Applied Computational Finance - assignment1

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1. Introduction

In this assignment, I'm going to use Monte Carlo scheme to price path dependent options. I choose Asian option to analyze, since this option is strong path dependent option. I will use Arithmetic average and Geometric average to calculate the option values of Asian option and I will also consider fixed and floating strike price, discrete and continuous sampling. In both cases I will use the Euler-Maruyama scheme for simulating.

2. Numerical procedure used of pricing Asian option

The Forward Euler-Maruyama method for GBM gives:

$$\frac{\delta S_t}{S_t} = \frac{S_{t+\delta t} - S_t}{S_t} \sim r\delta t + \sigma\phi\sqrt{\delta t}$$

Then we can price S_{t+dt} by using previous price S_t for all of cases:

$$S_{t+\delta t} \sim S_t\left(1 + r\delta t + \sigma\phi\sqrt{\delta t}\right)$$

2.1 Discrete and continuous sampling

We can choose different dt to simulate Asian price. In this assignment, I choose $dt =$ 0.01 and 0.002 to simulate discrete sample.

However, the function of continuous sampling is $\int_0^T S(t)dt$. And in the continuous case, Asian option with arithmetic average is given by:

$$A(0,T) = \frac{1}{T}\int_t^T S(t)dt$$

And geometric average is:

$$A(0,T) = exp(\frac{1}{T}\int_t^T ln\,(S(t))dt)$$

If we use Euler-Maruyama scheme, we can't integrate since there are dt in $S(t)$. So I use very small dt = 0.0001 and 0.0002 to monitor continuous sampling.

2.2 Fixed and floating strike

Fixed strike Asian call payoff:

$$C(T) = max(A(0,T) - K, 0)$$

Where A denotes the average price for the period [0, T], and K is the strike price.
The equivalent put option is given by:

$$P(T) = max(K - A(0,T),0)$$

The floating strike Asian call option has the payoff:

$$C(T) = (S(T) - kA(0,T),0)$$

Where $S(T)$ is the price at maturity and k is a weighting, usually 1 so often omitted from descriptions. In this assignment I choose I to pricing floating strike Asian option. The equivalent put option payoff is given by:

$$P(T) = (kA(0,T) - S(T),0)$$

2.3 Arithmetic and Geometric average

For the case of discrete monitoring (with monitoring at the times $0 = t_0, t_1, t_2, \dots, t_n = T$) we have the average given by:

$$A(0,T) = \frac{1}{n+1}\sum_{i=0}^{n} S(t_i)$$

There also exist Asian options with geometric average this is given by:

