Mesh generation for twin-screw compressors by spline-based parameterization using preconditioned Anderson acceleration

Delft University of Technology

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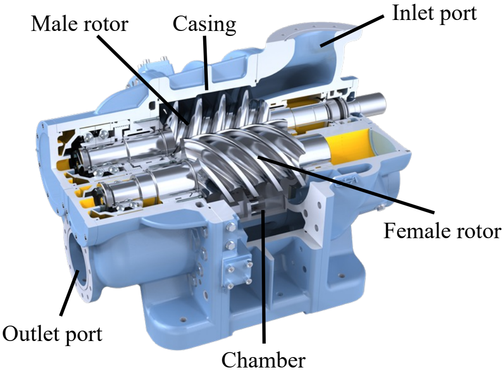
11–13 September 2023, City, University of London

Agenda

Research Motavition



Rotary Twin-Screw Compressor (source [a])

- **Structured mesh generation** is a crucial preprocessing step in the simulation-based analysis of positive displacement machines.

[a] https://www.gascompressors.co.uk/technologies/oil-flooded-screw-compressor/

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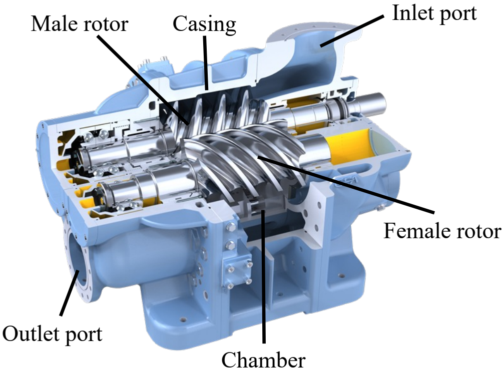

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- **Structured mesh generation** is a crucial preprocessing step in the simulation-based analysis of positive displacement machines.
- **IsoGeometric Analysis (IGA)** changes the workflow fundamentally.
- **Benefits:**
  - **Unified design and analysis;**
  - **Precise, efficient geometry;**
  - **No data type transition;**
  - **Continuous high-order fields;**
  - **Superior approximation.**

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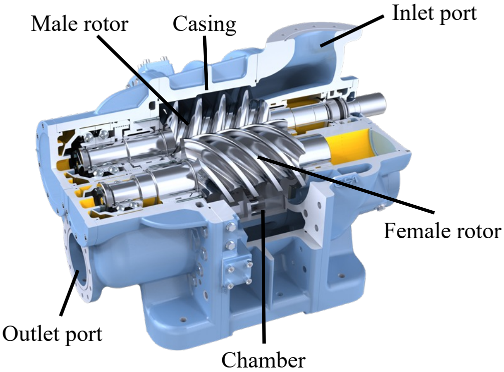

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  - **Superior approximation.**
- This research is on generating analysis-suitable, **high-order NURBS parameterizations** of compressor geometries.

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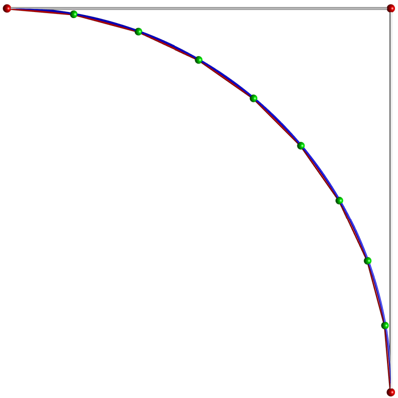
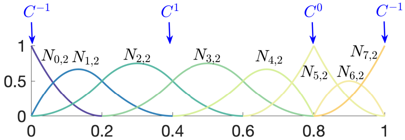
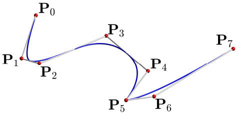
Why Non-Uniform Rational B-Splines (NURBS)?



3 control points vs. 10 grid points

- **Unified Representation**: Both standard (e.g., conics, quadrics) and free-form shapes;
- **Compact Geometry**: Efficient with fewer control points;
- **Fast Evaluations**: Stable and reliable;
- **Geometric Clarity**: Intuitive for designers;
- **Extensive Toolkit**: Including knot operations;
- Invariant under transformations.





