Visibility相位校准公式

Qizhi Huang

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1 坐标系

天球的XYZ坐标系：

+Z指向北天极，+X指向校准源所有赤经

地面的xyz坐标系：

+z指向天顶，+x指向当地的东方，+y指向当地的北方

2 基线坐标变换

地面上的基线坐标为(x, y, z)

xyz坐标系

（1）绕x轴转 $-(90-\mathrm{lat})$ 度，此时+z指向+Z，+x指向+Y，+y指向-X

（2）再绕z轴转 -90 度，此时+z指向+Z，+x指向+X，+y指向+Y

$$
\begin{aligned}
\begin{bmatrix} X_b \\ Y_b \\ Z_b \end{bmatrix} &= \begin{bmatrix} \cos(-90) & \sin(-90) & 0 \\ -\sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-90+\mathrm{lat}) & \sin(-90+\mathrm{lat}) \\ 0 & -\sin(-90+\mathrm{lat}) & \cos(-90+\mathrm{lat}) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\
&= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin(\mathrm{lat}) & -\cos(\mathrm{lat}) \\ 0 & \cos(\mathrm{lat}) & \sin(\mathrm{lat}) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\
&= \begin{bmatrix} -y\sin(\mathrm{lat}) + z\cos(\mathrm{lat}) \\ x \\ y\cos(\mathrm{lat}) + z\sin(\mathrm{lat}) \end{bmatrix} \qquad (1)
\end{aligned}
$$

3 被观测纬度圈在天球上的坐标

纬圈坐标 $(90-\mathrm{Dec}, \varphi)$，其中$\varphi \Rightarrow \mathrm{RA}$

由于+X指向校准源所有的赤经，于是校准源所在的 $\varphi_s = 0$

$$\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} = \begin{bmatrix} \sin(90-\mathrm{Dec})\cos(\varphi) \\ \sin(90-\mathrm{Dec})\sin(\varphi) \\ \cos(90-\mathrm{Dec}) \end{bmatrix} = \begin{bmatrix} \cos(\mathrm{Dec})\cos(\varphi) \\ \cos(\mathrm{Dec})\sin(\varphi) \\ \sin(\mathrm{Dec}) \end{bmatrix} \tag{2}$$

4 基线造成的相位延时

$$\begin{aligned} \mathrm{Phase} &= \begin{bmatrix} X_b \\ Y_b \\ Z_b \end{bmatrix} \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} \\ &= x\cos(\mathrm{Dec})\sin(\varphi) \\ &\quad + y\left[\sin(\mathrm{Dec})\cos(\mathrm{lat}) - \cos(\mathrm{Dec})\sin(\mathrm{lat})\cos(\varphi)\right] \\ &\quad + z\left[\sin(\mathrm{Dec})\sin(\mathrm{lat}) + \cos(\mathrm{Dec})\cos(\mathrm{lat})\cos(\varphi)\right] \end{aligned} \tag{3}$$

当 $z=0$ 时:

$$\mathrm{Phase} = x\cos(\mathrm{Dec})\sin(\varphi) + y\left[\sin(\mathrm{Dec})\cos(\mathrm{lat}) - \cos(\mathrm{Dec})\sin(\mathrm{lat})\cos(\varphi)\right] \tag{4}$$

当 $z=0$，φ 是小角（$\sin(\varphi) \approx \varphi$，$\cos(\varphi) \approx 1$）时:

$$\begin{aligned} \mathrm{Phase} &\approx x\cos(\mathrm{Dec})\varphi + y\left[\sin(\mathrm{Dec})\cos(\mathrm{lat}) - \cos(\mathrm{Dec})\sin(\mathrm{lat})\right] \\ &= x\cos(\mathrm{Dec})\varphi + y\sin(\mathrm{Dec}-\mathrm{lat}) \end{aligned} \tag{5}$$