$$A(0,T) = (\prod_{i=0}^{n} S(t_i))^{\frac{1}{n+1}}$$

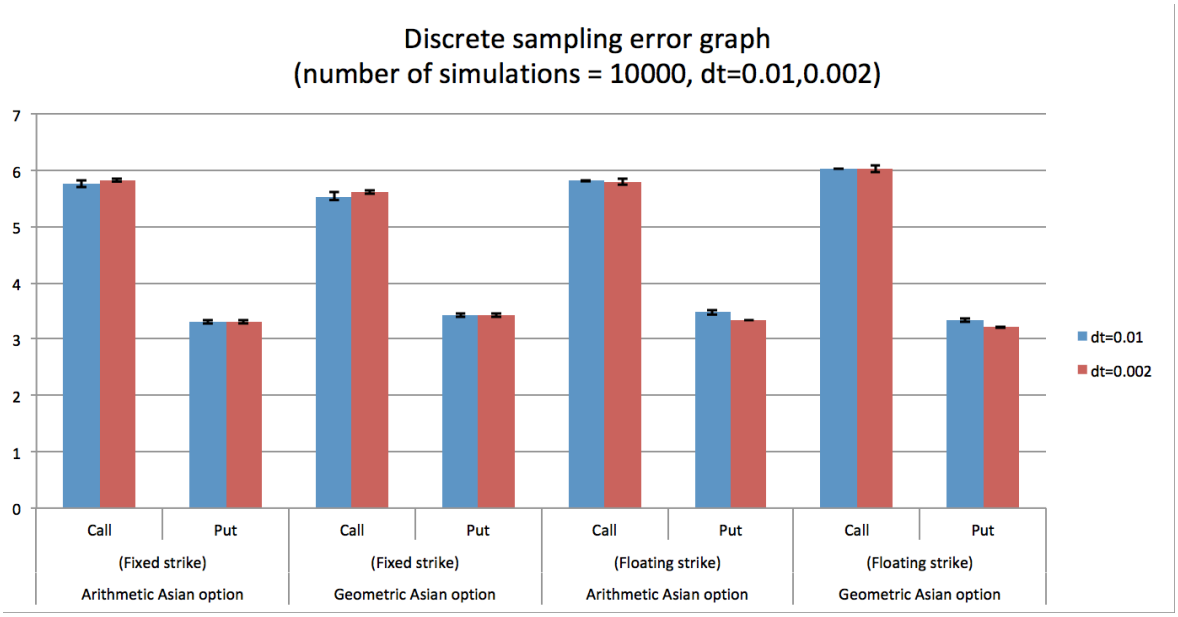
3. Results

Simulating discrete Asian option value

dt	Number of simulations	Arithmetic Asian option (Fixed strike)		Geometric Asian option (Fixed strike)		Arithmetic Asian option (Floating strike)		Geometric Asian option (Floating strike)	
		Call	Put	Call	Put	Call	Put	Call	Put
0.01	1	0	2.474	0	2.707	13.252	0	13.385	0
	100	4.371	3.063	4.191	3.174	5.738	3.853	5.912	3.736
	1000	5.806	3.176	5.579	3.289	6.076	3.032	6.299	2.914
	10000	5.759	3.308	5.539	3.430	5.821	3.475	6.032	3.345
0.002	1	0	6.877	0	7.184	0	2.437	0	2.130
	100	5.916	2.584	5.693	2.687	5.533	4.301	5.708	4.150
	1000	6.197	3.115	5.968	3.227	5.771	3.351	5.986	3.224
	10000	5.834	3.317	5.614	3.434	5.807	3.333	6.020	3.208

Standard deviation of discrete sampling

dt	Number of simulations	Arithmetic Asian option (Fixed strike)		Geometric Asian option (Fixed strike)		Arithmetic Asian option (Floating strike)		Geometric Asian option (Floating strike)	
		Call	Put	Call	Put	Call	Put	Call	Put
0.01	1	0	0	0	0	0	0	0	0
	100	0.673	1.202	0.649	1.224	0.410	0.374	0.398	0.360
	1000	0.189	0.432	0.181	0.433	0.193	0.050	0.200	0.049
	10000	0.068	0.034	0.065	0.036	0.014	0.036	0.010	0.034
0.002	1	0	0	0	0	0	0	0	0
	100	0.557	0.068	0.536	0.059	0.544	0.368	0.543	0.357
	1000	0.205	0.099	0.178	0.097	0.198	0.171	0.205	0.170
	10000	0.032	0.035	0.030	0.037	0.060	0.002	0.063	0.003