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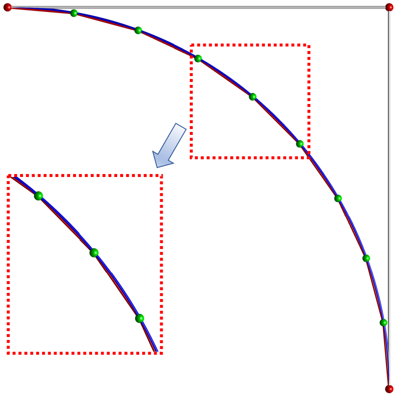

Close-up of the arc

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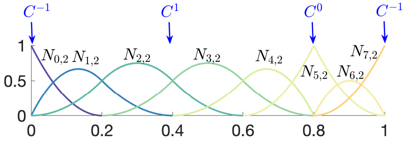

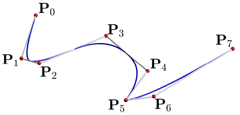

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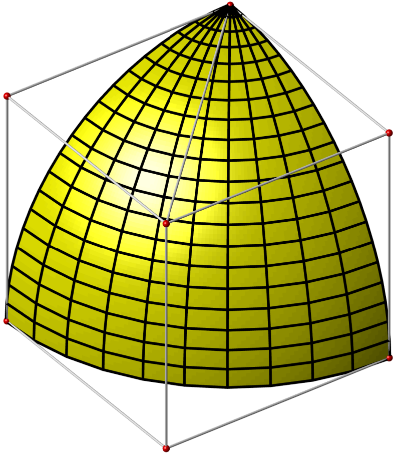

9 control points vs. 231 grid points

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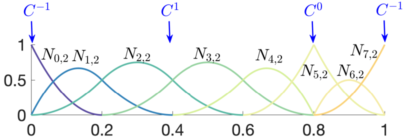
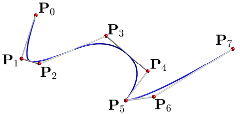
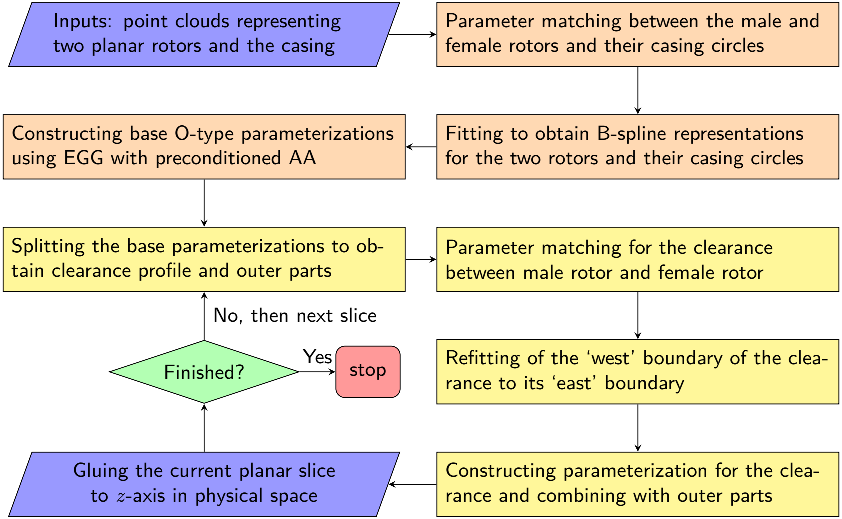




Main Workflow of Our Method



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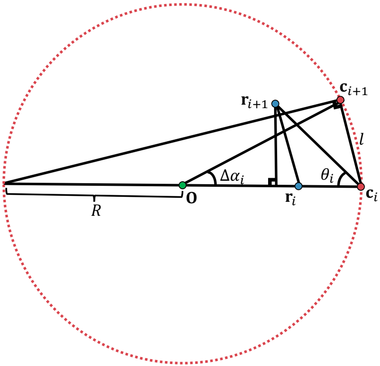
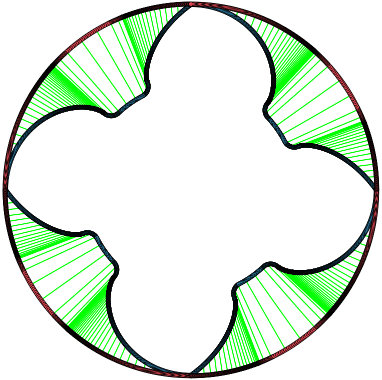
1. Research Background and Motivation
2. **PART I: Processing Steps**
3. PART II: Quasi-harmonic EGG with Preconditioned Anderson Acceleration
4. PART III: High-quality Clearance Parameterization
5. Application and Simulation
6. Conclusions and Outlook

