CMPSC 40, Section 9

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To Jiyu: remember to record the section.

1 Schedule

Practice Problems. - 20 min.
Any questions. - remaining time.

Reminder: log onto the ESCI Online system to take the ESCI survey for us! Your feedback will be useful and appreciated.

2 Master Theorem

Theorem 2.1 *If $T(n) = aT(\lceil n/b \rceil) + O(n^d)$ for some $a > 0, b > 1$ and $g \geq 0$, then*

$$T(n) = \begin{cases} O(n^d) & \text{if } d > \log_b a \\ O(n^d \log n) & \text{if } d = \log_b a \\ O(n^{\log_b a}) & \text{if } d < \log_b a \end{cases} \tag{1}$$

Determine the time complexity of the following recurrence relation:

1. $T(n) = 8T(n/2) + O(n^2)$
2. $T(n) = 2T(n/2) + O(n)$

Solution.

1. $O(n^3)$ (Normal Matrix Multiplication)
2. $O(n \log n)$ (Merge Sort)

3 Relations

Recall *reflexive*, *symmetric/anti-symmetric*, *transitive* relations.
A relation R on a set S is a subset of $S \times S$. Just pairs of the form (a, b) for $a, b \in S$.

reflexive: for any $a \in S$, (a, a) is in this relation.

symmetric: if (a, b) is in R, then (b, a) is also in R

anti-symmetric: if (a, b) is in R, and (b, a) is in R, $a = b$.
e.g. $a \geq b$, $b \geq a$, $a = b$.

$A \subseteq B$

transitive: if (a, b) is in R, (b, c) is in R, then we have (a, c).
e.g. $>$: if $a > b$, $b > c$, then we have $a > c$

Determine whether the following relations are reflexive, symmetric/anti-symmetric, transitive.

1. R is on the set of soccer players. $(a, b) : a$ is taller than b .
2. R is on the set of NHL players. $(a, b) : a$ and b received the same salary in the 2019–20 season.
3. R is on the set of NBA players. $(a, b) : a$ and b were in the same team at the end of the season.
4. R is on the set of time-complexity functions. $(f, g) : f$ is $O(g)$.
5. R is on the set of time-complexity functions. $(f, g) : f$ is $\Theta(g)$.

Recall that an *equivalence* relation is simultaneously reflexive, symmetric, and transitive.

Consider the following sets,

Let X be the set $X := \{\frac{1}{4}, \frac{1}{2}, 1, 2, 4\}$.
Let Y be the set $\{3,\ 6,\ 12\}$.

For $a, b \in Y$, we say a be X-equivalent to b if there exists an element $g \in X$ such that $g \cdot a = b$.

Prove that X-equivalence is an equivalence relation on Y.

Transitive Closure.

- Think of it as a graph.
- For example
- The set of all people is the set of vertices in a direct graph.
- if "a observers b", then the edge (a, b) is in this graph. Which is an edge with direction.
- For a graph G, you have (a, b) and (b, c) is in G. But it's not necessary that (a, c) is in this graph.
- a transitive closure of G, is just the graph G^* which builds on G. If (a, b) and (b, c) is in G, then (a, c) is in G^*