Appendix: Exploring Climate Change Through the Lens of Records Theory

A Record Theory Models

Table A.1: Some properties of the most common models in Record Theory

Model	**Mathematical Expression**	**Description**	**Probability to observe a record at time t**	**Expected number of records $E[N_T]$**
i.i.d	$X_t \sim F(.)$	X_t are identically distributed and independent from each other, with low number of records most of them concentrated in the beginning. Main properties of the model are distribution free.	$1/t$	$\approx log(T) + \varpi$ where $\varpi \approx 0.577$ is the Euler-Mascheroni constant.
DTRW	$X_t = X_{t-1} + \varepsilon_t$ where ε_t follows a continuous and symmetric distribution	Observations are correlated and not identically distributed, with varying number of records. Main properties of the model are distribution-free like below.	$\binom{2t}{t} 2^{-2t}$ $\approx \frac{1}{\sqrt{\pi T}}$	$(2T+1)\binom{2T}{T} 2^{-2T}$ $\approx \sqrt{\frac{4T}{\pi}}$
LDM	$X_t = Y_t + \theta \cdot t$ where $\{Y_t\}_{1 \leq t \leq T}$ are *i.i.d r.v*	The process is independent but not identically distributed. Records are more occurring depending on the trend $(\theta) > 0$ value.	$\frac{1-e^{-\theta/\beta}}{1-e^{-\theta t/\beta}} \xrightarrow[t\to\infty]{} 1 - e^{-\theta/\beta}$ *	$\sum_{k=1}^{T} \frac{1-e^{-\theta}}{1-e^{-\theta k}}$ *
Yang-Nevzorov	$X_t \sim F(.)^{\gamma^t}$ where γ^t $(\gamma \geq 1)$ are real constant and $F(.)$ is a distribution function.	The process is independent but not identically distributed; but depends on t. Records are more occurring depending on the power (γ) value. Main properties are distribution-free.	$\frac{\gamma^t}{\gamma} \cdot \frac{\gamma - 1}{\gamma^t - 1} \xrightarrow[t\to\infty]{} 1 - \frac{1}{\gamma}$	$\sum_{k=1}^{T} \frac{\gamma^k(\gamma-1)}{\gamma(\gamma^k - 1)}$

* *only in the case where Y_t follows Gumbel(α, β) distribution*

B Scoring System

Table B.1: Features, Definitions, and Interpretation in Climate Context

Factor	Feature	Definition	Formula	Interpretation in Climate Context
Frequency	Record Count (N_T)	Number of records in a given time frame.	$N_T = \sum_{t=1}^{T} \delta_t$ where $\delta_t = 1$ if X_t is a record, and 0 if not.	Measures the frequency of record-breaking events over time. A higher number of records suggests increasing climate variability: For temperature, frequent new records indicate a possible warming trend, and for precipitation, a rise in records may suggest a shift in rainfall patterns.
	Fitted Model (M)	The most suitable model among the four discussed in record theory (*i.i.d.*, DTRW, LDM, Yang-Nevzorov).	Statistical tests are performed. For more info check Arnold et al. (1998); Hamie et al. (2018) and Hoayek (2016).	Helps identify whether climate extremes follow a stationary (*i.i.d*) or non-stationary (LDM, YN, DTRW) behavior: - i.i.d model → Records occur less frequently over time, suggesting a relatively stable climate. - Other models → More frequent and intense records indicate a non-stationary climate, where warming trends or shifting precipitation patterns drive more extreme events.
	High-to-Low Records Ratio (R)	Ratio of the number of records in the Max-series to the Min-series.	$R = \frac{N_{T,max}}{N_{T,min}}$	Indicates asymmetry in the frequency of high (maximum) vs. low (minimum) records, reflecting warming or cooling trends in the climate. $R > 1$ (more high records) suggests a warming climate, with increasing record-high temperatures outpacing cold records.
Intensity	VMR of X_t	Variance-to-mean ratio of the series X_t, including all values.	$\text{VMR}_{X_t} = \frac{\text{Variance of } X_t}{\text{Mean of } X_t}$	Provides an overall measure of climate variability, incorporating both ordinary and record-breaking events. A high VMR indicates significant shifts in climate extremes. For temperature, this suggests intense swings, and for precipitation, high VMR reflects greater unpredictability.
	VMR of X_t Excluding Records	Variance-to-mean ratio of the series X_t, excluding record values.	$\text{VMR}_{X_t, t \neq L_n} = \frac{\text{Variance of } X_t}{\text{Mean of } X_t}, t \neq L_n$	Measures the variability of ordinary climate observations, excluding extreme records. High values indicate that even non-record temperatures or precipitation levels fluctuate significantly, suggesting an increasingly unstable climate.

