

The Bigger The Better II

Group 8-29 *

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1 Introduction

This project aims to find an algorithm to determine the side length of the largest square that can be inscribed inside a convex n -gon. It is a continuation from a previous project completed in 2021, The Bigger The Better. [1]

1.1 Rationale

Do note that the definition of inscribed is such that all vertices of the square lie on the sides of the polygon.

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1.2 Research Questions

1. What is the side length of the largest square that can be inscribed in a triangle?
2. What is the side length of the largest square that can be inscribed in a regular n -gon, given $n \neq 4$?
3. What is the side length of the largest square that can be inscribed in a convex n -gon?

1.3 Project Scope

This project will mainly focus on polygons which are convex. This allows many restrictions to be made.

2 Research Question 1

2.1 Introduction

The first research question aims to find out the side length of the largest square that can be inscribed in a triangle, given the side lengths of the triangle.

2.2 Key insights

1. It can be seen that no more than 2 vertices of a square can lie on a single side, as a square has at most 2 vertices lying on a single line.
2. We notice how a triangle has 3 sides, and a square has 4 vertices. In order for all the vertices to lie on the triangle, by pigeonhole principle, at least one side has at least 2 vertices lying on it.
3. Combining the first 2 insights, we can see that 2 sides of the triangle will have 1 vertices each lying on it, while the other side will have 2 vertices lying on it.

2.3 Solutions

$$\begin{aligned}
s &= \frac{c}{1 + \cot \angle A + \cot \angle B} \\
&= \frac{c \sin \angle A}{\sin \angle A + \cos \angle A + \cot \angle B \sin \angle A} \\
&= \frac{c \sin \angle A \sin \angle B}{\sin \angle A \sin \angle B + \cos \angle A \sin \angle B + \sin \angle A \cos \angle B} \\
&= \frac{c \sin \angle A \sin \angle B}{\sin \angle A \sin \angle B + \sin (\angle A + \angle B)} \\
&= \frac{c \sin \angle A \sin \angle B}{\sin \angle A \sin \angle B + \sin (180 - \angle C)} \\
&= \frac{c \sin \angle A \sin \angle B}{\sin \angle A \sin \angle B + \sin \angle C} \\
&= \frac{2Rc \sin \angle A \sin \angle B}{2R \sin \angle A \sin \angle B + 2R \sin \angle C} \\
&= \frac{ac \sin \angle B}{a \sin \angle B + c} \\
&= \frac{2Rac \sin \angle B}{2Ra \sin \angle B + 2Rc} \\
&= \frac{abc}{2Rc + ab} s_{max} = \max \left(\frac{abc}{2Rc + ab}, \frac{abc}{2Rb + aa}, \frac{abc}{2Ra + bc} \right)
\end{aligned}$$

References

- [1] H. Wu and X. C. Huo. “The Bigger The Better”. In: (2021). URL: <http://projectsday.hci.edu.sg/2021/05-Report/cat-08/8-02/index.pdf>.