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Yet another structured mesh generator for screw machines simulation

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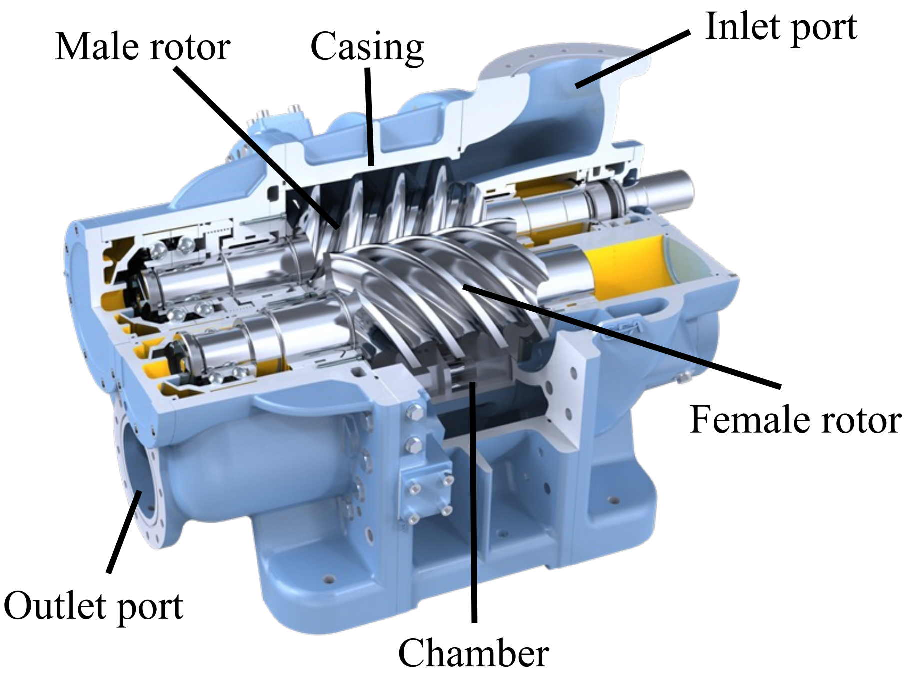
Outline

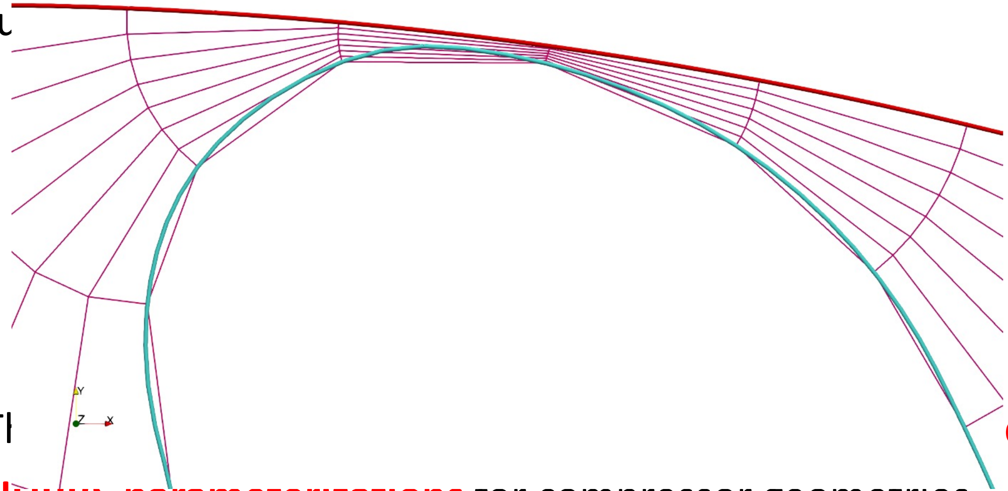
1. Background and motivation
2. Overview of the algorithm
3. Key components of mesh generator
4. Results
5. Conclusions and outlook

Research motivation



Rotary twin-screw compressor

- **Structured mesh generation** is a crucial preprocessing step in the simulation-based analysis of twin-screw machines.
- However, the existing mesh generators typically produce only **linear meshes with straight-sided cells;**
- **Iso-Geometric Analysis (IGA)** [1] changes the workflow fu[illegible]

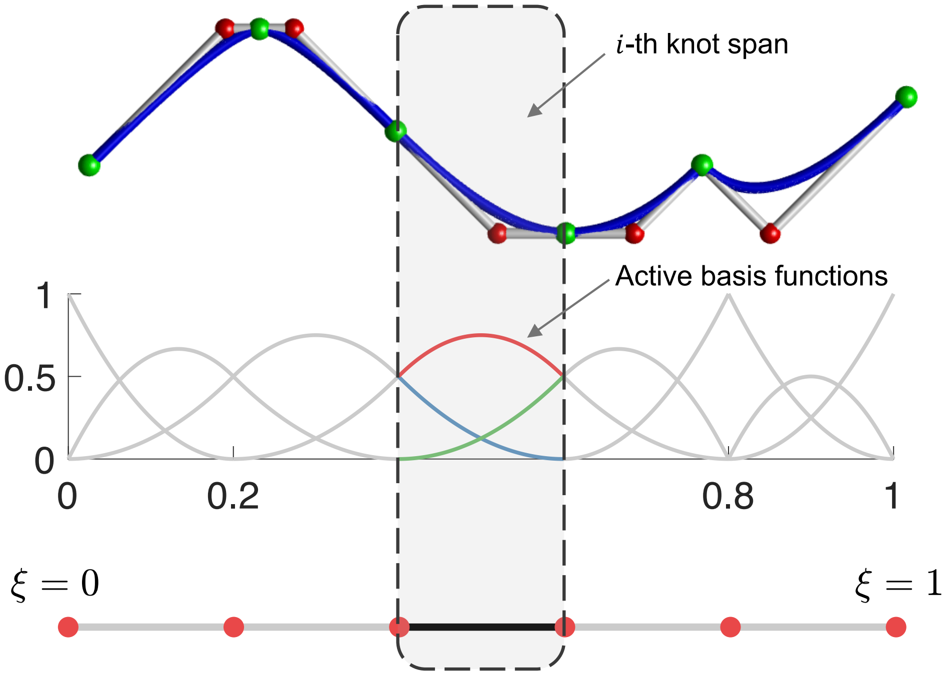


- Th[illegible] **der NURBS parameterizations** for compressor geometries.

Source: https://www.gascompressors.co.uk/technologies/oil-floodedscrew-compressor/

1. Hughes, T. J., Cottrell, J. A., & Bazilevs, Y. (2005). Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. CMAME, 194(39-41), 4135-4195.

Why Non-Uniform Rational B-Splines (NURBS)?

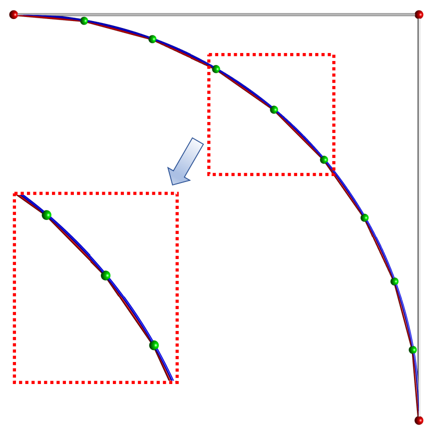


Piece-wise (rational) polynomial description

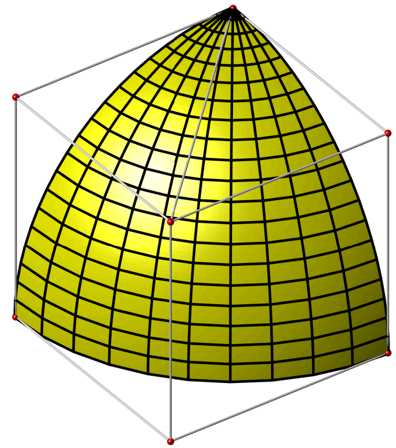
$$\mathcal{C}(\xi) = \sum_{i=0}^{n} \boldsymbol{P}_i \frac{\omega_i N_{i,p}(\xi)}{\sum_{i=0}^{n} \omega_i N_{i,p}(\xi)}, \xi \in \widehat{\Omega},$$

with $N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$

- **Compact geometry** with fewer control points;
- **Unified representation** for both standard (e.g., conics, quadrics) and free-form shapes;
- **Fast evaluations:** stable and reliable;
- **Geometric clarity:** Intuitive for designers;
- **Extensive toolkit:** knot operations, refinements, etc.