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Table B.1 (Continued)

Factor	**Feature**	**Definition**	**Formula**	**Interpretation in Climate Context**
	Average of Records Time ($\overline{L_n}$)	Arithmetic average of the time (e.g., year) of record occurrences.	$\overline{L_n} = Avg(L_1, \ldots, L_{N_T})$	Reflects the temporal distribution of record-breaking events and if records are occurring in rapid succession. In a stationary climate (*i.i.d* model), records are rare and mostly appear early in the dataset, leading to a low average record time. However, in non-stationary climates (DTRW, LDM or YN models with strong drift θ or power γ), records become more frequent and distributed throughout time, resulting in a higher average record time. This means records are occurring more often and intensively.
Trend	Average of Records Value ($\overline{R_n}$)	Arithmetic average of record values, normalized by dividing by the first trivial record.	$\overline{R_n} = Avg(\frac{R_1, R_2, \ldots, R_n}{R_1})$	Represents the relative magnitude of records over time, with higher averages signaling stronger increase in record-breaking events and thus steeper slope.
	Slope of Trend Line (α)	The slope of the trend line fitted to the time series.	Newman et al. (2010) show that $$\alpha = \frac{X_t - X_1}{t-1} \to \frac{\Delta X_t}{\sigma}$$ where σ is the standard deviation of the time series about its mean, and ΔX_t is the change in the variable associated with the linear trend during one time step (1 year).	Reflects the overall direction and intensity of climate trends, such as gradual warming or cooling. For temperature, a positive slope suggests a warming climate, and for precipitation, a steep positive slope could mean an increase in extreme rainfall, while a negative slope may indicate prolonged drought periods.
	Proximity between non-record values to record ones ($P_{X_t, X_{L_n}}$)	Closeness of non-record values (X_t) to preceding record values ($R_n = X_{L_n}$)	For each non-record X_t, $t > L_n$, $P_{X_t,X_{L_n}} =$ $$X_t - \frac{X_1 + \ldots + X_{L_n}}{N_t}$$ The Proximity is measured for all X_t in a given time frame and then averaged arithmetically.	Captures how ordinary climate measures (e.g., daily temperatures) behave relative to past record-breaking events. In other words, it assesses whether non-record temperatures or precipitation events are approaching past extremes. For temperature, high proximity values suggest that even non-record temperatures are nearing previous record highs, signaling a generalized warming trend. For precipitation, high values may indicate that regular rainfall amounts are approaching previous extreme records, suggesting more frequent heavy rains.

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Table B.1 (Continued)

Factor	Feature	Definition	Formula	Interpretation in Climate Context
	Proximity between record values (P_{R_n,R_m})	Closeness of records (R_n) to preceding records,	For each record R_n, $n > m$, $P_{R_n,R_m} =$ $$R_n - \frac{R_1 + \ldots + R_m}{m}.$$ The Proximity is measured for all R_n in a given time frame and then averaged arithmetically.	Indicates how closely clustered records are. Smaller values mean that new records are close to previous ones which means a slow increase in the overall trend.
	VMR of Record Values (VMR_{R_n})	Variance-to-mean ratio of record values.	$\text{VMR}_{R_n} =$ $$\frac{\text{Variance of records}}{\text{Mean of records}}$$	Highlights variability in record-breaking events, with high VMR indicating large differences between consecutive records and a stronger trend.
	Percentage Increase of Records (ΔR_n)	Ratio of the last record to the first record, regardless of intermediate fluctuations.	$\Delta R_n = \frac{R_{N_T}}{R_1} - 1$	Quantifies the overall magnitude of change in records over time, indicating long-term trends in records. For example, a high ΔR_n for temperature suggests that the hottest records today are significantly hotter than past records, confirming strong warming trends.

