B Scoring System

Table B.1: Features, Definitions, and Interpretation in Climate Context

Factor	Feature	Definition	Formula	Interpretation in Climate Context
Frequency	Record Count (N_T)	Number of records in a given time frame.	$N_T = \sum_{t=1}^{T} \delta_t$ where $\delta_t = 1$ if X_t is a record, and 0 if not.	Measures the frequency of record-breaking events over time. A higher number of records suggests increasing climate variability: For temperature, frequent new records indicate a possible warming trend, and for precipitation, a rise in records may suggest a shift in rainfall patterns.
	Fitted Model (M)	The most suitable model among the four discussed in record theory (*i.i.d.*, DTRW, LDM, Yang-Nevzorov).	Statistical tests are performed. For more info check Arnold et al. (1998); Hamie et al. (2018) and Hoayek (2016).	Helps identify whether climate extremes follow a stationary (*i.i.d*) or non-stationary (LDM, YN, DTRW) behavior: - i.i.d model → Records occur less frequently over time, suggesting a relatively stable climate. - Other models → More frequent and intense records indicate a non-stationary climate, where warming trends or shifting precipitation patterns drive more extreme events.
	High-to-Low Records Ratio (R)	Ratio of the number of records in the Max-series to the Min-series.	$R = \frac{N_{T,max}}{N_{T,min}}$	Indicates asymmetry in the frequency of high (maximum) vs. low (minimum) records, reflecting warming or cooling trends in the climate. $R > 1$ (more high records) suggests a warming climate, with increasing record-high temperatures outpacing cold records.
Intensity	VMR of X_t	Variance-to-mean ratio of the series X_t, including all values.	$\text{VMR}_{X_t} = \frac{\text{Variance of } X_t}{\text{Mean of } X_t}$	Provides an overall measure of climate variability, incorporating both ordinary and record-breaking events. A high VMR indicates significant shifts in climate extremes. For temperature, this suggests intense swings, and for precipitation, high VMR reflects greater unpredictability.
	VMR of X_t Excluding Records	Variance-to-mean ratio of the series X_t, excluding record values.	$\text{VMR}_{X_t, t \neq L_n} = \frac{\text{Variance of } X_t}{\text{Mean of } X_t}, t \neq L_n$	Measures the variability of ordinary climate observations, excluding extreme records. High values indicate that even non-record temperatures or precipitation levels fluctuate significantly, suggesting an increasingly unstable climate.

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Table B.1 (Continued)

Factor	Feature	Definition	Formula	Interpretation in Climate Context
	Average of Records Time ($\overline{L_n}$)	Arithmetic average of the time (e.g., year) of record occurrences.	$\overline{L_n} = Avg(L_1, \ldots, L_{N_T})$	Reflects the temporal distribution of record-breaking events and if records are occurring in rapid succession. In a stationary climate (*i.i.d* model), records are rare and mostly appear early in the dataset, leading to a low average record time. However, in non-stationary climates (DTRW, LDM or YN models with strong drift θ or power γ), records become more frequent and distributed throughout time, resulting in a higher average record time. This means records are occurring more often and intensively.
Trend	Average of Records Value ($\overline{R_n}$)	Arithmetic average of record values, normalized by dividing by the first trivial record.	$\overline{R_n} = Avg(\frac{R_1, R_2, \ldots, R_n}{R_1})$	Represents the relative magnitude of records over time, with higher averages signaling stronger increase in record-breaking events and thus steeper slope.
	Slope of Trend Line (α)	The slope of the trend line fitted to the time series.	Newman et al. (2010) show that $$\alpha = \frac{X_t - X_1}{t-1} \rightarrow \frac{\Delta X_t}{\sigma}$$ where σ is the standard deviation of the time series about its mean, and ΔX_t is the change in the variable associated with the linear trend during one time step (1 year).	Reflects the overall direction and intensity of climate trends, such as gradual warming or cooling. For temperature, a positive slope suggests a warming climate, and for precipitation, a steep positive slope could mean an increase in extreme rainfall, while a negative slope may indicate prolonged drought periods.
	Proximity between non-record values to record ones ($P_{X_t, X_{L_n}}$)	Closeness of non-record values (X_t) to preceding record values ($R_n = X_{L_n}$)	For each non-record X_t, $t > L_n$, $P_{X_t, X_{L_n}} =$ $$X_t - \frac{X_1 + \ldots + X_{L_n}}{N_t}$$ The Proximity is measured for all X_t in a given time frame and then averaged arithmetically.	Captures how ordinary climate measures (e.g., daily temperatures) behave relative to past record-breaking events. In other words, it assesses whether non-record temperatures or precipitation events are approaching past extremes. For temperature, high proximity values suggest that even non-record temperatures are nearing previous record highs, signaling a generalized warming trend. For precipitation, high values may indicate that regular rainfall amounts are approaching previous extreme records, suggesting more frequent heavy rains.

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Table B.1 (Continued)

Factor	Feature	Definition	Formula	Interpretation in Climate Context
	Proximity between record values (P_{R_n,R_m})	Closeness of records (R_n) to preceding records,	For each record R_n, $n > m$, $P_{R_n,R_m} =$ $$R_n - \frac{R_1 + \ldots + R_m}{m}.$$ The Proximity is measured for all R_n in a given time frame and then averaged arithmetically.	Indicates how closely clustered records are. Smaller values mean that new records are close to previous ones which means a slow increase in the overall trend.
	VMR of Record Values (VMR_{R_n})	Variance-to-mean ratio of record values.	$\text{VMR}_{R_n} =$ $$\frac{\text{Variance of records}}{\text{Mean of records}}$$	Highlights variability in record-breaking events, with high VMR indicating large differences between consecutive records and a stronger trend.
	Percentage Increase of Records (ΔR_n)	Ratio of the last record to the first record, regardless of intermediate fluctuations.	$\Delta R_n = \frac{R_{N_T}}{R_1} - 1$	Quantifies the overall magnitude of change in records over time, indicating long-term trends in records. For example, a high ΔR_n for temperature suggests that the hottest records today are significantly hotter than past records, confirming strong warming trends.

References

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