# Inter-Diagonal Leg-Pair Phase Asymmetry in the Trotting Gait can Reduce Lateral Drift for a Quadrupedal Robot when Climbing Sloped Inclines with discrete footholds

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***Abstract*—Quadrupedal robots typically rely on symmetric trotting gaits optimized for flat or uniformly sloped terrain; however, real-world environments contain discrete, slanted footholds that impose inherently asymmetric contact conditions. This study investigates whether intentional deviations from the typical phase difference ($\pi$) between diagonal leg pairs in the trotting gait can reduce lateral drift and improve climbing performance on sloped surfaces with discrete footholds. A custom-built 2-DOF per-leg quadruped robot was used to systematically vary the inter-diagonal phase offset $\Delta\Phi_\alpha$ while maintaining symmetric coordination within-pairs. Experiments conducted in 5° and 10° inclines measured lateral drift, vertical distance climbed, and time per trial, with trajectory characteristics observed, under multiple phase-offset conditions. The results show that controlled phase asymmetry produces predictable steering effects and can substantially reduce drift on mild slopes but becomes ineffective on steeper inclines where foothold interaction forces dominate. These findings indicate that asymmetric gait parameters can serve as lightweight gait-level steering corrections for low-level slopes and may complement higher-level foothold or posture adaptation strategies for more challenging terrain.**

## I. Introduction

### A. Research question

Our chosen specific terrain is a rectangular board with discrete, slanted footholds, with adjustable incline, and our research question as follows: Can applying an offset to the inter-diagonal leg-pair phase, deviating from the canonical for symmetric gaits, reduce lateral drift for a quadrupedal robot navigating a sloped incline with slanted footholds?

This research addresses a critical knowledge gap in legged robotics. While symmetric quadrupedal gaits such as trotting, walking, and bounding have been extensively characterized for flat terrain and uniform continuous slopes, real-world climbing environments present a challenge that is fundamentally different–discrete footholds with varying orientations that create bilateral asymmetries in contact mechanics. Natural terrain rarely provides perfectly horizontal or uniform footholds. When a robot encounters slanted footholds on a slope, the left and right legs experience different friction coefficients, contact angles, and load distributions that violate the left-right symmetry assumptions in classical gait design.

Our research is novel for three reasons: Firstly, no prior work has systematically investigated bilateral temporal and spatial asymmetry as a deliberate control strategy for slanted foothold compensation. Existing papers assume symmetric timing despite asymmetric terrain.

Secondly, the interaction effects between foothold slant direction, terrain slope angle, and gait asymmetry parameters remain underexplored. We don't know how timing offsets and phase deviations should adapt as foothold geometry changes.

Finally, current gait controllers consider asymmetry as noise to be suppressed rather than as an adaptive control mechanism. We hope to re-conceptualize asymmetry as a purposeful strategy for heterogeneous terrain.

Understanding the dynamics of gait asymmetry should allow engineers to make better informed design decisions when deploying legged robots in unstructured environments such as disaster response, planetary exploration, and other applications where terrain cannot be engineered for symmetric gaits.

### B. Literature review

Recent advances in quadrupedal robotics have significantly enhanced mechanical design and locomotion control; however, optimizing gait for uneven or sloped terrain remains an unresolved challenge. Traditional gait generation methods often rely on symmetric and periodic inter-limb coordination—such as wave, trot, or bound gaits—which, although energetically efficient and dynamically stable on flat ground, tend to degrade in performance as terrain incline or irregularity increases.

Estremera and González de Santos (2002) addressed these limitations by proposing free gaits that relax symmetry and periodicity constraints to improve adaptability on uneven terrain [1]. Their method introduced variable leg sequencing and adaptive foothold selection guided by local stability margins, resulting in discontinuous free gaits—such as free-crab, free-turning, and free-spinning—that incorporate temporal and spatial phase variability between contralateral legs. These asymmetric adaptations allowed the SILO4 quadruped robot to maintain static stability and trajectory control across irregular surfaces, achieving minimal performance loss compared to symmetric baselines. Although this work established that asymmetric, non-periodic gaits can improve terrain adaptability, it did not quantify how specific bilateral temporal and spatial phase deviations affect stability margins or climbing efficiency on sloped or slanted surfaces.

