The Bigger The Better II

Group 8-29 *

2022

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1 Introduction

This project aims to find an algorithm to determine the side length of the largest square that can be inscribed inside a convex n-gon. It is a continuation from a previous project completed in 2021, The Bigger The Better. [1]

1.1 Rationale

Do note that the definition of inscribed is such that all vertices of the square lie on the sides of the polygon.

1.2 Research Questions

1. What is the side length of the largest square that can be inscribed in a triangle?
2. What is the side length of the largest square that can be inscribed in a regular n-gon, given $n \neq 4$?
3. What is the side length of the largest square that can be inscribed in a convex n-gon?

1.3 Project Scope

This project will mainly focus on polygons which are convex. This allows many restrictions to be made.

2 Literature Review

3 Research Question 1

3.1 Introduction

The first research question aims to find out the side length of the largest square that can be inscribed in a triangle, given the side lengths of the triangle.

3.2 Key Insights

1. It can be seen that no more than 2 vertices of a square can lie on a single side, as a square has at most 2 vertices lying on a single line.

2. We notice how a triangle has 3 sides, and a square has 4 vertices. In order for all the vertices to lie on the triangle, by pigeonhole principle, at least one side has at least 2 vertices lying on it.

3. Combining the first 2 insights, we can see that 2 sides of the triangle will have 1 vertices each lying on it, while the other side will have 2 vertices lying on it.

3.3 Solutions

A figure has been constructed for the purposes of illustrating the following proof.

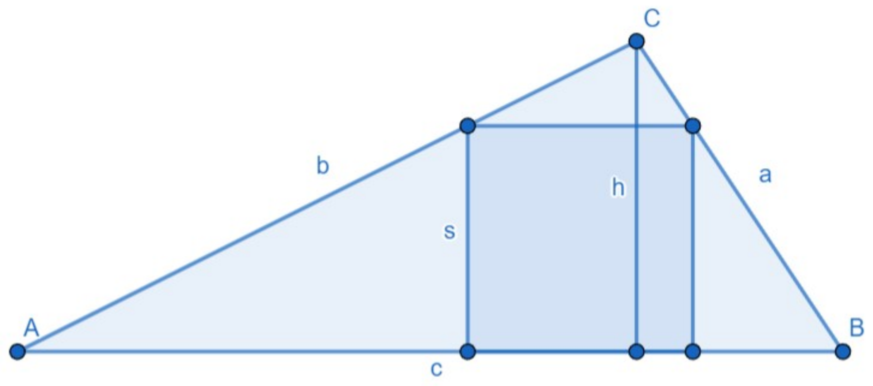

Figure 1: The figure for RQ1.

We see that side c can be formed with s as well as $s\cot\angle A$ and $s\cot\angle B$.

$$
\begin{aligned}
c &= s + s\cot\angle A + s\cot\angle B \\
s &= \frac{c}{1+\cot\angle A+\cot\angle B} \\
&= \frac{c\sin\angle A}{\sin\angle A+\cos\angle A+\cot\angle B\sin\angle A} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\cos\angle A\sin\angle B+\sin\angle A\cos\angle B} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\sin\left(\angle A+\angle B\right)} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\sin\left(180-\angle C\right)} \\
&= \frac{c\sin\angle A\sin\angle B}{\sin\angle A\sin\angle B+\sin\angle C} \\
&= \frac{2Rc\sin\angle A\sin\angle B}{2R\sin\angle A\sin\angle B+2R\sin\angle C} \\
&= \frac{ac\sin\angle B}{a\sin\angle B+c} \\
&= \frac{2Rac\sin\angle B}{2Ra\sin\angle B+2Rc} \\
&= \frac{abc}{2Rc+ab}
\end{aligned} \tag{1}
$$

Since each of the sides of the triangle, a, b and c can be the longest side, we can take the maximum of the three combinations, hence

$$s_{max} = \max\left(\frac{abc}{2Rc + ab}, \frac{abc}{2Rb + ac}, \frac{abc}{2Ra + bc}\right) \tag{2}$$

For obtuse triangles, we notice that only 1 placement exist, when the square lies on the longest side. We have:

$$s_{max} = \frac{abc}{2Rc + ab} \tag{3}$$

where c is the longest side.

References

[1] H. Wu and X. C. Huo. "The Bigger The Better". In: (2021). URL: `http://projectsday.hci.edu.sg/2021/05-Report/cat-08/8-02/index.pdf`.