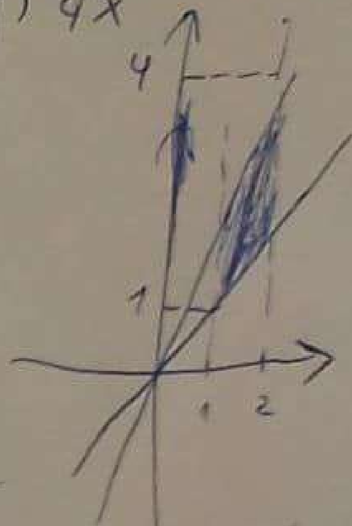
Чегулин М3113

№ 1

$$\int_1^2 dx \int_x^{2x} f(x,y)\,dy = \int_a^b dy \int f(x,y)\,dx$$



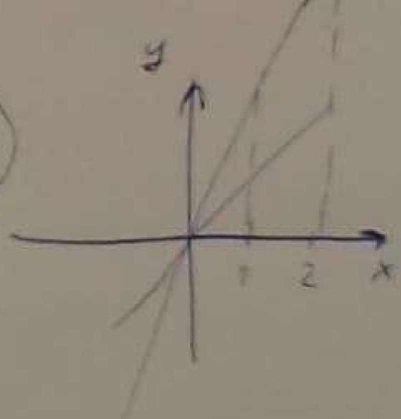
1) $y = x \Rightarrow x = y$

2) $y = 2x \Rightarrow x = 0{,}5y$

~~$x \le y \le 2x \Rightarrow 0{,}5y \le x \le y$~~

$1 \le x \le 2 \Rightarrow 1 \le y \le 4$

$$\int_1^2 dx \int_x^{2x} f(x,y)\,dy = \int_1^4 dy \int_x^{0{,}5x} f(x,y)\,dx$$



№ 4

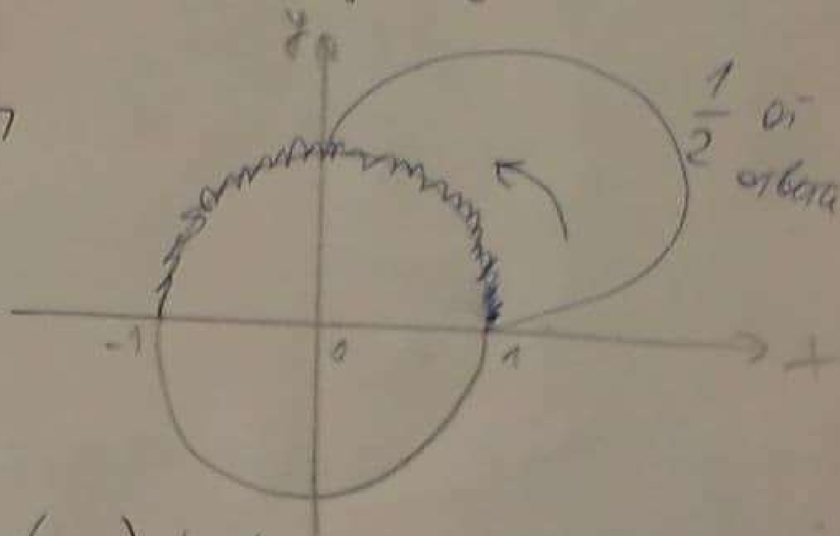
$$\begin{cases} x = \cos t \\ y = \sin t \end{cases} \quad (0 \le t \le \pi)$$

$$\rho = y$$

$$x^2 + y^2 = 1$$

$$x = \pm\sqrt{1-y^2}$$

$$\cancel{y = \sqrt{1-x^2}}$$



$$m = \iint\limits_{L} \rho(x,y)\,dx\,dy =$$

$$\boxed{y = \sqrt{1-x^2}}$$

$$= 2\int_0^1 dx \cdot \sqrt{1-x^2} =$$

$$\left| \begin{aligned} t &= \sqrt{1-x^2} \\ dt &= \frac{1}{-2\sqrt{1-x^2}}\,dx \end{aligned} \right.$$

