IGA-suitable parameterization: Advancements inside and outside G+Smo

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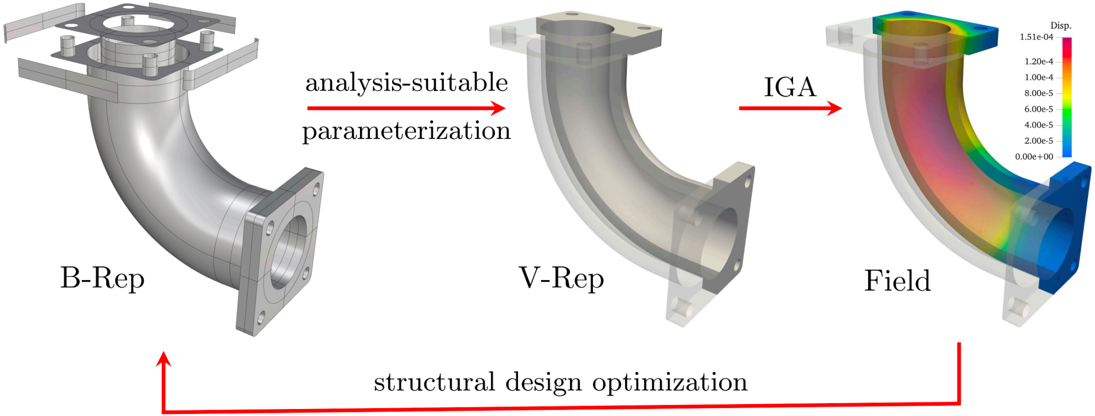
Agenda

1. Domain parameterization problem
2. Inside `G+Smo`
3. Applications
4. Outside `G+Smo`
5. Conclusions

Agenda

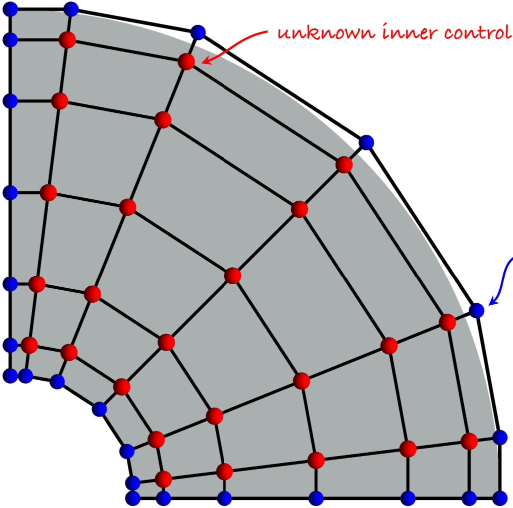
Domain parameterization for isogeometric analysis

- CAD models are usually represented by **boundary representation** (B-Rep);
- However, IGA requires an internal **spline-based parameterization** (V-Rep).



Design-analysis-optimization workflow in product development

Problem statement: domain parameterization



$$\boldsymbol{x}(\boldsymbol{\xi}) = \underbrace{\sum_{i\in\mathcal{I}_I} \mathbf{P}_i R_i(\boldsymbol{\xi})}_{\textbf{unknown}} + \underbrace{\sum_{j\in\mathcal{I}_B} \mathbf{P}_j R_j(\boldsymbol{\xi})}_{\textbf{known}}.$$

GOAL: to construct **unknown inner control points $\mathbf{P}_i$** (or basis functions $R_i(\boldsymbol{\xi})$) such that $\boldsymbol{x}$ ensures **bijectivity** and exhibits **optimal orthogonality** and **uniformity**.

- **Parameterization quality** significantly affects **downstream analysis!**

Agenda

Inside G+Smo: as_parameterization_example.cpp

Class Name	**Brief Description**
gsBarrierCore<d,T>	Core class for AS-parameterizations using various approaches.
gsBarrierPatch<d,T>	Data and geometry preprocessing for gsBarrierCore<d,T>.
gsHLBFGS<T>	Wrapper for the Hybrid Low-storage BFGS optimization solver.
AndersonAcceleration<T>	Anderson acceleration solver and its preconditioned variants.
preAAParam<T>	Parameters for the preconditioned AA solver.

- Example file: as_parameterization_example.cpp
- Test inputs: filedata/breps
    - 2D case: filedata/breps/2D/duck_BRep.xml
    - 3D case: filedata/breps/3D/duck_BRep.xml
    - multi-patch case: filedata/breps/other/TUDflame.xml, credits to Hugo Verhelst.

Inside G+Smo: as_parameterization_example.cpp

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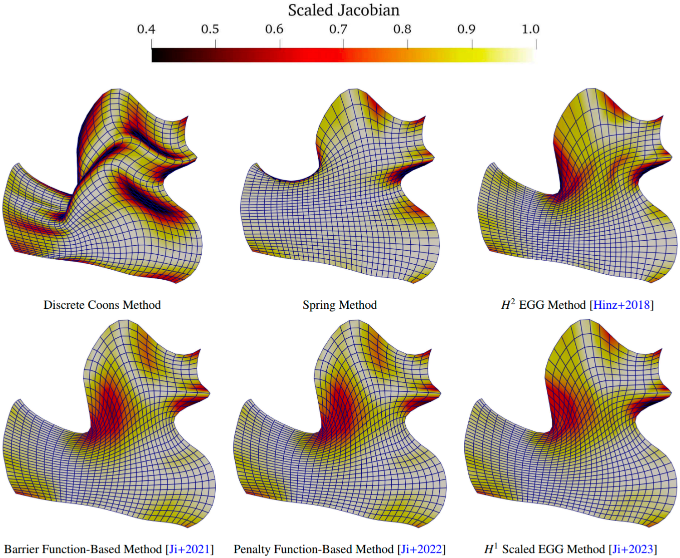
  - 3D case: filedata/breps/3D/duck_BRep.xml
  - multi-patch case: filedata/breps/other/TUDflame.xml, credits to Hugo Verhelst.
- **Many available parameterization methods in G+Smo**
  - Not get too much into algorithm details today. ;-)

Available methods in G+Smo



- Algebraic methods:
  - Coons patch
  - Spring patch
- Optimization-based methods:
  - Barrier-function-based [1]
  - Penalty-function-based [2]
- PDE-based methods:
  - Elliptic grid generation [3]
  - Improved EGG [4]

