高斯模糊/平滑/滤波
Gaussian Blur/Smoothing/Filter

Qizhi Huang

November 26, 2016

用$\mu = 0$的高斯函数

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{x^2}{2\sigma^2}} \tag{1}$$

对数据进行平滑，可以有效地压低数据中的高斯噪声。

原数据：σ_{in}，高斯滤波器：σ_{filter}，平滑数据：σ_{out}

$$\sigma_{\text{out}} \approx \frac{\sigma_{\text{in}}}{\sqrt{\sigma_{\text{filter}} \cdot 2\sqrt{\pi}}} \tag{2}$$

Equation (2) above is right (checked by python program). Figure below is wrong (also wiki: https://en.wikipedia.org/wiki/Gaussian_blur)

2

In Gaussian blur the value of each output pixel is calculated as a weighted sum of all input pixels:

$$\text{out}(x, y) = \sum_{j=-\infty}^{\infty}\sum_{i=-\infty}^{\infty} \frac{1}{2\pi\sigma_G^2} e^{-\frac{i^2+j^2}{2\sigma_G^2}}\, \text{in}(x+i, y+j).$$

We want to calculate the variance $\text{Var}[\text{out}(x, y)] = \sigma_f^2$ based on the variances $\text{Var}[\text{in}(x+i, y+i)] = \sigma_0^2$. We require that $\text{in}(x+i, y+i)$ are independent, which you should mention in your question. Based on the identity (variance of linear combination):

$$Var\left[\sum_i c_i X_i\right] = \sum_i c_i^2 Var[X_i],$$

where c_i are constants and X_i are independent random variables,

$$\sigma_f^2 = \sigma_0^2 \sum_{j=-\infty}^{\infty}\sum_{i=-\infty}^{\infty} \left(\frac{1}{2\pi\sigma_G^2} e^{-\frac{i^2+j^2}{2\sigma_G^2}}\right)^2.$$

For a large σ_G, the squared Gaussian is rather smooth and the sum can be approximated by an integral as:

$$\sigma_f^2 \approx \sigma_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\frac{1}{2\pi\sigma_G^2} e^{-\frac{i^2+j^2}{2\sigma_G^2}}\right)^2 \mathrm{d}i\,\mathrm{d}j = \frac{\sigma_0^2}{4\pi\sigma_G^2}$$

$$\implies \sigma_f \approx \frac{\sigma_0}{2\sigma_G\sqrt{\pi}},$$

which is the same result as in Wikipedia. You can calculate the integral using Wolfram Alpha.

Note that the noise needs not be Gaussian for the above to apply.