$$Y = i\omega C_0 + 2w^2 Y_{11}^2 d_{31}^2 \left\{ \frac{1}{D} \right\}$$

其中分母 $D$ 是复数:

$$D = \left( Z_0 \frac{Al}{2} + R_{l/2} \right) + i \left( \frac{\pi Z_0 \Delta f_n}{2 f_{n(\mathrm{air})}} + X_{l/2} \right)$$

假设 $Z_0$ 是复数:

$$Z_0 = -\frac{AY_{11}wh}{\omega} + i\frac{BY_{11}wh}{\omega}$$

代入 $D$:

$$D = \left( \left( -\frac{AY_{11}wh}{\omega} + i\frac{BY_{11}wh}{\omega} \right) \frac{Al}{2} + R_{l/2} \right) + i \left( \frac{\pi \left( -\frac{AY_{11}wh}{\omega} + i\frac{BY_{11}wh}{\omega} \right) \Delta f_n}{2 f_{n(\mathrm{air})}} + X_{l/2} \right)$$

展开并分离实部和虚部:

$$D = \left( -\frac{A^2 Y_{11} whl}{2\omega} + R_{l/2} - \frac{\pi B Y_{11} wh \Delta f_n}{2\omega f_{n(\mathrm{air})}} \right) + i \left( \frac{ABY_{11}whl}{2\omega} - \frac{\pi A Y_{11} wh \Delta f_n}{2\omega f_{n(\mathrm{air})}} + X_{l/2} \right)$$

我们需要最小化分母 $D$ 的模量:

$$|D| = \sqrt{a_D^2 + b_D^2}$$

其中:

$$a_D = -\frac{A^2 Y_{11} whl}{2\omega} + R_{l/2} - \frac{\pi B Y_{11} wh \Delta f_n}{2\omega f_{n(\mathrm{air})}}$$

$$b_D = \frac{ABY_{11}whl}{2\omega} - \frac{\pi A Y_{11} wh \Delta f_n}{2\omega f_{n(\mathrm{air})}} + X_{l/2}$$

为了最小化 $|D|$，我们需要使 $b_D = 0$。这是因为，当 $b_D \neq 0$ 时， $|D|$ 将由于 $b_D$ 的存在而增大。

将 $b_D$ 设为零，我们得到:

$$\frac{ABY_{11}whl}{2\omega} - \frac{\pi A Y_{11} wh \Delta f_n}{2\omega f_{n(\mathrm{air})}} + X_{l/2} = 0$$

解这个方程得到:

$$\Delta f_n = \frac{\frac{ABY_{11}whl}{2\omega} + X_{l/2}}{\frac{\pi A Y_{11} wh}{2\omega f_{n(\mathrm{air})}}}$$

$$\Delta f_n = \frac{ABY_{11}whl f_{n(\mathrm{air})}}{\pi A Y_{11} wh} + \frac{2\omega f_{n(\mathrm{air})} X_{l/2}}{\pi A Y_{11} wh}$$

$$\Delta f_n = \frac{Bl f_{n(\mathrm{air})}}{\pi} + \frac{2\omega f_{n(\mathrm{air})} X_{l/2}}{\pi A Y_{11} wh}$$