

APPENDIX: EXPLORING CLIMATE CHANGE THROUGH THE LENS OF RECORDS THEORY

A Record Theory Models

Table A.1: Some properties of the most common models in Record Theory

Model	Mathematical Expression	Description	Probability to observe a record at time t	Expected number of records $E[N_T]$
<i>i.i.d</i>	$X_t \sim F(\cdot)$	X_t are identically distributed and independent from each other, with low number of records most of them concentrated in the beginning. Main properties of the model are distribution free.	$1/t$	$\approx \log(T) + \varpi$ where $\varpi \approx 0.577$ is the Euler-Mascheroni constant.
DTRW	$X_t = X_{t-1} + \varepsilon_t$ where ε_t follows a continuous and symmetric distribution	Observations are correlated and not identically distributed, with varying number of records. Main properties of the model are distribution-free like below.	$\binom{2t}{t} 2^{-2t}$ $\approx \frac{1}{\sqrt{\pi T}}$	$(2T+1) \binom{2T}{T} 2^{-2T}$ $\approx \sqrt{\frac{4T}{\pi}}$
LDM	$X_t = Y_t + \theta \cdot t$ where $\{Y_t\}_{1 \leq t \leq T}$ are <i>i.i.d r.v</i>	The process is independent but not identically distributed. Records are more occurring depending on the trend (θ) > 0 value.	$\frac{1 - e^{-\theta/\beta}}{1 - e^{-\theta t/\beta}}$ $\xrightarrow{t \rightarrow \infty}$ $1 - e^{-\theta/\beta_*}$	$\sum_{k=1}^T \frac{1 - e^{-\theta_*}}{1 - e^{-\theta k}}$
Yang-Nevzorov	$X_t \sim F(\cdot)^{\gamma^t}$ where γ^t ($\gamma \geq 1$) are real constant and $F(\cdot)$ is a distribution function.	The process is independent but not identically distributed; but depends on t. Records are more occurring depending on the power (γ) value. Main properties are distribution-free.	$\frac{\gamma^t}{\gamma} \cdot \frac{\gamma-1}{\gamma^t-1}$ $\xrightarrow{t \rightarrow \infty}$ $1 - \frac{1}{\gamma}$	$\sum_{k=1}^T \frac{\gamma^k(\gamma-1)}{\gamma(\gamma^k-1)}$

* only in the case where Y_t follows Gumbel(α, β) distribution