$$Y = i\omega C_0 + 2w^2 Y_{11}^2 d_{31}^2 \left(\frac{1}{D} \right)$$

其中分母 D 是复数:

$$D = a + ib$$

其中

$$a = Z_0 \frac{Al}{2} + R_{l/2}$$

$$b = \frac{\pi Z_0 \Delta f_n}{2 f_{n(\mathrm{air})}} + X_{l/2}$$

$$|D| = \sqrt{a^2 + b^2}$$

$$\frac{d|D|}{d(\Delta f_n)} = \frac{d}{d(\Delta f_n)} \left(\sqrt{a^2 + b^2} \right)$$

首先求 b 对 Δf_n 的导数:

$$b = \frac{\pi Z_0 \Delta f_n}{2 f_{n(\mathrm{air})}} + X_{l/2}$$

$$\frac{db}{d(\Delta f_n)} = \frac{\pi Z_0}{2 f_{n(\mathrm{air})}}$$

使用链式法则:

$$\frac{d|D|}{d(\Delta f_n)} = \frac{1}{2\sqrt{a^2 + b^2}} \cdot 2b \cdot \frac{db}{d(\Delta f_n)}$$

$$\frac{d|D|}{d(\Delta f_n)} = \frac{b}{\sqrt{a^2 + b^2}} \cdot \frac{\pi Z_0}{2 f_{n(\mathrm{air})}}$$

设置导数为零以找到极值:

$$\frac{b}{\sqrt{a^2 + b^2}} \cdot \frac{\pi Z_0}{2 f_{n(\mathrm{air})}} = 0$$

由于 $\frac{\pi Z_0}{2 f_{n(\mathrm{air})}} \neq 0$，我们必须有:

$$b = 0$$

约束条件

我们知道:

$$b = \frac{\pi Z_0 \Delta f_n}{2 f_{n(\mathrm{air})}} + X_{l/2}$$

将 b 设为零:

$$\frac{\pi Z_0 \Delta f_n}{2 f_{n(\mathrm{air})}} + X_{l/2} = 0$$

解这个方程，得到：

$$\frac{\Delta f_n}{f_{n(\mathrm{air})}} = \frac{-2X_{l/2}}{\pi Z_0}$$