Table B.2: EFA Factor loadings and Model Fit for each country's dataset

Feature	**IDN**	**LBN**	**FR**	**EGY**	**NOR**	**GRL***
Record count	0.8	0.8	0.7	0.7	0.8	0.8
Fitted Model	0.9	0.9	0.5	0.7	0.7	-0.7
High-to-low records Ratio	0.8	0.7	0.7	0.6	0.6	0.7
Average of Records value	0.9	0.9	0.8	0.8	0.9	0.8
Slope of Trend line	0.6	0.8	0.7	0.8	0.7	0.6
Closeness of Xt to record values	0.6	0.9	0.9	0.9	0.8	0.8
Closeness of each record to previous record values	0.7	0.8	0.8	0.8	0.7	0.7
VMR of record values	0.9	0.9	0.9	0.9	0.9	0.9
Percentage increase of records	0.9	0.9	0.8	0.8	0.9	0.8
VMR of Xt excluding records	0.9	0.9	0.9	0.9	0.6	-0.7
VMR of Xt	0.8	0.9	0.8	0.9	0.4	0.9
Average of records time	0.6	0.7	0.6	0.8	0.7	0.8
Mean item complexity	1.6	**1.4**	**1.4**	1.5	**1.4**	1.6
Cumulative Proportion of variance explained (%)	82	82.5	72.4	**84.3**	68.3	83
RMSR	**0.03**	**0.03**	0.08	**0.03**	0.09	0.04
Tucker Lewis Index of factoring reliability	0.544	0.550	0.331	**0.801**	0.164	0.421
RMSEA	0.375	0.355	0.431	**0.282**	0.434	0.417
BIC	222.86	191.47	331.92	**91.49**	334.39	295.27
Correlation Matrix fit	0.9852	0.983	0.9498	**0.9885**	0.926	0.988

* *Since Greenland's temperatures are consistently negative, they are multiplied by -1 to prevent negative coefficients for certain features. Record definition will be inverted.*
Note: Same colored cells correspond to EFA grouping the same features into the same factor. In addition, EFA was also performed on precipitation data but there were no major differences in the results.

C Descriptive Statistics

Table C.1: Descriptive analysis of the variables for the stations included from 1901 to 2021

Station		Mean	Sd	Range	CV (%)	Kurtosis	Skew	Normality[1] (*p_value*)	KPSS (*p_value*)
Temperature (°C)									
Jambi	Mean	27.16	0.46	1.84	1.7	(0.69)	0.52	0.00*	0.01*
	Max	31.74	0.54	1.88	1.4	(0.58)	0.45	0.00*	0.01*
	Min	22.63	0.50	2.02	2.2	(0.53)	0.65	0.00*	0.01*
Cairo	Mean	21.24	0.65	3.84	3.1	0.46	0.20	0.38	0.01*
	Max	27.68	0.64	3.66	2.32	0.32	0.03	0.72	0.01*
	Min	14.85	0.69	4.15	4.62	0.60	0.38	0.08	0.01*
Beirut	Mean	15.72	0.78	4.46	4.97	0.07	0.25	0.71	0.01*
	Max	20.98	0.79	4.66	3.78	0.15	0.24	0.71	0.01*
	Min	10.52	0.78	4.26	7.47	(0.03)	0.27	0.67	0.01*
Paris	Mean	11.01	0.76	3.72	6.87	(0.33)	0.43	0.08	0.01*
	Max	24.8	1.52	6.92	6.14	(0.81)	0.21	0.06	0.02*
	Min	(0.76)	1.63	7.67	(214.9)	0.08	(0.69)	0.00*	0.10
Oslo	Mean	4.49	0.92	4.17	20.37	(0.73)	(0.08)	0.17	0.02*
	Max	8.59	0.90	4.06	10.51	(0.76)	(0.09)	0.23	0.04*
	Min	0.41	0.97	4.37	237.3	(0.64)	(0.09)	0.21	0.02*
GRL	Mean	(19.27)	0.73	3.42	(3.78)	(0.36)	0.2	0.55	0.01*
	Max	(15.39)	0.73	3.54	(4.73)	(0.34)	0.15	0.77	0.01*
	Min	(23.19)	0.74	3.44	(3.18)	(0.39)	0.24	0.35	0.01*

[1] *Assessed through Shapiro-Wilk's test*
* *Significane at 5% level*

Table C.2: Descriptive analysis of the variables for the stations included from 1901 to 2021

