

网格点和初始化

选五个点，根据初值条件：

- $j = 1$ 到 $j = 5$ ，初始条件 $u^0 = [1, 1, 0.5, 0, 0]$

数值格式和计算步骤

1. 左偏心格式（迎风）
 - 更新方程： $u_j^{k+1} = u_j^k - \frac{1}{2}(u_j^k - u_{j-1}^k)$
2. 右偏心格式（背风）
 - 更新方程： $u_j^{k+1} = u_j^k - \frac{1}{2}(u_{j+1}^k - u_j^k)$
3. Lax-Wendroff格式
 - 更新方程： $u_j^{k+1} = u_j^k - \frac{1}{2}(u_{j+1}^k - u_{j-1}^k) + \frac{1}{8}(u_{j+1}^k - 2u_j^k + u_{j-1}^k)$

就你书上132页公式

$$k = 0$$

左偏心格式

- $u_1^1 = 1$
- $u_2^1 = 1 - 0.5 \times (1 - 1) = 1$
- $u_3^1 = 0.5 - 0.5 \times (0.5 - 1) = 0.75$
- $u_4^1 = 0 - 0.5 \times (0 - 0.5) = 0.25$
- $u_5^1 = 0$

右偏心格式

- $u_1^1 = 1 - 0.5 \times (1 - 1) = 1$
- $u_2^1 = 1 - 0.5 \times (0.5 - 1) = 1.25$
- $u_3^1 = 0.5 - 0.5 \times (0 - 0.5) = 0.75$
- $u_4^1 = 0 - 0.5 \times (0 - 0) = 0$
- $u_5^1 = 0$

LW格式

- $u_1^1 = 1$
- $u_2^1 = 1 - 0.5 \times (0.5 - 1) + 0.125 \times (0.5 - 2 \times 1 + 1) = 1 - 0.25 + 0.0625 = 0.8125$
- $u_3^1 = 0.5 - 0.5 \times (0 - 0.5) + 0.125 \times (0 - 2 \times 0.5 + 1) = 0.75 - 0.125 = 0.625$
- $u_4^1 = 0 - 0.5 \times (0 - 0) + 0.125 \times (0 - 2 \times 0 + 0.5) = 0.0625$
- $u_5^1 = 0$

$$k = 1$$

- 左偏心：

$$u^1 = [1, 1, 0.75, 0.25, 0]$$

- 右偏心：

$$u^1 = [1, 1.25, 0.75, 0, 0]$$

左偏心格式

- $u_1^2 = 1$
- $u_2^2 = 1 - 0.5 \times (1 - 1) = 1$
- $u_3^2 = 0.75 - 0.5 \times (0.75 - 1) = 0.875$
- $u_4^2 = 0.25 - 0.5 \times (0.25 - 0.75) = -0.125$
- $u_5^2 = 0$

右偏心格式

利用方程：

$$u_j^{k+1} = u_j^k - \frac{1}{2}(u_{j+1}^k - u_j^k)$$

- $u_1^2 = 1$
- $u_2^2 = 1 - 0.5 \times (1.25 - 1) = 0.875$
- $u_3^2 = 1.25 - 0.5 \times (0.75 - 1.25) = 1.5$
- $u_4^2 = 0.75 - 0.5 \times (0 - 0.75) = 1.125$
- $u_5^2 = 0$

LW

懒得算了，这是python

```
u_previous = [1, 1, 0.75, 0.25, 0]
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```
u_lw = u_previous.copy()
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for j in range(1, len(u_previous)-1):
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```
    u_lw[j] = (u_previous[j] - 0.5 * (u_previous[j+1] - u_previous[j-1]) +  
              0.125 * (u_previous[j+1] - 2*u_previous[j] + u_previous[j-1]))
```

```
u_lw
```

$$u^2 = [1, 1.09375, 1.09375, 0.65625, 0]$$