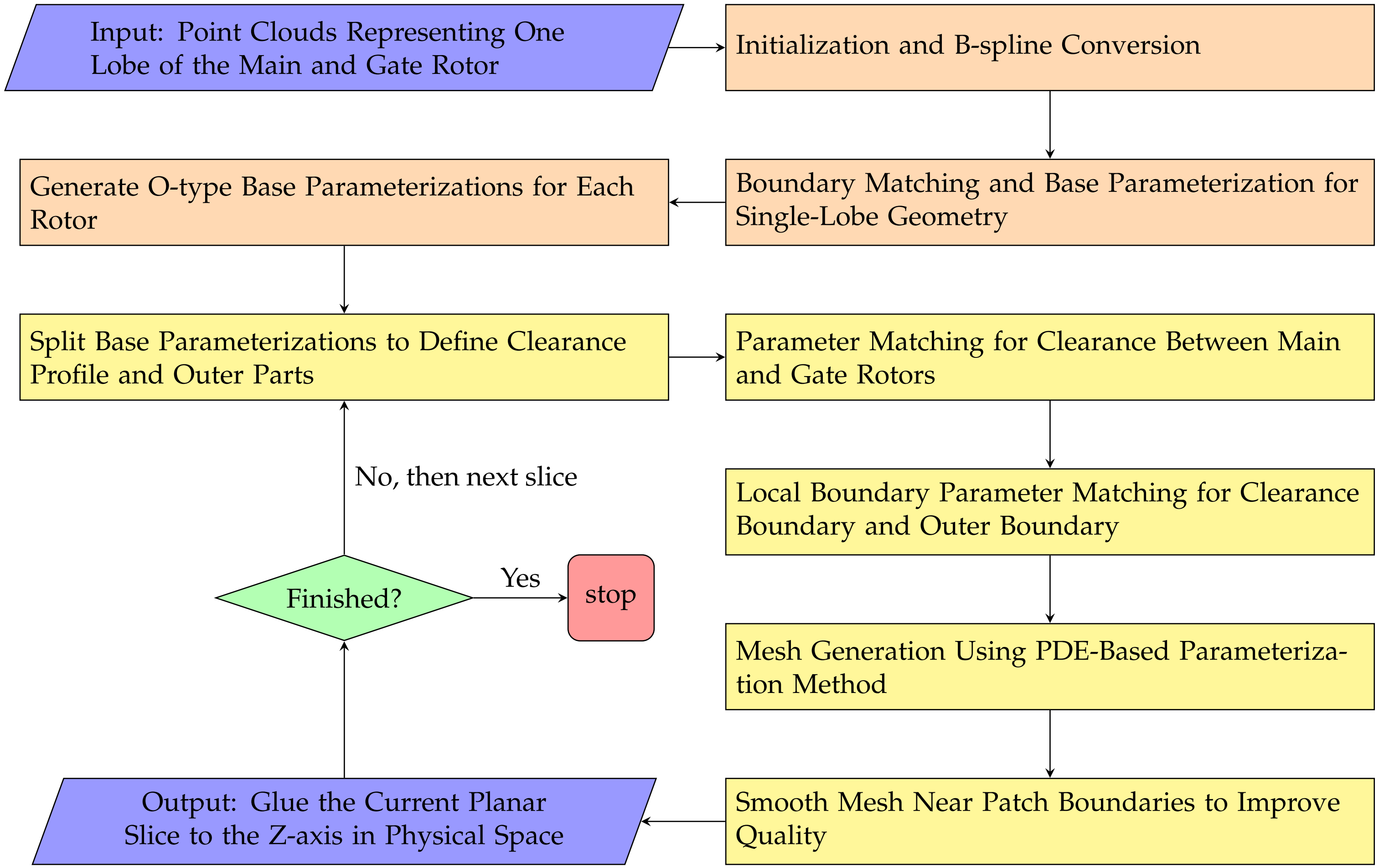


3 control points vs.
10 grid points



9 control points vs.
231 grid points

Overview of the algorithm

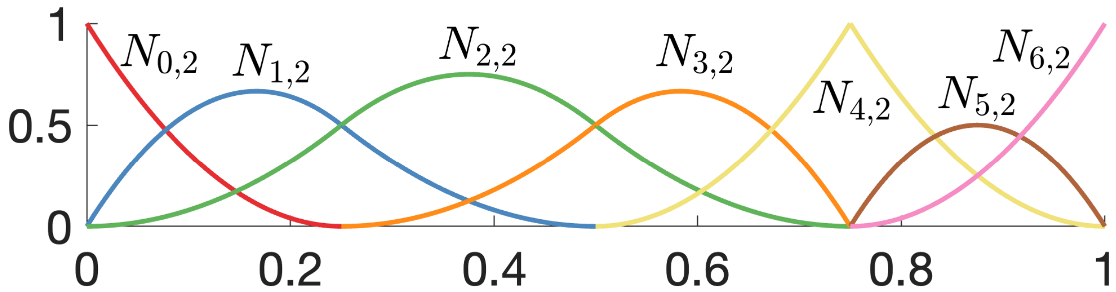


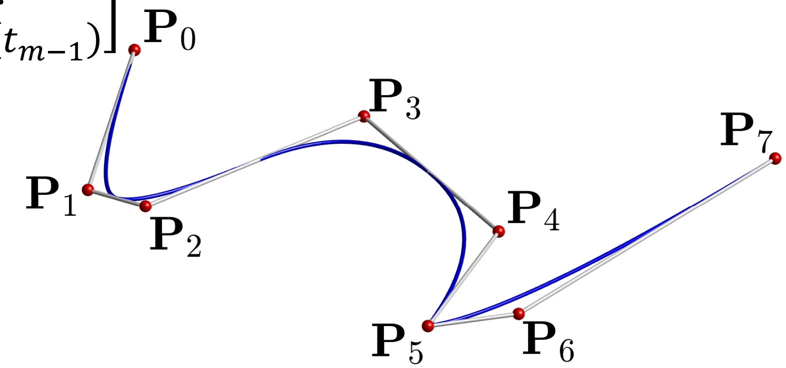
- Generate the B-spline representation of the casing curve for each rotor;
- Convert the input point cloud into B-splines using a **constrained least squares B-spline fitting** method;
  - Point cloud parameterization using **chord-length parameterization method;**
  - Control points $\{P_i\}_{i=0}^{n}$ are determined by solving the following linear system

$$\begin{bmatrix} \mathbf{A}^\top\mathbf{A} & I_{2\times2} \\ I_{2\times2} & O \end{bmatrix}\begin{bmatrix} P_f \\ P_c \end{bmatrix} = \begin{bmatrix} \mathbf{A}^\top r_f \\ r_c \end{bmatrix},$$

where the $(m-1)\times(n-1)$ configuration matrix is

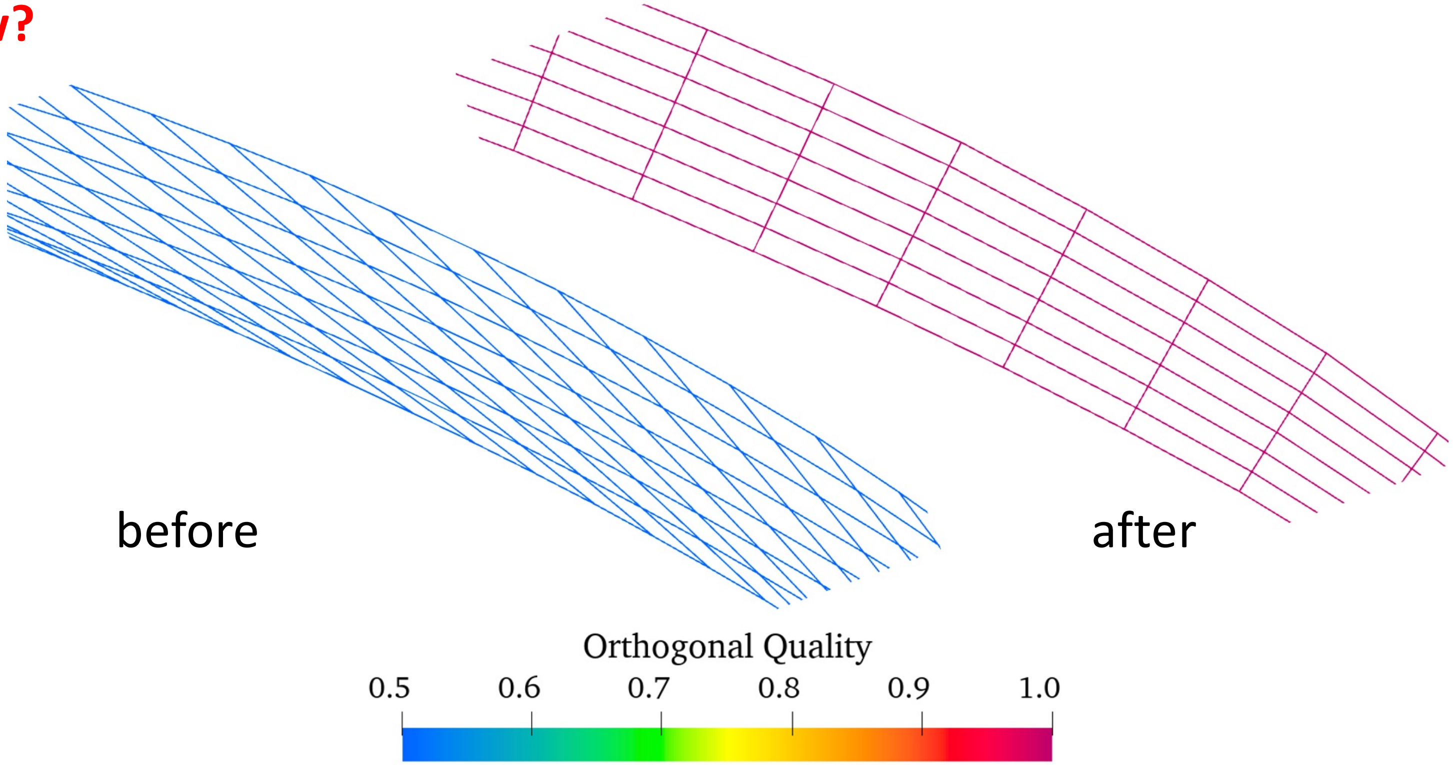
$$\mathbf{A} = \begin{bmatrix} N_1(t_1) & N_2(t_1) & \cdots & N_{n-1}(t_1) \\ N_1(t_2) & N_2(t_2) & \cdots & N_{n-1}(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ N_1(t_{m-1}) & N_2(t_{m-1}) & \cdots & N_{n-1}(t_{m-1}) \end{bmatrix}.$$



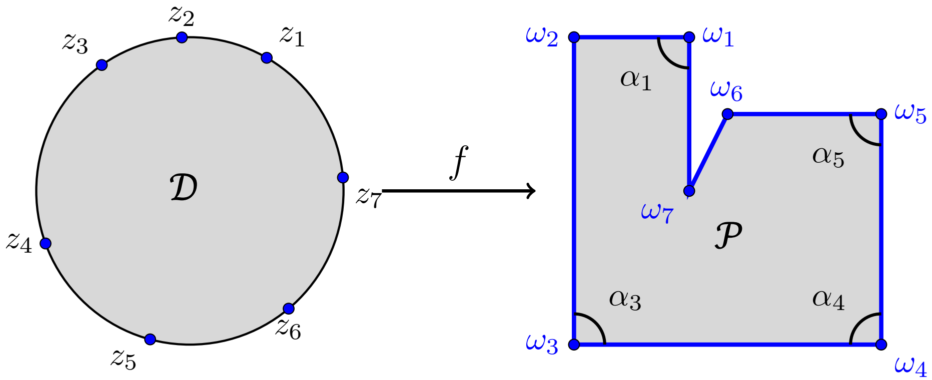


Role of Boundary reparameterization

- **Parameter speed** of the boundary curves significantly affects the mesh quality;
- Mesh quality is greatly improved by using the **boundary reparameterization technique.**
- **So, how?**

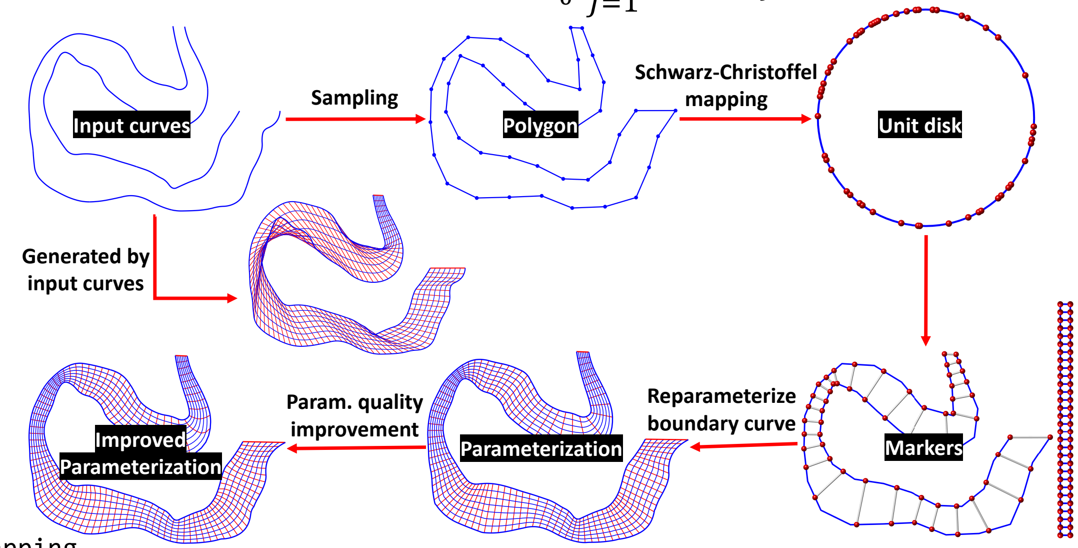


Boundary parameter matching using Schwarz-Christoffel mapping



- **Riemann mapping theorem**: ∃ analytic function f **with non-zero derivative** such that $f(\mathcal{D}) = \mathcal{P}$.
- **Schwarz-Christoffel formula**