Station		**Mean**	**Sd**	**Range**	**CV** (%)	**Kurtosis**	**Skew**	**Normality**[1] *(p_value)*	**KPSS** *(p_value)*
					Precipitation (mm)				
Jambi	Mean	230.23	15.25	86.77	6.62	0.26	(0.28)	0.51	0.10
	Max	306.78	18.13	89.51	5.91	(0.25)	0.37	0.18	0.10
	Min	150.22	25.45	132.34	16.94	0.12	(0.13)	0.21	0.10
Cairo	Mean	1.82	0.48	2.16	26.63	(0.37)	0.36	0.06	0.08
	Max	6.03	2.32	10.92	38.5	0.31	0.78	0.00*	0.09
	Min	0.20	0.05	0.40	27.4	6.7	2.03	0.00*	0.10
Beirut	Mean	58.81	13.22	68.59	22.47	(0.04)	0.01	0.40	0.10
	Max	197.82	53.61	312.62	27.1	1.02	0.4	0.02*	0.10
	Min	0.04	0.12	1.1	292.63	53.64	6.66	0.00*	0.06
Paris	Mean	52.19	8.01	41.61	15.35	(0.32)	(0.04)	0.81	0.10
	Max	118.47	18.79	85.15	15.86	0.28	0.64	0.00*	0.10
	Min	29.19	10.4	49.46	35.62	(0.11)	0.12	0.50	0.10
Oslo	Mean	86.22	9.73	41.39	11.28	(0.64)	0.34	0.016	0.10
	Max	142.71	19.6	105.8	13.73	0.55	0.43	0.04*	0.10
	Min	42.22	12.35	54.77	29.25	(0.59)	0.07	0.20	0.02*
GRL	Mean	36.64	2.89	14.55	7.88	(0.26)	0.26	0.48	0.04*
	Max	51.56	6.4	33.71	12.42	(0.15)	0.34	0.24	0.10
	Min	23.9	3.11	15.39	13.02	(0.15)	0.31	0.35	0.06

