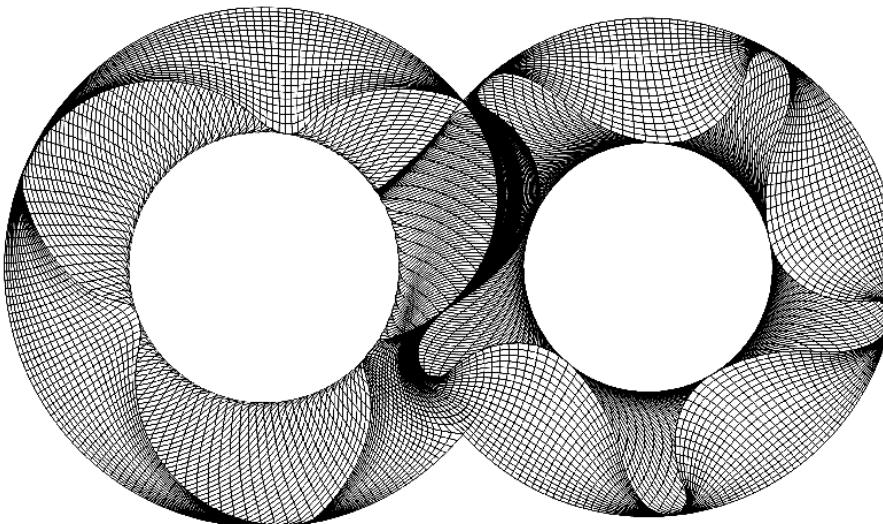
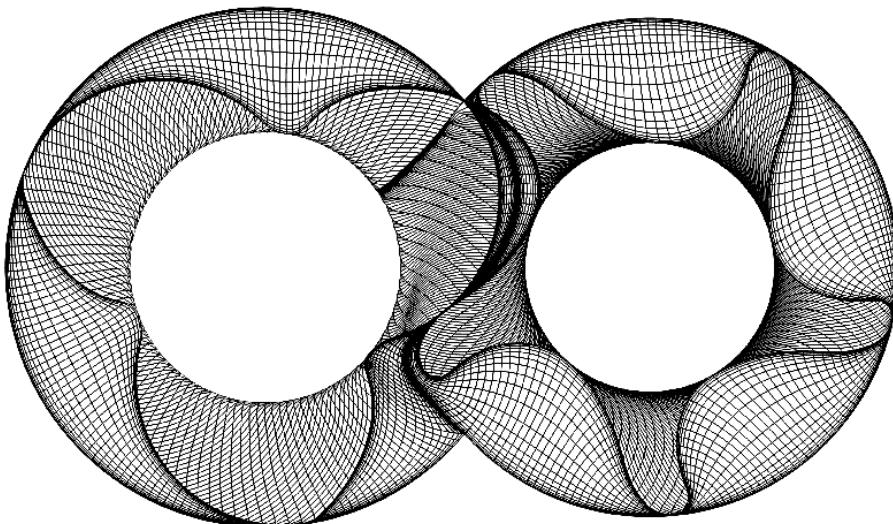
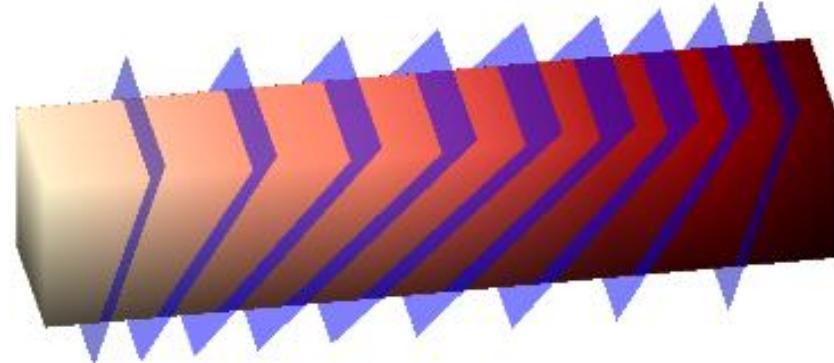




