The Bigger The Better II

Group 8-29 *

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Contents

1 Introduction

This project aims to find an algorithm to determine the side length of the largest square that can be inscribed inside a convex n-gon. It is a continuation from a previous project completed in 2021, The Bigger The Better. [1]

1.1 Rationale

Do note that the definition of inscribed is such that all vertices of the square lie on the sides of the polygon.

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1.2 Research Questions

1. What is the side length of the largest square that can be inscribed in a triangle?

2. What is the side length of the largest square that can be inscribed in a regular n-gon, given $n \neq 4$?

3. What is the side length of the largest square that can be inscribed in a convex n-gon?

1.3 Project Scope

This project will mainly focus on polygons which are convex. This allows many restrictions to be made.

2 Research Question 1

2.1 Introduction

The first research question aims to find out the side length of the largest square that can be inscribed in a triangle, given the side lengths of the triangle.

2.2 Key insights

1. It can be seen that no more than 2 vertices of a square can lie on a single side, as a square has at most 2 vertices lying on a single line.

2. We notice how a triangle has 3 sides, and a square has 4 vertices. In order for all the vertices to lie on the triangle, by pigeonhole principle, at least one side has at least 2 vertices lying on it.

3. Combining the first 2 insights, we can see that 2 sides of the triangle will have 1 vertices each lying on it, while the other side will have 2 vertices lying on it.

2.3 Solutions

$$
\begin{aligned}
s &= \frac{c}{1+\cot\angle A+\cot\angle B} \\
&= \frac{c\sin\angle A}{\sin\angle A+\cos\angle A+\cot\angle B\sin\angle A} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\cos\angle A\sin\angle B+\sin\angle A\cos\angle B} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\sin\left(\angle A+\angle B\right)} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\sin\left(180-\angle C\right)} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\sin\angle C} \\
&= \frac{2Rc\sin\angle A\sin\angle B}{2R\sin\angle A\sin\angle B+2R\sin\angle C} \\
&= \frac{ac\sin\angle B}{a\sin\angle B+c} \\
&= \frac{2Rac\sin\angle B}{2Ra\sin\angle B+2Rc} \\
&= \frac{abc}{2Rc+ab}
\end{aligned}
$$

$$s_{max} = \max\left(\frac{abc}{2Rc+ab}, \frac{abc}{2Rb+aa}, \frac{abc}{2Ra+bc}\right)$$

References

[1] H. Wu and X. C. Huo. “The Bigger The Better”. In: (2021). URL: `http://projectsday.hci.edu.sg/2021/05-Report/cat-08/8-02/index.pdf`.