PART I: Processing Steps

- **Boundary Regularization**: Ensures high-quality parameterization.
- **Conversion**: Employs end-points constrained least-squares B-spline fitting.
- **B-spline Utilization**: Uses B-spline rather than original point clouds for subsequent operations. **Enables precise geometry error control.**





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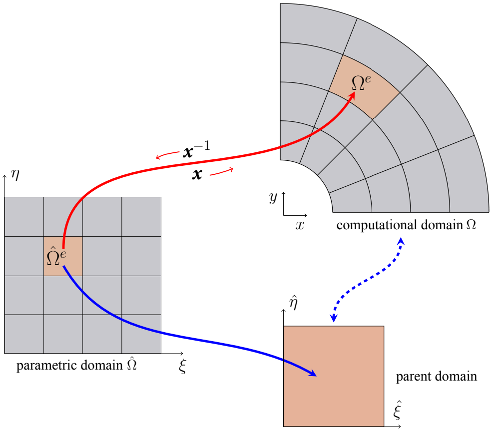
PART II: Quasi-harmonic EGG with Preconditioned Anderson Acceleration

- Main Contributions:
  - **Improved Quality!! Quasi-harmonic EGG in H^1 Sobolev Space**
    A novel approach for generating high-quality parameterizations.
  - **Enhanced Efficiency!! Preconditioned Anderson Acceleration solver**
    An advanced method to efficiently solve the underlying nonlinear systems.

Elliptic Grid Generation (EGG) method

- To compute a quasi-harmonic mapping $\boldsymbol{x} : \hat{\Omega} \to \Omega$ by solving:

$$\begin{cases} \nabla \cdot (\boldsymbol{A}(\boldsymbol{x})\nabla\xi(\boldsymbol{x})) = 0 \\ \nabla \cdot (\boldsymbol{A}(\boldsymbol{x})\nabla\eta(\boldsymbol{x})) = 0 \end{cases} \quad \text{s.t. } \boldsymbol{x}^{-1}|_{\partial\Omega} = \partial\hat{\Omega}.$$



- The **existence and uniqueness** of the harmonic mapping $\boldsymbol{x}^{-1}$ is guaranteed if [a]:
  - The curvature of $\hat{\Omega}$ is non-positive;
  - The boundary $\hat{\Omega}$, when considered with respect to the metric on Ω, is convex.
- The unique solution $\boldsymbol{x}^{-1}$ offers a **one-to-one mapping** (with the Jacobian $\boldsymbol{J}$ not vanishing), which is ensured by the Radó-Kneser-Choquet theorem. [b]

[a] Ref.: Eells, J., & Lemaire, L., (1978). A report on harmonic maps. Bulletin of the London mathematical society, 10(1):1–68.

[b] Ref.: Duren, P., & Hengartner, W., (1997). Harmonic mappings of multiply connected domains. Pac. J. Math. 180, 201–220.

Elliptic Grid Generation (EGG) & Its Discretizations

- $\boldsymbol{A}(\boldsymbol{x}) = \boldsymbol{I}$ yields the nonlinear vector-valued second-order PDE [J.P. Hinz 2020]

$$\forall R_i \in \Sigma_0 : \begin{cases} \int_{\hat{\Omega}} \mathbf{R}\ \widetilde{\mathcal{L}}x\ d\hat{\Omega} = \mathbf{0}, \\ \int_{\hat{\Omega}} \mathbf{R}\ \widetilde{\mathcal{L}}y\ d\hat{\Omega} = \mathbf{0}, \end{cases} \quad \text{s.t. } \mathbf{x}|_{\partial\hat{\Omega}} = \partial\Omega,$$

where

$$\widetilde{\mathcal{L}} = \frac{\mathcal{L}}{g_{11} + g_{22}}, \quad \text{and } \mathcal{L} = g_{22}\frac{\partial^2}{\partial\xi^2} - 2g_{12}\frac{\partial^2}{\partial\xi\partial\eta} + g_{11}\frac{\partial^2}{\partial\eta^2}.$$