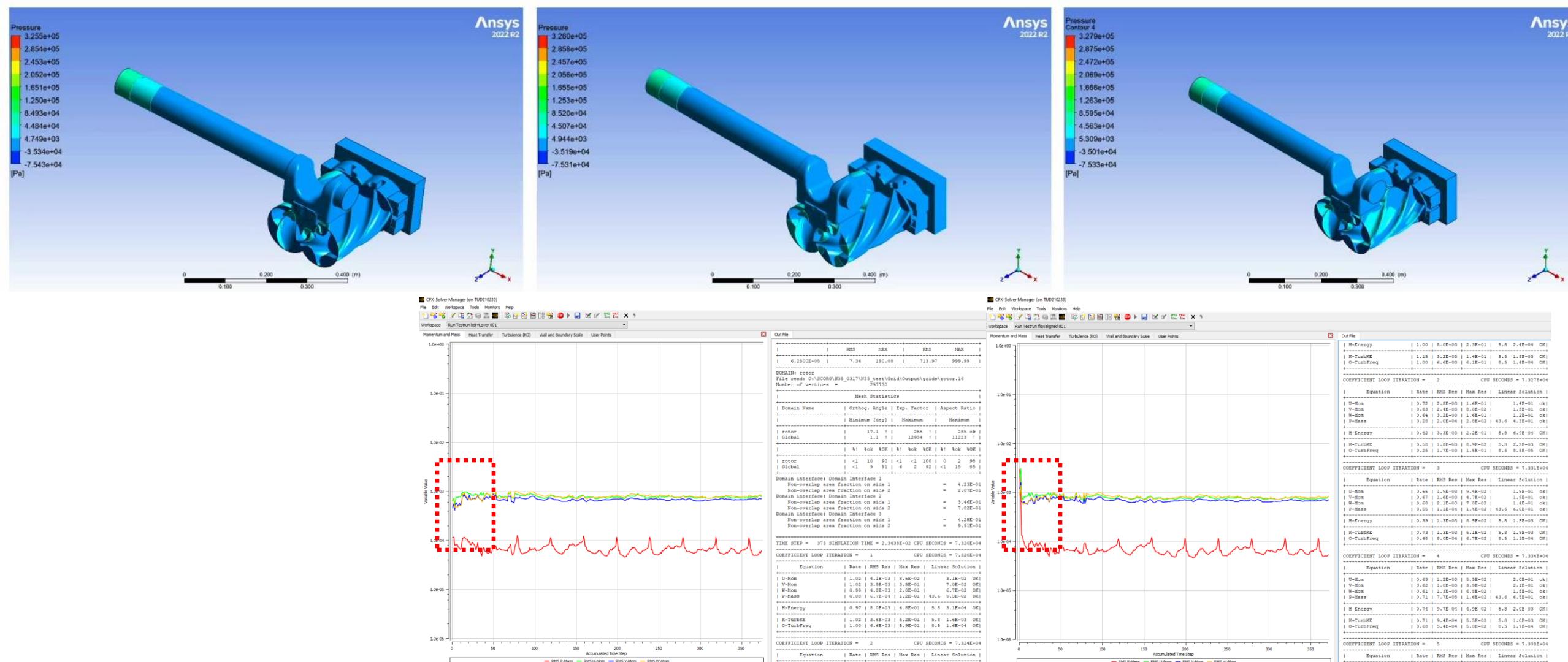
- Boundary layer mesh

$$\begin{cases} \xi = \hat{\xi} \\ \eta = \frac{\tanh(\alpha(2\hat{\eta} - 1))}{2 \tanh(\hat{\eta})} + \frac{1}{2} \end{cases}$$