5 纬圈上的波束（beam）

一维高斯波束（θ 从 $-$ 到 0 再到 $+$）：

$$\mathrm{Beam} = \exp\left\{-\frac{\theta^2}{2\sigma^2}\right\} \tag{6}$$

在天球坐标系XYZ中

$$\begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix} = \begin{bmatrix} \sin(90-\mathrm{Dec})\cos(\varphi) \\ \sin(90-\mathrm{Dec})\sin(\varphi) \\ \cos(90-\mathrm{Dec}) \end{bmatrix} = \begin{bmatrix} \cos(\mathrm{Dec})\cos(\varphi) \\ \cos(\mathrm{Dec})\sin(\varphi) \\ \sin(\mathrm{Dec}) \end{bmatrix} \tag{7}$$

将其旋转到被观测纬圈上：

绕+X轴转 $90-\mathrm{Dec}$ 度

$$
\begin{aligned}
\begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(90-\mathrm{Dec}) & \sin(90-\mathrm{Dec}) \\ 0 & -\sin(90-\mathrm{Dec}) & \cos(90-\mathrm{Dec}) \end{bmatrix} \begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin(\mathrm{Dec}) & \cos(\mathrm{Dec}) \\ 0 & -\cos(\mathrm{Dec}) & \sin(\mathrm{Dec}) \end{bmatrix} \begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix} \\
&= \begin{bmatrix} \cos(\mathrm{Dec})\cos(\varphi) \\ \sin(\mathrm{Dec})\cos(\mathrm{Dec})\sin(\varphi) + \sin(\mathrm{Dec})\cos(\mathrm{Dec}) \\ -\cos^2(\mathrm{Dec})\sin(\varphi) + \sin^2(\mathrm{Dec}) \end{bmatrix} \\
&= \begin{bmatrix} \cos(\mathrm{Dec})\cos(\varphi) \\ \sin(\mathrm{Dec})\cos(\mathrm{Dec})\,[1+\sin(\varphi)] \\ 1-\cos^2(\mathrm{Dec})\,[1+\sin(\varphi)] \end{bmatrix} = \begin{bmatrix} \sin(\theta_g)\cos(\varphi_g) \\ \sin(\theta_g)\sin(\varphi_) \\ \cos(\theta_g) \end{bmatrix} \qquad (8)
\end{aligned}
$$

可求得：

$$
\theta_g = \arccos\left\{1-\cos^2(\mathrm{Dec})\,[1+\sin(\varphi)]\right\}, \qquad \theta_g(\mathrm{center}) = \arccos\left\{\sin^2(\mathrm{Dec})\right\} \qquad (9)
$$

于是，在纬圈上的一维高斯波束为：

$$
\begin{aligned}
\mathrm{Beam} &= \exp\left\{-\frac{[\theta_g-\theta_g(\mathrm{center})]^2}{2\sigma^2}\right\} \\
&= \exp\left\{-\frac{\left[\ \arccos\left\{1-\cos^2(\mathrm{Dec})\,[1+\sin(\varphi)]\right\} - \arccos\left\{\sin^2(\mathrm{Dec})\right\}\ \right]^2}{2\sigma^2}\right\} \qquad (10)
\end{aligned}
$$

当 φ 是小角（$\sin(\varphi)\approx\varphi$，$\cos(\varphi)\approx 1$）时：

$$
\mathrm{Beam} = \exp\left\{-\frac{[\ \cos(\mathrm{Dec})\varphi\ ]^2}{2\sigma^2}\right\} \qquad (11)
$$