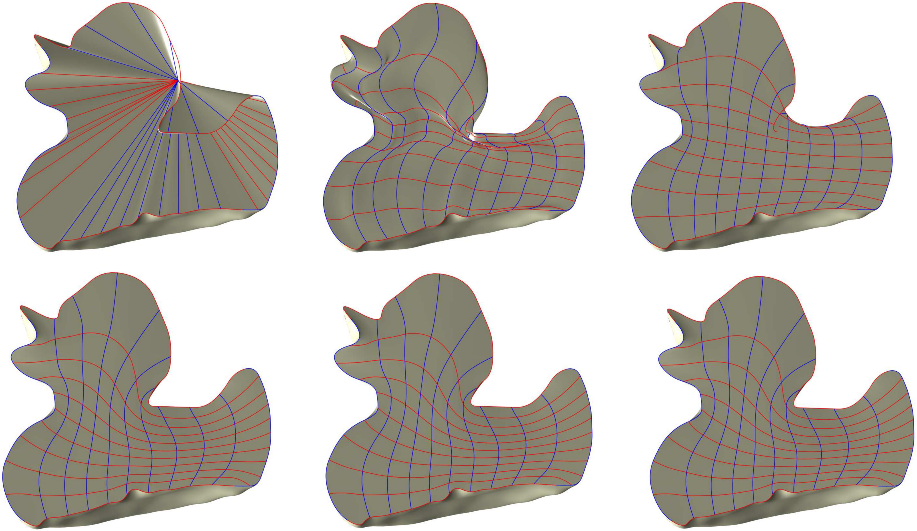
[1] Ji, Y. et al. (2021). Constructing high-quality ...

[2] Ji, Y. et al. (2022). Penalty function-based ...

[3] Hinz, J. et al. (2018). Elliptic grid generation ...

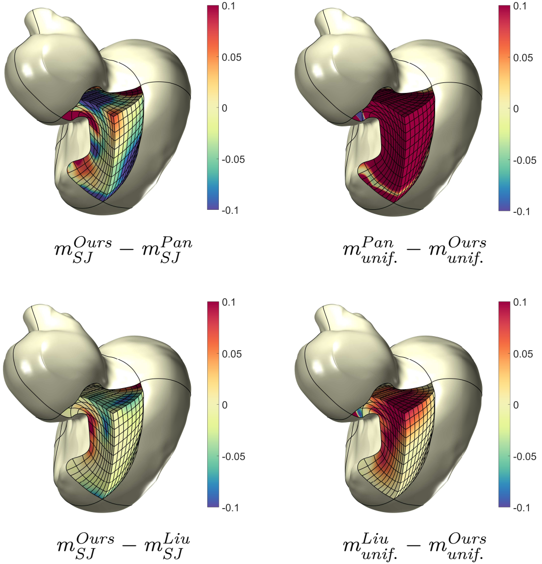
[4] Ji, Y. et al. (2023). On an improved PDE-based ...

Strong similarity from different initializations



Same point Discrete Coons Smoothness energy

Parameterization Quality Comparison



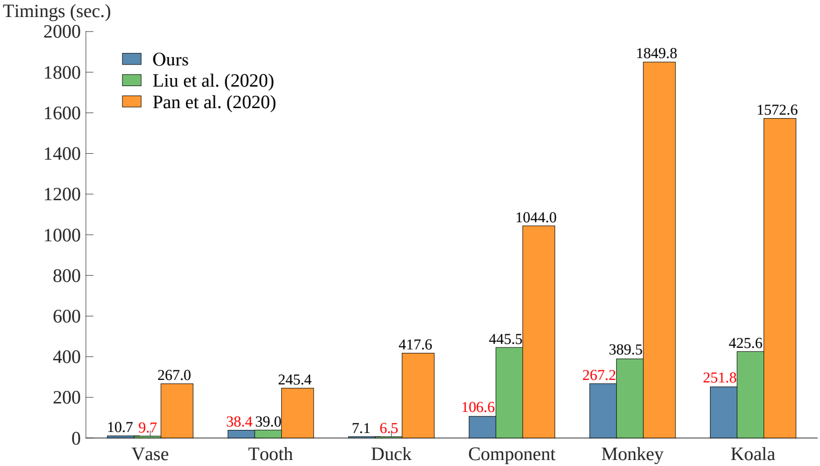
- Comparisons to Pan et al. 2020 [1] and Liu et al. 2020 [2]
- Positive values (red regions) indicate our method performs better

[1] Pan, M., Chen, F., & Tong, W. (2020). Volumetric spline parameterization for isogeometric analysis. Computer Methods in Applied Mechanics and Engineering, 359, 112769.

[2] Liu, H., Yang, Y., Liu, Y., & Fu, X. M. (2020). Simultaneous interior and boundary optimization of volumetric domain parameterizations for IGA. Computer Aided Geometric Design, 79, 101853.

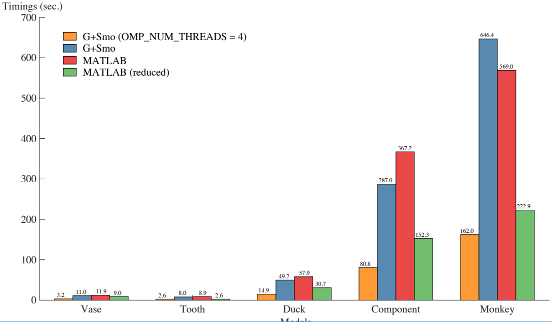
Efficiency Comparison

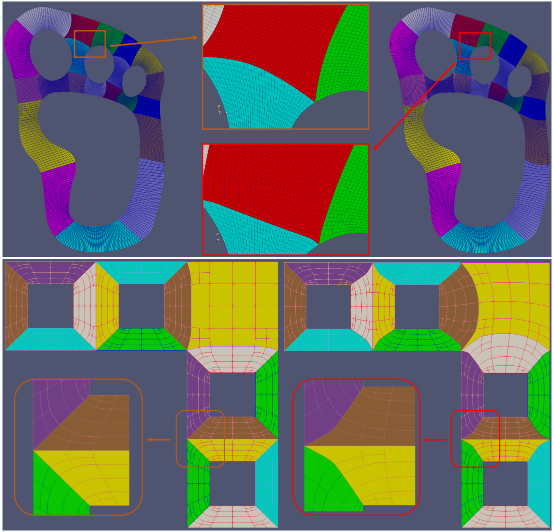
- Significantly **faster** than Pan et al. (2020);
- **Large-Scale Models:** Outperforms Liu et al. (2020);
- **Implementation:** MATLAB (ours) vs. C++ (Pan et al. & Liu et al.).