$$= 2\left(-\frac{2}{3}\left(\sqrt{1-x^2}\right)^3\right)\Big|_0^1 =$$

$$\int \sqrt{1-x^2}\,dx = \int -2\,t^2\,dt = \frac{-2}{3}t^3$$

$= \quad \frac{4}{3}$

Ответ: $\frac{4}{3}$

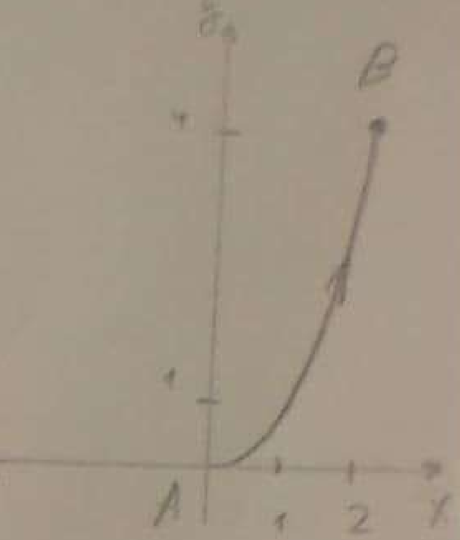
Ответ: $\frac{4}{3}$

№5

$$\int_L xy^2\,dx + (x+y)\,dy$$

L: $y = x^2$

из точки (0;0) в (2;4).



$$\int_{AB} xy^2\,dx + (x+y)\,dy = \int_0^2 \left(x(x^2)^2 + (x+x^2)\cdot y'_x\right)dx$$

$$= \int_0^2 \left(x^5 + 2x^2 + 2x^3\right)dx = \left(\frac{1}{6}x^6 + \frac{1}{2}x^4 + \frac{2}{3}x^3\right)\Big|_0^2$$

$$= \frac{1}{6}\cdot 2^6 + \frac{1}{2}\cdot 2^4 + \frac{2}{3}\cdot 2^3 = \frac{64+48+32}{6} = \frac{144}{6} = 24$$

Ответ: 24

№ 2

$$\iiint\limits_V \frac{y^2 z \, dx\, dy\, dz}{\sqrt{(x^2+y^2)^3}} = \int_0^{\frac{1}{2}} dx \int_0^{\sqrt{3}\cdot x} dy \int_0^{3(x^2+y^2)} dz \cdot \frac{y^2 z}{\sqrt{(x^2+y^2)^3}}$$

$V: \; y \geq 0 \quad y \leq \sqrt{3}\cdot x$

$$\left.\begin{aligned} z &= 3(x^2+y^2) \\ z &= 3 \end{aligned}\right\} \Rightarrow 3(x^2+y^2) = 3$$