Building on the role of asymmetry in gait optimization, Alqaham, Cheng, and Gan (2024) explored energy-optimal asymmetrical gait selection for quadrupedal robots under varying speed and directional constraints [2]. Their results demonstrated that energy consumption can be minimized by introducing temporal phase offsets between legs, contingent on the robot's mechanical structure and center of mass. This suggests that optimal gait symmetry is context-dependent and should be adjusted according to the robot's morphology and environmental conditions. The study focused primarily on energy optimization over flat terrain and did not investigate how asymmetrical phasing influences stability and balance during locomotion on inclined or uneven surfaces, where lateral destabilization is more prominent.

Addressing slope-specific locomotion, Guo et al. (2019) proposed practical methods for climbing steep slopes using a 60° inclined ramp as a testbed. Their technique relied on an intermittent crawl gait combined with an attitude adjustment algorithm that used IMU feedback to maintain the center of mass (CoM) within the support polygon. They also refined the Normalized Energy Stability Margin (NESM) to determine the most stable climbing posture. Simulation and experimental results showed that maximizing the support polygon improved balance and static stability on steep slopes. While effective on uniform inclines, this approach assumed consistent foothold geometry and did not integrate temporal or spatial asymmetry between bilateral leg pairs. Consequently, it does not generalize to irregular or slanted footholds, where adaptive asymmetry could further enhance performance. [3]

A related simulation study by Sellers and Hirasaki (2018) demonstrated that when quadrupedal gait optimization considers only energetic cost, it produces highly symmetric footfall patterns that lack lateral stability [4]. Introducing a second objective for stability led to systematic phase deviations and asymmetric interlimb coordination, yielding gaits that were less energy-efficient but more stable and biologically realistic. The study validated the concept of asymmetry-driven stability only through simulation. It did not experimentally verify how controlled bilateral phase offsets affect climbing stability or performance on real sloped terrains.

Similarly, Majithiya and Dave (2021) developed a static gait scheme for maintaining a horizontal body posture while climbing slopes. Their algorithm employed a two-phase discontinuous gait, ensuring the center of gravity remained within the support polygon during each leg movement. Multibody simulations confirmed positive static stability with negligible body rotation ($<0.01°$) even on inclined planes. Although effective for static slope climbing, the method assumed uniform slope geometry and lacked dynamic adaptability. It did not consider bilateral timing asymmetry or spatial phase deviations, both of which may become critical on slanted or discontinuous footholds.[5]

From a control perspective, Song, Zhu, and Xu (2023) developed a Central Pattern Generator (CPG) framework based on delay-coupled Van der Pol (VDP) oscillators, demonstrating that phase delays and temporal asymmetries among oscillators can produce a continuum of stable gaits, including walk, trot, pace, and bound [6]. They showed that introducing time delays induces Hopf bifurcations, enabling flexible transitions between gaits and maintaining dynamic stability. Their results confirmed that spatiotemporal asymmetry can improve robustness and adaptability on irregular terrains. While the VDP-CPG model theoretically supports gait asymmetry, it has not been experimentally validated on sloped or slanted footholds. The authors did not quantify how bilateral phase asymmetry correlates with stability metrics or climbing performance in real environments.

In a more recent contribution, Li et al. (2024) presented an adaptive control algorithm for quadruped robots navigating unknown high-slope terrains [7]. Their method utilized terrain estimation and active posture adjustment to maintain the CoM within the support polygon. The algorithm improved stability under uncertain slope conditions through real-time adaptation.

Despite its adaptability, the study maintained symmetrical gait patterns and focused on posture control rather than interlimb timing or phase asymmetry. The effect of deliberately imposed bilateral asymmetry on stability and climbing capability remains unaddressed.