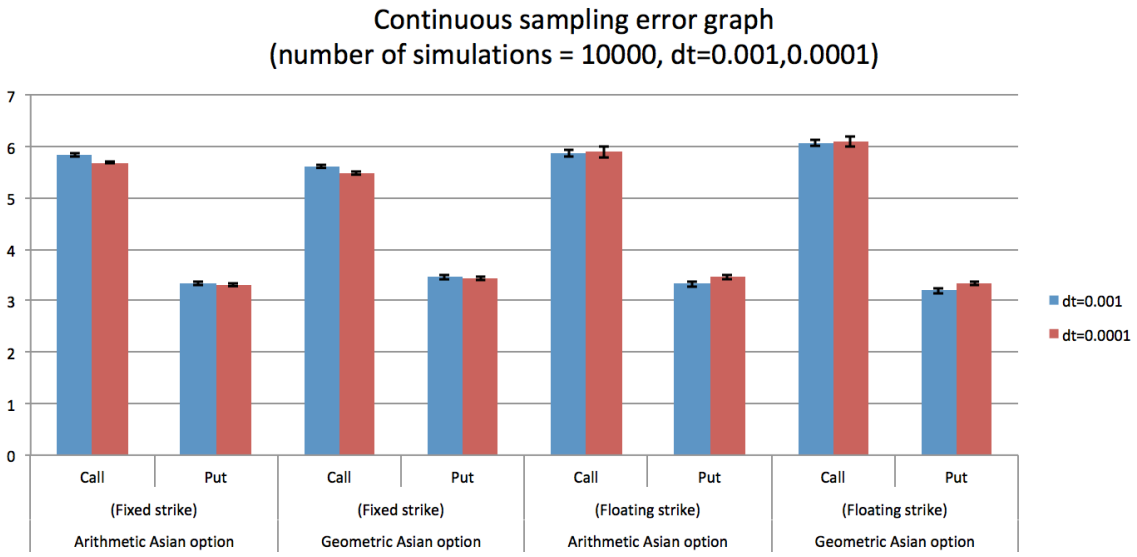


Simulating continuous Asian option value

dt	Number of simulations	Arithmetic Asian option (Fixed strike)		Geometric Asian option (Fixed strike)		Arithmetic Asian option (Floating strike)		Geometric Asian option (Floating strike)	
		Call	Put	Call	Put	Call	Put	Call	Put
0.001	1	5.017	0	4.916	0	0	6.674	0	6.571
	100	4.856	3.797	4.647	3.959	4.798	4.849	4.970	4.652
	1000	5.337	3.194	5.128	3.321	5.815	3.587	6.018	3.456
	10000	5.838	3.337	5.617	3.455	5.863	3.327	6.077	3.202
0.0001	1	3.332	0	2.527	0	14.948	0	15.753	0
	100	5.652	3.837	5.465	3.972	4.441	4.140	4.623	3.999
	1000	5.970	3.331	5.737	3.447	6.012	3.239	6.235	3.113
	10000	5.692	3.321	5.477	3.439	5.888	3.456	6.096	3.330

Standard deviation of continuous sampling

dt	Number of simulations	Arithmetic Asian option (Fixed strike)		Geometric Asian option (Fixed strike)		Arithmetic Asian option (Floating strike)		Geometric Asian option (Floating strike)	
		Call	Put	Call	Put	Call	Put	Call	Put
0.001	1	0	0	0	0	0	0	0	0
	100	0.591	0.298	0.573	0.309	0.626	0.280	0.651	0.277
	1000	0.272	0.113	0.245	0.117	0.430	0.120	0.457	0.115
	10000	0.032	0.035	0.032	0.036	0.060	0.058	0.063	0.056
0.0001	1	0	0	0	0	0	0	0	0
	100	0.580	0.611	0.552	0.608	0.727	0.210	0.750	0.204
	1000	0.187	0.440	0.180	0.454	0.195	0.300	0.201	0.287
	10000	0.023	0.035	0.023	0.036	0.100	0.035	0.100	0.034



4. Observations and conclusion

1. The results show that as the number of simulations increases, Monte Carlo simulations by using Euler-Maruyama scheme monitor more accurate arithmetic and geometric Asian option value.
2. Regardless of arithmetic or geometric average, floating or fixed strike price, option value of call option always higher than put option. Moreover, most of put option values are around 3, and most of call option values are around 6.
3. With the same type of strike price, whatever it is continuous or discrete, geometric put option value always larger than arithmetic put option value, conversely, geometric call option value always smaller than arithmetic call option value.
4. When simulate path number equal to 1, we can discover that fixed strike results cannot influent floating strike result, because when call option with value zero under fixed strike, both of call option equal to zero and put option equal to zero are possible in the situation of floating strike.
5. All of standard deviations are very small. I only use standard deviation of 10000 simulation's results of discrete and continuous Asian option to make error graph, because the number of simulations is larger, the results of option value are more accurate. Error graphs show that both of continuous and discrete sample have very small deviation, which means bias is very small for all of cases.