G+Smo Implementation

- In our released G+Smoimplementation, **3-4x speed-up**;
- Suitable for **multi-patch** and **THB-spline** parameterization;
- **Precomputation and fast matrix assembly in progress**.





Elliptic Grid Generation (EGG) & Its H^2 Discretization

- To compute a harmonic mapping $\boldsymbol{x}: \ \hat{\Omega} \rightarrow \Omega$ by solving Laplace equations:

$$\begin{cases} \nabla \cdot \nabla \xi(x, y) = 0 \\ \nabla \cdot \nabla \eta(x, y) = 0 \end{cases} \quad \text{s.t. } \boldsymbol{x}^{-1}|_{\partial\Omega} = \partial\hat{\Omega}.$$

- **Existence and uniqueness**, and unique solution $\boldsymbol{x}^{-1}$ offers a **one-to-one mapping** (Radó-Kneser-Choquet theorem).

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- Nonlinear vector-valued second-order PDE [1]

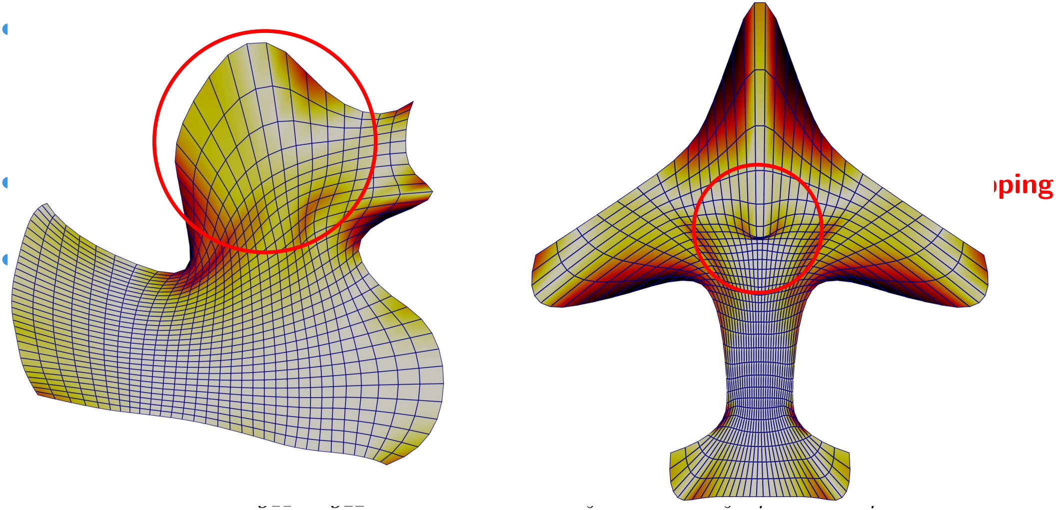
$$\forall R_i \in \Sigma_0 : \begin{cases} \int_{\hat{\Omega}} \mathbf{R}\ \widetilde{\mathcal{L}} x\ d\hat{\Omega} = \mathbf{0}, \\ \int_{\hat{\Omega}} \mathbf{R}\ \widetilde{\mathcal{L}} y\ d\hat{\Omega} = \mathbf{0}, \end{cases} \quad \text{s.t. } \boldsymbol{x}|_{\partial\hat{\Omega}} = \partial\Omega,$$

where

$$\widetilde{\mathcal{L}} = \frac{\mathcal{L}}{g_{11} + g_{22}}, \quad \text{and } \mathcal{L} = g_{22}\frac{\partial^2}{\partial\xi^2} - 2g_{12}\frac{\partial^2}{\partial\xi\partial\eta} + g_{11}\frac{\partial^2}{\partial\eta^2}.$$

[1] Hinz, J. P. (2020). PDE-Based Parameterization Techniques for Isogeometric Analysis Applications. TU Delft doctoral dissertation.

Elliptic Grid Generation (EGG) & Its H^2 Discretization



- **Non-uniform** elements appear, may even be **non-bijective**.

[1] Hinz, J. P. (2020). PDE-Based Parameterization Techniques for Isogeometric Analysis Applications. TU Delft doctoral dissertation.

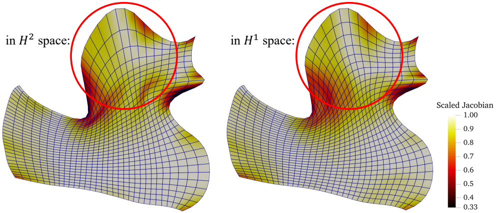
Quasi-Hamonic Mapping

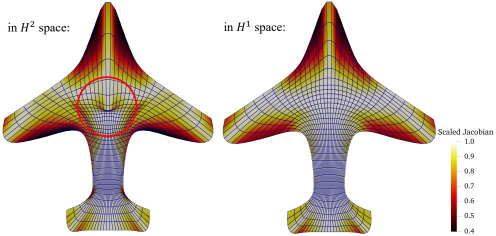
- We introduce a monitor function $\mathbb{A}$ into our governing equation:

$$-\nabla \cdot (\mathbb{A} \nabla \boldsymbol{\xi}) = \mathbf{0}.$$

where the monitor function is defined as

$$\mathbb{A} = \begin{bmatrix} \frac{1}{|\mathcal{J}|} & 0 \\ 0 & \frac{1}{|\mathcal{J}|} \end{bmatrix}.$$





Discretization in Sobolev space H^1

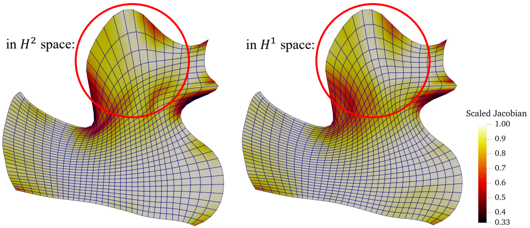
- Variational formulation in Sobolev space H^1 reads