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- Metric tensor $\boldsymbol{A}(\mathbf{x}) = \mathrm{diag}\left(1/|\mathcal{J}|, 1/|\mathcal{J}|\right)$ yields the variational formulation in the Sobolev space H^1:

$$\forall R_i \in \Sigma_0 : \begin{cases} \mathbf{F}^x_{H^1} = \mathbf{0}, \\ \mathbf{F}^y_{H^1} = \mathbf{0}, \end{cases} \quad \text{s.t. } \mathbf{x}|_{\partial\hat{\Omega}} = \partial\Omega.$$

  where

$$\mathbf{F}^x_{H^1} = \int_{\hat{\Omega}} \nabla_{\mathbf{x}}\mathbf{R} \cdot \nabla_{\mathbf{x}}\xi\ d\hat{\Omega}, \quad \mathbf{F}^y_{H^1} = \int_{\hat{\Omega}} \nabla_{\mathbf{x}}\mathbf{R} \cdot \nabla_{\mathbf{x}}\eta\ d\hat{\Omega},$$

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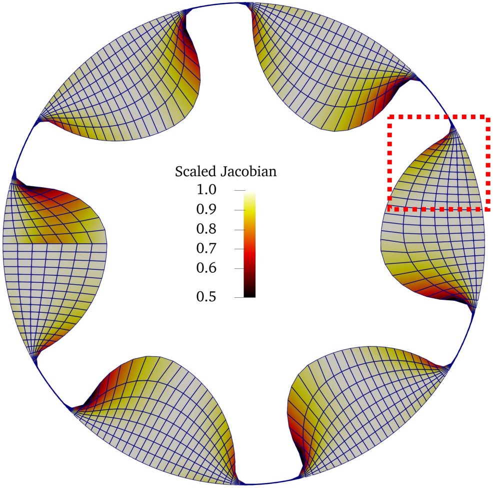
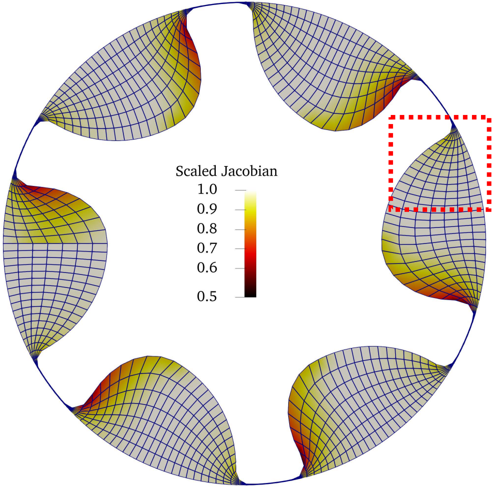


Hinz 2020's result



Our method with better orthogonality

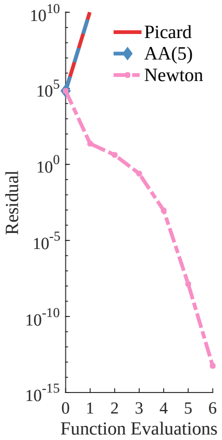
Anderson Acceleration (D.G. Anderson, 1965)

- AA(m) linearly recombines m **previous iterates** in a manner that approximately **minimizes the linearized fixed-point residual $\mathcal{F}$ in a least-squares fashion**.

Algorithm AA: Anderson Acceleration

Given $\boldsymbol{u}_0$ and window size $m \geq 1$;
Set $\boldsymbol{u}_1 = \mathcal{G}(\boldsymbol{u}_0)$;
for $k = 1, 2, \ldots, itmax$ *until* $\|\mathcal{F}_k\| < tol$ **do**
    Set $m_k = \min\{m, k\}$;
    Determine $\boldsymbol{\alpha}^{(k)} = \left(\alpha_0^{(k)}, \alpha_1^{(k)}, \ldots, \alpha_{m_k}^{(k)}\right)^{\mathrm{T}}$ that solves
    $$\underset{\boldsymbol{\alpha}=(\alpha_0,\alpha_1,\ldots,\alpha_{m_k})^{\mathrm{T}}}{\arg\min} \left\| \sum_{i=0}^{m_k} \mathcal{F}_{k-m_k+i}\boldsymbol{\alpha} \right\|_2^2, \quad \text{s.t.} \quad \sum_{i=0}^{m_k} \alpha_i = 1;$$
    Update $\boldsymbol{u}_{k+1} = \sum_{i=0}^{m_k} \alpha_i^{(k)} \mathcal{G}_{k-m_k+i}$;

Bratu problem (64×64):



Enhancing Anderson Acceleration with Preconditioning

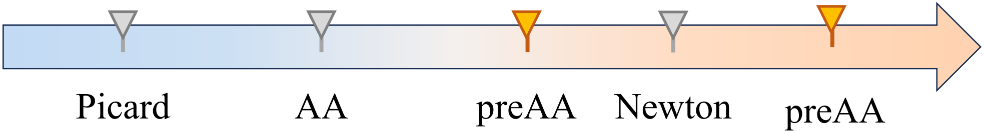
Preconditioning Strategy

We introduce a preconditioning strategy for the fixed-point iteration scheme:

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k - \boldsymbol{\mathcal{M}}_k^{-1} \boldsymbol{\mathcal{F}}_k,$$

where $\boldsymbol{\mathcal{M}}_k$ is a non-singular matrix, known as the **preconditioner** at iteration k.

