Isogeometric analysis-suitable parametrization for complex fluid simulations

Delft University of Technology, the Netherlands

Ye Ji (纪 野), Matthias Möller

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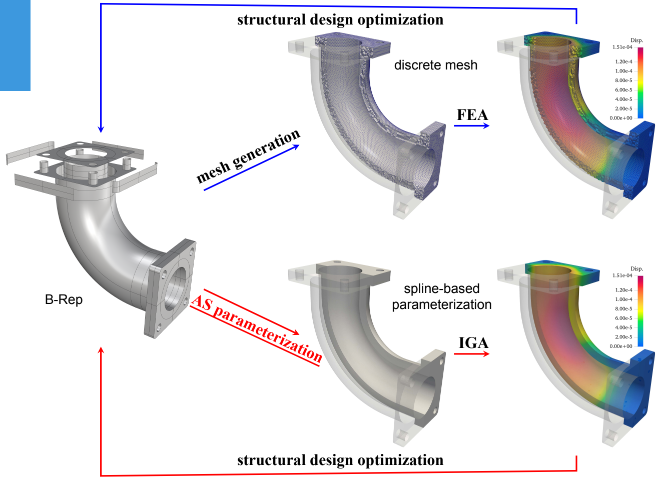
Agenda

1. **Research background and motivation**
2. IGA-suitable parametrization approaches
3. Real-world applications
4. Conclusions and outlook

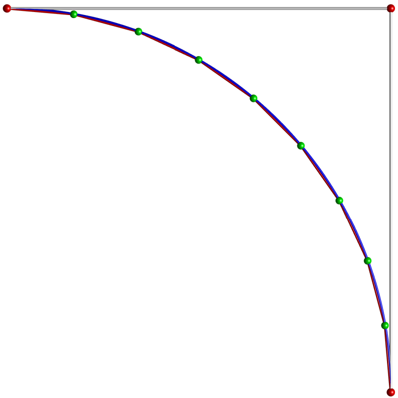
IsoGeometric Analysis (IGA)



Design-analysis-optimization product development workflow

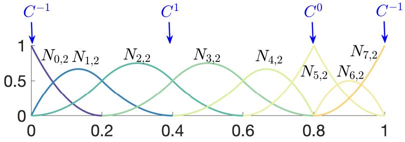
- Introduced by T.J.R. Hughes et al., 2005.
- **Key idea:** Approximate physical fields with the **same basis functions** used in CAD model representation.
- Advantages over traditional FEA:
  - **Unified design and analysis;**
  - **Precise and efficient geometry;**
  - **No data type transformation;**
  - **Simplified mesh refinement;**
  - **Continuous high-order fields;**
  - **Superior approximation.**
- Broad applications: fluid simulations, shell analysis, fluid-structure interaction, shape and topology optimization, etc.

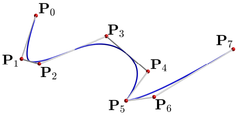
Why Non-Uniform Rational B-Splines (NURBS)?



3 control points vs. 10 grid points

- **Unified Representation**: Both standard (e.g., conics, quadrics) and free-form shapes;
- **Compact Geometry**: Efficient with fewer control points;
- **Fast Evaluations**: Stable and reliable;
- **Geometric Clarity**: Intuitive for designers;
- **Extensive Toolkit**: Including knot operations;
- Invariant under transformations.





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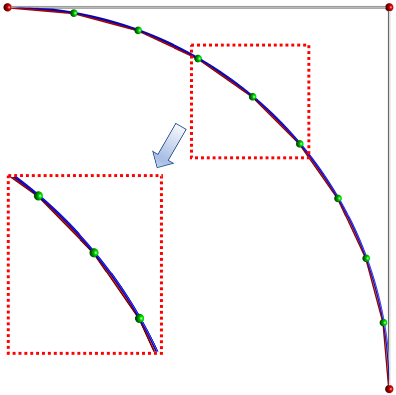

Close-up of the circle arc

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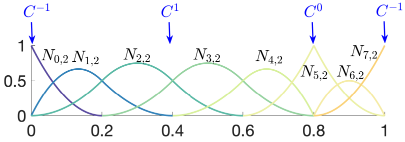

