① 12 вариант.

$X \sim N(a, d)$; $n = 200$; $S_n^{(2)} = 40$

$$I = \left(\hat{\theta}_n - \frac{t_\gamma}{\sqrt{n \cdot I(\hat{\theta}_n)}} , \hat{\theta}_n + \frac{t_\gamma}{\sqrt{n \cdot I(\hat{\theta}_n)}} \right)$$

$I(a) = 1/d$, $I(d) = 1/2d^2$

$\gamma = 0.9 \Rightarrow t_\gamma = 1.65$

$\hat{\theta}_n = S_n^{(2)} = 40.$

$I(\hat{\theta}_n) = \frac{1}{2 \cdot 40^2} = 1/3200$

$$I = \left(40 - \frac{1.65}{\sqrt{200 \cdot 1/3200}} , 40 + \frac{1.65}{\sqrt{200 \cdot 1/3200}} \right)$$

$I = (33.4, 46.6)$

$H_0 : d = d_0 = 50$

$H_1 : d < d_0$

$\alpha = 0.05$, $\gamma = 1 - 0.05 = 0.95 \Rightarrow c_\gamma = 1.65$

$\theta_0 = d_0$

$I(\theta_0) = 1/5000$

$Z_n = \sqrt{n I(\theta_0)} \, (\hat{\theta}_n - \theta_0) = -2$

$$\psi_{n,\alpha}(x_1, \dots, x_n) = \begin{cases} 1, & Z_n \le -c_\gamma \\ 0, & Z_n > c_\gamma \end{cases}$$

-2 & -1.65

$-2 \le -1.65 \Rightarrow$ принимаем H_1

Ответ: $(33.4, 46.6)$, H_1