直角坐标旋转和uvw坐标

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使用右手螺旋定则：

右手拇指指向被绕轴的正方向，剩下四指向手心旋转的方向为旋转角的正方向。

转动矩阵：

$$\text{新坐标} = \text{转动矩阵}R \cdot \text{旧坐标} \tag{1}$$

P点在直角坐标系xyz中的坐标为(x, y, z)。

P点不动，直角坐标系xyz绕x轴转动α角，变成新坐标系$xy'z'$，P点在新坐标系中的坐标为$(xy'z')$，有转动矩阵：

$$\begin{bmatrix} x \\ y' \\ z' \end{bmatrix} = R_x^c \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \Rightarrow \qquad R_x^c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \tag{2}$$

P点不动，直角坐标系xyz绕y轴转动α角，变成新坐标系$x'yz'$，P点在新坐标系中的坐标为$(x'yz')$，有转动矩阵：

$$R_y^c = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \tag{3}$$

P点不动，直角坐标系xyz绕z轴转动α角，变成新坐标系$x'y'z$，P点在新坐标系中的坐标为$(x'y'z)$，有转动矩阵：

$$R_z^c = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

如果直角坐标系xyz不动，P点分别绕x轴、y轴、z轴转α角得到新点P'，求P'在xyz的坐标，可认为是P点不动，反方向旋转坐标系$-\alpha$角。P'的旋转矩阵为

$$R_x^p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \qquad R_y^p = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \qquad R_z^p = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

4 Geometric Relationships and Polarimetry

In this chapter we start to examine some of the practical aspects of interferometry. These include baselines, antenna mounts and beamshapes, and the response to polarized radiation, all of which involve geometric considerations and coordinate systems. The discussion is concentrated on earth-based arrays with tracking antennas, which illustrate the principles involved, although the same principles apply to other systems such as those that include one or more antennas in earth orbit.

4.1 ANTENNA SPACING COORDINATES AND (u, v) LOCI

Various coordinate systems are used to specify the relative positions of the antennas in an array, and of these one of the more convenient for terrestrial arrays is shown in Fig. 4.1. A right-handed Cartesian coordinate system is used where X and Y are measured in a plane parallel to the earth's equator, X in the meridian plane (defined as the plane through the poles of the earth and the reference point in the array), Y is measured toward the east, and Z toward the north pole. In terms of hour angle H and declination δ, the coordinates (X, Y, Z) are measured toward $(H = 0, \delta = 0)$, $(H = -6^h, \delta = 0)$, and $(\delta = 90°)$, respectively. If $(X_\lambda, Y_\lambda, Z_\lambda)$ are the components of $\mathbf{D}_\lambda$ in the (X, Y, Z) system, the components (u, v, w) are given by

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \sin H & \cos H & 0 \\ -\sin\delta\cos H & \sin\delta\sin H & \cos\delta \\ \cos\delta\cos H & -\cos\delta\sin H & \sin\delta \end{bmatrix} \begin{bmatrix} X_\lambda \\ Y_\lambda \\ Z_\lambda \end{bmatrix}. \tag{4.1}$$

Here (H, δ) are usually the hour angle and declination of the phase reference position. (In VLBI observations it is customary to set the X axis in the Greenwich meridian, in which case H is measured with respect to that meridian rather than a local one.) The elements of the transformation matrix given above are the direction cosines of the (u, v, w) axes with respect to the (X, Y, Z) axes and can easily be derived from the relationships in Fig. 4.2. Another method of specifying the baseline vector is in terms of its length, D, and the hour angle and declination, (h, d), of the intersection of the baseline direction with the northern celestial

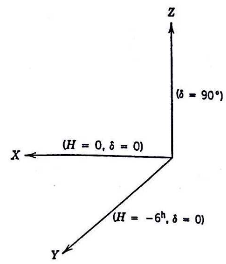

Figure 4.1 The (X, Y, Z) coordinate system for specification of relative positions of antennas. Directions of the axes specified are in terms of hour angle H and declination δ.

hemisphere. The coordinates in the (X, Y, Z) system are then given by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = D \begin{bmatrix} \cos d \cos h \\ -\cos d \sin h \\ \sin d \end{bmatrix}. \tag{4.2}$$

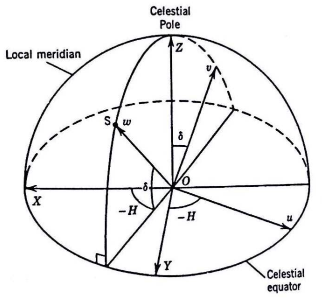

Figure 4.2 Relationships between the (X, Y, Z) and (u, v, w) coordinate systems. The (u, v, w) system is defined for observation in the direction of the point S, which has hour angle and declination H and δ. As shown, S is in the eastern half of the hemisphere and H is therefore negative. The direction cosines in the transformation matrix in Eq. (4.1) follow from the relationships in this diagram. The relationship in Eq. (4.2) can also be derived if we let S represent the direction of the baseline and put the baseline coordinates (h, d) for (H, δ).