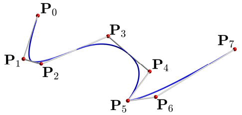

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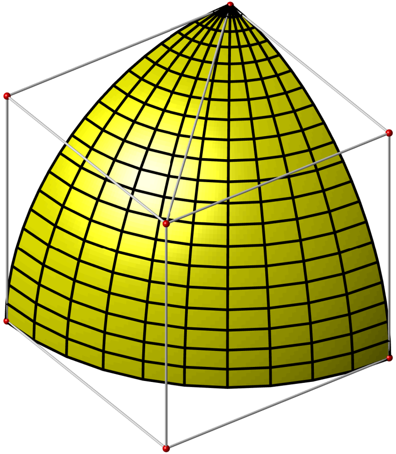

9 control points vs. 231 grid points

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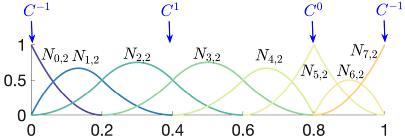
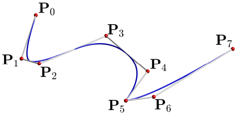
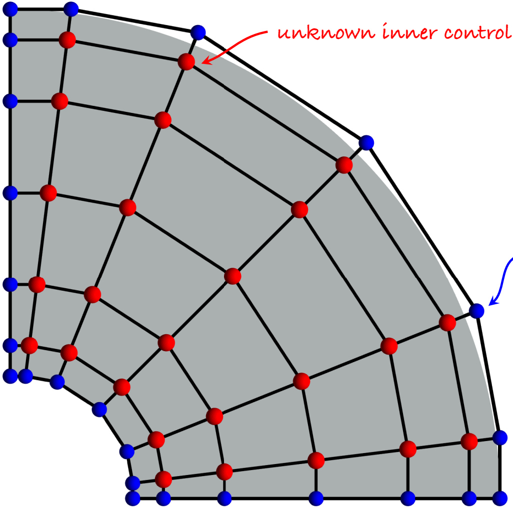




Problem Statement: Domain Parameterization



$$\boldsymbol{x}(\boldsymbol{\xi}) = \underbrace{\sum_{i \in \mathcal{I}_I} \mathbf{P}_i R_i(\boldsymbol{\xi})}_{\textbf{unknown}} + \underbrace{\sum_{j \in \mathcal{I}_B} \mathbf{P}_j R_j(\boldsymbol{\xi})}_{\textbf{known}}.$$

GOAL: To construct the **unknown inner control points $\mathbf{P}_i$** (or basis functions $R_i(\boldsymbol{\xi})$) such that $\boldsymbol{x}$ ensures **bijectivity** and exhibits **minimal angle and volume distortion**.

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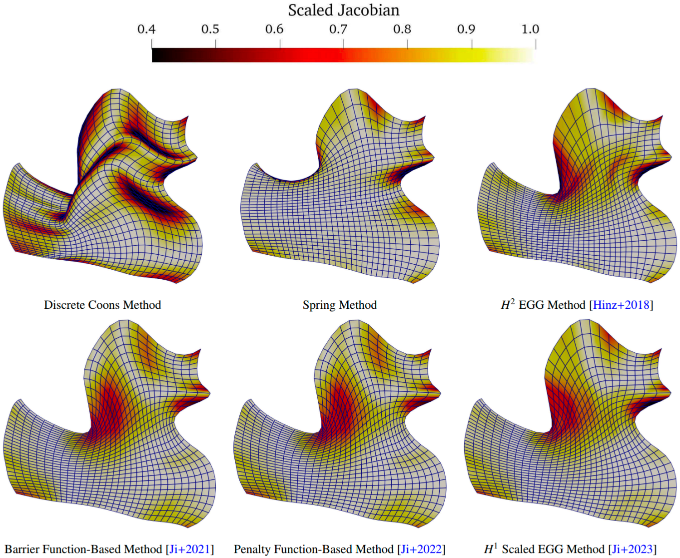
Different parameterization approaches



- Algebraic methods:
  - Coons patch
  - Spring patch
- Optimization-based methods:
  - Barrier-function-based [1]
  - Penalty-function-based [2]
- PDE-based methods:
  - Elliptic grid generation [3]
  - Improved EGG [4]

[1] Ji, Y. et al. (2021). Constructing high-quality ...