- **Ideal preconditioner:** Close to $jac(\boldsymbol{\mathcal{F}})$, yet computationally inexpensive.
- More flexibility, e.g., constant $\alpha \boldsymbol{I}$, diagonal Jacobian $diagJac(\boldsymbol{\mathcal{F}})$, upper (lower) triangular Jacobian $TriU(jac(\boldsymbol{\mathcal{F}}))$ and block-diagonal Jacobian $diagBlockJac(\boldsymbol{\mathcal{F}})$.
- **Delayed Update Strategy:** Reduces frequent preconditioner updates.



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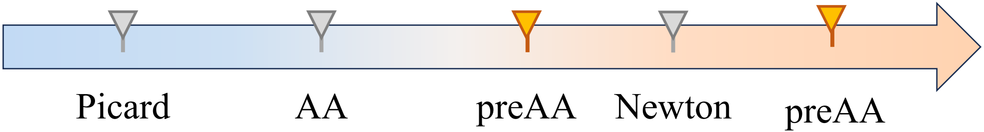

PreAA: Preconditioned Anderson Acceleration

Algorithm PreAA: preAA

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Set $\boldsymbol{u}_1 = \boldsymbol{\mathcal{G}}(\boldsymbol{u}_0)$;
for $k = 1, 2, \ldots, itmax$ *until* $\|\boldsymbol{\mathcal{F}}(\mathbf{u})\| < tol$ **do**

```
// Update preconditioner
```

if k *is evenly divisible by* N_{update} **then**
Update preconditioning matrix $\boldsymbol{\mathcal{M}}_k$;
Set $m_k = \min\{m, k\}$;
Compute $\boldsymbol{\mathcal{E}}_k$ by solving $\boldsymbol{\mathcal{M}}_k \boldsymbol{\mathcal{E}}_k = -\boldsymbol{\mathcal{F}}_k$;
Determine $\boldsymbol{\alpha}^{(k)} = \left(\alpha_0^{(k)}, \alpha_1^{(k)}, \ldots, \alpha_{m_k}^{(k)}\right)^{\mathrm{T}}$ that solves

$$\operatorname*{arg\,min}_{\boldsymbol{\alpha}=(\alpha_0,\alpha_1,\ldots,\alpha_{m_k})^{\mathrm{T}}} \left\| \sum_{i=0}^{m_k} \boldsymbol{\mathcal{E}}_{k-m_k+i} \boldsymbol{\alpha} \right\|_2^2, \quad \text{s.t.} \quad \sum_{i=0}^{m_k} \alpha_i = 1.$$

Update $\boldsymbol{u}_{k+1} = \sum_{i=0}^{m_k} \alpha_i^{(k)} \left(\boldsymbol{u}_{k-m_k+i} + \boldsymbol{\mathcal{E}}_{k-m_k+i}\right)$;

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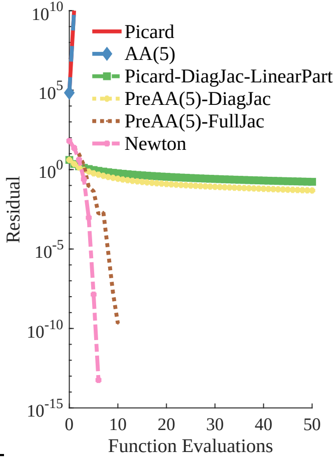
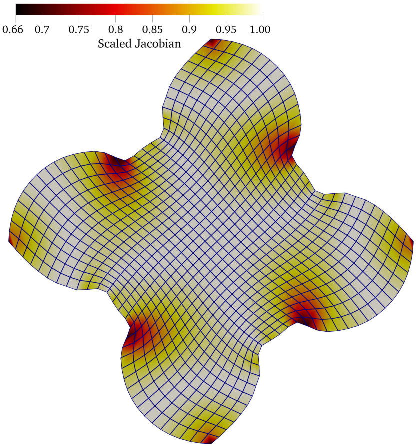
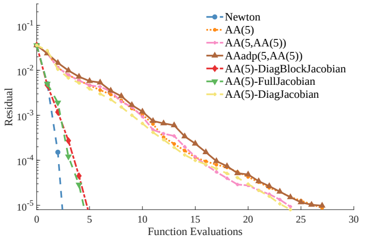
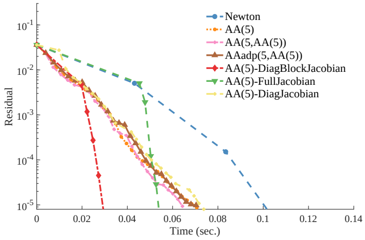


