CS:4420 Artificial Intelligence
Spring 2018

Homework 4

Due: Friday, April 6 by 11:59pm

This is a written assignment to be done individually. Write your solutions in a text editor or word processor and submit on ICON a printout of the file in PDF format. Make sure you write your name at the beginning of the file.

1 Domain Modeling in First-Order Logic

1. Do Problem 8.10 from the textbook.

2. For this problem and the next you should use only the relation and function symbols in Figure 1. *Do not introduce your own symbols.*

   (a) Translate each of the following FOL sentences in *good, natural* English (they should have no x's or y's). Note that, for readability, square brackets are also used as parentheses.

   i. $\forall x\ [Person(x) \Rightarrow \exists y\ (Person(y) \wedge Needs(x, y))]$
   ii. $\forall x\ [Loves(Mary, x) \Rightarrow Loves(John, x)]$
   iii. $\forall x\ [Person(x) \Rightarrow \exists y\ (Has(x, y) \wedge Heart(y))]$
   iv. $\forall x\ [Person(x) \Rightarrow \exists y\ \exists z\ (Parent(x, y) \wedge Parent(x, z) \wedge \neg(y = z))]$
   v. $\forall x\ \forall y\ \forall z\ \forall u\ [(Person(x) \wedge Parent(x, y) \wedge Parent(x, z) \wedge Parent(x, u)) \Rightarrow (u = z \vee u = y)]$
   vi. $\forall s\ [(Student(s) \wedge Likes(s, AI)) \Rightarrow Likes(s, CS4420)]$
   vii. $\neg[\exists x\ \forall y\ (Person(y) \Rightarrow Likes(y, x))]$
   viii. $\exists x\ \exists y\ [Bug(x) \wedge Program(y) \wedge Wrote(John, y) \wedge In(x, y)]$
   ix. $\exists x\ [Bug(x) \wedge \forall y\ (Program(y) \wedge Wrote(John, y) \Rightarrow In(x, y))]$
   x. $\neg\exists y\ Needs(Mary, y)$
   xi. $\exists x\ [Student(x) \wedge \neg Awake(x) \wedge \forall y\ ((Student(y) \wedge x \neq y) \Rightarrow Awake(y))]$
   xii. $\forall x\ \forall y\ [Student(x) \wedge American(x) \wedge Wine(y) \Rightarrow \neg Drink(x, y)]$
   xiii. $\forall x\ Parent(x, mother(x))$
   xiv. $\neg\forall x\ (Person(x) \Rightarrow Knows(x, mother(x)))$

Predicate	Intended Meaning
American(x)	x is American
Bug(x)	x is a (software) bug
Class(x)	x is a class
Drinks(x,y)	x drinks y
From(x,y)	x is from y
Good(x)	x is good
Grandparent(x,y)	y is a grandparent of x
Has(x,y)	x has y
Heart(x)	x is a heart
In(x,y)	x is in y
Knows(x,y)	x knows y
Likes(x,y)	x likes y
Loves(x,y)	x loves y
Museum(x)	x is a museum

Predicate	Intended Meaning
Needs(x,y)	x needs y
Parent(x,y)	y is a biological parent of x
Person(x)	x is a person
Program(x)	x is a program
Student(x)	x is a student
Teaches(x,y)	x teaches y
Tease(x,y,z)	x teases y at time z
Time(x)	x is a time
Visited(x,y)	x visited y
Wants(x,y)	x wants y
Wine(x)	x is a kind of wine
Wrote(x,y)	x wrote y

Function	Intended Meaning
mother(x)	the biological mother of x

Figure 1:

(b) Translate each of the following English statements to FOL. Use only logical symbols from the set $\{\forall, \exists, \wedge, \vee, \neg, \Rightarrow, \Leftrightarrow, =\}$. You can use the constant symbols *Fred*, *Jane*, *France*, *Louvre*, with the expected meaning.
Make sure you use parentheses to avoid ambiguous readings of your sentences.

i. Students love museums.
ii. Not every student likes a good museum.
iii. Some Americans like wines from France.
iv. Americans who love wines from France dislike American wines.
v. Students do not like the classes taught by Fred.
vi. Jane visited all the museums in France except the Louvre.
vii. Fred knows any museum visited by Jane.
viii. Everyone knows someone from France.
ix. Fred drinks all kinds of wine.
x. Fred drinks only wine.
xi. Fred knows someone who always writes buggy programs.
xii. Fred teases everybody all of the time.
xiii. Everybody has exactly two parents.
xiv. Not everyone knows someone with a French mother.
xv. Those who know Jane's mother love her.
xvi. Fred's maternal grandmother teaches a class that Jane loves.
xvii. You cannot dislike people you love.
xviii. Jane only loves people with a good heart.
xix. No one has something that everybody wants.

2 Validity and Entailment in FOL with equality

For the problems below it is helpful to recall that all interpretations in FOL are assumed to have a non-empty domain.

1. Let Γ be the knowledge base $\{Spouse(Jim, Laura), \neg(Jim = George)\}$ and let φ be the sentence $\neg Spouse(George, Laura)$.

   (a) Argue informally but convincingly that Γ does not entail α.

   (b) Provide enough sentences that when added to Γ ensure that $\Gamma \models \alpha$.

2. For each FOL sentence below say whether it is valid or not *and briefly explain why.* Specifically, for each valid sentence argue informally but precisely why every possible interpretation makes the sentence true; for each invalid sentence describe an interpretation that makes the sentence false.

   (a) $\forall x \, \forall y \, (x = y \Rightarrow y = x)$

   (b) $\forall x \, \forall y \, [(x < y) \Rightarrow \neg(x = y)]$

   (c) $\exists x \, \exists y \, x = y$

   (d) $\forall x \, \exists y \, \neg(x = y)$

   (e) $(\forall x \, P(x)) \Rightarrow \exists y \, P(y)$

   (f) $(\exists x \, P(x)) \Rightarrow \forall y \, P(y)$

   (g) $(\forall x \, P(x)) \Rightarrow P(f(g(a, b)))$

   (h) $(P(a) \wedge \forall x \, \forall x \, x = y) \Rightarrow \forall x \, P(x)$

3 Optional, Extra credit

Do Problem 8.21 from the textbook.