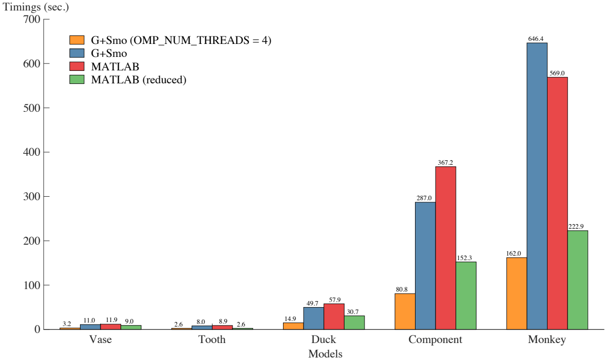
[2] Ji, Y. et al. (2022). Penalty function-based ...

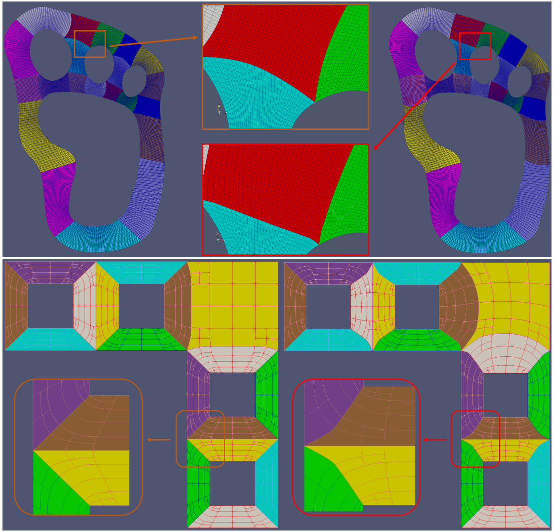
[3] Hinz, J. et al. (2018). Elliptic grid generation ...

[4] Ji, Y. et al. (2023). On an improved PDE-based ...

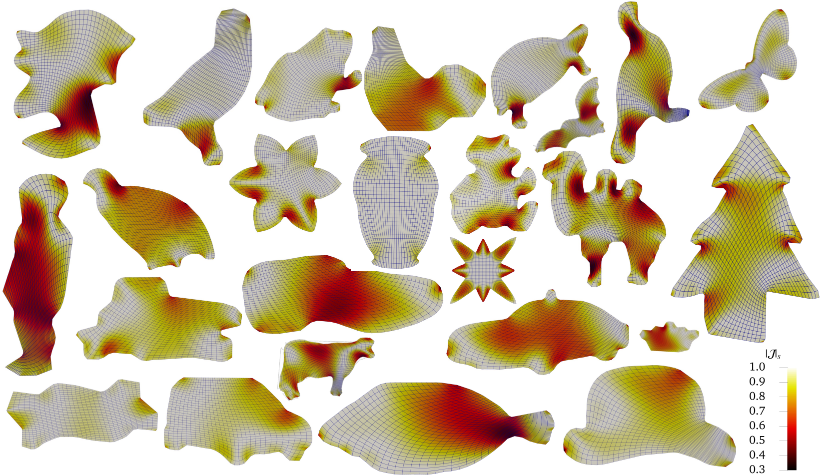
Open-sourced G+Smo Implementation

- Open-source implementation available:
  G+Smo: https://github.com/gismo/gismo;
- Supports **multi-patch** and **THB-spline** parameterization.



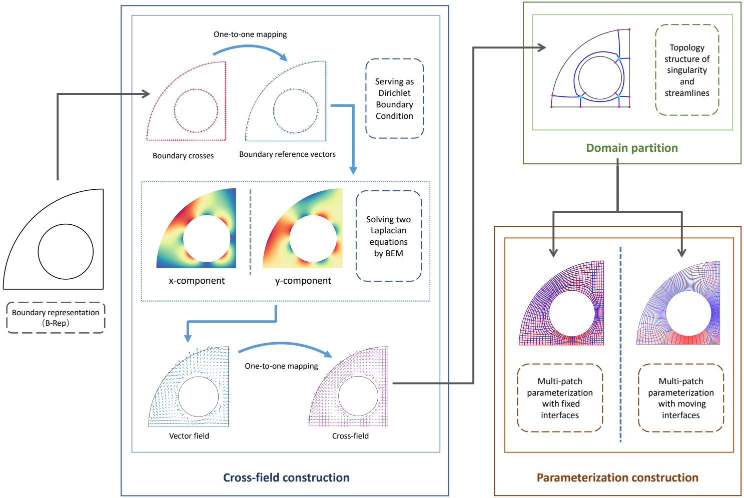


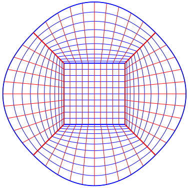
Planar Parameterization Test Dataset (977 models)