[1] *Assessed through Shapiro-Wilk's test*
[*] *Significane at 5% level*

Figure C.1: Annual mean temperature

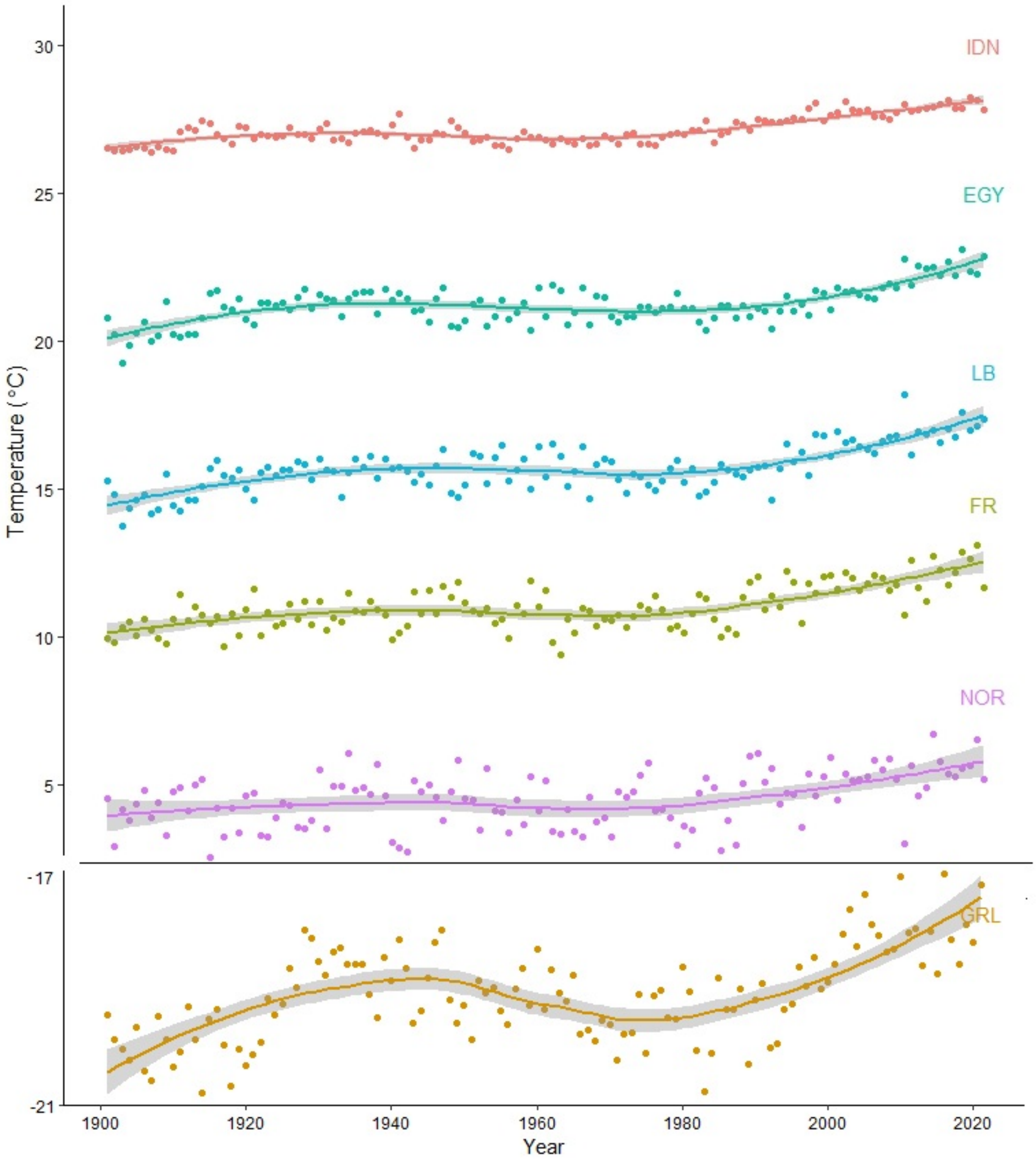
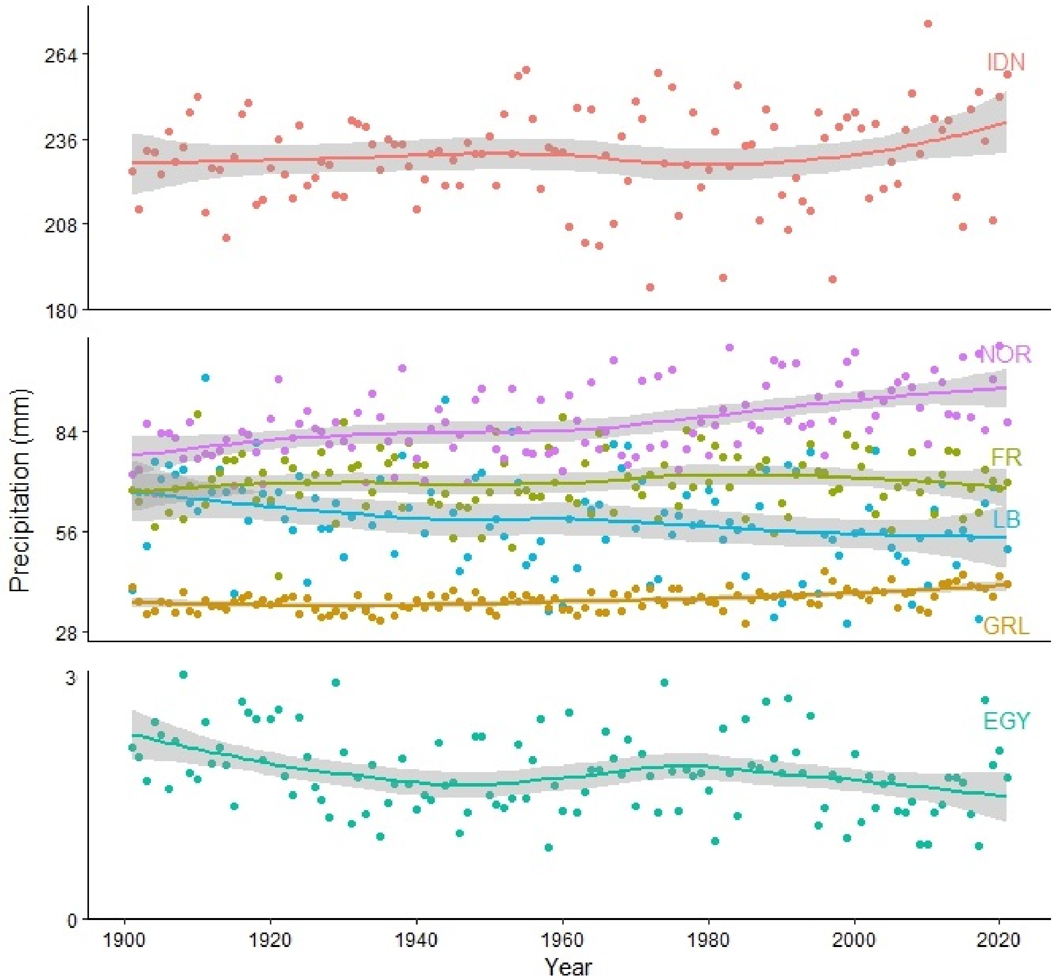