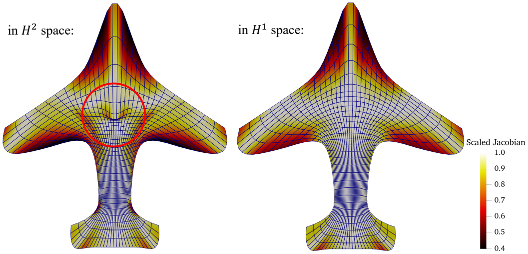
$$\forall R_i \in \Sigma_0 : \begin{cases} \mathbf{F}^x_{H^1} = \mathbf{0}, \\ \mathbf{F}^y_{H^1} = \mathbf{0}, \end{cases} \quad \text{s.t. } \mathbf{x}|_{\partial\hat{\Omega}} = \partial\Omega.$$

where

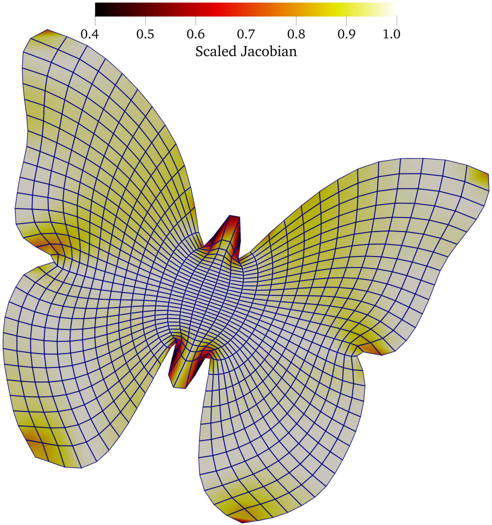
$$\mathbf{F}^x_{H^1} = \int_{\hat{\Omega}} \nabla_{\mathbf{x}} \mathbf{R} \cdot \nabla_{\mathbf{x}} \xi \; d\hat{\Omega}, \quad \mathbf{F}^y_{H^1} = \int_{\hat{\Omega}} \nabla_{\mathbf{x}} \mathbf{R} \cdot \nabla_{\mathbf{x}} \eta \; d\hat{\Omega}.$$

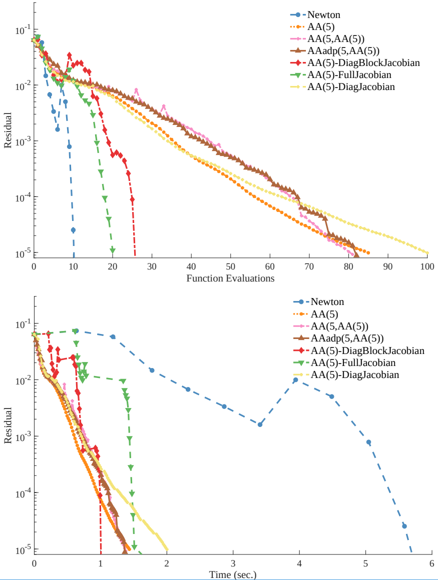
- Needs to solve a **nonlinear system**.



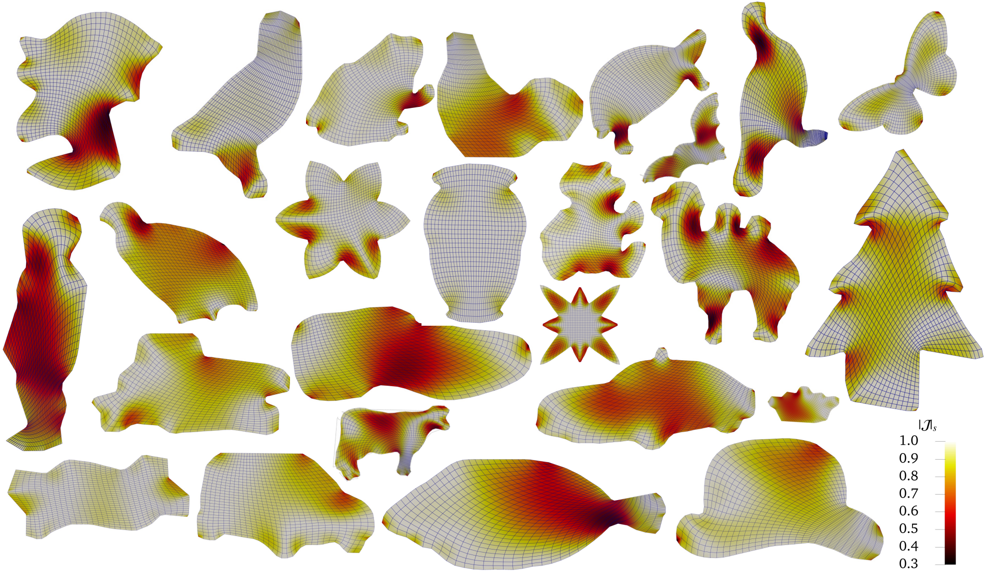


Preconditioned Anderson acceleration solver

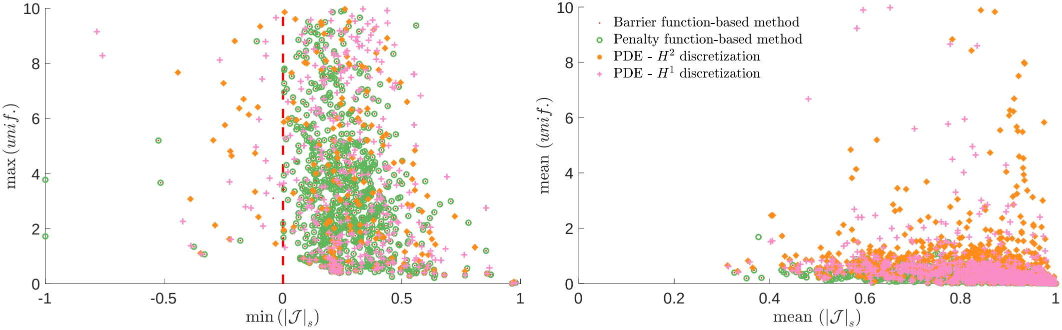




Planar Parameterization Test Dataset (977 models)



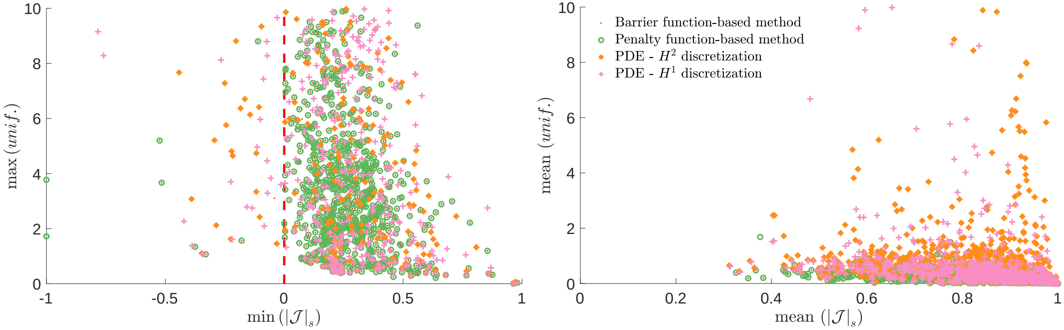
Effectiveness and Quality Assessment



Success rates:

