

# The Bigger The Better II

Group 8-29 \*

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Rationale . . . . .	2
1.2	Research Questions . . . . .	2
1.3	Project Scope . . . . .	2
<b>2</b>	<b>Literature Review</b>	<b>2</b>
<b>3</b>	<b>Research Question 1</b>	<b>2</b>
3.1	Introduction . . . . .	2
3.2	Key Insights . . . . .	2
3.3	Solutions . . . . .	3

## 1 Introduction

This project aims to find an algorithm to determine the side length of the largest square that can be inscribed inside a convex  $n$ -gon. It is a continuation from a previous project completed in 2021, The Bigger The Better. [1]

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## 1.1 Rationale

Do note that the definition of inscribed is such that all vertices of the square lie on the sides of the polygon.

## 1.2 Research Questions

1. What is the side length of the largest square that can be inscribed in a triangle?
2. What is the side length of the largest square that can be inscribed in a regular  $n$ -gon, given  $n \neq 4$ ?
3. What is the side length of the largest square that can be inscribed in a convex  $n$ -gon?

## 1.3 Project Scope

This project will mainly focus on polygons which are convex. This allows many restrictions to be made.

# 2 Literature Review

## 3 Research Question 1

### 3.1 Introduction

The first research question aims to find out the side length of the largest square that can be inscribed in a triangle, given the side lengths of the triangle.

### 3.2 Key Insights

1. It can be seen that no more than 2 vertices of a square can lie on a single side, as a square has at most 2 vertices lying on a single line.
2. We notice how a triangle has 3 sides, and a square has 4 vertices. In order for all the vertices to lie on the triangle, by pigeonhole principle, at least one side has at least 2 vertices lying on it.

3. Combining the first 2 insights, we can see that 2 sides of the triangle will have 1 vertices each lying on it, while the other side will have 2 vertices lying on it.

### 3.3 Solutions

$$\begin{aligned}
s &= \frac{c}{1 + \cot \angle A + \cot \angle B} \\
&= \frac{c \sin \angle A}{\sin \angle A + \cos \angle A + \cot \angle B \sin \angle A} \\
&= \frac{c \sin \angle A \sin \angle B}{\sin \angle A \sin \angle B + \cos \angle A \sin \angle B + \sin \angle A \cos \angle B} \\
&= \frac{c \sin \angle A \sin \angle B}{\sin \angle A \sin \angle B + \sin (\angle A + \angle B)} \\
&= \frac{c \sin \angle A \sin \angle B}{\sin \angle A \sin \angle B + \sin (180 - \angle C)} \\
&= \frac{c \sin \angle A \sin \angle B}{\sin \angle A \sin \angle B + \sin \angle C} \\
&= \frac{2Rc \sin \angle A \sin \angle B}{2R \sin \angle A \sin \angle B + 2R \sin \angle C} \\
&= \frac{ac \sin \angle B}{a \sin \angle B + c} \\
&= \frac{2Rac \sin \angle B}{2Ra \sin \angle B + 2Rc} \\
&= \frac{abc}{2Rc + ab}
\end{aligned}$$

Figure 1: The solution for RQ1 for acute triangles and right triangles.

Since each of the sides of the triangle,  $a$ ,  $b$  and  $c$  can be the longest side, we can take the maximum of the three combinations, hence

$$s_{max} = \max \left( \frac{abc}{2Rc + ab}, \frac{abc}{2Rb + ac}, \frac{abc}{2Ra + bc} \right) \quad (1)$$

For obtuse triangles, we notice that only 1 placement exist, when the square lies on the longest side. We have:

$$s_{max} = \frac{abc}{2Rc + ab} \quad (2)$$

, where  $c$  is the longest side.

## References

- [1] H. Wu and X. C. Huo. “The Bigger The Better”. In: (2021). URL: <http://projectsday.hci.edu.sg/2021/05-Report/cat-08/8-02/index.pdf>.