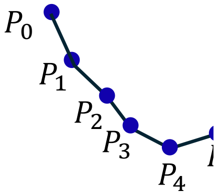
$$f(z) = f(z_0) + C\int_{z_0}^{z}\prod_{j=1}^{n}\left(1-\frac{\zeta}{z_j}\right)^{\beta_j} d\zeta$$



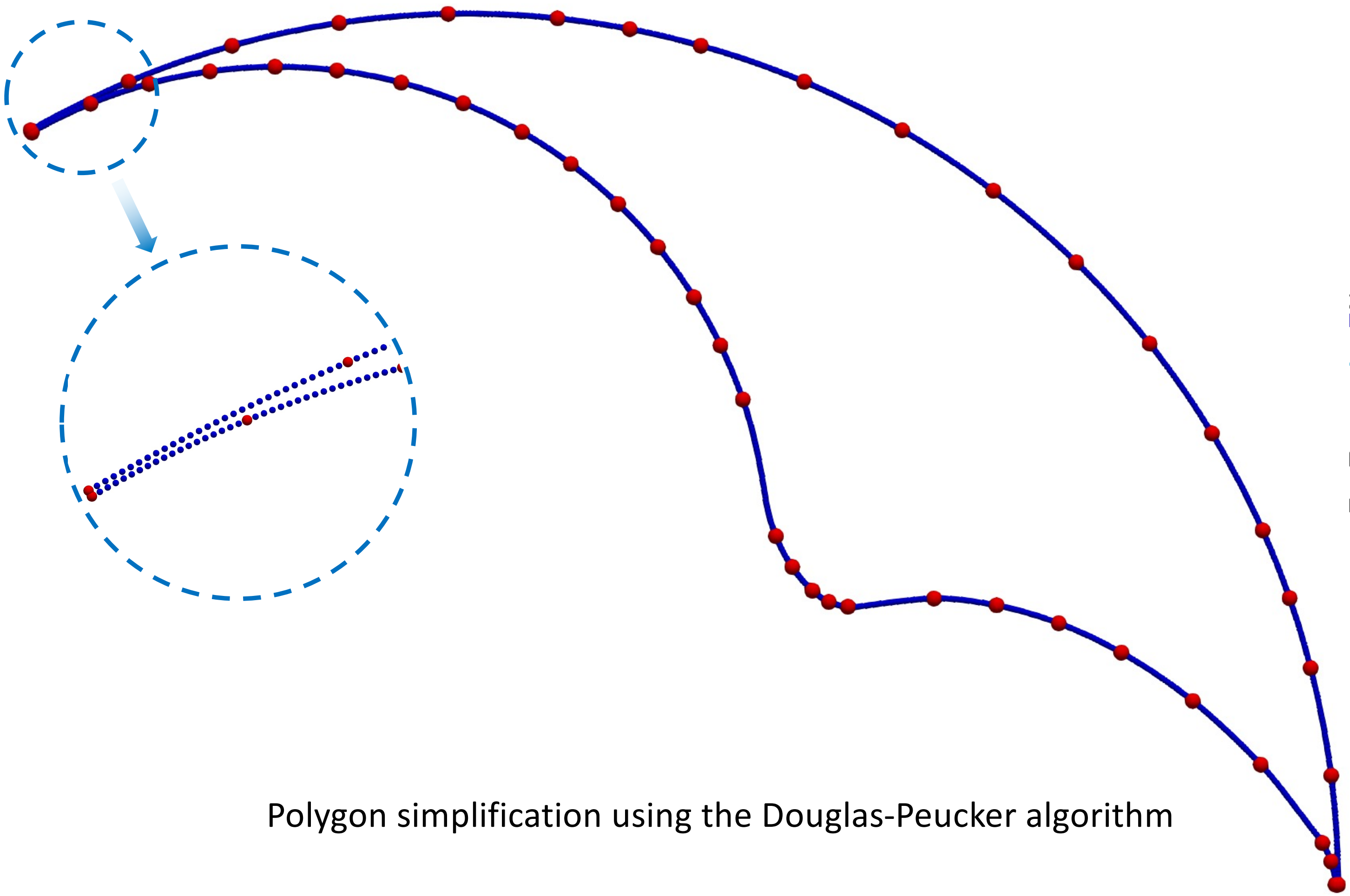
- Solving the Schwarz-Christoffel parameter problem for $\{z_j\}$ numerically allows us to compute sets of markers on the two opposite curves that can be used to reparametrize one curve w.r.t. the other.
- Solving the parameter problem is far from easy, **the CRDT algorithm**[1] is adopted and implemented.

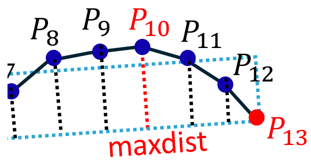
1. Driscoll, T. A., & Vavasis, S. A. (1998). Numerical conformal mapping using cross-ratios and Delaunay triangulation. SIAM Journal on Scientific Computing, 19(6), 1783-1803.

- **Number of**
  - Step 1
  - Step 2



(a) Inp





nd Searching
e points

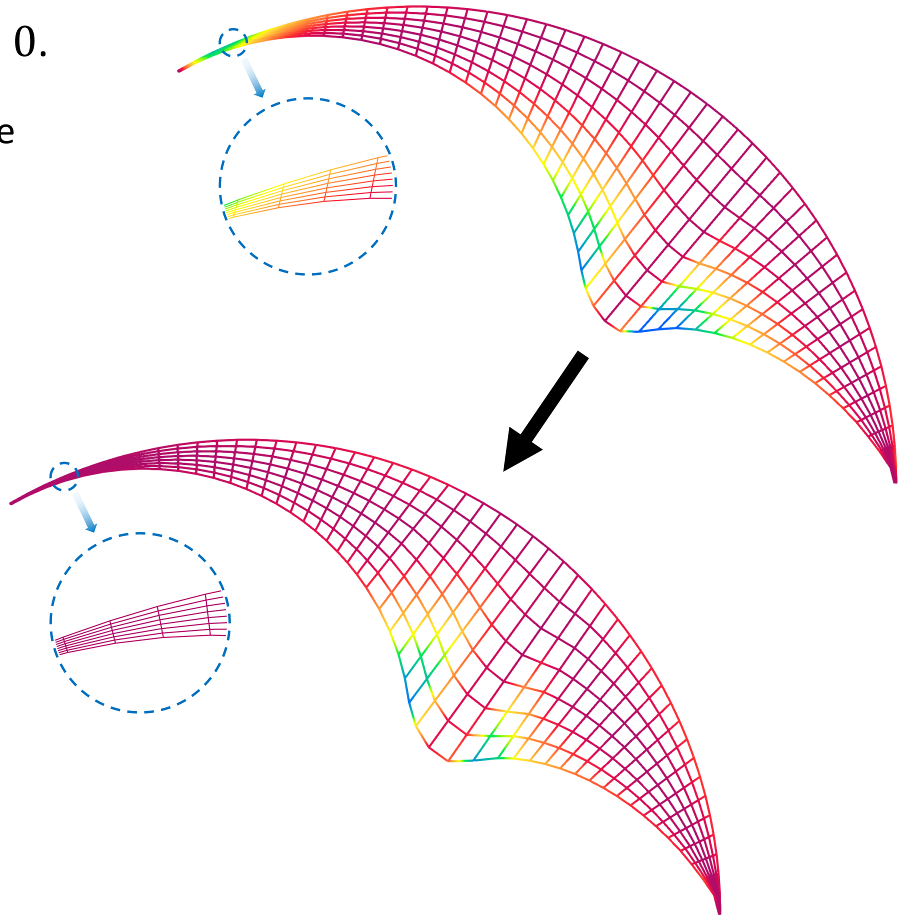
Polygon simplification using the Douglas-Peucker algorithm

Boundary reparameterization

- **Theorem**[1]: B-spline and NURBS basis functions are invariant under scaling and translation of the knot vector, i.e., $N_{i,p}^{\Xi}(\xi) = N_{i,p}^{\hat{\Xi}}(s\xi + t)$ with $\hat{\Xi} = s\Xi + t$, $s > 0$.
  - For each pair of markers $\left(P_i^{East}, P_i^{West}\right)$ identify the pair of the parameters $\left(\xi_i^{East}, \xi_i^{West}\right)$ by the **closest point projection method**, i.e., solving the nonlinear equation

$$(C^*(\xi) - P_i^*) \cdot \frac{\partial C^*(\xi)}{\partial \xi} = 0$$

  using Newton's method;
  - Without loss of generality, align the segment of curve C^{East} defined over the parameter interval $[\xi_i^{West}, \xi_i^{East}]$ with C^{West} by applying an affine transformation;
- Parametrically changed, while **geometrically consistent.**

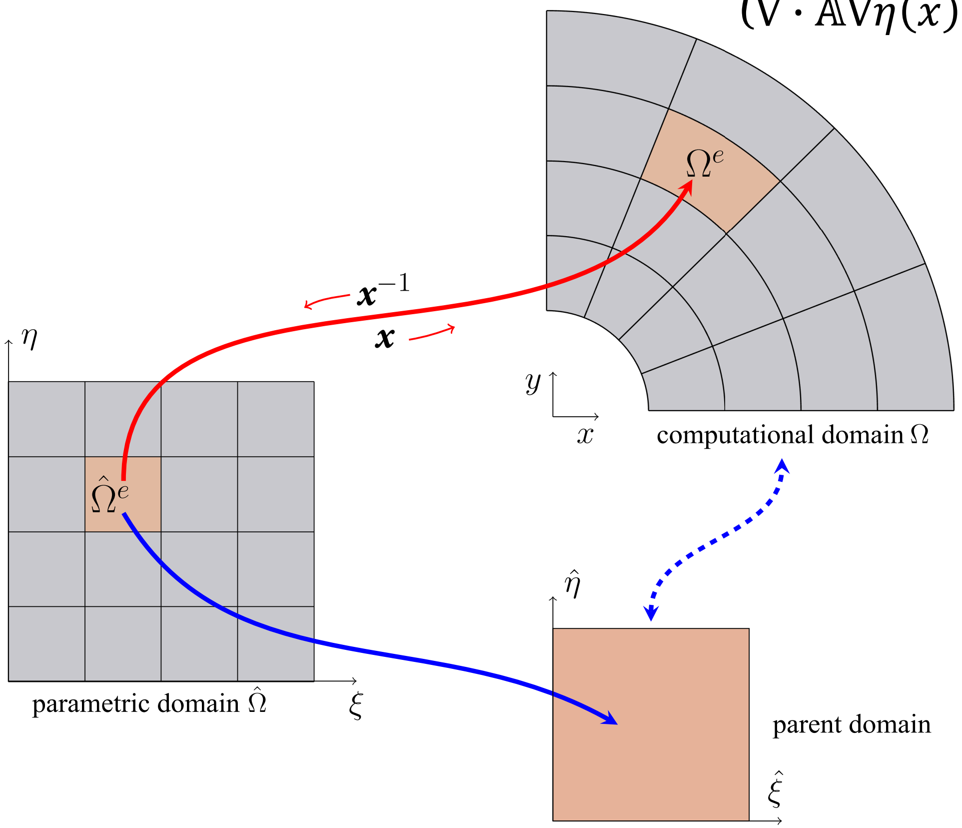


1. Ji, Y., Möller, M., Yu, Y., & Zhu, C. (2024). Boundary parameter matching for isogeometric analysis using Schwarz-Christoffel mapping. *Engineering with Computers*, 1-19.