- PDE - H^2 discretization [Hinz+2018]: $608/977 \simeq 62.23\%$;
- PDE - H^1 discretization [Ji+2023]: $721/977 \simeq 73.80\%$;

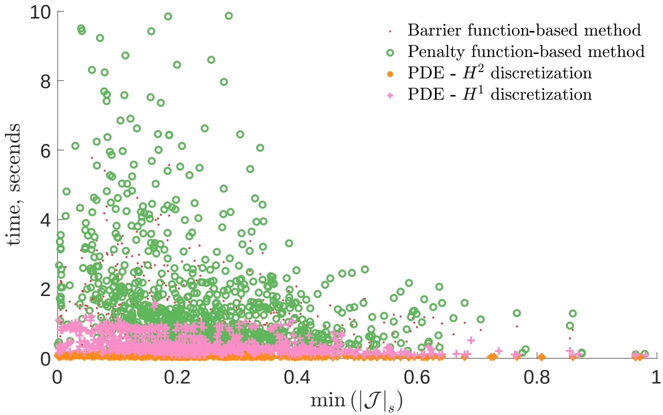
Effectiveness and Quality Assessment



Success rates:

- PDE - H^2 discretization [Hinz+2018]: $608/977 \simeq 62.23\%$;
- PDE - H^1 discretization [Ji+2023]: $721/977 \simeq 73.80\%$;
- Barrier function-based method [Ji+2021]: $961/977 \simeq$ **98.36%**;
- Penalty function-based method [Ji+2022]: $956/977 \simeq 97.85\%$.

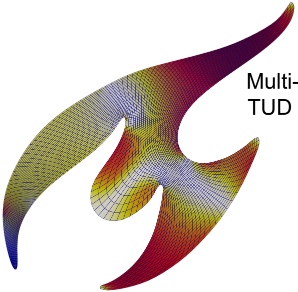
Computation Time



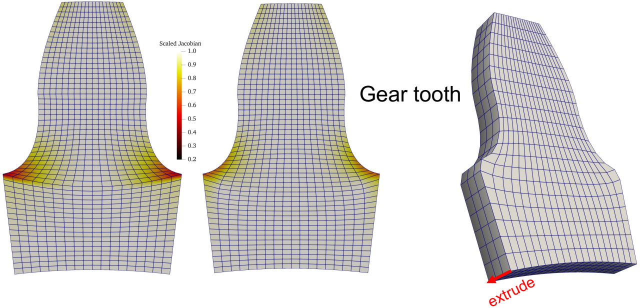
- PDE-based $\sim$ 0.2 sec., optimization based $\sim$ 2 sec. on my laptop.
- PDE-based methods demonstrate higher efficiency.

Agenda

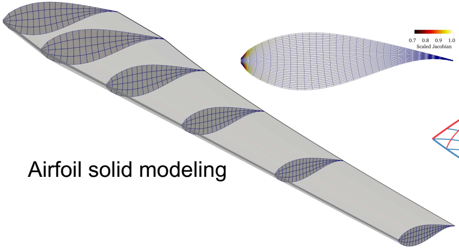
as_parameterization_example.cpp is helping people!



Multi-patch TUD flame

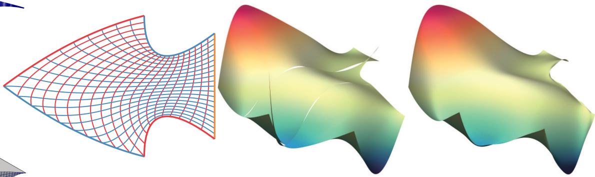


Gear tooth



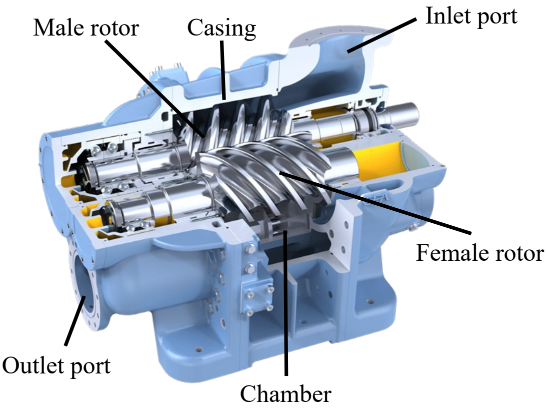
Airfoil solid modeling

Multi-sided surface modeling

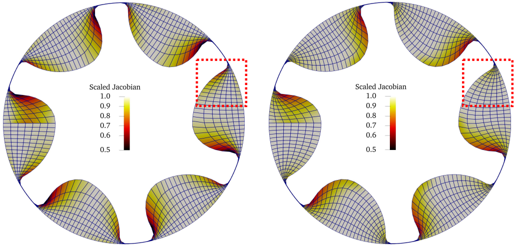


discontinuous stress field (C^0 multi-patch NURBS) **continuous** stress field (multi-sided surface)

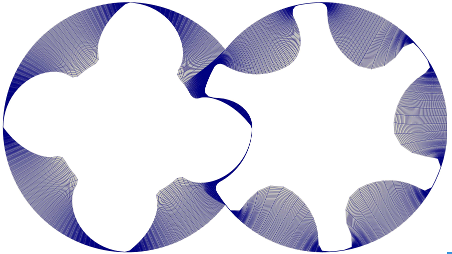
Rotary Twin-Screw Compressors



Rotary Twin-Screw Compressor (source [a])



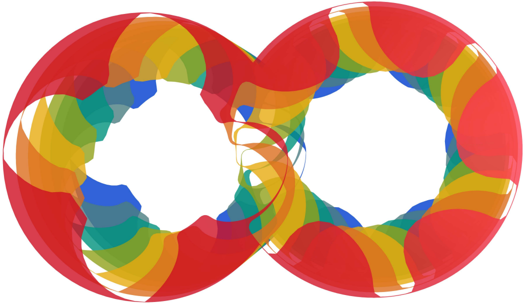
Hinz's (2020) Ours

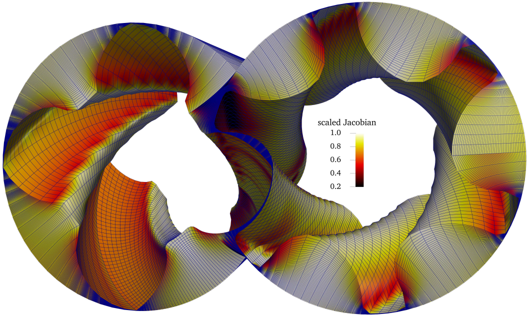


[a] https://www.gascompressors.co.uk/technologies/oil-flooded-screw-compressor/

Volumetric Completion via Spline Lofting

- Complete volumetric parameterization by lofting computed slices.
- With the **PreAA** solver, achieve parameterization for twin-screw machines **in just 3 seconds** on my personal laptop.