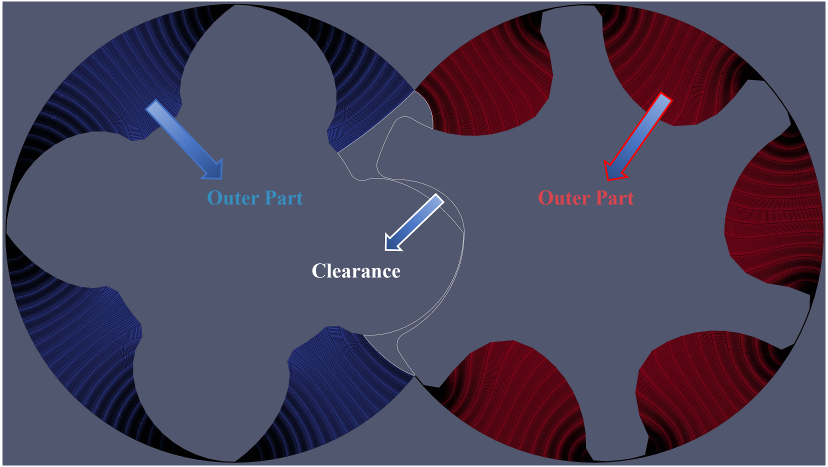
Male rotor example: performance comparisons







Dual C-type Parameterizations from Base Rotors



- By splitting at the CUSP-points for each slice $\xi = \xi_{i_3}$, we obtain dual C-type parameterizations for the outer parts and the clearance part.

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PART III: High-quality Clearance Parameterization

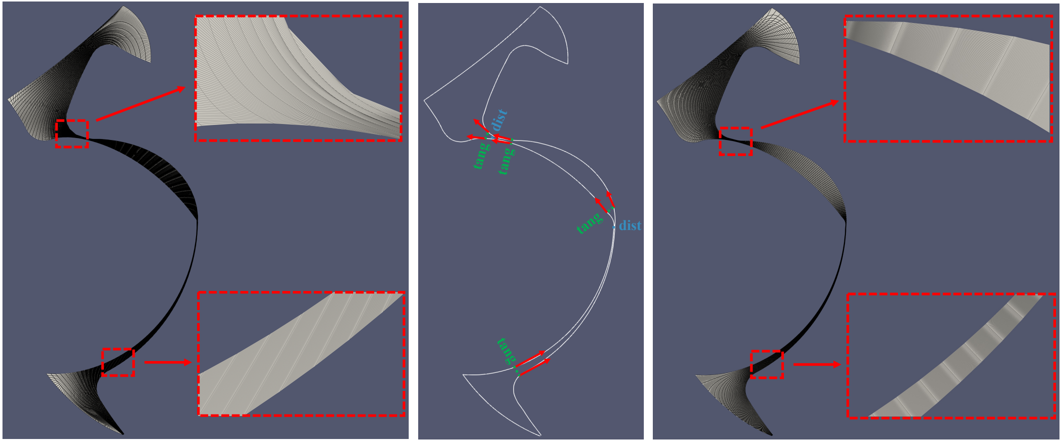
Objective: Generate a high-quality parameterization for the clearance part at every slice.