Figure C.2: Annual mean precipitation



D Results

Figure D.1: Score plots for each country using temperature as climate variable

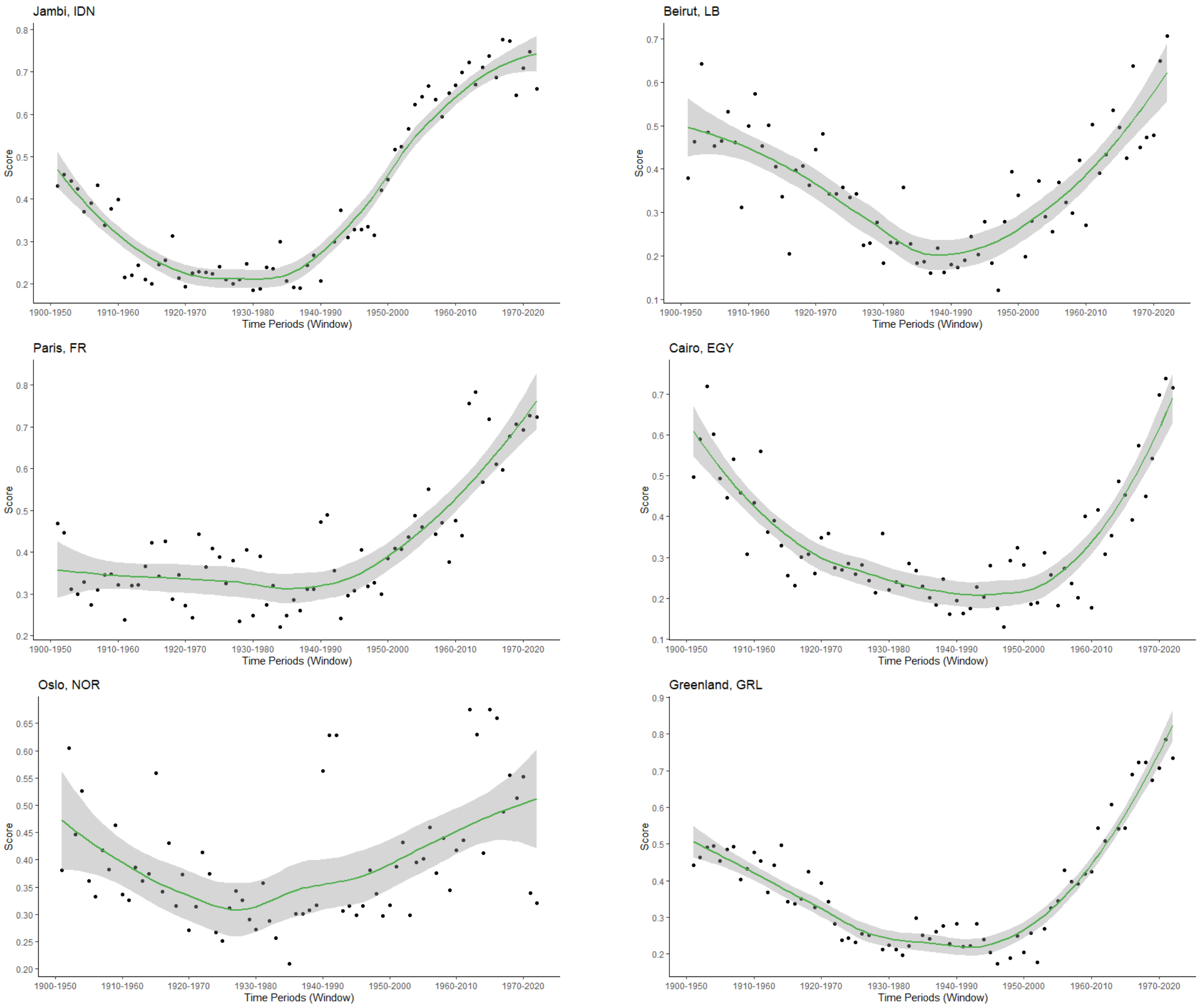