The source of biological inspiration was this paper by Frigon and Gossard, who demonstrated that the mammalian spinal central pattern generator had intrinsic temporal asymmetry, where cycle period changes predominantly through extension phase modulation while flexion remains relatively consistent. The extensor-dominated pattern persisted without supraspinal input or sensory feedback, indicating that asymmetry is embedded within the circuitry of the spinal cord. Extension phases consumed about 67% of the cycle period, compared to 30% for flexion phases, suggesting that implementing temporal asymmetries in robotic CPGs could provide biologically-grounded cycle timing adjustments for slopes requiring longer ground contact. The experiments studied neural signals in paralyzed cats without real limb movement or ground contact, so the findings do not directly show whether these timing patterns actually improve stability during physical locomotion. The observed asymmetry was also between stance and swing phases, not between left and right legs, and the study did not measure stability or climbing ability. [8]

Fukuoka et al. discovered that diverse quadrupedal gaits emerge spontaneously from leg loading feedback applied to a trot-based CPG network. The tilt of the body during the diagonal leg-pair stance created differential loading that fed back negatively to flexor activation, which produced phase shifts between originally synchronized legs. Their mechanism generated nine distinct gaits that included lateral and diagonal walks, left and right-lead canters, and transverse gallops for each diagonal pair. The gaits transitioned naturally with speed, suggesting that load-sensing could enable robots to automatically adjust gait patterns based on weight distribution shifts on inclined terrains. Some limitations of their study were that their model only operated in the sagittal plane on flat terrain, and the body tilt came from speed-dependent oscillations rather than from asymmetric loading as inclines would impose. They also did not quantify stability metrics or climbing performance, so it is uncertain whether the emergent asymmetric patterns actually improved robustness. [9]

Collectively, these studies suggest that asymmetric gait coordination—whether through phase delays, load feedback, or adaptive timing—may enhance locomotor stability and adaptability. However, most prior work either (1) focused on flat or uniform slopes, (2) investigated posture and support polygon control rather than inter-limb coordination, or (3) explored asymmetry only theoretically or biologically, without quantitative robotic implementation.

Despite much evidence that asymmetry can improve adaptability, no existing study has systematically investigated how intentional bilateral asymmetry—through temporal timing offsets and spatial phase deviations—affects the stability, lateral drift, and climbing efficiency of quadrupedal robots on sloped terrain with slanted footholds.

This research therefore aims to experimentally and computationally evaluate whether deliberately designed inter-leg-pair asymmetry in the trotting gait can reduce lateral drift for quadrupedal robots navigating complex, inclined environments.

### C. Hypothesis

We hypothesize that introducing inter-leg-pair asymmetry by applying offsets to the nominal $\pi$, will enable a quadrupedal robot to climb sloped, discretely slanted terrains at an incline of 5-10°, with greater directional stability than traditional symmetric gaits, measured by a reduction in absolute lateral drift. By intentionally desynchronizing inter-diagonal leg timing and phase relationships, we expect the robot to better compensate for uneven foothold orientations and asymmetric friction forces that arise on slanted surfaces.

The hypothesis is directly testable through controlled robotic experiments in which phase deviation ($\Delta\Phi$), and terrain slope ($\beta$) are systematically varied. Dependent variables with which performance will be measured include lateral drift (cm/trial), and the vertical component of the distance the robot travelled, as well as time elapsed before success or failure. The control condition will consist of a symmetric trot gait traversing an identical terrain surface. The robot's experimental success will be established through reaching a vertical distance of 65 cm (designated by a "finish line"), and yielding improvements in lateral drift reduction.

Our hypothesis is grounded in biological and physical principles. Research on cats demonstrates that asymmetric control of cycle period is an inherent property of the spinal locomotor rhythm generator that can be modified by external factors [8]. This neural flexibility, combined with the load-feedback mechanism where "load differences between legs led to phase differences between their CPGs," [9] suggests that bilateral timing asymmetries are not pathological deviations, but represent a control strategy available to the locomotor system.

Studies of cats on split-belt treadmills showed that animals maintained coordination during left-right asymmetric conditions by "adjusting cycle and phase (stance and swing) durations between the slow and fast sides" [8]. While split-belt walking imposes external speed asymmetry, slanted footholds impose mechanical loading asymmetry, and both require bilateral temporal adjustments.