- **Boundary Matching:**
  - Match parameters for the west and east boundaries;
  - Refit the west boundary after matching.
- **Clearance Reparameterization:**

  Apply the proposed quasi-harmonic EGG parameterization technique to generate high-quality clearance parameterization.
- **Volumetric Completion:**

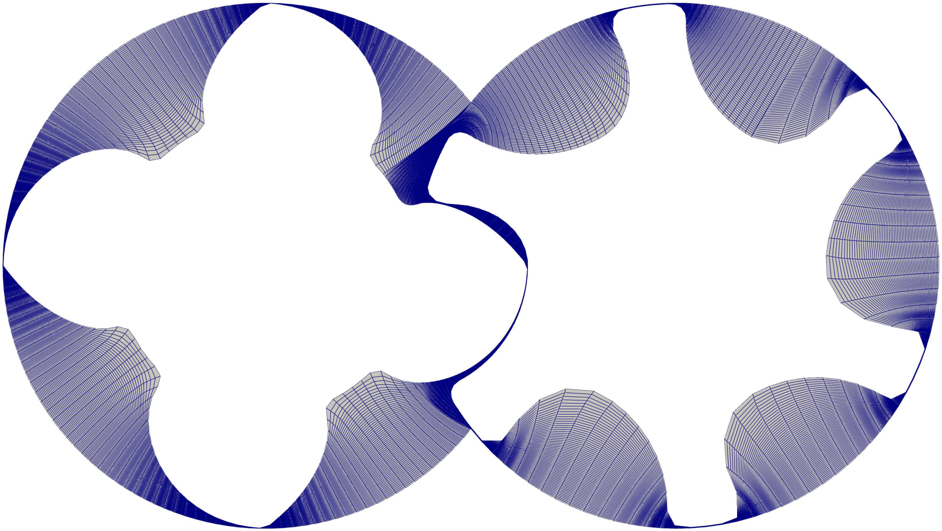
  Upon obtaining all slice parameterizations, employ the spline lofting technique for the final volumetric parameterization.

Parameter Matching Technique



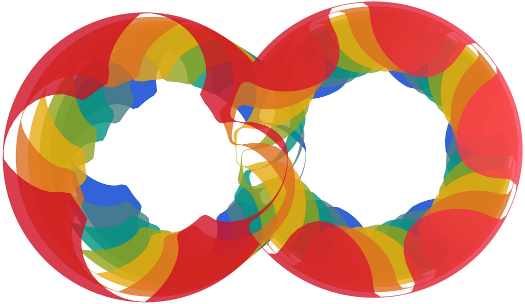
Before ⟶ Our Parameter Matching Strategy ⟶ **After**

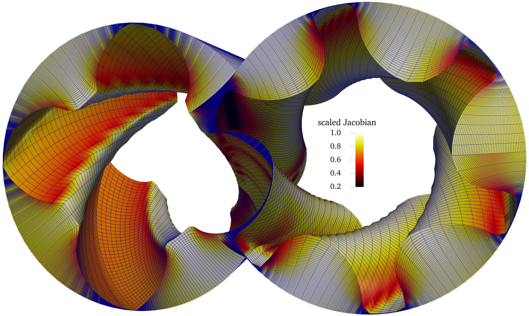
Resulting Parameterization Slices



Volumetric Completion via Spline Lofting

- Complete volumetric parameterization by lofting computed slices.
- With the **PreAA** solver, achieve parameterization for twin-screw machines **in just 3 seconds** on my personal laptop.





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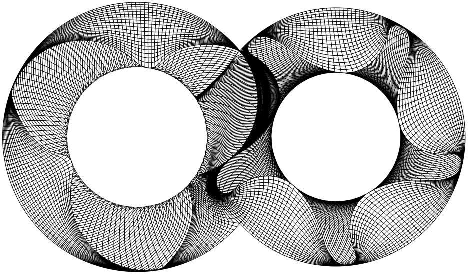
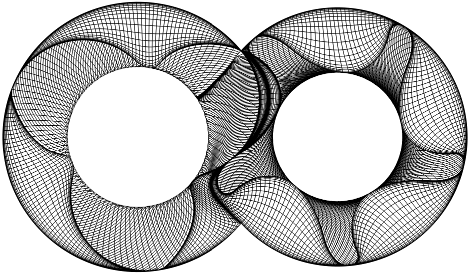
Discretization I: Boundary Layer Mesh

- **ALL YOU NEED** is just **down-sampling**.
- Employ a simple expansion transformation:

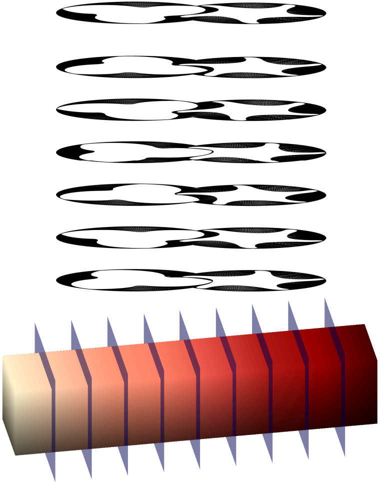
$$\begin{cases} \xi = \hat{\xi}, \\ \eta = \dfrac{\tanh\left(\alpha(2\hat{\eta}-1)\right)}{2\tanh(\hat{\eta})} + \dfrac{1}{2}, \end{cases}$$

where α represents the expansion factor.