Elliptic Grid Generation (EGG) method

- To compute a quasi-harmonic mapping $x\colon\ \Omega \rightarrow \widehat{\Omega}$ by solving

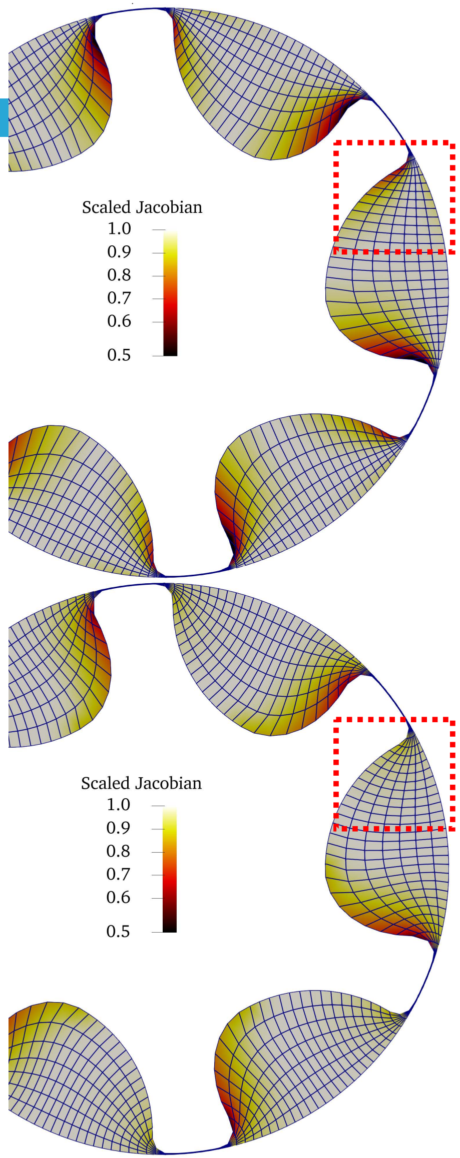
$$\begin{cases} \nabla \cdot \mathbb{A}\nabla\xi(x) = 0 \\ \nabla \cdot \mathbb{A}\nabla\eta(x) = 0 \end{cases} \quad \text{s.t. } \mathrm{x}^{-1}_{\partial\Omega} = \partial\widehat{\Omega}$$



- The **existence and uniqueness** [1] of the harmonic mapping x^{-1} is guaranteed if
  - The curvature of $\widehat{\Omega}$ is non-positive;
  - The boundary $\partial\widehat{\Omega}$ of $\widehat{\Omega}$, when considered with respect to the metric on Ω, is convex.
- The **unique solution x^{-1} offers a one-to-one mapping** (with non-vanishing Jacobian J), which is ensured by the Radó-Kneser-Choquet theorem[2].

1. Eells, J., & Lemaire, L., (1978). A report on harmonic maps. Bulletin of the London mathematical society, 10(1):1–68.
2. Duren, P., & Hengartner, W., (1997). Harmonic mappings of multiply connected domains. Pac. J. Math. 180, 201-220.

Elliptic Grid Generation (EGG) method – cont'd



- $\mathbb{A}(x) = I$ yields the following nonlinear vector-valued second-order PDE [1]

$$\forall N_i \in \left(S_{p,q}^{\Xi,\boldsymbol{H}}\right)^0 : \begin{cases} \int\int_\Omega R \cdot \tilde{\mathcal{L}} y d\Omega = 0 \\ \int_\Omega R \cdot \tilde{\mathcal{L}} y d\Omega = 0 \end{cases} \quad \text{s.t. } x\Big|_{\partial\hat{\Omega}} = \partial\Omega,$$

where the differentiation operator $\tilde{\mathcal{L}} = \frac{\mathcal{L}}{g_{11}+g_{22}}$, and $\mathcal{L} = g_{22}\frac{\partial^2}{\partial\xi^2} - 2g_{12}\frac{\partial^2}{\partial\xi\partial\eta} + g_{11}\frac{\partial^2}{\partial\eta^2}$.

- $\mathbb{A}(x) = \mathrm{DIAG}(\frac{1}{|J|}, \frac{1}{|J|})$ yields the variational formulation in Sobolev space H^1 [2]:

$$\forall\ N_i \in \left(\mathbb{S}_{p,q}^{\Xi,H}\right)^0 : \begin{cases} \int\int_\Omega \nabla \mathrm{R} \cdot \nabla\xi(x) d\Omega = 0 \\ \int_\Omega \nabla \mathrm{R} \cdot \nabla\eta(x) d\Omega = 0 \end{cases} \quad \text{s.t. } x\Big|_{\partial\hat{\Omega}} = \partial\Omega,$$

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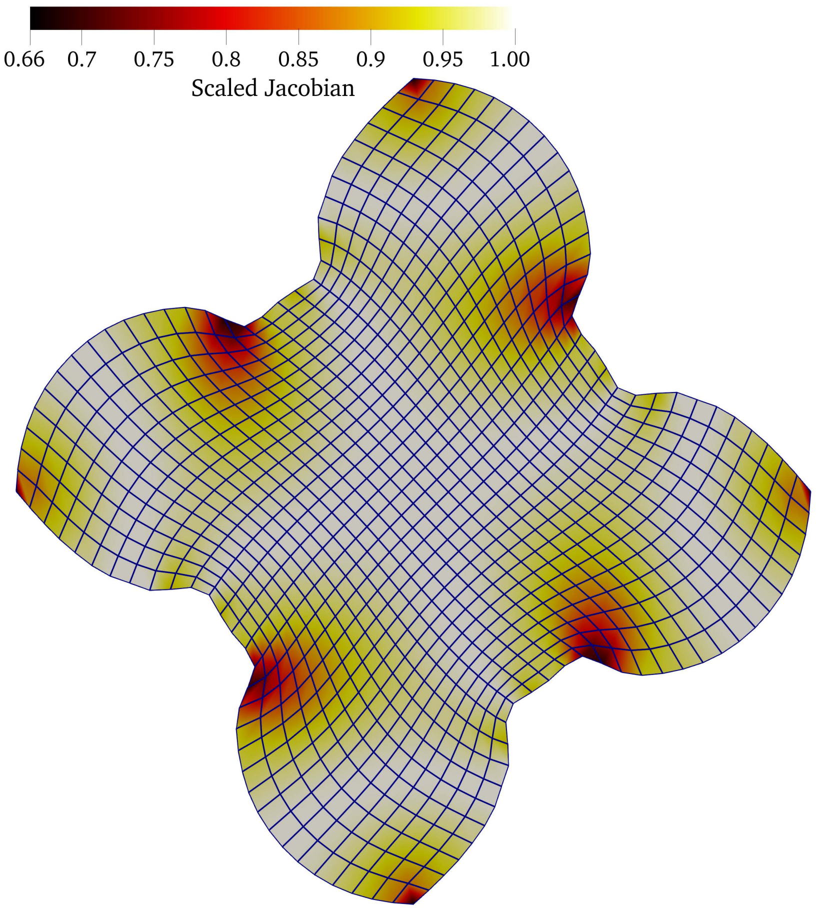
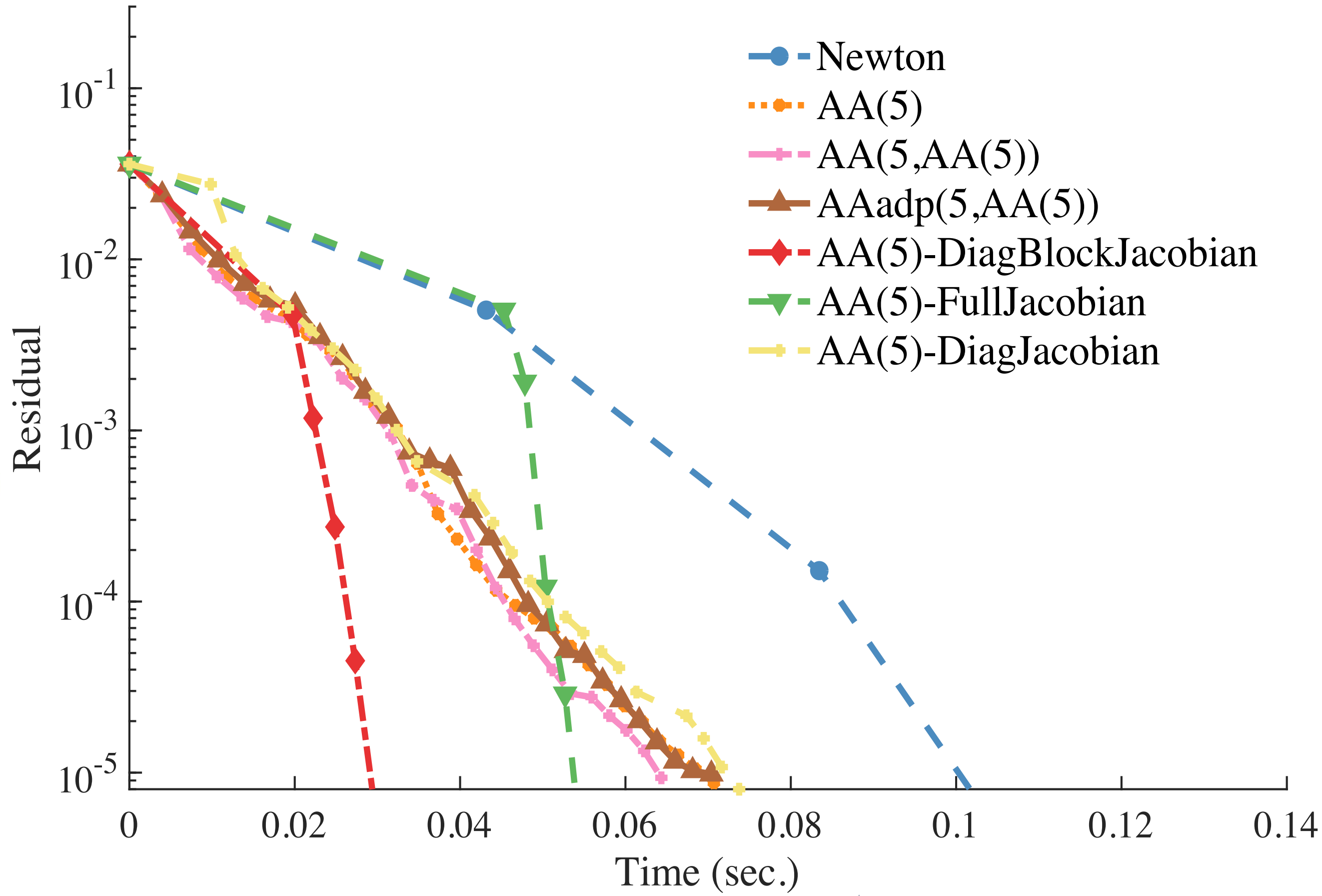
1. Hinz, J., Möller, M., & Vuik, C. (2018). Spline-based parameterization techniques for twin-screw machine geometries. In IOP Conference Series: Materials Science and Engineering.
2. Ji, Y., Chen, K., Möller, M., & Vuik, C. (2023). On an improved PDE-based elliptic parameterization method for isogeometric analysis using preconditioned Anderson acceleration. Computer Aided Geometric Design, 102, 102191.