Building on the load-feedback principle, slanted footholds create specific mechanical demands. Slanted contact surfaces alter the effective friction coefficient and normal force distribution; asymmetric stance durations could prevent slip, and on slopes, asymmetric timing could proactively shift the center of mass toward the uphill side by modulating when each leg bears weight, using the load-feedback mechanism to maintain lateral balance.

## II. Experiment plan

### A. Robot Design

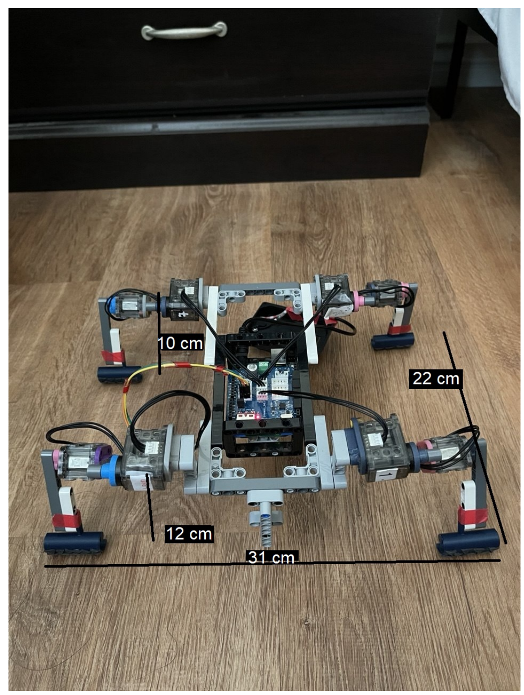


Fig. 1: Robot Design

Figure 1 shows the quadrupedal robot platform designed specifically to enable systematic investigation of bilateral gait asymmetry on sloped terrain with discrete, slanted footholds. The body frame consists of a rectangular chassis measuring 19 cm in length, 10 cm in width, and 6 cm in height, constructed from Lego pieces and PLA to provide lightweight rigidity while allowing rapid design iteration. After including the motors and legs, the robot's total dimensions are 22 cm in length, 31 cm in width, with the forward legs at 12 cm, and the back legs at 10 cm in height. The central compartment houses the Arduino Uno microcontroller.

The robot employs four legs arranged in a standard quadrupedal configuration with two forelimbs and two hindlimbs. The hip width, defined as the lateral spacing between left and right hip motors, measures 22 cm, while the longitudinal hip-to-hip distance in the sagittal plane is 16 cm. The height of the robot in this posture is 12 cm. Each leg consists of a proximal hip joint and a distal knee joint, connected by two rigid links, with the 2 degrees of freedom spanning the fore/aft and dorsal/ventral axes, the robot consisting of 8 Dynamixel servo motors in total. This two-degree-of-freedom design represents a deliberate simplification that constrains leg motion to the sagittal plane, eliminating hip abduction and adduction to reduce computational complexity and state space dimensionality while still providing sufficient freedom to test the core research hypothesis regarding gait asymmetries.

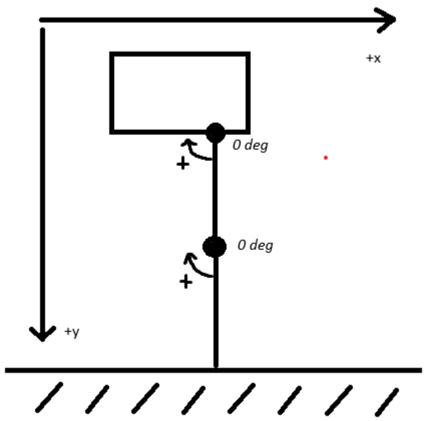


Fig. 2: Robot Morphology and Kinematics.