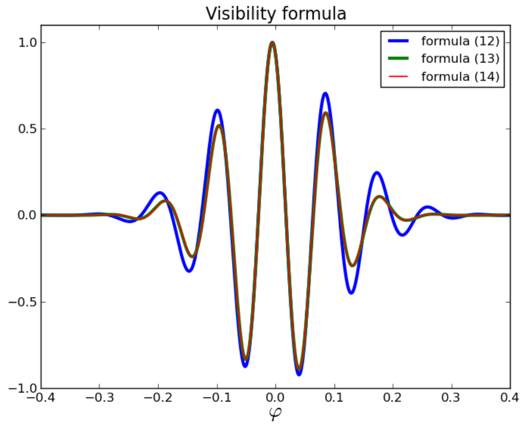
6 纬圈上的一维visibility近似公式

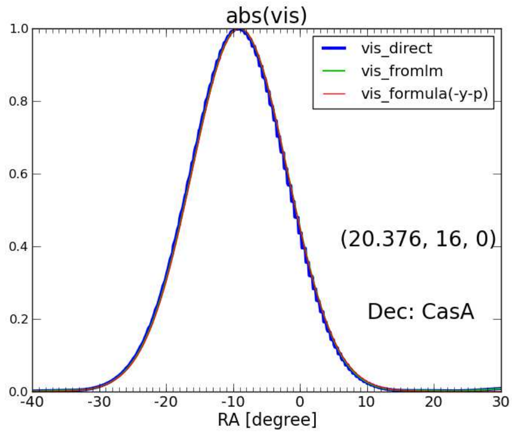
$$
\begin{aligned}
\mathrm{Visibility} &= \mathrm{Beam}\cdot\exp\{i\cdot\mathrm{Phase}\} \\
&= \exp\left\{-\frac{\left[\ \arccos\left\{1-\cos^2(\mathrm{Dec})\,[1+\sin(\varphi)]\right\} - \arccos\left\{\sin^2(\mathrm{Dec})\right\}\ \right]^2}{2\sigma^2}\right\} \\
&\quad \bullet \exp\left\{i\frac{2\pi}{\lambda}\left\{\,x\cos(\mathrm{Dec})\sin(\varphi) + y\,[\sin(\mathrm{Dec})\cos(\mathrm{lat}) - \cos(\mathrm{Dec})\sin(\mathrm{lat})\cos(\varphi)]\,\right\}\right\} \qquad (12) \\
&= \exp\left\{-\frac{[\cos(\mathrm{Dec})\sin(\varphi)\,]^2}{2\sigma^2}\right\}\cdot\exp\left\{i\frac{2\pi}{\lambda}\left[\,x\cos(\mathrm{Dec})\sin(\varphi) + y\sin(\mathrm{Dec}-\mathrm{lat})\,\right]\right\} \qquad (13) \\
&= \exp\left\{-\frac{[\cos(\mathrm{Dec})\varphi]^2}{2\sigma^2}\right\}\cdot\exp\left\{i\frac{2\pi}{\lambda}\left[\,x\cos(\mathrm{Dec})\varphi + y\sin(\mathrm{Dec}-\mathrm{lat})\,\right]\right\} \qquad (14)
\end{aligned}
$$