- Flow-aligned discretization



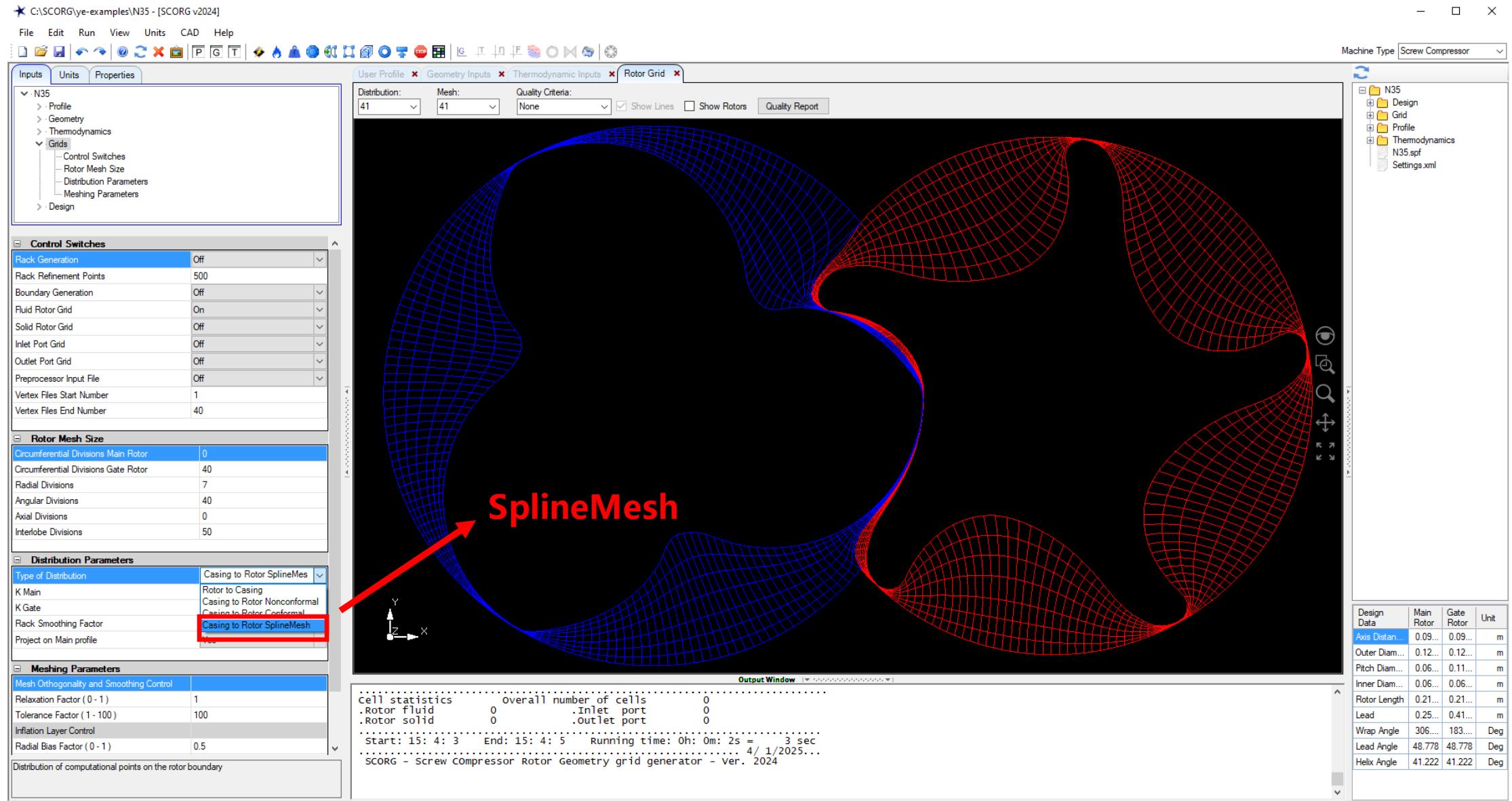
# Simulation results using ANSYS CFX



SCORG™

Boundary layer mesh

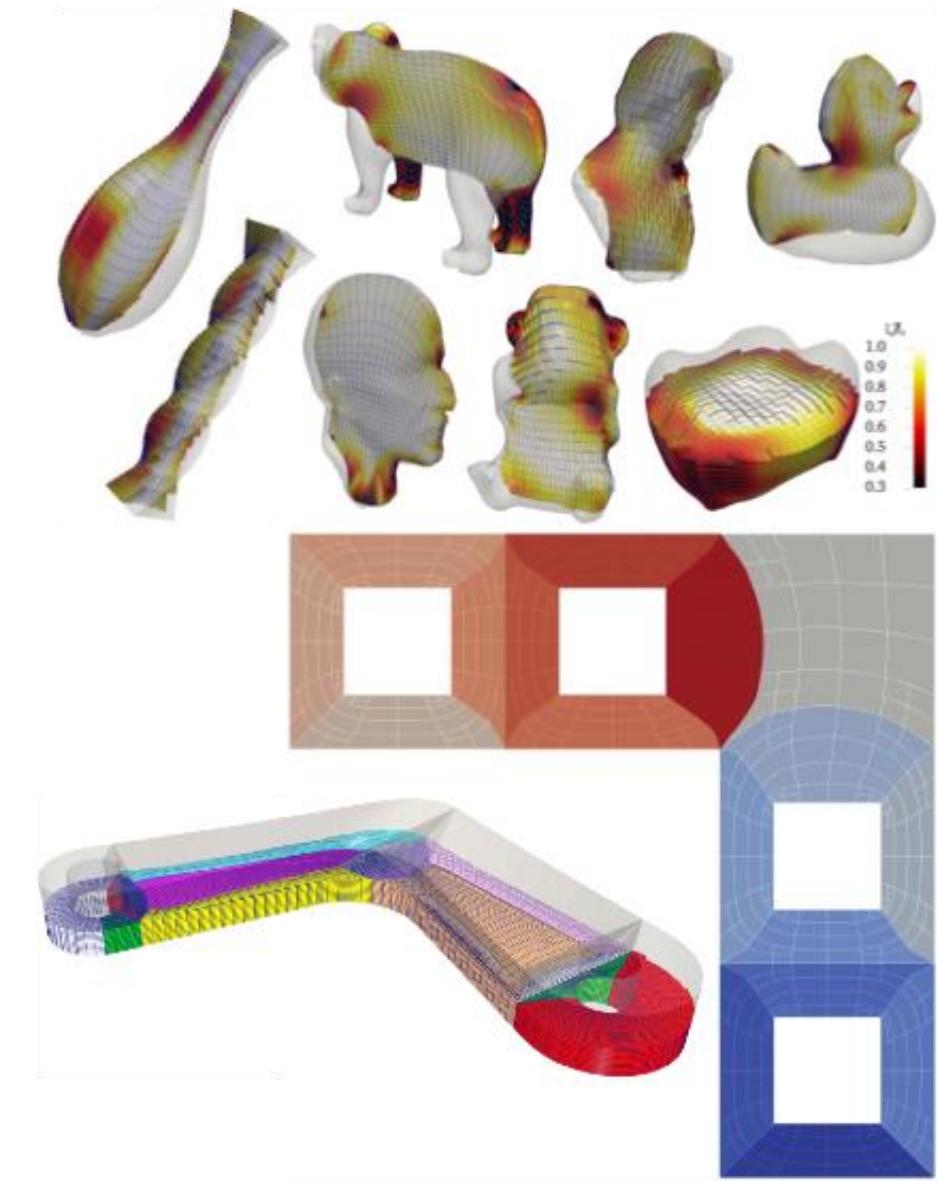
Flow-aligned mesh



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- **G+Smo offers** three major classes of **parameterization methods**;
- Demonstrates **improved robustness and efficiency** over existing methods;
- Applicability in both **academic** and **real-world industry scenarios**;
  
- Future Work:
  - Integrating newly developed and ongoing methods into G+Smo for broader usability;
  - User-friendly **graphical user interface**.



# IGA 2025

Eindhoven, The Netherlands

14-17 September 2025



EINDHOVEN  
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TECHNOLOGY

*Clemens Verhoosel  
Harald van Brummelen  
Matthias Möller  
Deepesh Toshniwal*



iacm



# Many thanks for your attention!

## Q&A

If you are interested in my research, please feel free to contact me! ;-)

-  Email: [y.ji-1@tudelft.nl](mailto:y.ji-1@tudelft.nl)
-  GitHub: [jiyess](https://github.com/jiyess)
-  Homepage: <https://jiyess.github.io/>