The hip joint, $\theta_1$, provides forward and backward rotation in the sagittal plane. The upper link, analogous to a femur, measures 6.6 cm in length. The knee joint, $\theta_2$, provides flexion and extension of the lower link. The lower link, analogous to a tibia, measures 8.5 cm in length and terminates in a 3D-printed, semi-cylindrical foot design (Figure 3) to provide compliance and friction during ground contact. $\theta_1$ is measured clockwise from the positive y-axis, with $\theta_1$ and $\theta_2$ equal to zero degrees when the distal end of both links points towards the ventral side of the robot in series at full extension (Figure 2). The combined leg length of about 15 cm provides adequate reach to span between discrete footholds while maintaining reasonable ground clearance during the swing phase. The robot's total standing height is approximately 12 cm, adjustable based on the knee flexion angle, and each foot can reach a parabolic workspace, adjustable in stride length, longitudinal offset, curve steepness, maximum y clamping, and rotation.

This morphology addresses the research question through several intentional choices that balance hardware constraints with experimental requirements. 2-DOF legs enable simple, inter-diagonal leg-pair asymmetry implementation, as with sagittal plane motion only, we can independently control the timing and phase of each leg while maintaining relative planar stability. This allows clean isolation of phase deviation ($\Delta\phi$) effects without overcomplicating with 3-D dynamics. The leg length proportions provide sufficient reach for sloped terrain navigation while the compact footprint, with our previously defined lateral hip spacing, ensures the robot can place feet on individual discrete footholds without interference between adjacent legs. Our longitudinal hip spacing provides adequate stability while allowing the robot to meet the longitudinal, 20 cm dimensional constraint. Additionally, the motors are restricted to operating within the ventral space of the robot rather than swinging dorsally to reset as in the fast phase of a Buehler Clock, so as to not disrupt the center of mass and perturb the asymmetric gait.

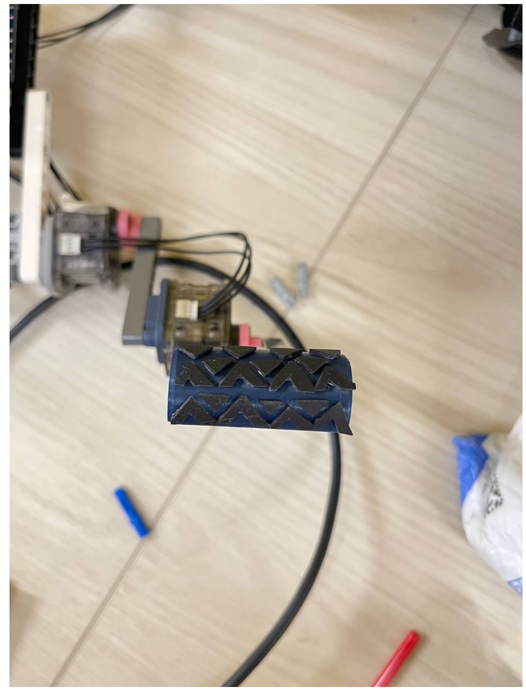

Fig. 3: Robot Foot Design

While the morphological design of our quadrupedal robot employs conventional two-degree-of-freedom legs similar to many platforms in literature, the novel contribution of this work lies in the kinematic control strategy that systematically investigates the inter-diagonal-pair phase relationship ($\Delta\Phi_\alpha$) as a mechanism for compensating terrain-induced lateral drift on sloped surfaces with asymmetric footholds. Most existing quadrupedal gait controllers assume or enforce bilateral symmetry, where left and right legs maintain mirror-image phase relationships, or in the case of the trotting gait, the LF-RR and RF-LR pairs individually move at the same time (intra-diagonal phase difference of 0), but the pairs themselves are offset by an inter-diagonal phase offset of $\pi$. The trot gait implementations described by Fukuoka *et al.* and Song, Zhu, and Xu both generate symmetric patterns where diagonal leg pairs move synchronously and the phase difference between diagonal pairs equals exactly $\pi$ radians. Even adaptive gait systems such as the CPG-based controllers with sensory feedback maintain symmetry as an implicit constraint, allowing asymmetry to emerge only as transient perturbations rather than as a deliberate control strategy.