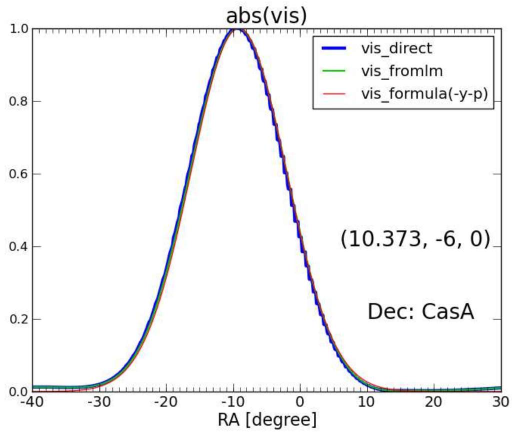
"vis_formula" below is

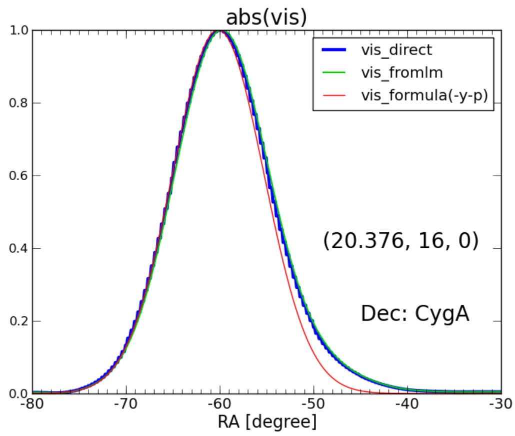
$$\text{vis\_formula} = \exp\left\{-\frac{[\cos(\text{Dec})\varphi]^2}{2\sigma^2}\right\}\cdot\exp\left\{-i\frac{2\pi}{\lambda}\left[x\cos(\text{Dec})\varphi - y\sin(\text{Dec}-\text{lat})\right]\right\} \tag{15}$$

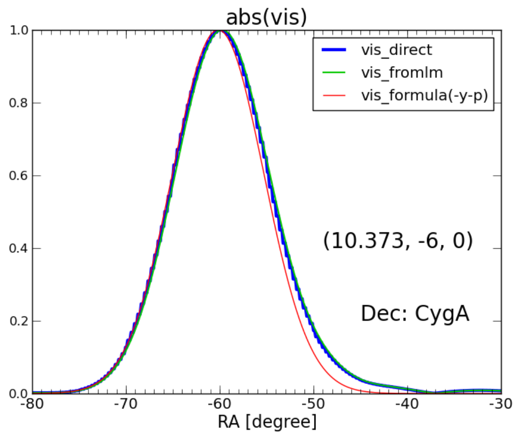
Use equation (15) to calibrate the phase, then can use my map-making program to make the map.