$$x^2+y^2 = 1$$

$$x^2+3x^2 = 1$$

$$\Downarrow$$

$$x = \frac{1}{2}$$

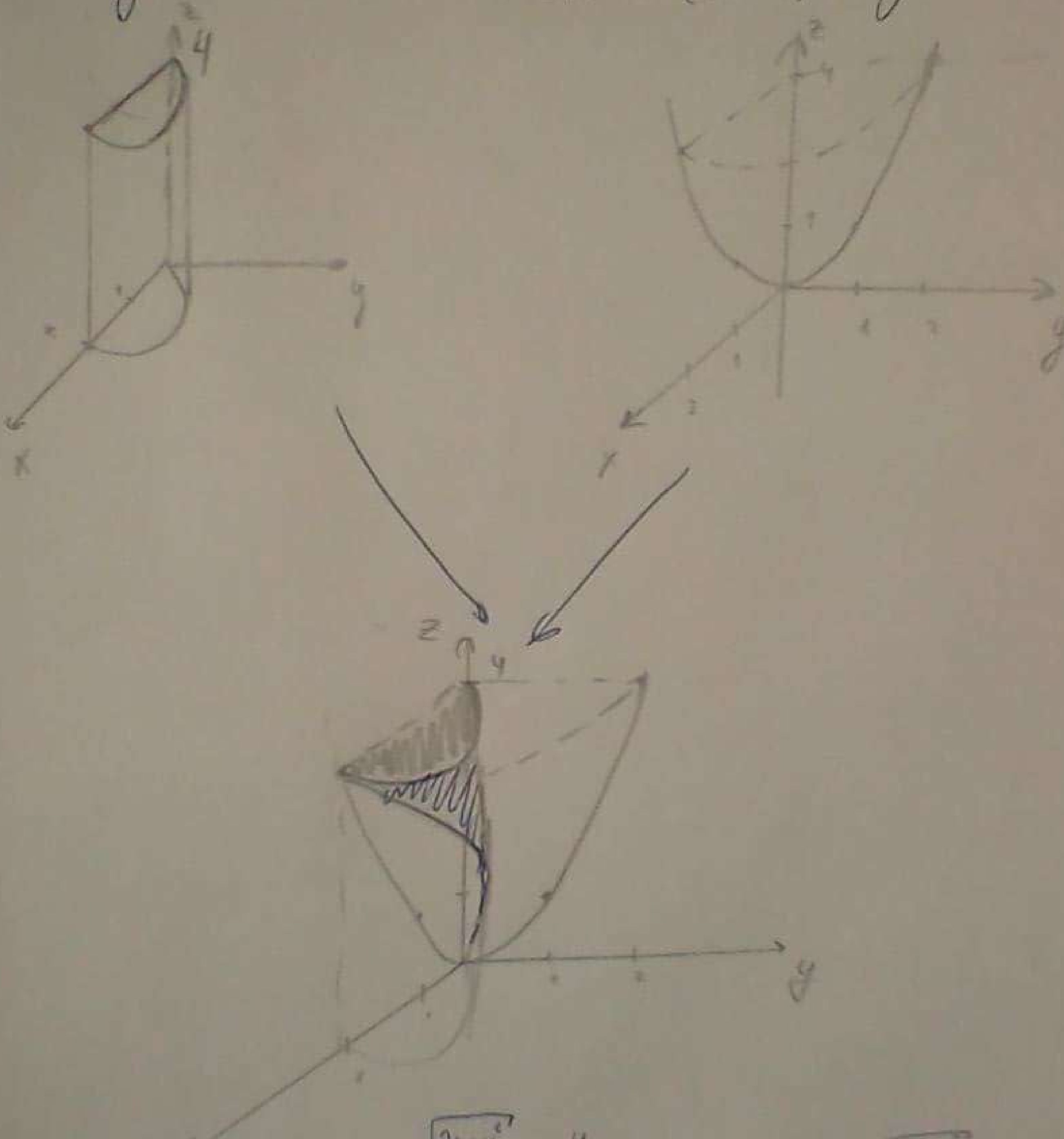
$$= \int_0^{0,5} dx \int_0^{x\sqrt{3}} dy \cdot \left(\left(\frac{y^2}{(\sqrt{x^2+y^2})^3} \right) \cdot \frac{1}{2} z^2 \right) \Bigg|_0^{3(x^2+y^2)} =$$

$$= \int_0^{0,5} dx \int_0^{x\sqrt{3}} dy \cdot \frac{3}{2} (x^2+y^2)^2 \cdot (x^2+y^2)^{-\frac{3}{2}} \cdot y^2 =$$

$$= \int_0^{0,5} dx \int_0^{x\sqrt{3}} dy \cdot \frac{3}{2} \sqrt{x^2+y^2} \cdot y^2$$

N3

$$x^2+y^2=2x \Rightarrow \cancel{x-1} \quad (x-1)^2+y^2=1$$



$$\int_0^2 dx \int_0^{\sqrt{2x-x^2}} dy \int_0^4 dz = \int_0^2 dx \int_0^{\sqrt{2x-x^2}} 4z\, dy =$$

$$\int_0^2 dx \int_0^{\sqrt{2x-x^2}} dy \int_0^{(x^2+y^2)} dz = \int_0^2 dx \int_0^{\sqrt{2x-x^2}} dy \cdot (x^2+y^2) =$$

$$= \int_0^2 dx \left(x^2 y + y^3\right)\Big|_0^{\sqrt{2x-x^2}} = \int_0^2 dx \cdot x^2 \cdot \sqrt{2x-x^2}$$

$$= \left(\frac{\sqrt{-(x-2)x}\left(6x^3-2x^2-5x-15\right)+15\arcsin(x-1)}{24}\right)\Bigg|_0^2 =$$

$$= \frac{5\pi}{8}$$