The Bigger The Better II

Group 8-29 *

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Contents

1 Introduction

This project aims to find an algorithm to determine the side length of the largest square that can be inscribed inside a convex n-gon. It is a continuation from a previous project completed in 2021, The Bigger The Better. [1]

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1.1 Definitions

placement

A valid location, size and rotation of the square such that all vertices of the square lie on the edges of the polygon.

inscribed

All vertices of the square must lie on the edges of the polygon.

RQ Research Question

1.2 Research Questions

1. What is the side length of the largest square that can be inscribed in a triangle?
2. What is the side length of the largest square that can be inscribed in a regular n-gon, given $n \neq 4$?
3. What is the side length of the largest square that can be inscribed in a convex n-gon?

1.3 Project Scope

This project will only focus on convex polygons.

2 Literature Review

3 Research Question 1

3.1 Introduction

RQ1 aims to find out the side length of the largest square that can be inscribed in a triangle, given the side lengths of the triangle.

3.2 Key Insights

1. It can be seen that no more than two vertices of a square can lie on a single side, as a square has at most two co-linear vertices.

2. We notice how a triangle has three sides, and a square has four vertices. In order for all the vertices to lie on the triangle, using the Pigeonhole Principle, there will be at least one side with two vertices lying on it.

3. Combining the above insights, there will be one vertex of the square each lying on two sides of the triangle, with the other two vertices of the square lying on the latter side of the triangle.

3.3 Solutions

A figure has been constructed for the purposes of illustrating the following proof.

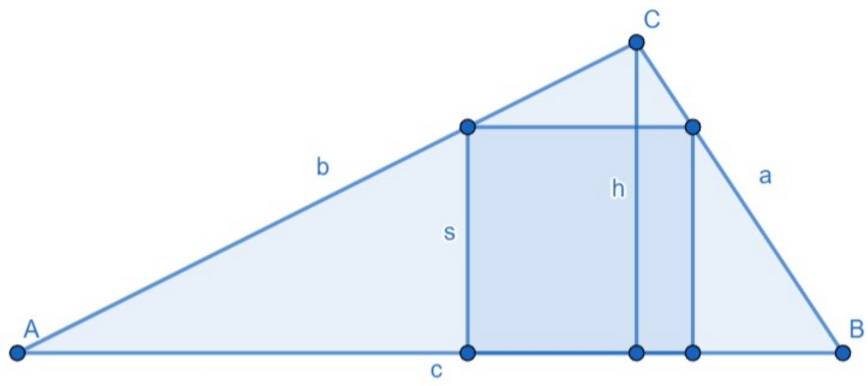

Figure 1: The figure for RQ1.

We see that side c can be formed with s as well as $s\cot A$ and $s\cot\angle B$.

$$\begin{aligned}
c &= s + s\cot\angle A + s\cot\angle B \\
s &= \frac{c}{1+\cot\angle A+\cot\angle B} \\
&= \frac{c\sin\angle A}{\sin\angle A+\cos\angle A+\cot\angle B\sin\angle A} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\cos\angle A\sin\angle B+\sin\angle A\cos\angle B} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\sin(\angle A+\angle B)} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\sin(180-\angle C)} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\sin\angle C} \\
&= \frac{2Rc\sin\angle A\sin\angle B}{2R\sin\angle A\sin\angle B+2R\sin\angle C} \\
&= \frac{ac\sin\angle B}{a\sin\angle B+c} \\
&= \frac{2Rac\sin\angle B}{2Ra\sin\angle B+2Rc} \\
&= \frac{abc}{2Rc+ab}
\end{aligned}$$

Since each of the sides of the triangle, a, b and c can be the longest side, we can take the maximum of the three combinations, hence

$$s_{\max} = \max\left(\frac{abc}{2Rc+ab}, \frac{abc}{2Rb+ac}, \frac{abc}{2Ra+bc}\right) \tag{1}$$

For obtuse triangles, we notice that only one placement exists, when the square lies on the longest side. We have:

$$s = \frac{abc}{2Rc+ab} \tag{2}$$

where c is the longest side.

References

[1] H. Wu and X. C. Huo. “The Bigger The Better”. In: (2021). URL: http://projectsday.hci.edu.sg/2021/05-Report/cat-08/8-02/index.pdf.