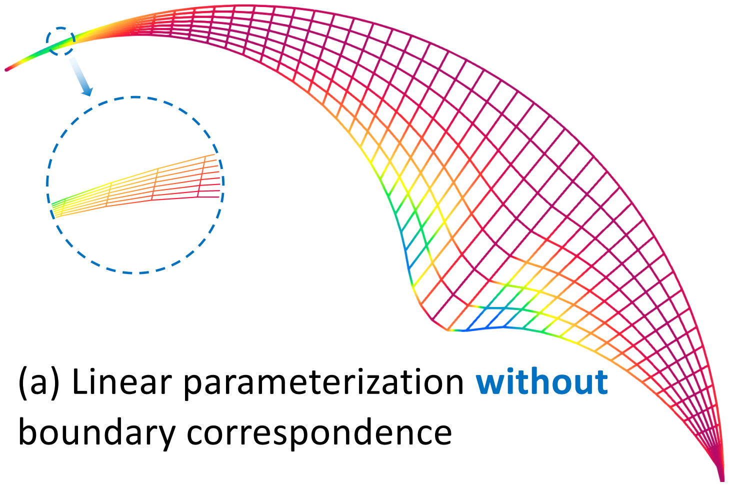
Preconditioned Anderson Acceleration solver



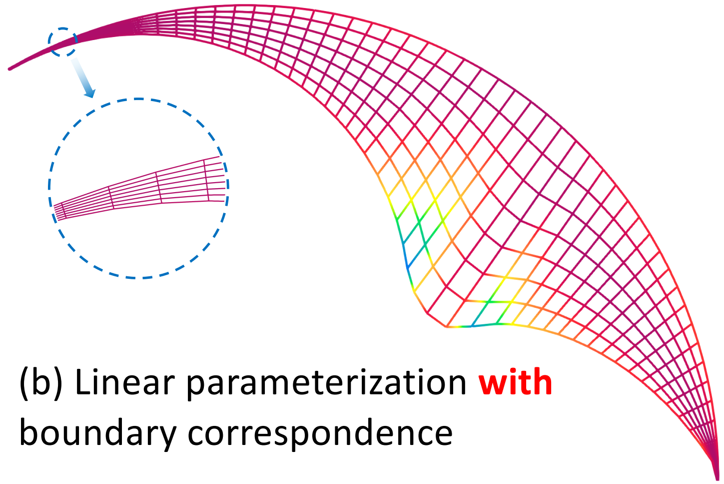


Picard AA preAA Newton preAA

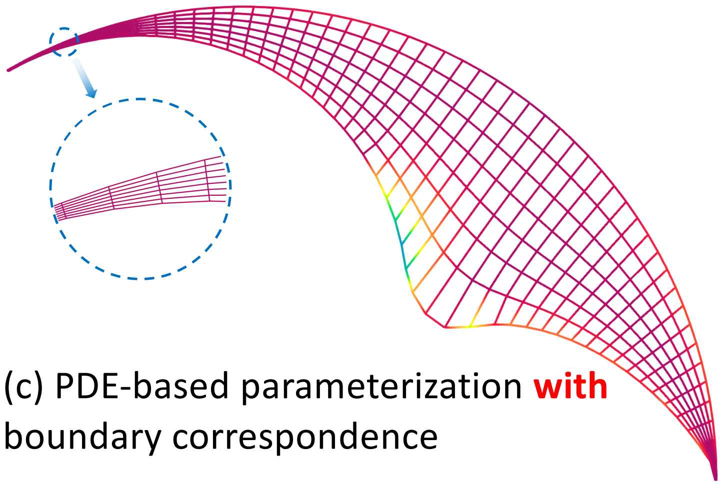
Comparison of different parameterization methods



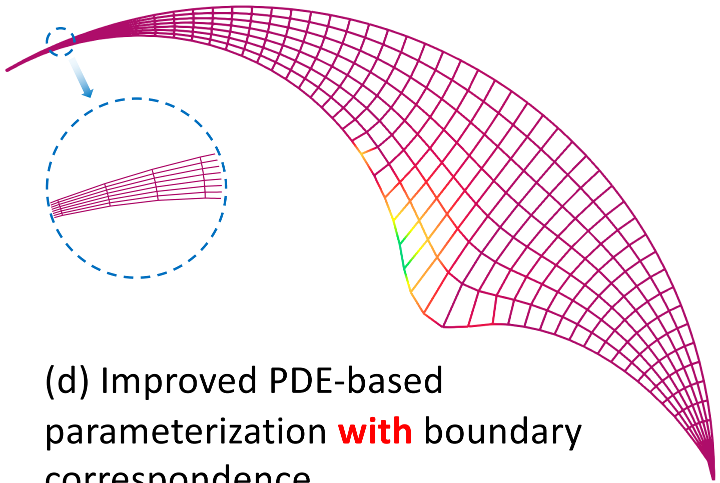
(a) Linear parameterization **without** boundary correspondence



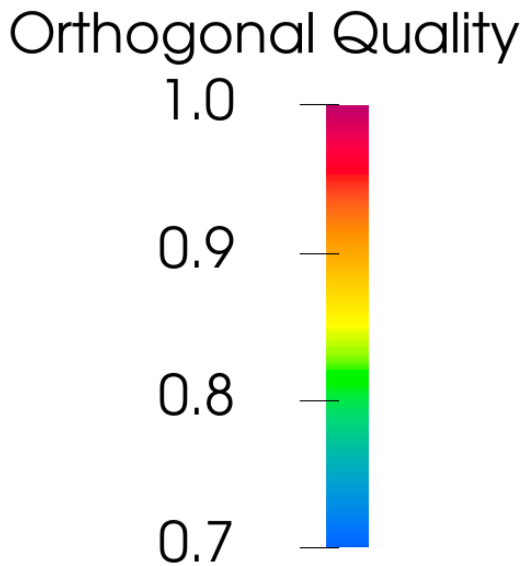
(b) Linear parameterization **with** boundary correspondence



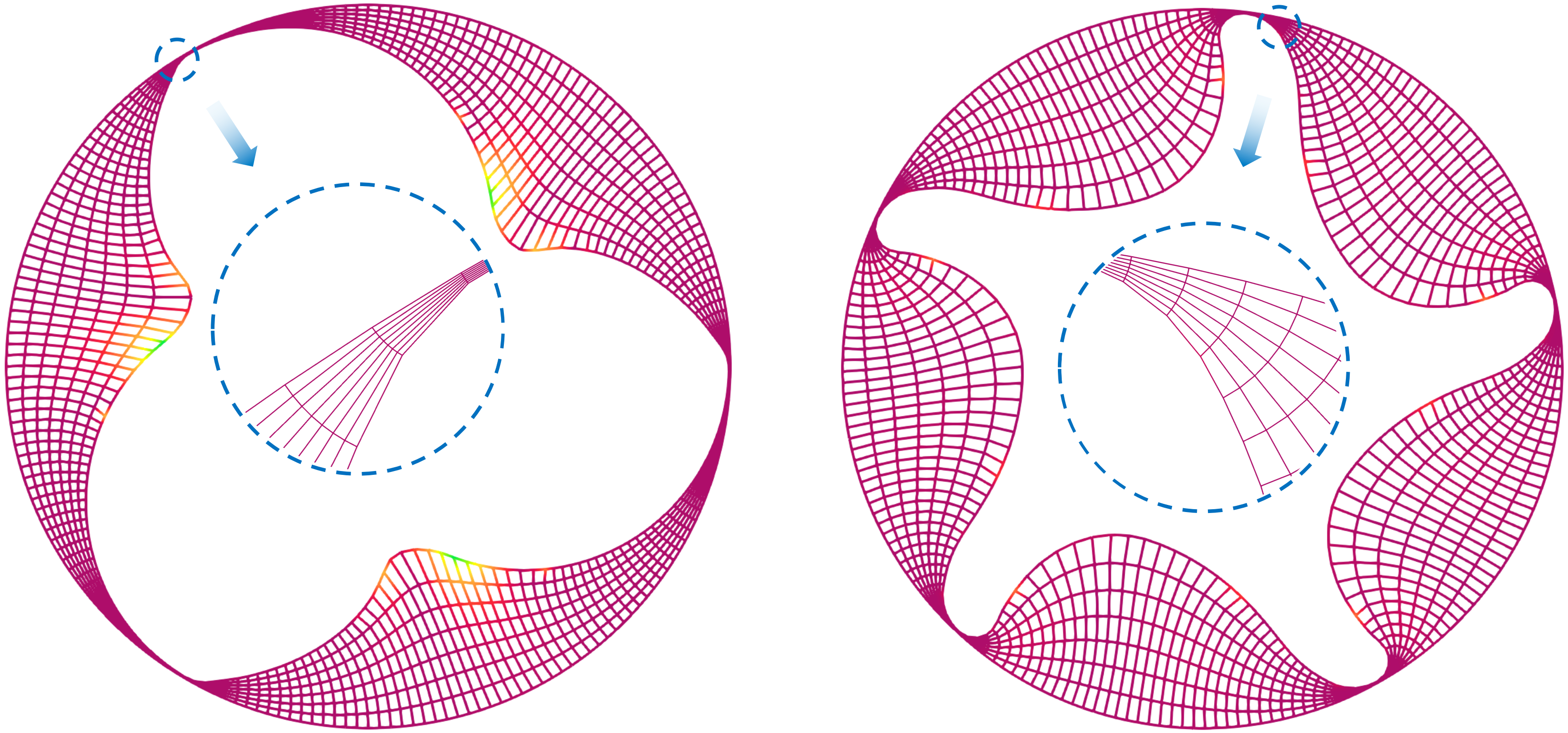
(c) PDE-based parameterization **with** boundary correspondence