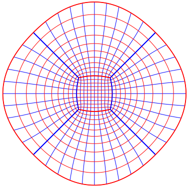


- PDE-based method - **higher computational efficiency** (≈0.2 sec. vs. ≈2 sec.) while **lower success rates** than optimization-based method (≈78% vs. ≈98%).

Multi-patch Parameterization using Cross-field

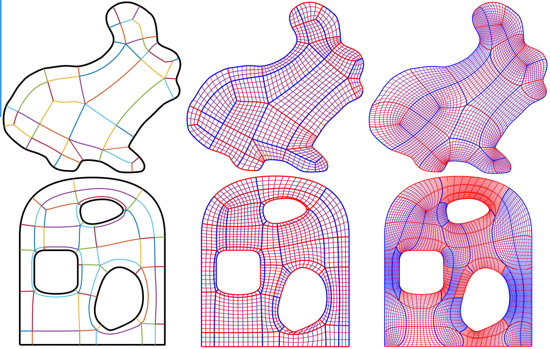


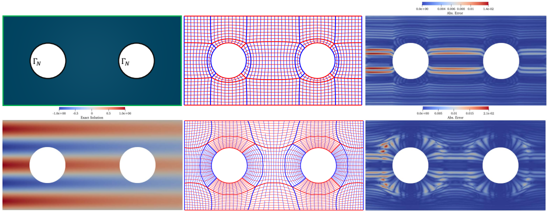




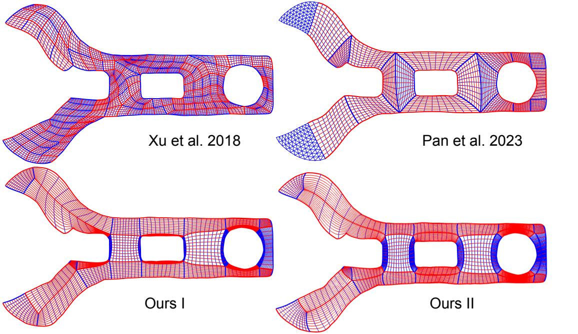
[1] Collin, A., Sangalli, G., & Takacs, T. (2016). Analysis-suitable G^1 multi-patch parametrizations ... *Computer Aided Geometric Design*, 47, 93-113.
[2] Zhang, Y., Ji, Y., & Zhu, C. G. (2024). Multi-patch parameterization method for isogeometric analysis using singular structure of cross-field. *Computers & Mathematics with Applications*, 162, 61-78.

