16-642 Manipulation, Estimation, and Control
Fall 2022
Midterm Exam

Instructions:
Open note, open book, open internet.
You may use any resource except for other humans.
Please show all work. Intermediate steps must be legible to receive credit.
Your solution must be submitted as a single pdf file to Canvas by
10:10am on Thursday, October 13, 2022

1. (15 points) Write the scalar ODE below in state space form.

$$\dddot{y} + \gamma\ddot{y} - (\dot{y} - y^2)^2 = u.$$

2. (15 points) Derive the conditions under which the observer state estimate $\hat{x}(t)$ converges to the actual state $x(t)$, where $\hat{x}(t)$ is found by integrating the equation

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K_o(C\hat{x}(t) - y(t)),$$

and $x(t)$, $u(t)$, and $y(t)$ are the state, input, and output to the usual continuous time state space system:

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t).$$

3. (15 points) It is desired for a closed loop system to have a step response with a 1% settling time t_s of 2 seconds and a percent overshoot M_p of 5%. Sketch the region of the complex plane where the closed loop poles can be placed in order to meet these requirements.

4. Consider the nonlinear system

$$\dot{x} = \begin{bmatrix} x_2^2 + x_1 \cos x_2 - u \\ x_2 + (x_1 + 1)x_1 + x_1 \sin x_2 + u \end{bmatrix}$$

   (a) (5 points) Verify that there is an equilibrium point at $x = 0$.

   (b) (15 points) Linearize the system about the equilibrium point at the origin.

   (c) (5 points) What can you say about the stability of the nonlinear system for the equilibrium point at 0?

5. (5 points) Is the equilibrium point at zero stable for the unforced state space system below stable? Explain your answer.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -3 & -6 \end{bmatrix} x$$

6. (10 points) Is the system below controllable? Explain your answer.

$$\dot{x} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

7. (15 points) Consider the linear state space system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -3 & -6 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 4 & 5 & 6 \\ 0 & 0 & 7 & 8 & 9 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u,$$

where the full state can be measured. Use eigenvalue placement to design a feedback control law $u = -Kx$ so that the resulting closed loop system is asymptotically stable. (hint: this is trickier than it looks, as you will quickly discover if you try to blindly use the MATLAB place command. You may find some solace in the block diagonal structure of the A matrix, the answers to the previous two problems, and the general advice to let go of the things you cannot control.)