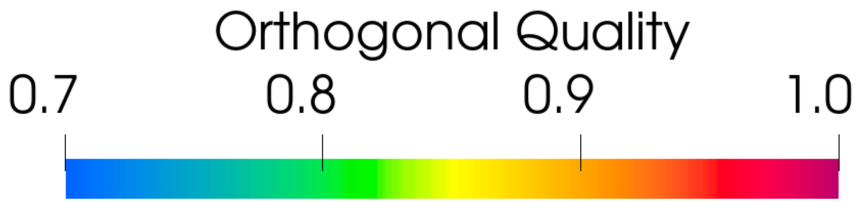
(d) Improved PDE-based parameterization **with** boundary correspondence



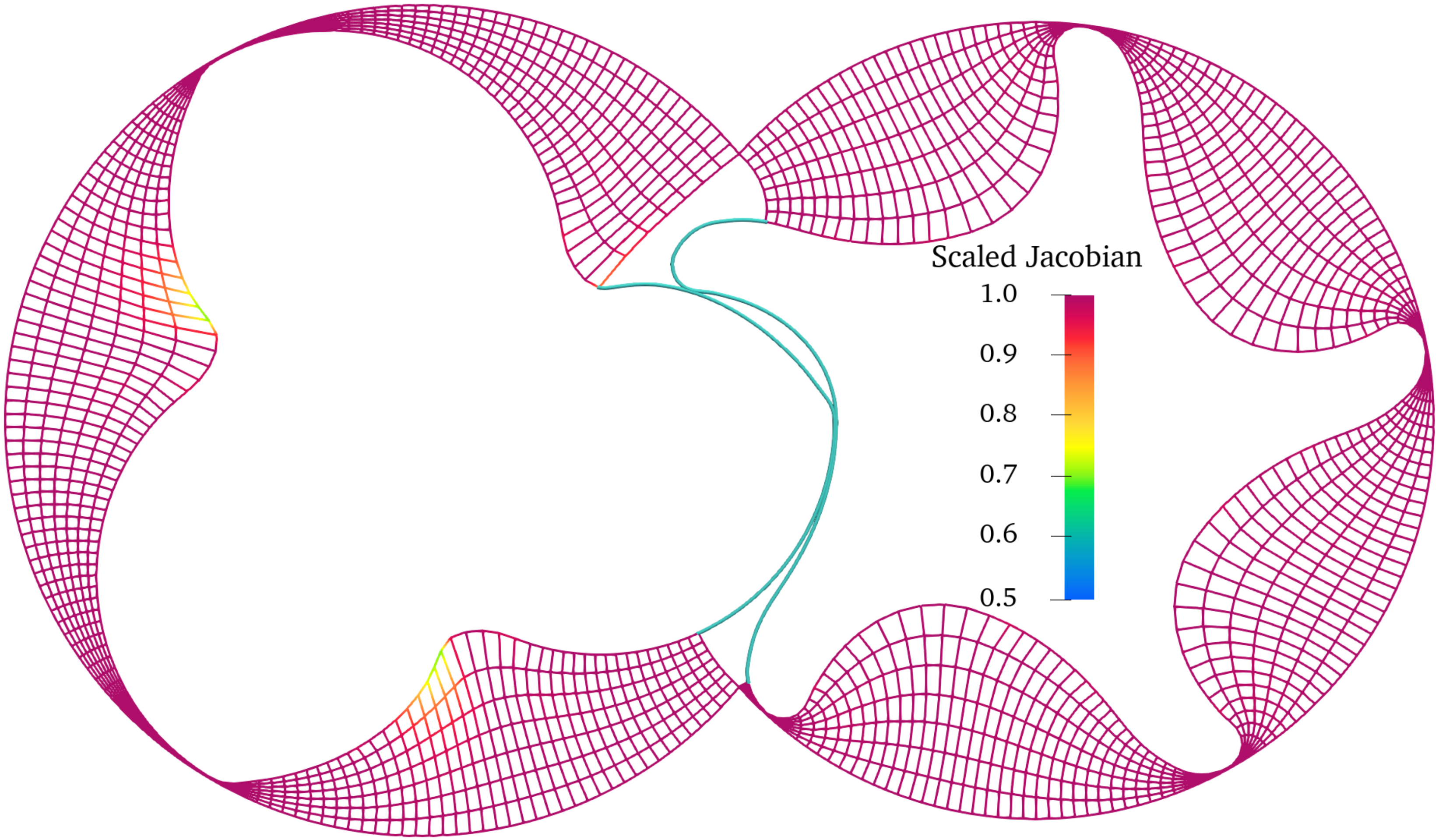
Base parameterizations for main/gate rotors



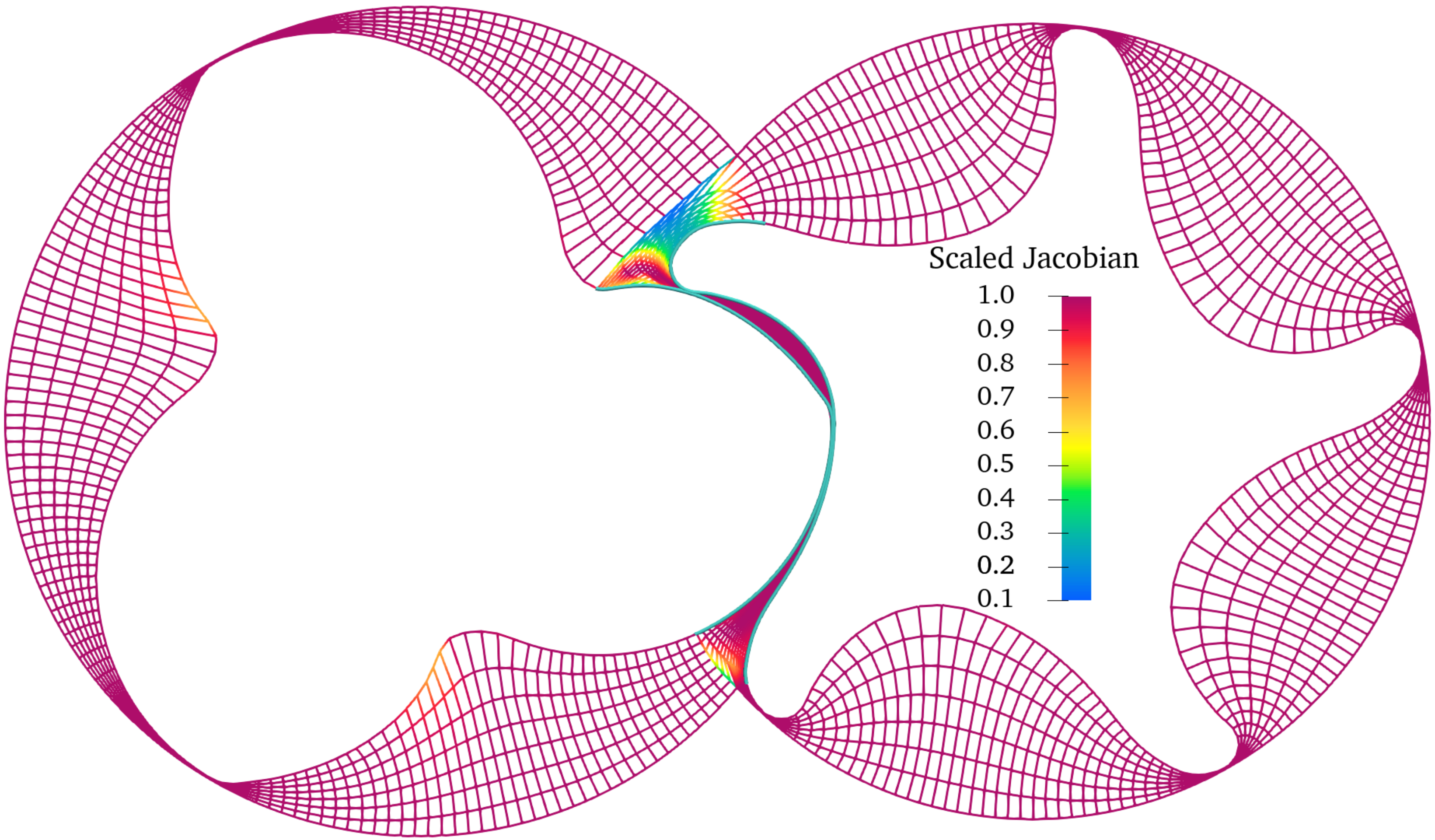
- Obtain the complete base parameterization for each rotor by rotating the one-lobe geometry.
    - using inherent symmetry of the geometry



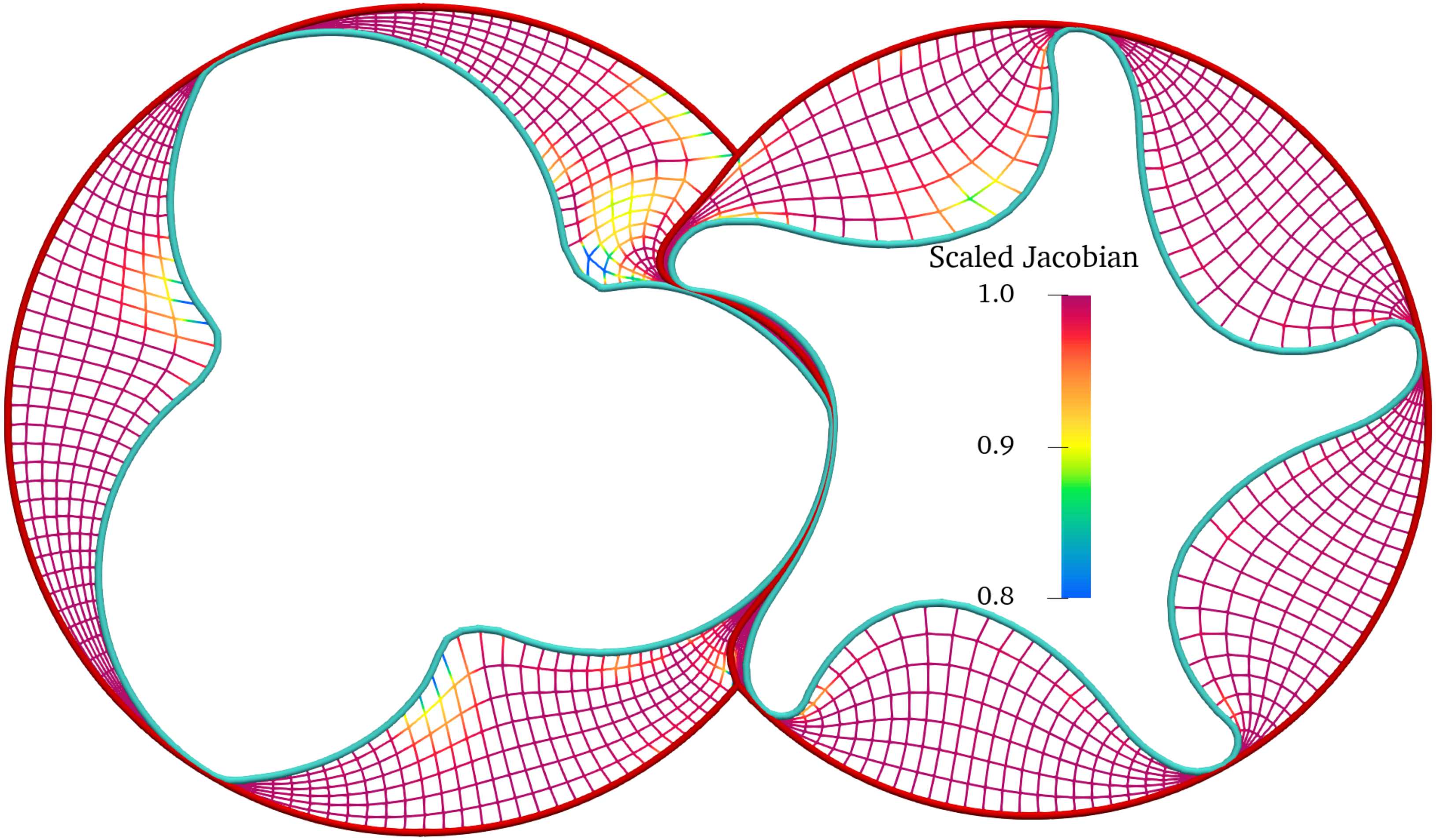
Splitting of the base parameterization at CUSP points