Figure D.2: Score plots for each country using Precipitation as climate variable

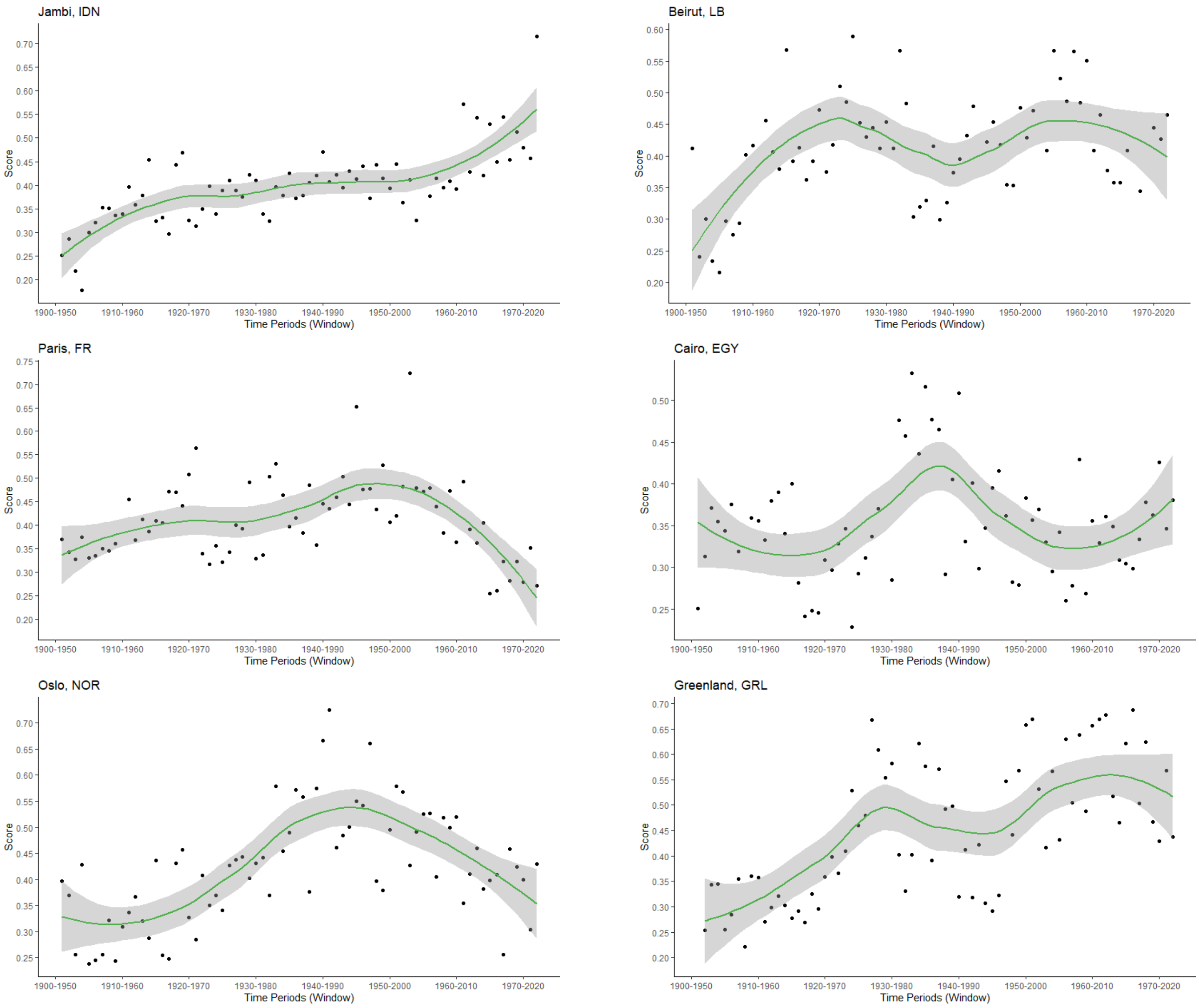
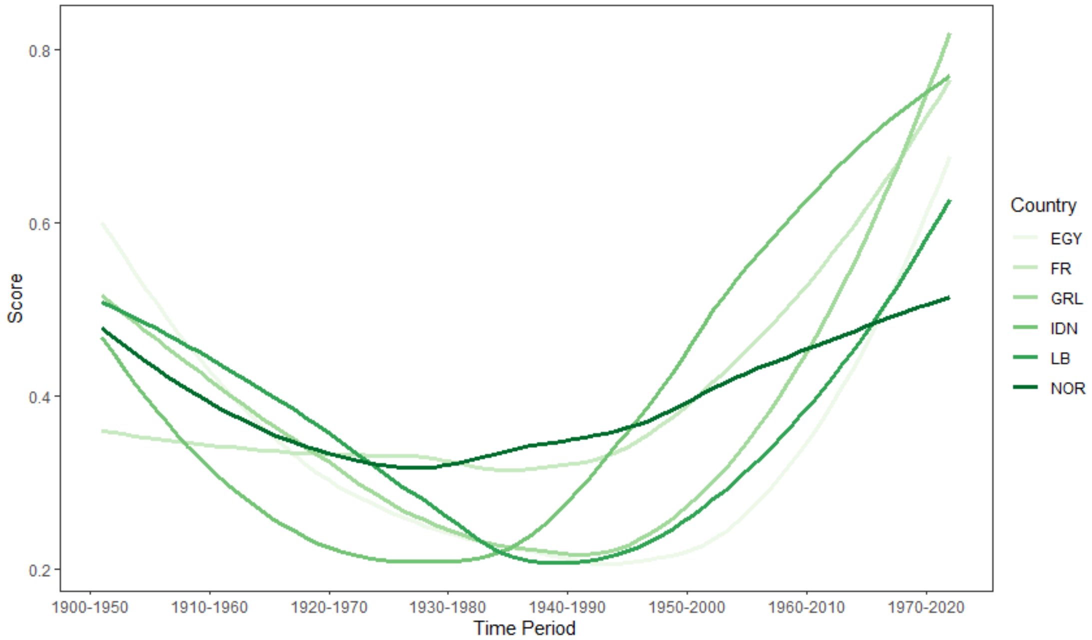
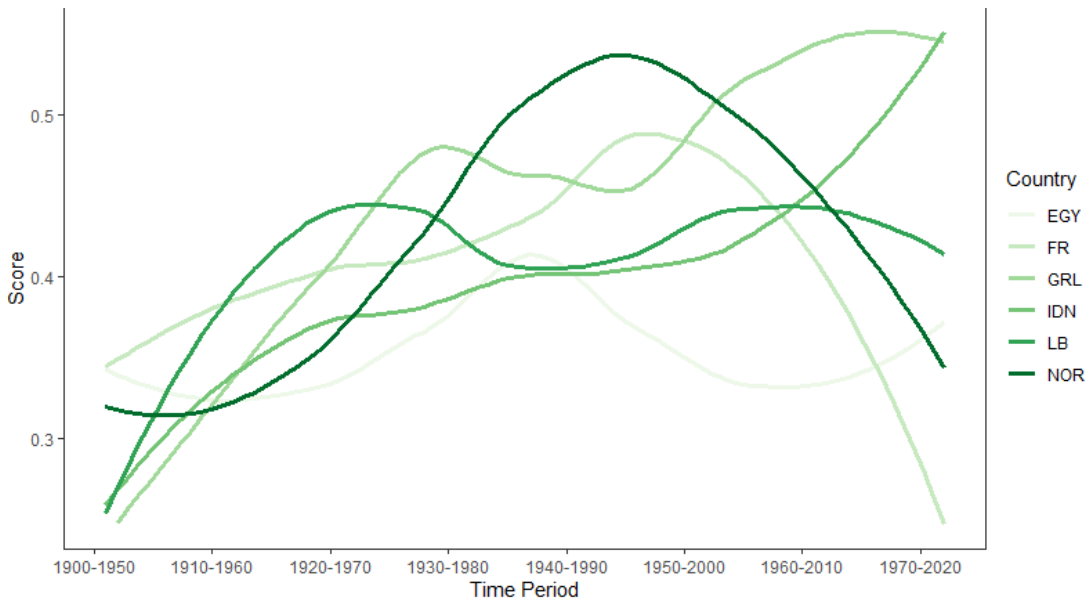


Figure D.3: Score plots for all countries using Temperature and Precipitation as climate variables

(a) Temperature



(b) Precipitation



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