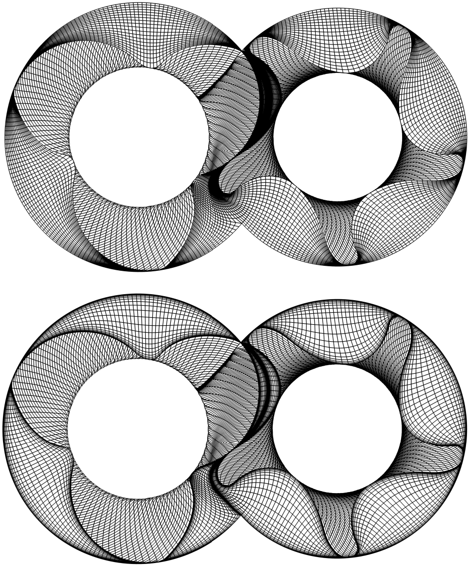


Discretization I: Boundary Layer Mesh

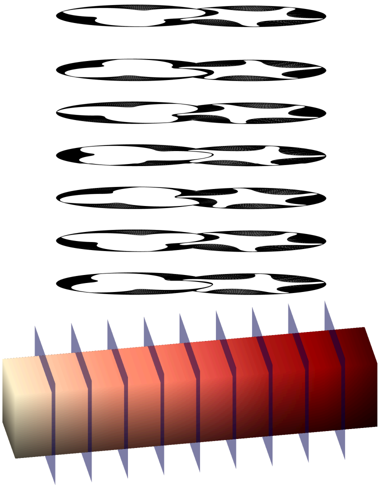
- `bsp.eval()` **is ALL YOU NEED**.
- Employ a simple expansion transformation:

$$\begin{cases} \xi = \hat{\xi}, \\ \eta = \dfrac{\tanh\left(\alpha(2\hat{\eta}-1)\right)}{2\tanh(\hat{\eta})} + \dfrac{1}{2}, \end{cases}$$

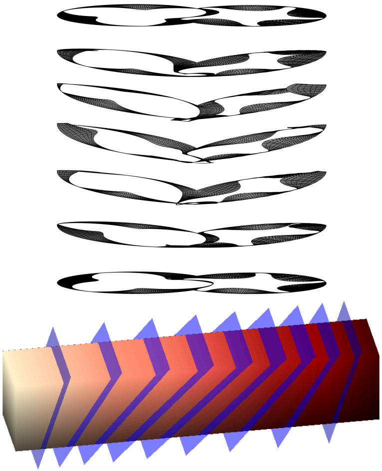
where α represents the expansion factor.



Discretization II: Flow-aligned hexahedral mesh



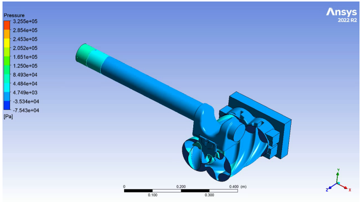
generic discretization

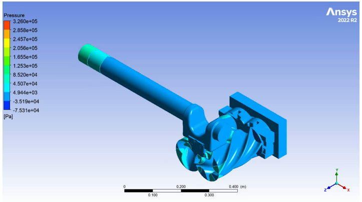


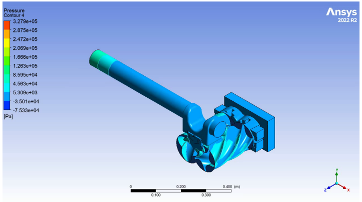
flow-aligned discretization

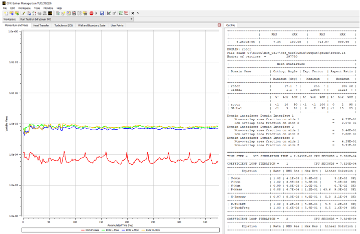
Simulation using ANASYS CFX

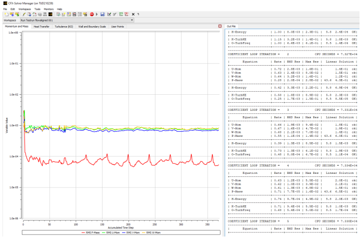
- Mesh density: $198 \times 95 \times 8$ for the male rotor and $200 \times 95 \times 8$ for the female rotor.









