$$Y = i\omega C_0 + 2 w^2 Y_{11}^2 d_{31}^2 iggl\{ rac{1}{D} iggr\}$$

其中分母 D 是复数:

$$D = \left(Z_0 rac{Al}{2} + R_{l/2}
ight) + i igg(rac{\pi Z_0 \Delta f_n}{2 f_{n(\mathrm{air})}} + X_{l/2}igg)$$

假设 Z_0 是复数:

$$Z_0 = -rac{AY_{11}wh}{\omega} + irac{BY_{11}wh}{\omega}$$

代入 D:

$$D = \left(\left(-rac{AY_{11}wh}{\omega} + irac{BY_{11}wh}{\omega}
ight)rac{Al}{2} + R_{l/2}
ight) + i\left(rac{\pi\left(-rac{AY_{11}wh}{\omega} + irac{BY_{11}wh}{\omega}
ight)\Delta f_n}{2f_{n(ext{air})}} + X_{l/2}
ight)$$

展开并分离实部和虚部:

$$D = \left(-rac{A^2Y_{11}whl}{2\omega} + R_{l/2} - rac{\pi BY_{11}wh\Delta f_n}{2\omega f_{n(ext{air})}}
ight) + i \left(rac{ABY_{11}whl}{2\omega} - rac{\pi AY_{11}wh\Delta f_n}{2\omega f_{n(ext{air})}} + X_{l/2}
ight)$$

我们需要最小化分母 D 的模量:

$$|D|=\sqrt{a_D^2+b_D^2}$$

其中:

$$a_D = -rac{A^2Y_{11}whl}{2\omega} + R_{l/2} - rac{\pi BY_{11}wh\Delta f_n}{2\omega f_{n(\mathrm{air})}}$$

$$b_D = rac{ABY_{11}whl}{2\omega} - rac{\pi AY_{11}wh\Delta f_n}{2\omega f_{n(ext{air})}} + X_{l/2}$$

为了最小化 |D|,我们需要使 $b_D=0$ 。这是因为,当 $b_D\neq 0$ 时, |D| 将由于 b_D 的存在而增大。

将 b_D 设为零, 我们得到:

$$rac{ABY_{11}whl}{2\omega}-rac{\pi AY_{11}wh\Delta f_n}{2\omega f_{n(\mathrm{air})}}+X_{l/2}=0$$

解这个方程得到:

$$\Delta f_n = rac{rac{ABY_{11}whl}{2\omega} + X_{l/2}}{rac{\pi AY_{11}wh}{2\omega f_{n(ext{air})}}} \ \Delta f_n = rac{ABY_{11}whlf_{n(ext{air})}}{\pi AY_{11}wh} + rac{2\omega f_{n(ext{air})}X_{l/2}}{\pi AY_{11}wh}$$

$$\Delta f_n = rac{Blf_{n(\mathrm{air})}}{\pi} + rac{2\omega f_{n(\mathrm{air})} X_{l/2}}{\pi A Y_{11} w h}$$