Results and Comparisons

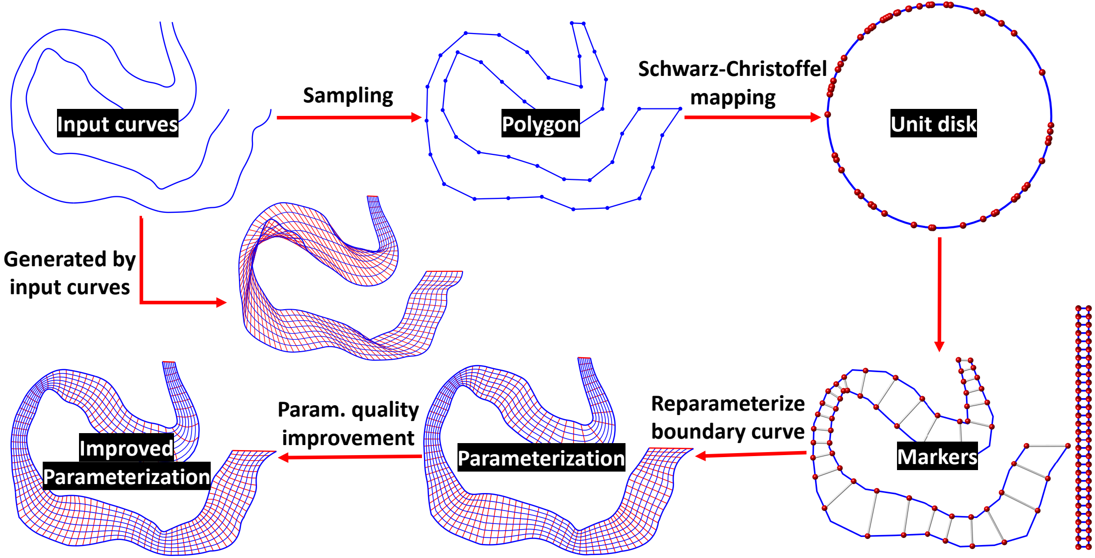




Model	#Patch	Method	$\lvert\mathcal{J}\rvert_s$		*unif.*		
			min.	avg.	min.	avg.	max.
rabbit	33	Coons	-0.8593	0.9628	0.7030	**0.9410**	1.0524
		fixed-I	**0.2204**	**0.9504**	0.6103	0.9544	**0.9982**
		moving-I	0.02918	0.9283	**0.0000**	0.9550	1.0000
3 holes	46	Coons	-0.5492	0.9710	0.8008	0.9573	1.0958
		fixed-I	**0.1545**	**0.9716**	0.8007	0.9573	0.9978
		moving-I	0.1461	0.9361	**0.6791**	**0.9571**	**0.9968**



Parameter matching via Schwarz-Christoffel mapping



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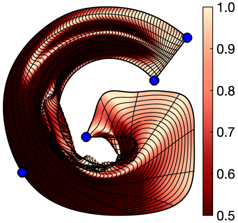
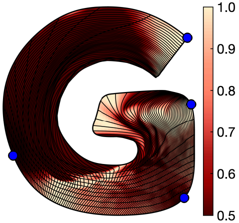
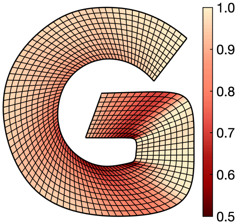


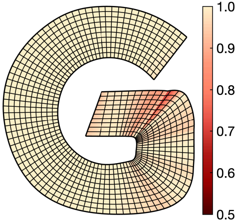
Optimal transport [1]



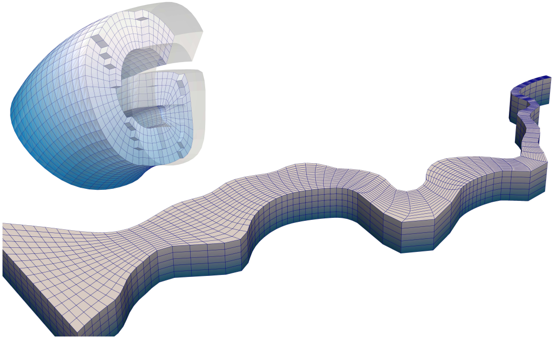
Deep learning [2]



Input [3]



Ours [3]



[1] Zheng, Y., Pan, M., & Chen, F. (2019). Boundary correspondence of planar domains for isogeometric analysis based on optimal mass transport. *Computer-Aided Design*, 114, 28-36.

[2] Zhan, Z., Zheng, Y., Wang, W., & Chen, F. (2023). Boundary Correspondence for Iso-Geometric Analysis Based on Deep Learning. Communications in Mathematics and Statistics, 11(1), 131-150.

[3] Ji, Y., Möller M., Yu Y., & C. Zhu Boundary parameter matching for isogeometric analysis using Schwarz-Christoffel mapping. Submited.

