$$Z_0 = -\frac{AY_{11}wh}{\omega} + i\frac{BY_{11}wh}{\omega}$$

代入 D

将 Z_0 代入 D:

$$D = \left(\left(-\frac{AY_{11}wh}{\omega} + i\frac{BY_{11}wh}{\omega}\right)\frac{Al}{2} + R_{l/2}\right) + i\left(\frac{\pi\left(-\frac{AY_{11}wh}{\omega} + i\frac{BY_{11}wh}{\omega}\right)\Delta f_n}{2f_{n(\mathrm{air})}} + X_{l/2}\right)$$

展开并分离实部和虚部:

$$D = \left(-\frac{A^2Y_{11}whl}{2\omega} + i\frac{ABY_{11}whl}{2\omega} + R_{l/2}\right) + i\left(-\frac{\pi AY_{11}wh\Delta f_n}{2\omega f_{n(\mathrm{air})}} + i\frac{\pi BY_{11}wh\Delta f_n}{2\omega f_{n(\mathrm{air})}} + X_{l/2}\right)$$

为了简化表达式，我们将 i 乘进虚部项中，然后重新组合实部和虚部:

$$D = \left(-\frac{A^2Y_{11}whl}{2\omega} + R_{l/2}\right) + i\left(\frac{ABY_{11}whl}{2\omega} - \frac{\pi AY_{11}wh\Delta f_n}{2\omega f_{n(\mathrm{air})}} + X_{l/2}\right) - \frac{\pi BY_{11}wh\Delta f_n}{2\omega f_{n(\mathrm{air})}}$$

重新组合实部和虚部

将分母 D 的实部和虚部重新组合:

实部 a_D:

$$a_D = -\frac{A^2Y_{11}whl}{2\omega} + R_{l/2} - \frac{\pi BY_{11}wh\Delta f_n}{2\omega f_{n(\mathrm{air})}}$$

虚部 b_D:

$$b_D = \frac{ABY_{11}whl}{2\omega} - \frac{\pi AY_{11}wh\Delta f_n}{2\omega f_{n(\mathrm{air})}} + X_{l/2}$$

最小化分母的模量

我们需要最小化分母 D 的模量:

$$|D| = \sqrt{a_D^2 + b_D^2}$$

对 a_D 和 b_D 的导数求极值

为了找到极值，我们对 $|D|$ 求导数:

$$\frac{\partial|D|}{\partial\Delta f_n} = \frac{a_D\frac{\partial a_D}{\partial\Delta f_n} + b_D\frac{\partial b_D}{\partial\Delta f_n}}{\sqrt{a_D^2 + b_D^2}}$$

设导数为零，我们得到:

$$a_D\frac{\partial a_D}{\partial\Delta f_n} + b_D\frac{\partial b_D}{\partial\Delta f_n} = 0$$

计算导数

我们计算 a_D 和 b_D 对 Δf_n 的导数:

$$\frac{\partial a_D}{\partial \Delta f_n} = -\frac{\pi B Y_{11} w h}{2\omega f_{n(\mathrm{air})}}$$

$$\frac{\partial b_D}{\partial \Delta f_n} = -\frac{\pi A Y_{11} w h}{2\omega f_{n(\mathrm{air})}}$$

代入得到:

$$a_D\left(-\frac{\pi B Y_{11} w h}{2\omega f_{n(\mathrm{air})}}\right) + b_D\left(-\frac{\pi A Y_{11} w h}{2\omega f_{n(\mathrm{air})}}\right) = 0$$

简化后得到:

$$-\frac{\pi Y_{11} w h}{2\omega f_{n(\mathrm{air})}}(a_D B + b_D A) = 0$$

解决方程

要使上式为零，我们有两种情况:

1. $\frac{\pi Y_{11} w h}{2\omega f_{n(\mathrm{air})}} \neq 0$ 总是成立，所以必须有:

$$a_D B + b_D A = 0$$

代入 a_D 和 b_D 的表达式:

$$\left(-\frac{A^2 Y_{11} w h l}{2\omega} + R_{l/2} - \frac{\pi B Y_{11} w h \Delta f_n}{2\omega f_{n(\mathrm{air})}}\right)B + \left(\frac{A B Y_{11} w h l}{2\omega} - \frac{\pi A Y_{11} w h \Delta f_n}{2\omega f_{n(\mathrm{air})}} + X_{l/2}\right)A = 0$$

解这个方程

将上式展开:

$$-\frac{A^2 B Y_{11} w h l}{2\omega} + B R_{l/2} - \frac{\pi B^2 Y_{11} w h \Delta f_n}{2\omega f_{n(\mathrm{air})}} + \frac{A^2 B Y_{11} w h l}{2\omega} - \frac{\pi A^2 Y_{11} w h \Delta f_n}{2\omega f_{n(\mathrm{air})}} + A X_{l/2} = 0$$

简化得到:

$$B R_{l/2} - \frac{\pi B^2 Y_{11} w h \Delta f_n}{2\omega f_{n(\mathrm{air})}} - \frac{\pi A^2 Y_{11} w h \Delta f_n}{2\omega f_{n(\mathrm{air})}} + A X_{l/2} = 0$$

$$B R_{l/2} + A X_{l/2} - \frac{\pi Y_{11} w h \Delta f_n}{2\omega f_{n(\mathrm{air})}}(B^2 + A^2) = 0$$

$$B R_{l/2} + A X_{l/2} = \frac{\pi Y_{11} w h \Delta f_n}{2\omega f_{n(\mathrm{air})}}(B^2 + A^2)$$

$$\Delta f_n = \frac{2\omega f_{n(\mathrm{air})}(B R_{l/2} + A X_{l/2})}{\pi Y_{11} w h (B^2 + A^2)}$$