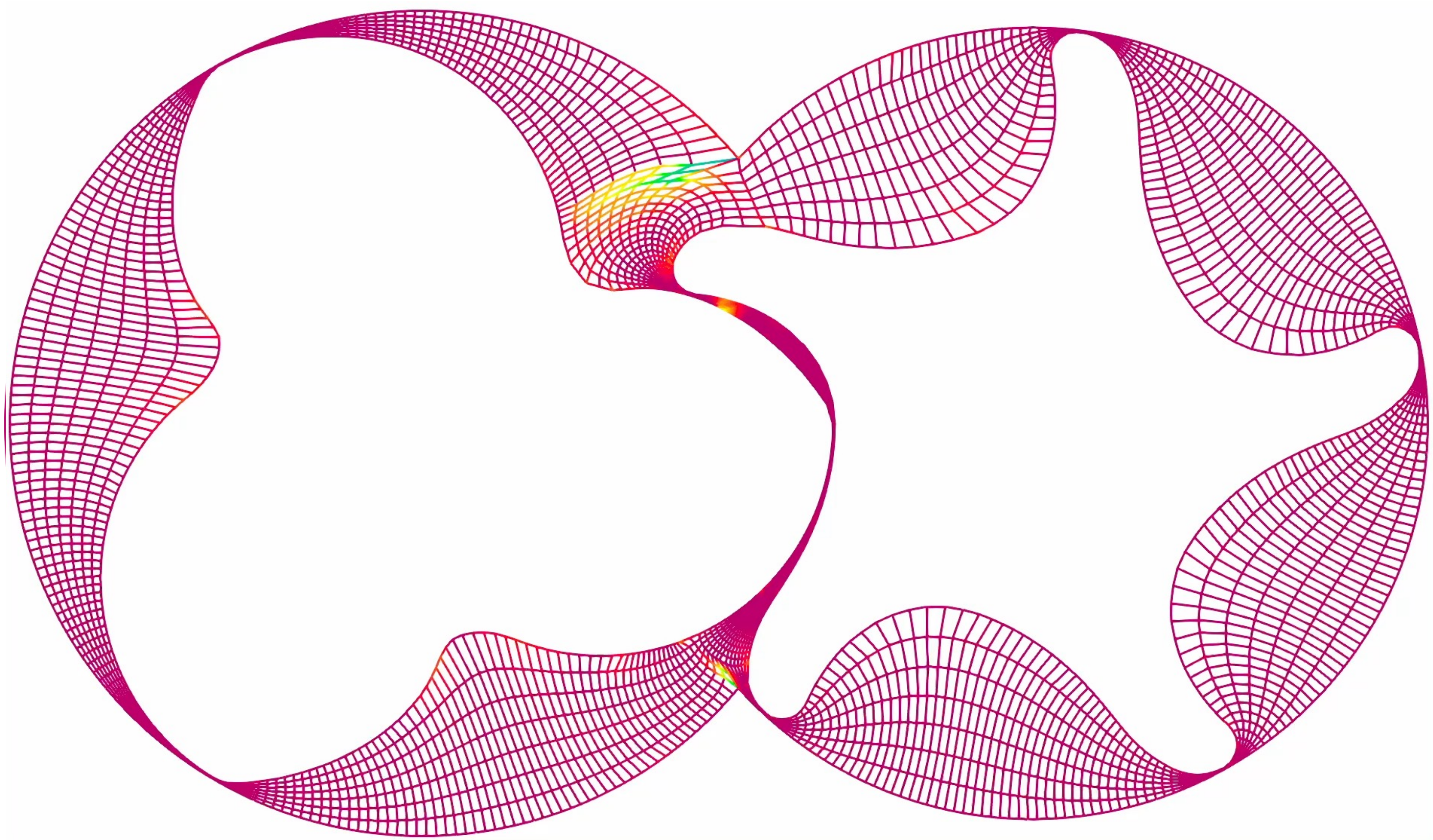
Resulting mesh without smoothing



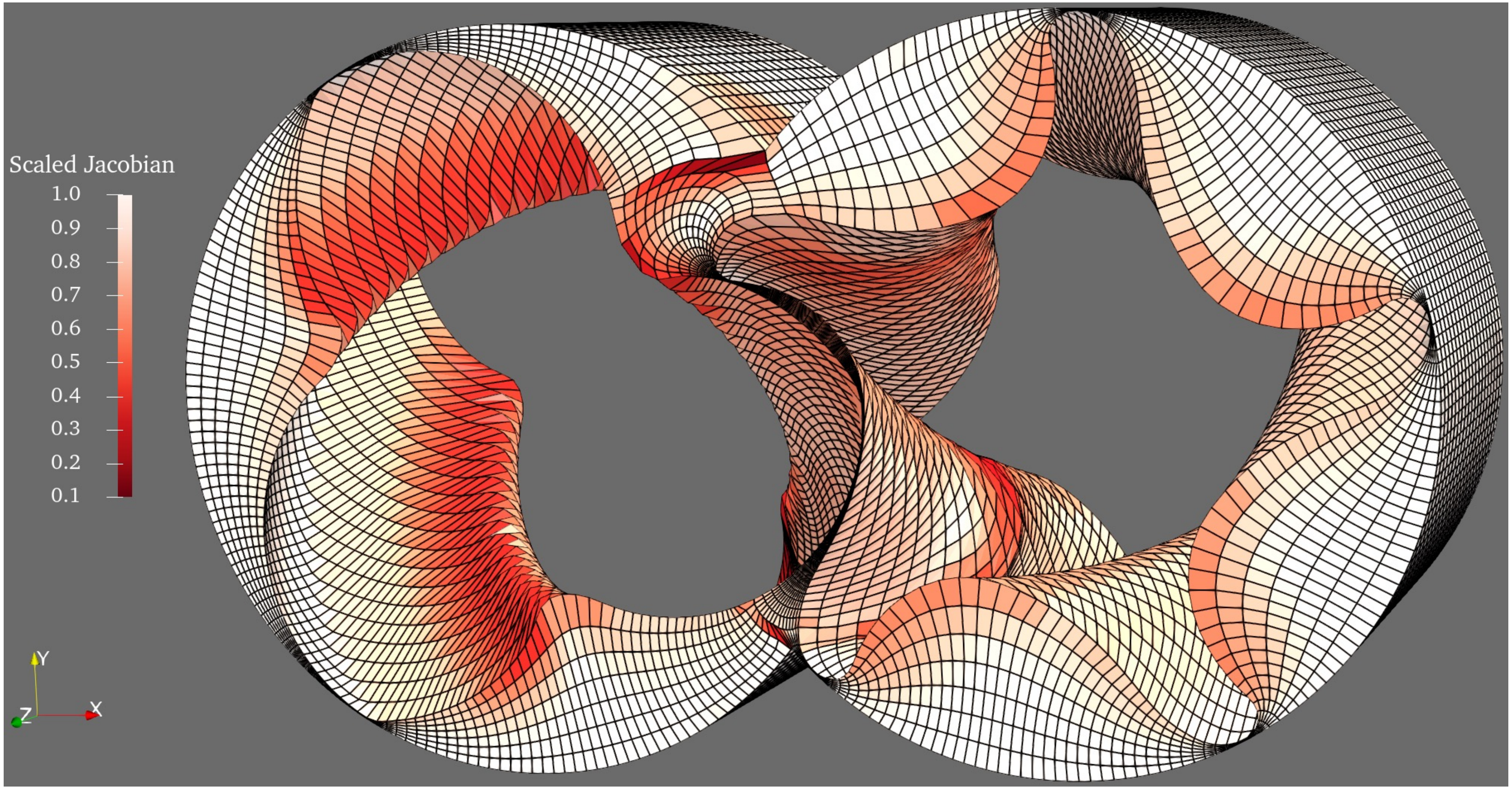
Resulting mesh for a representative slice before smoothness improvement



Resulting mesh for a representative slice after smoothness improvement



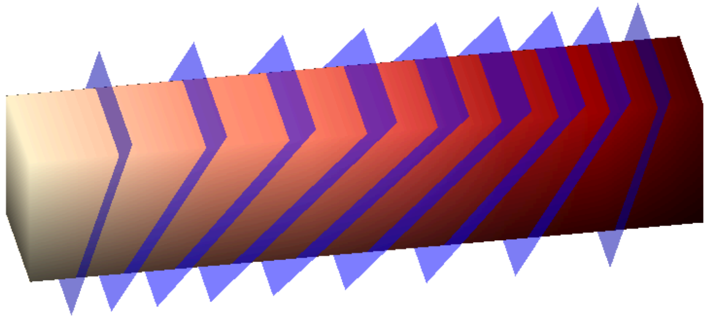
Generated structured hexahedral mesh



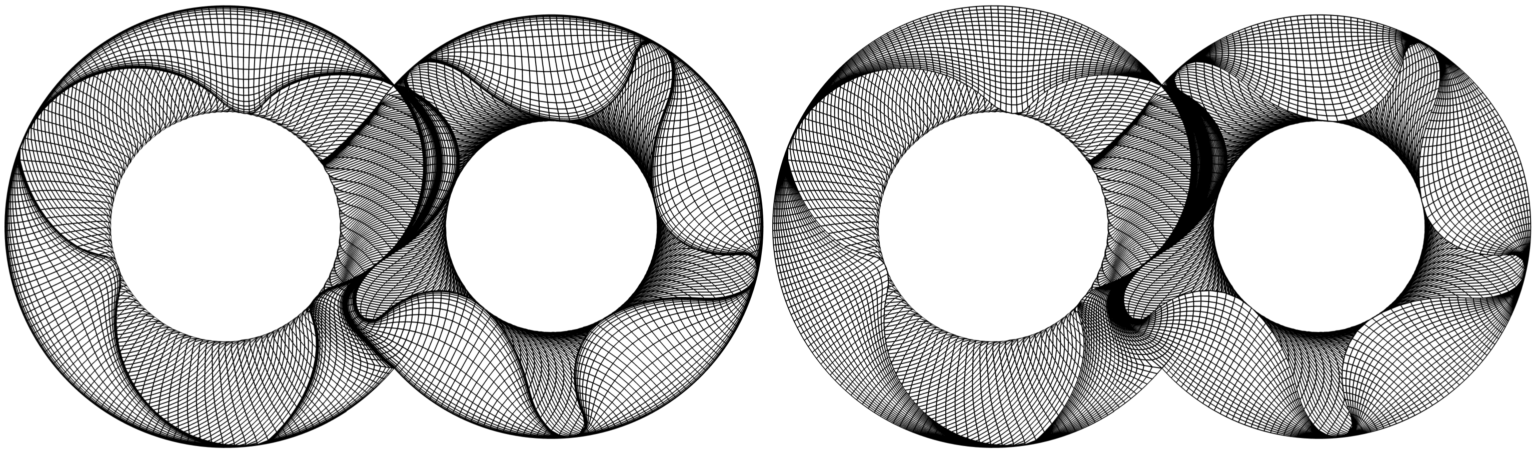
Customized mesh from spline-based parameterization