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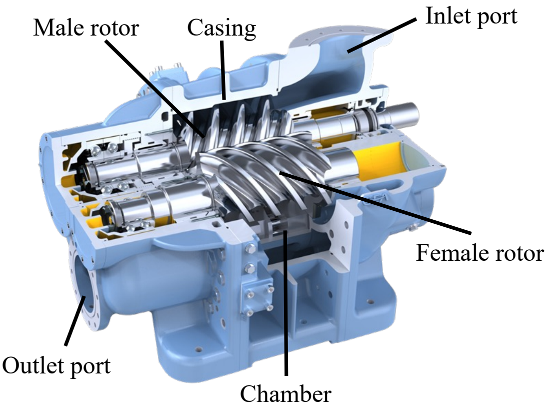
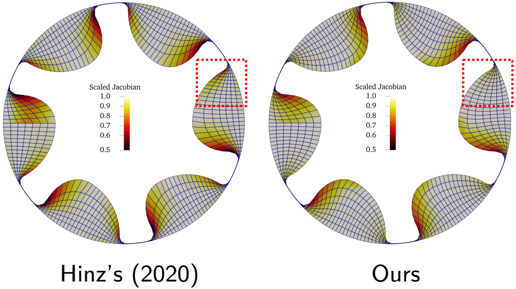
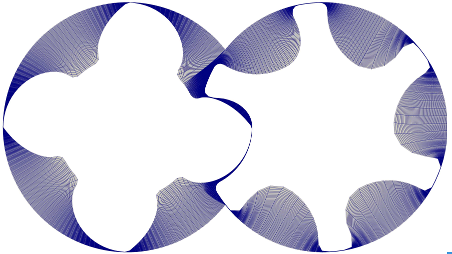
Industrial use case: rotary twin-screw compressors



Rotary Twin-Screw Compressor (source [a])

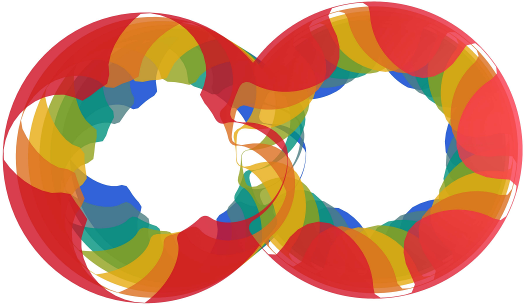


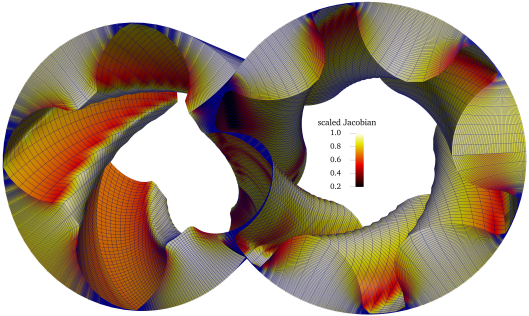


[a] https://www.gascompressors.co.uk/technologies/oil-flooded-screw-compressor/

Volumetric Completion via Lofting

- Complete volumetric parameterization by lofting computed slices.



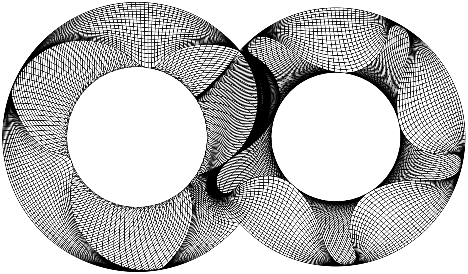


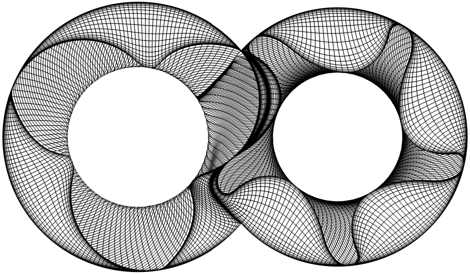
Discretization I: Boundary Layer Mesh

- **Spline evaluation is ALL YOU NEED**.
- Employ a simple expansion transformation:

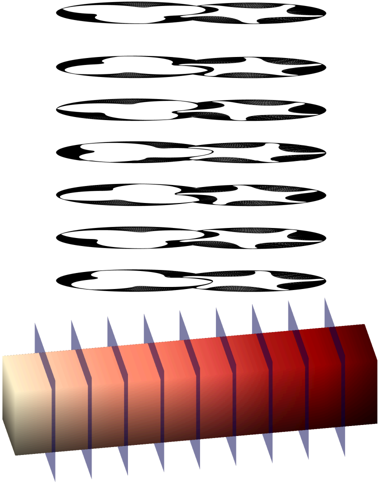
$$\begin{cases} \xi = \hat{\xi}, \\ \eta = \dfrac{\tanh\left(\alpha(2\hat{\eta}-1)\right)}{2\tanh(\hat{\eta})} + \dfrac{1}{2}, \end{cases}$$

where α represents the expansion factor.