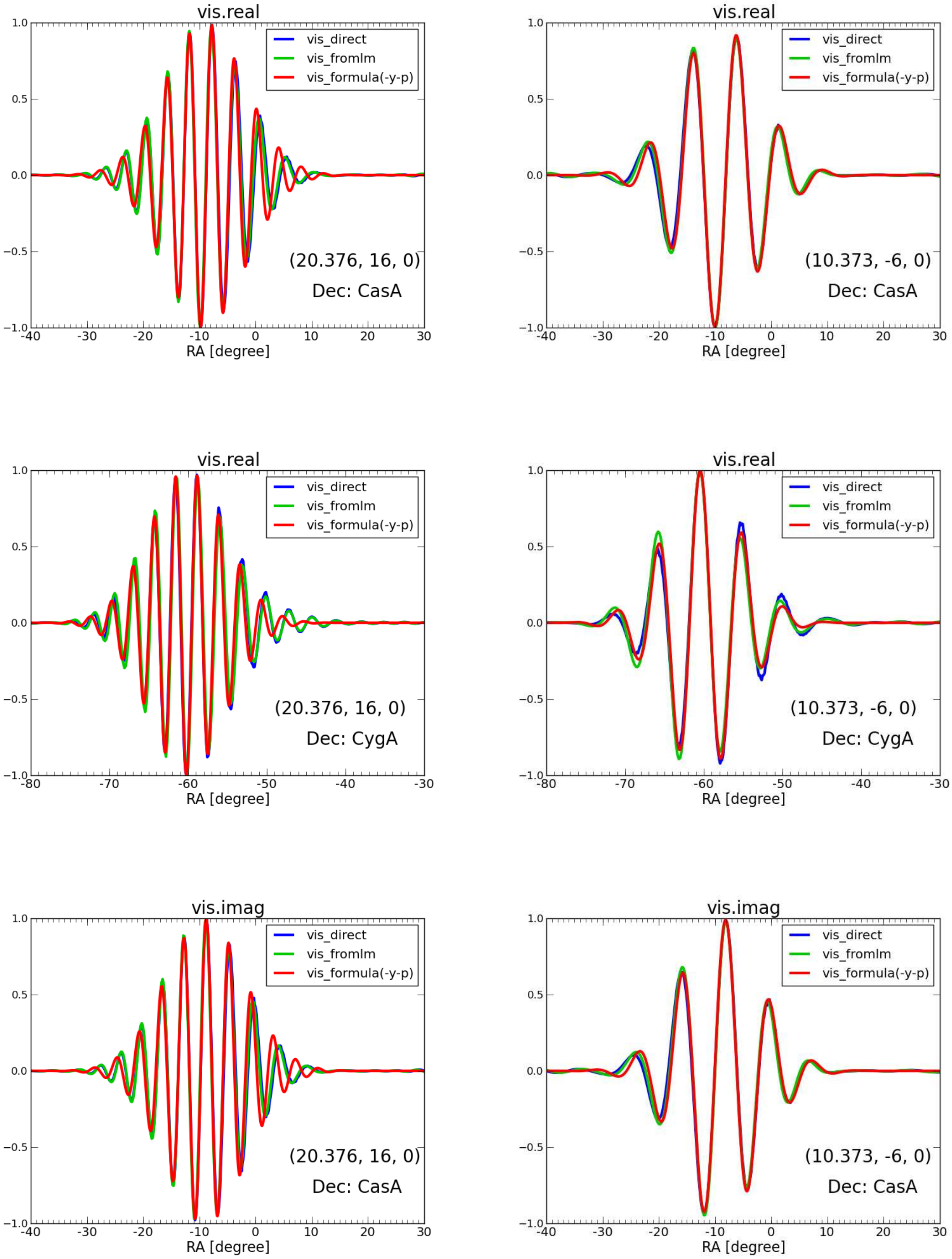








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vis_direct
vis_fromlm
vis_formula(-y-p)
(20.376, 16, 0)
Dec: CasA
RA [degree]
vis.real
vis_direct
vis_fromlm
vis_formula(-y-p)
(10.373, -6, 0)
Dec: CasA
RA [degree]
vis.real
vis_direct
vis_fromlm
vis_formula(-y-p)
(20.376, 16, 0)
Dec: CygA
RA [degree]
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vis_direct
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vis_formula(-y-p)
(10.373, -6, 0)
Dec: CygA
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vis_direct
vis_fromlm
vis_formula(-y-p)
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Dec: CasA
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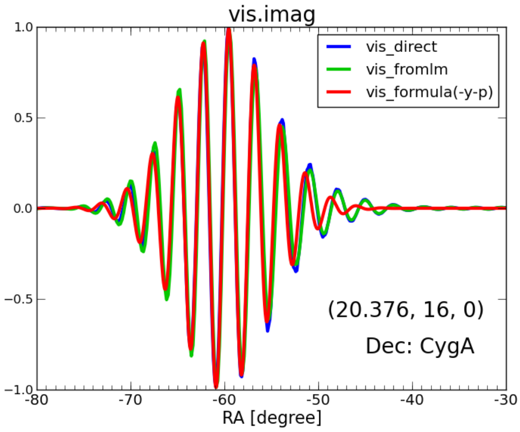
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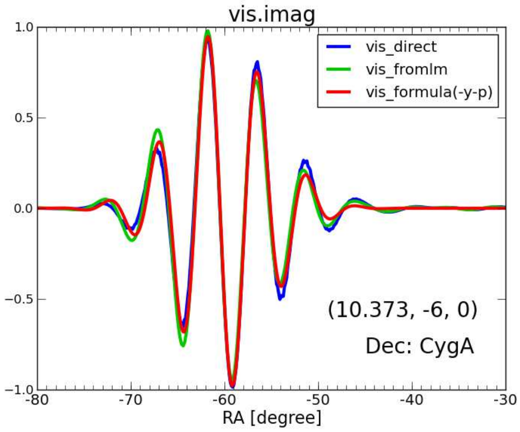
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