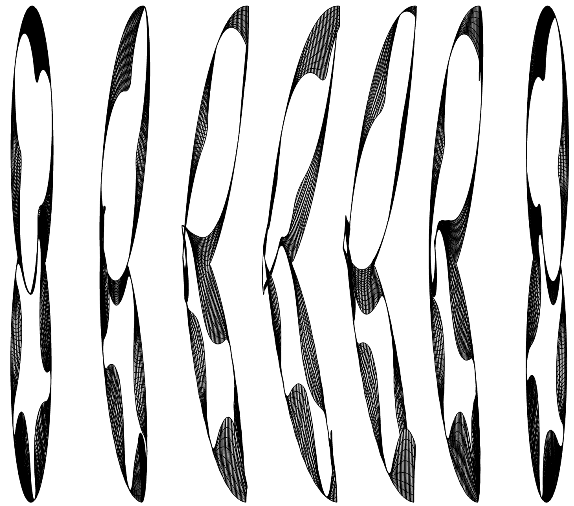
- Boundary layer mesh

$$\begin{cases} \xi = \hat{\xi} \\ \eta = \dfrac{tanh\big(\alpha(2\hat{\eta} - 1)\big)}{2\,tanh(\hat{\eta})} + \dfrac{1}{2} \end{cases}$$

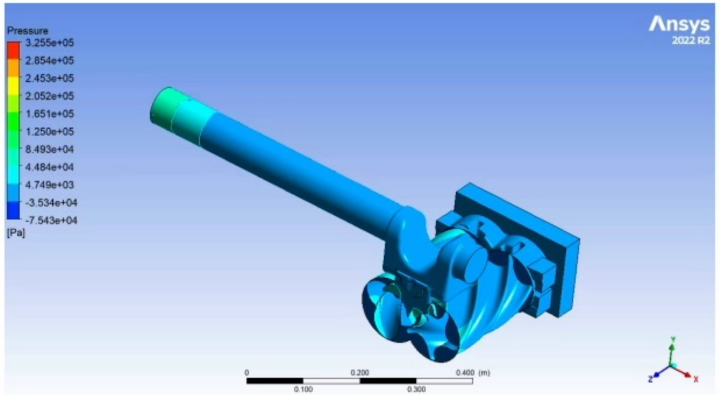


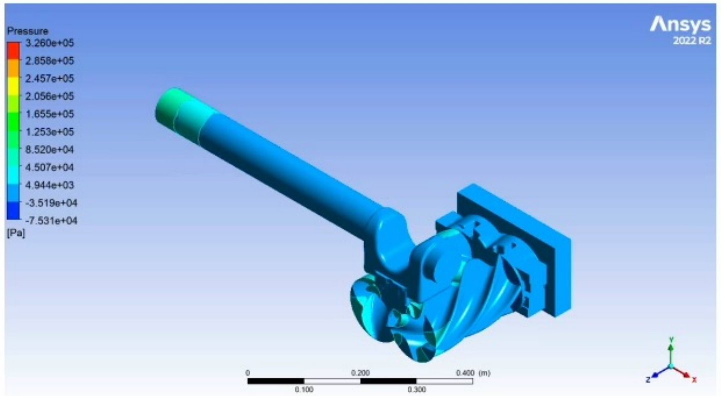
- Flow-aligned discretization.

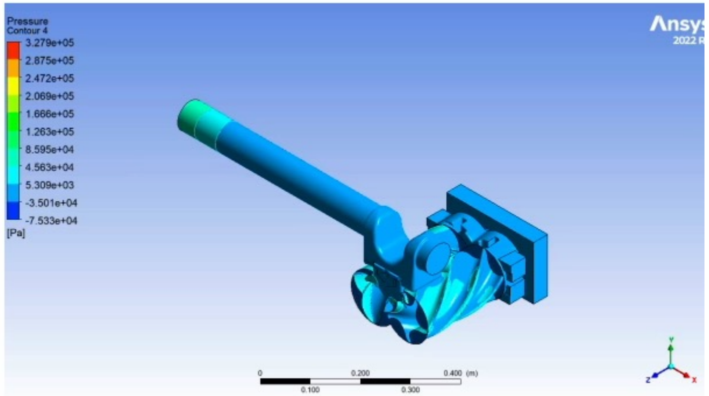


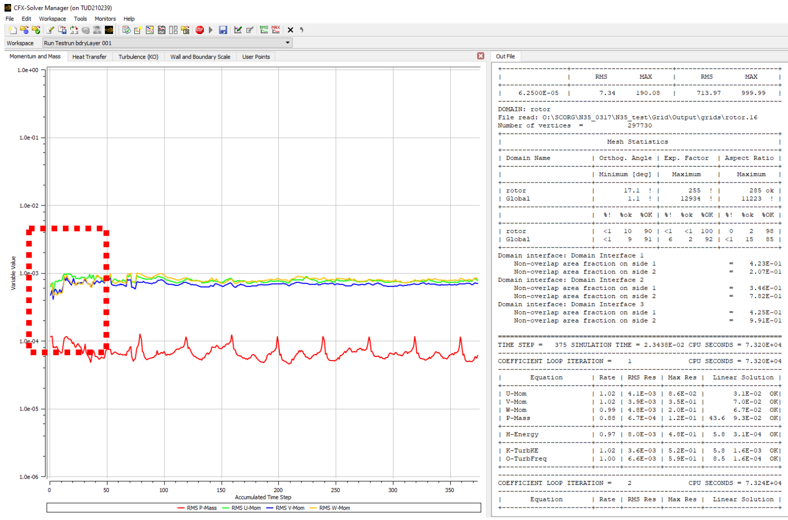


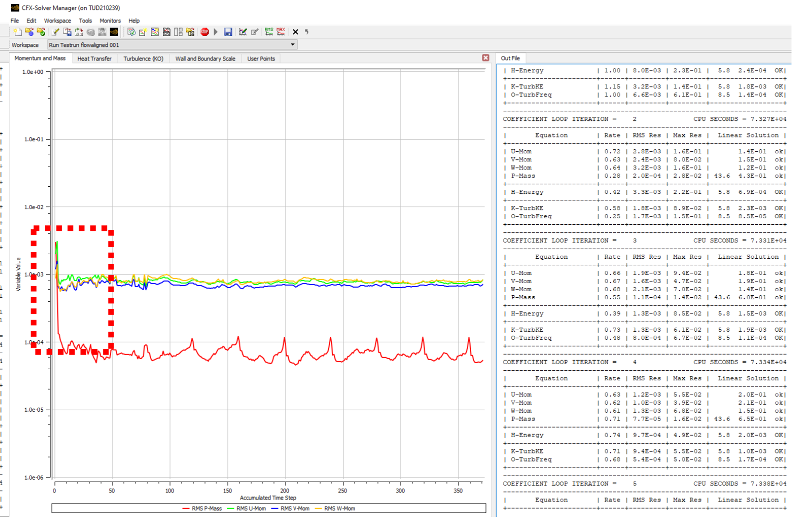
Simulation results using ANSYS CFX











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Boundary layer mesh

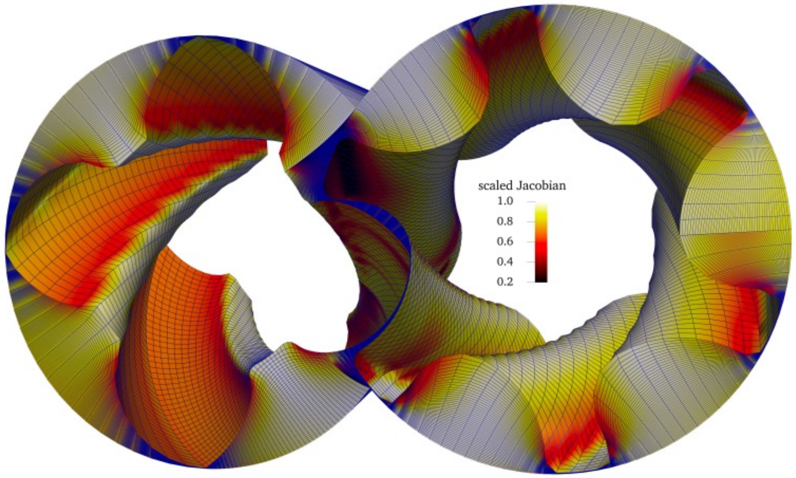
Flow-aligned mesh

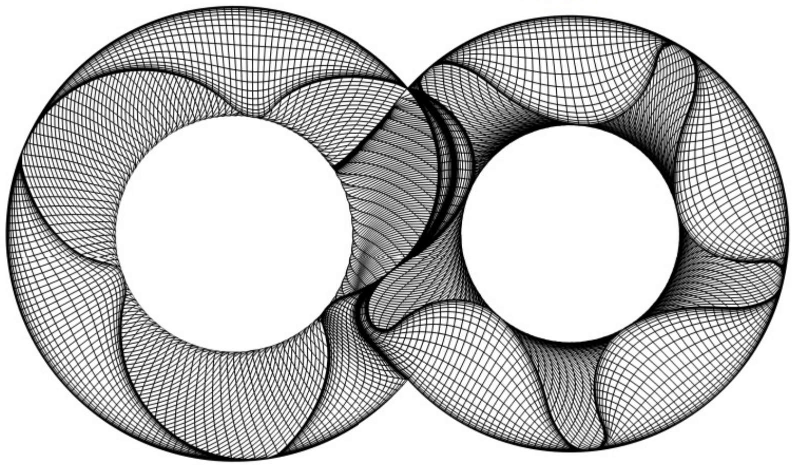
Conclusions

- **High-order spline-based mesh generator** for twin-screw compressors;
- **Boundary reparameterization** using Schwarz-Christoffel map;
- **Isogeometric analysis-based grid generation technique** enhanced by the preconditioned AA solver.

Limitations and future work

- **Boundary parameter matching** remains time-consuming;
- **Local smoothing techniques;**
- **Further simulation testing.**





Many thanks for your attention!

Q&A

If you are interested in my research, please feel free to contact me! ;-)

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- **GitHub:** jiyess
- **Homepage:** https://jiyess.github.io/