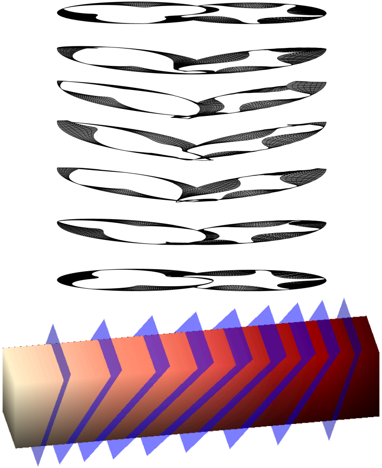




Discretization II: Flow-Aligned Hex Mesh



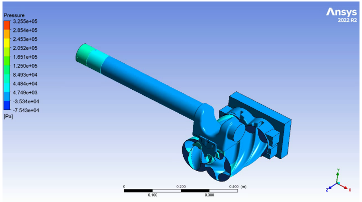
Ordinary discretization

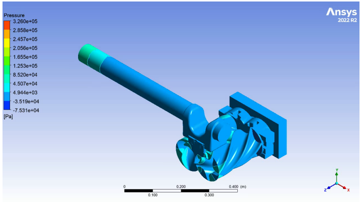


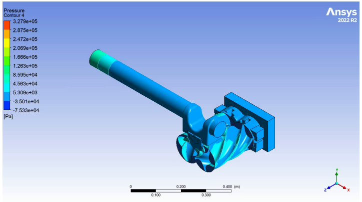
Flow-aligned discretization

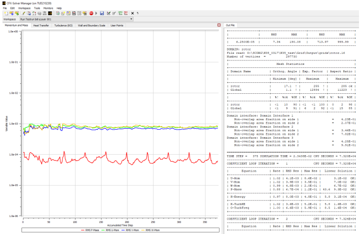
Simulation using ANASYS CFX

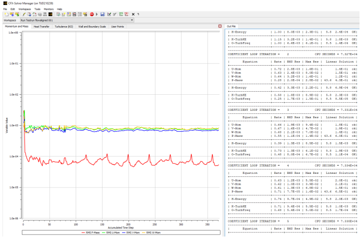
- Mesh density: $198 \times 95 \times 8$ for the male rotor and $200 \times 95 \times 8$ for the female rotor.











SCORG ™ Boundary Layer Mesh **Flow-aligned Mesh**

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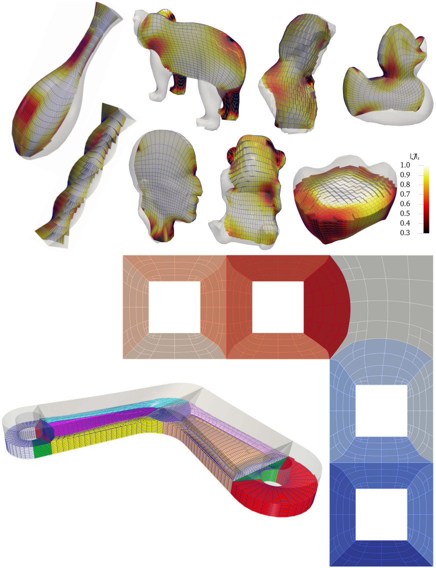
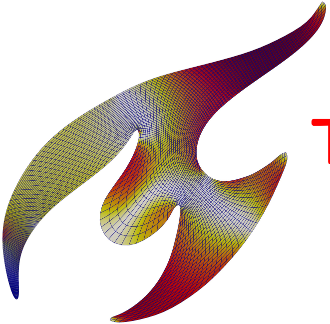
Conclusions and outlook

- Proposed several **IGA-suitable parameterization methods**;
- **Open-source implementation** available in `G+Smo`;
- Demonstrated **applicability** in **real-world industry scenarios** - complex fluid simulations;

Future Work:

- **Adaptive methods**: Further enhance efficiency and robustness for specific PDEs.





Thanks for Your Attention!

Q&A.

If you are interested in our research, please feel free to contact us! ;-)

- Email: y.ji-1@tudelft.nl
- Homepage: https://jiyess.github.io
- GitHub: jiyess