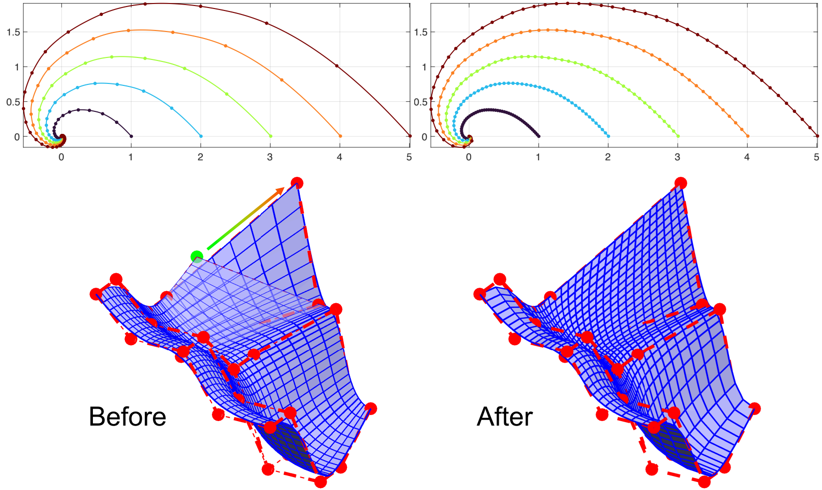


SCORG ™ Boundary Layer Mesh **Flow-aligned Mesh**

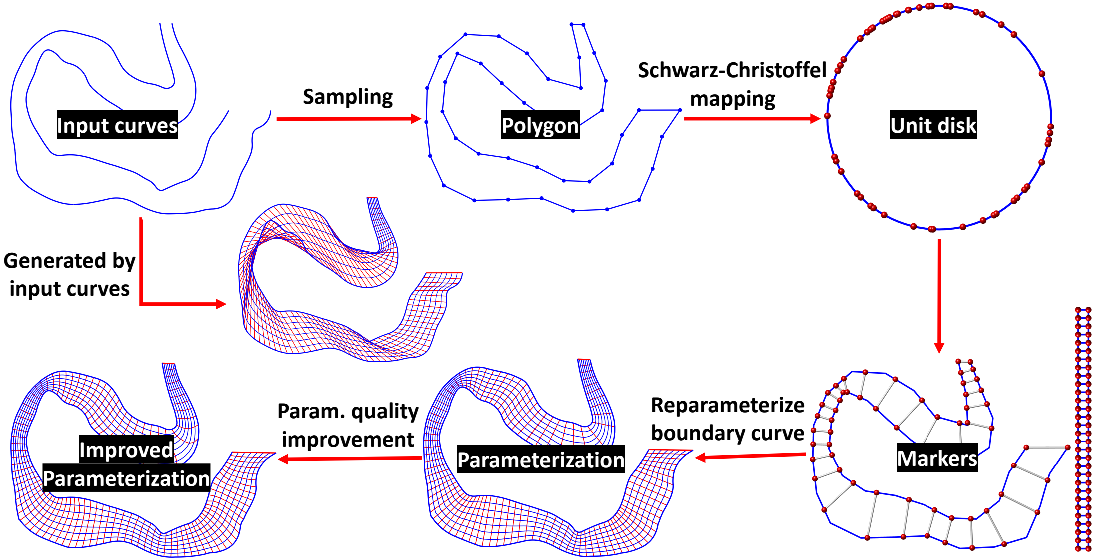
Agenda

I: Extension to curve/surface reparameterization

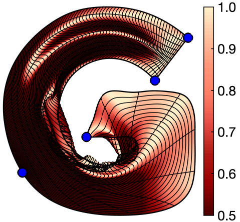
- Optimizing parametric representation while **keeping the underlying geometry**.
- Ensures the resulting geometry **remains in NURBS form**.



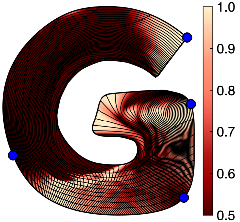
II: Parameter matching via Schwarz-Christoffel mapping



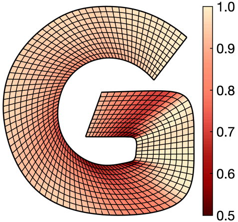
Results and Comparisons



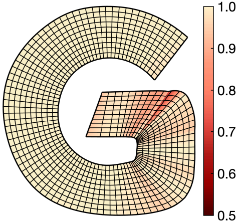
Optimal transport [1]



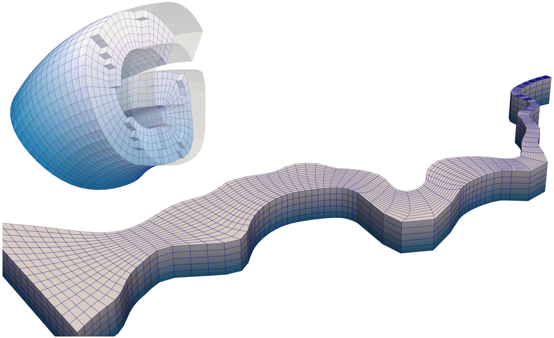
Deep learning [2]



Input [3]



Ours [3]

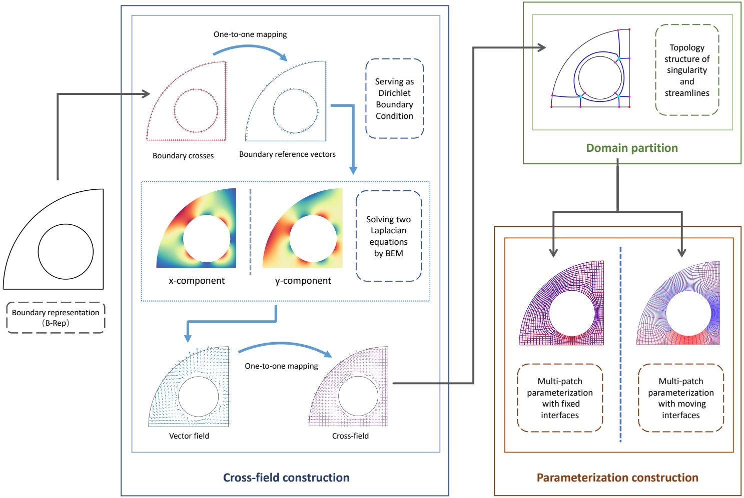


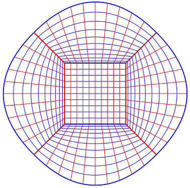
[1] Zheng, Y., Pan, M., & Chen, F. (2019). Boundary correspondence of planar domains for isogeometric analysis based on optimal mass transport. *Computer-Aided Design*, 114, 28-36.

[2] Zhan, Z., Zheng, Y., Wang, W., & Chen, F. (2023). Boundary Correspondence for Iso-Geometric Analysis Based on Deep Learning. Communications in Mathematics and Statistics, 11(1), 131-150.

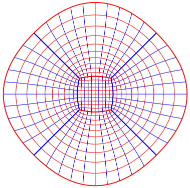
[3] Ji, Y., Möller M., Yu Y., & C. Zhu Boundary parameter matching for isogeometric analysis using Schwarz-Christoffel mapping. Submited.

III: Multi-patch Parameterization using Cross-field





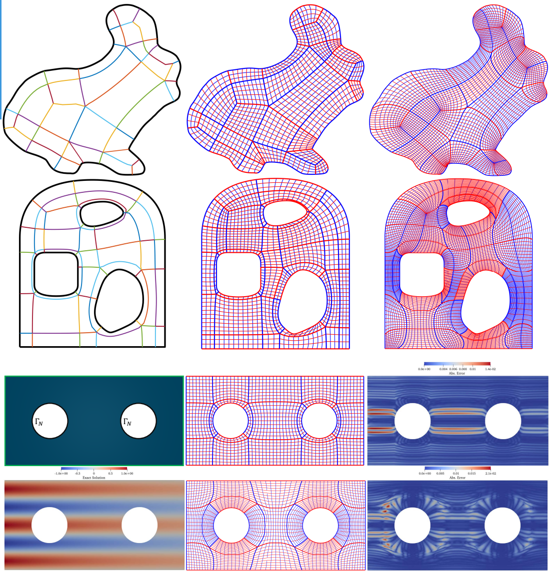
AS-G^1 [1]