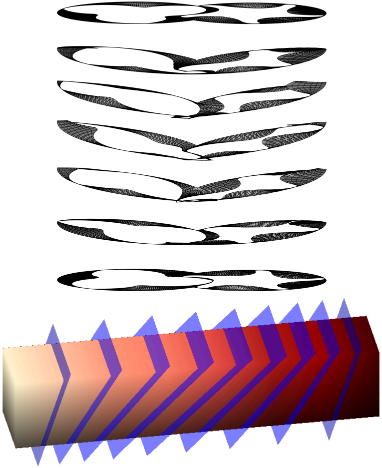




Discretization II: Flow-Aligned Hex Mesh



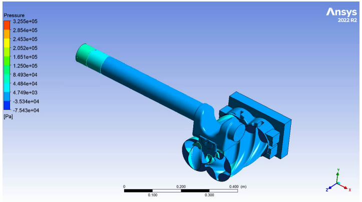
generic discretization

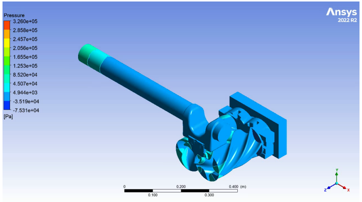


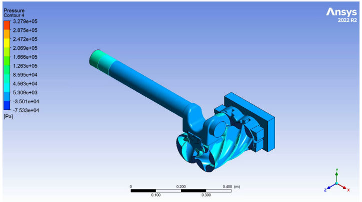
flow-aligned discretization

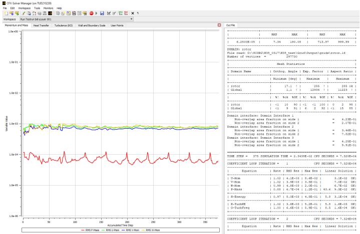
Simulation using ANSYS CFX

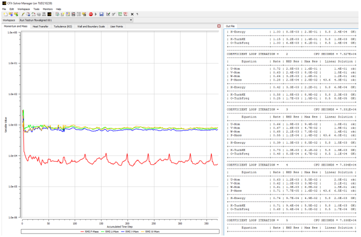
- Mesh density: $198 \times 95 \times 8$ for the male rotor and $200 \times 95 \times 8$ for the female rotor.









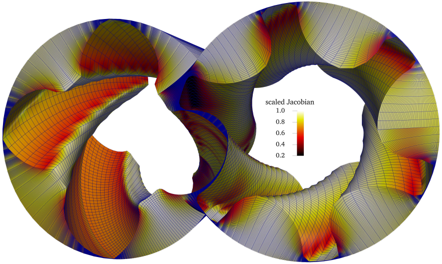


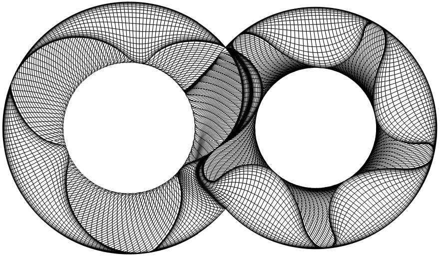
SCORG ™ Boundary Layer Mesh **Flow-aligned Mesh**

Agenda

Conclusions and Outlook

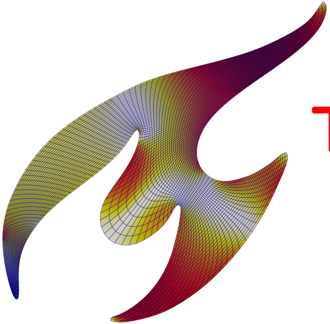
- **Spline-based mesh generation workflow** is built up for twin-screw compressors.
- **Innovative Techniques:** Integration of quasi-harmonic Elliptic Grid Generation (EGG) and PreAA nonlinear solver.
- **Easy conversion** into high-quality structured meshes.





Future Work:

- **Advanced Parameter Matching method.**
- **Extension to Complex Geometries** for industrial needs.



Thanks for Your Attention!

Q&A.

If interested in my research, please feel free to contact me! ;-)

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