$$Z_0 = -rac{AY_{11}wh}{\omega} + irac{BY_{11}wh}{\omega}$$

代入 D

将 Z<sub>0</sub>代入 D:

$$D = \left(\left(-rac{AY_{11}wh}{\omega} + irac{BY_{11}wh}{\omega}
ight)rac{Al}{2} + R_{l/2}
ight) + i\left(rac{\pi\left(-rac{AY_{11}wh}{\omega} + irac{BY_{11}wh}{\omega}
ight)\Delta f_n}{2f_{n( ext{air})}} + X_{l/2}
ight)$$

展开并分离实部和虚部:

$$D = \left(-rac{A^2Y_{11}whl}{2\omega} + irac{ABY_{11}whl}{2\omega} + R_{l/2}
ight) + iigg(-rac{\pi AY_{11}wh\Delta f_n}{2\omega f_{n(\mathrm{air})}} + irac{\pi BY_{11}wh\Delta f_n}{2\omega f_{n(\mathrm{air})}} + X_{l/2}igg)$$

为了简化表达式, 我们将 i 乘进虚部项中, 然后重新组合实部和虚部:

$$D = \left(-rac{A^2Y_{11}whl}{2\omega} + R_{l/2}
ight) + i \left(rac{ABY_{11}whl}{2\omega} - rac{\pi AY_{11}wh\Delta f_n}{2\omega f_{n( ext{air})}} + X_{l/2}
ight) - rac{\pi BY_{11}wh\Delta f_n}{2\omega f_{n( ext{air})}}$$

#### 重新组合实部和虚部

将分母 D 的实部和虚部重新组合:

实部  $a_D$ :

$$a_D = -rac{A^2Y_{11}whl}{2\omega} + R_{l/2} - rac{\pi BY_{11}wh\Delta f_n}{2\omega f_{n(\mathrm{air})}}$$

虚部  $b_D$ :

$$b_D = rac{ABY_{11}whl}{2\omega} - rac{\pi AY_{11}wh\Delta f_n}{2\omega f_{n( ext{air})}} + X_{l/2}$$

### 最小化分母的模量

我们需要最小化分母 D 的模量:

$$|D| = \sqrt{a_D^2 + b_D^2}$$

## 对 $a_D$ 和 $b_D$ 的导数求极值

为了找到极值, 我们对 |D| 求导数:

$$rac{\partial |D|}{\partial \Delta f_n} = rac{a_D rac{\partial a_D}{\partial \Delta f_n} + b_D rac{\partial b_D}{\partial \Delta f_n}}{\sqrt{a_D^2 + b_D^2}}$$

设导数为零, 我们得到:

$$a_{D}rac{\partial a_{D}}{\partial \Delta f_{n}}+b_{D}rac{\partial b_{D}}{\partial \Delta f_{n}}=0$$

# 计算导数

我们计算  $a_D$  和  $b_D$  对  $\Delta f_n$  的导数:

$$egin{aligned} rac{\partial a_D}{\partial \Delta f_n} &= -rac{\pi B Y_{11} w h}{2 \omega f_{n ext{(air)}}} \ rac{\partial b_D}{\partial \Delta f_n} &= -rac{\pi A Y_{11} w h}{2 \omega f_{n ext{(air)}}} \end{aligned}$$

代入得到:

$$a_Digg(-rac{\pi B Y_{11} w h}{2\omega f_{n(\mathrm{air})}}igg) + b_Digg(-rac{\pi A Y_{11} w h}{2\omega f_{n(\mathrm{air})}}igg) = 0$$

简化后得到:

$$-rac{\pi Y_{11}wh}{2\omega f_{n(\mathrm{air})}}(a_DB+b_DA)=0$$

### 解决方程

要使上式为零, 我们有两种情况:

1.  $\frac{\pi Y_{11}wh}{2\omega f_{n(\text{air})}} \neq 0$  总是成立,所以必须有:

$$a_D B + b_D A = 0$$

代入  $a_D$  和  $b_D$  的表达式:

$$igg(-rac{A^2Y_{11}whl}{2\omega}+R_{l/2}-rac{\pi BY_{11}wh\Delta f_n}{2\omega f_{n( ext{air})}}igg)B+igg(rac{ABY_{11}whl}{2\omega}-rac{\pi AY_{11}wh\Delta f_n}{2\omega f_{n( ext{air})}}+X_{l/2}igg)A=0$$

### 解这个方程

将上式展开:

$$-\frac{A^2BY_{11}whl}{2\omega} + BR_{l/2} - \frac{\pi B^2Y_{11}wh\Delta f_n}{2\omega f_{n(\text{air})}} + \frac{A^2BY_{11}whl}{2\omega} - \frac{\pi A^2Y_{11}wh\Delta f_n}{2\omega f_{n(\text{air})}} + AX_{l/2} = 0$$

简化得到:

$$egin{split} BR_{l/2} - rac{\pi B^2 Y_{11} w h \Delta f_n}{2 \omega f_{n( ext{air})}} - rac{\pi A^2 Y_{11} w h \Delta f_n}{2 \omega f_{n( ext{air})}} + A X_{l/2} = 0 \ BR_{l/2} + A X_{l/2} - rac{\pi Y_{11} w h \Delta f_n}{2 \omega f_{n( ext{air})}} (B^2 + A^2) = 0 \ BR_{l/2} + A X_{l/2} = rac{\pi Y_{11} w h \Delta f_n}{2 \omega f_{n( ext{air})}} (B^2 + A^2) \ \Delta f_n = rac{2 \omega f_{n( ext{air})} (BR_{l/2} + A X_{l/2})}{\pi Y_{11} w h (B^2 + A^2)} \end{split}$$