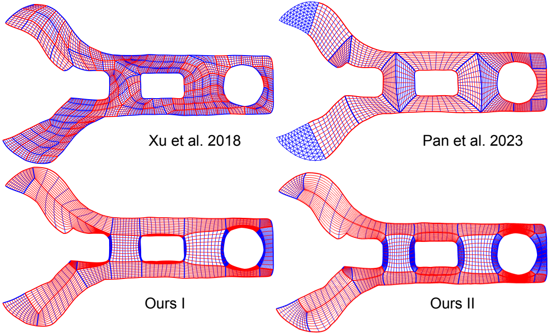
Ours [2]

[1] Collin, A., Sangalli, G., & Takacs, T. (2016). Analysis-suitable G^1 multi-patch parametrizations ... *Computer Aided Geometric Design*, 47, 93-113.
[2] Zhang, Y., Ji, Y., & Zhu, C. (2024). Multi-patch parameterization method for isogeometric analysis using singular structure of cross-field. *Computers and Mathematics with Applications*, accepted.

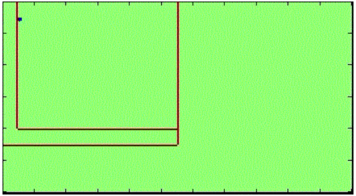
Results and Comparisons

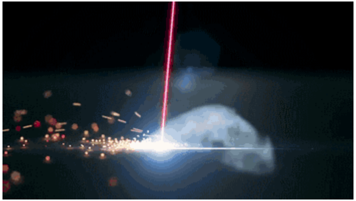


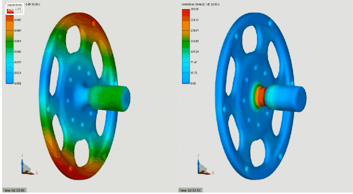
Model	#Patch	Method	$\lvert\mathcal{J}\rvert_s$ min.	$\lvert\mathcal{J}\rvert_s$ avg.	*unif.* min.	*unif.* avg.	*unif.* max.
rabbit	33	Coons	-0.8593	0.9628	0.7030	0.9410	1.0524
		fixed-I	0.2204	0.9504	0.6103	0.9544	0.9982
		moving-I	0.02918	0.9283	0.0000	0.9550	1.0000
3 holes	46	Coons	-0.5492	0.9710	0.8008	0.9573	1.0958
		fixed-I	0.1545	0.9716	0.8007	0.9573	0.9978
		moving-I	0.1461	0.9361	0.6791	0.9571	0.9968



IV: r-adaptivity - Anisotropic phenomena in physics


Wave Propagation. source


Laser Printing. source

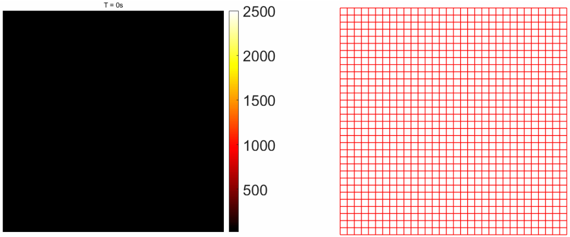

Stress Concentration. source

- **Localized and anisotropic features extensively exist** in physical phenomena;
- Isotropic parameterizations are not efficient for such problems;
- **Anisotropic parameterizations (r-adaptivity)**
  – Enhance per-DOF accuracy while keeping constant total DOFs.

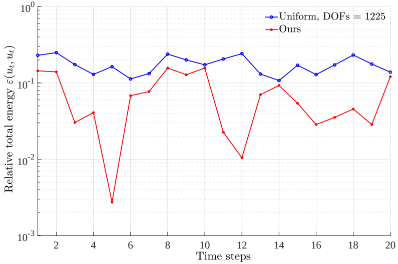
Application to Time-dependent Dynamic PDE

- Consider a 2D **linear heat transfer problem** with a moving Gaussian heat source [1]:

$$\begin{cases} C_p\rho\frac{\partial u(\mathbf{x},t)}{\partial t} - \nabla\cdot(\kappa\nabla u(\mathbf{u},t)) = f(\mathbf{x},t) & \text{in } \Omega\times T \\ u(\mathbf{x},t) = u_0 & \text{in } \Omega \\ \kappa\nabla u(\mathbf{x},t) = 0 & \text{on } \partial\Omega\times T \end{cases}$$



$u(\mathbf{x},t)$ and the parameterizations



Errors vs. time instants t

[1] Carraturo, M., Giannelli, C., Reali, A., & Vázquez, R. (2019). Suitably graded THB-spline refinement and coarsening: Towards an adaptive isogeometric analysis of additive manufacturing processes. Computer Methods in Applied Mechanics and Engineering, 348, 660-679.

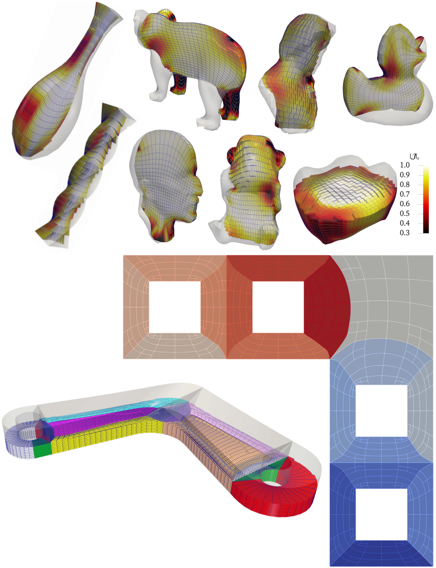
Agenda

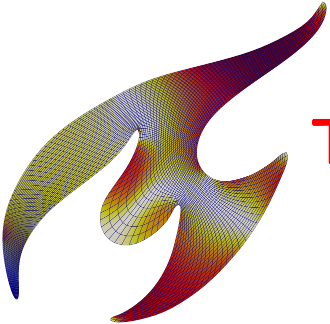
Conclusions

- G+Smo integrates **three major classes** of parameterization methods;
- Demonstrates **improved robustness and efficiency** over existing methods;
- Applicability in **real-world industry scenarios**;
- Many developed methods are still **not** in G+Smo.

Future Work:

- **Incorporation of our developed and currently under-development methods** into G+Smo;
- **User-friendly** graphical user interface.





Thanks for Your Attention!

Q&A.

If interested in my research, please feel free to contact me! ;-)

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