The kinematic approach in this research treats the inter-diagonal phase offset $\Delta\Phi_\alpha$ as a continuously variable control parameter that can deviate from the canonical $\pi$ value, investigating how these deviations affect the robot's ability to maintain straight trajectories on challenging terrain. This represents a departure from the discrete gait classification paradigm (walk, trot, pace, gallop) toward a continuous gait space where intermediate phase relationships create hybrid patterns that may offer performance advantages for specific terrain geometries. This contrasts with the "free gaits" approach of Estremera and González de Santos, which relaxed periodicity constraints and allowed variable leg sequencing but did not systematically investigate how specific quantitative asymmetry parameters affect performance on geometrically characterized terrain. Their free-crab and free-turning gaits introduced temporal variability adaptively in response to stability margins, but did not test controlled bilateral phase offsets as independent variables across parametric terrain variations.

Previous work on gait transitions by Righetti and Ijspeert explored smooth interpolation between different gait patterns through CPG parameter modulation, but focused on speed-dependent transitions on flat terrain rather than terrain-dependent adaptations on slopes. Similarly, the energy-optimal asymmetrical gait selection work by Alqaham, Cheng, and Gan introduced temporal phase offsets between legs to minimize energetic cost, but focused on flat terrain locomotion and did not address how asymmetry compensates for bilateral differences in contact mechanics imposed by slanted footholds.

The hypothesized benefit of varying $\Delta\Phi_\alpha$ lies in its ability to modulate the temporal overlap between the two diagonal support phases, which directly influences how the center of mass trajectory evolves during the gait cycle on asymmetric terrain. When $\Delta\Phi_\alpha = \pi$ in a symmetric trot, the two diagonal pairs alternate precisely, creating equal-duration periods where the left-front and right-rear legs support the body versus when the right-front and left-rear legs provide support. On terrain with slanted footholds that create bilateral asymmetries in traction and contact mechanics, this symmetric alternation may produce oscillating lateral forces that accumulate into net drift over multiple gait cycles.

By adjusting $\Delta\Phi_\alpha$ away from $\pi$, the robot can shift the relative durations and timing of the two support phases, potentially allowing the diagonal pair that experiences more favorable contact conditions to bear weight for a longer portion of the gait cycle while the less-favored diagonal pair contributes less to forward propulsion. For example, if the foothold slant creates better traction for the left-front and right-rear diagonal pair because their contact angles are more favorable, increasing $\Delta\Phi_\alpha$ slightly above $\pi$ would extend the stance overlap for this pair while reducing the overlap for the right-front and left-rear pair, biasing the locomotion strategy toward exploiting the better-supported diagonal.

On sloped terrain where lateral drift can continuously accumulate, the increased support periods from $\Delta\Phi_\alpha < \pi$ might provide more opportunity for corrective lateral weight shifts, while the reduced support from $\Delta\Phi_\alpha > \pi$ might allow quicker adaptation to unexpected foothold variations. The optimal value likely depends on the specific terrain slope ($\beta$), as steeper slopes increase both the difficulty of maintaining balance—favoring longer support periods—and the penalty for any lateral slip, favoring quicker, more decisive weight transfers.

Our coordinate frame is defined as follows: the origin is at the hip joint, the positive $x$-axis is toward the direction of travel, the positive $y$-axis points ventrally, and the $z$-axis is lateral, following the right-hand rule convention. The endpoint foot position relative to the hip in our coordinate frame is given by:

$$x = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) \tag{1}$$

$$y = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) \tag{2}$$

Here, the joint angle $\theta_1$ is measured clockwise from the positive $y$-axis, representing the relative orientation of the femur in space, with $\theta_1 = 0^\circ$ corresponding to the femur pointing directly downward. $\theta_2$ represents the relative knee angle measured clockwise from the distal end of the femur to the tibia, with $\theta_2 = 0^\circ$ corresponding to a fully extended leg where femur and tibia are collinear. Positive values of $\theta_2$ indicate knee flexion with the tibia rotated clockwise relative to the femur's longitudinal axis.

The Jacobian matrix relates joint velocities to foot velocities and is essential for trajectory planning and singularity avoidance. Taking partial derivatives of the forward kinematics equations yields:

$$\begin{aligned} J &= \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{bmatrix} \\ &= \begin{bmatrix} L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \\ -L_1 \sin(\theta_1) - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \end{bmatrix} \end{aligned} \tag{3}$$

The determinant of the Jacobian is:

$$\det(J) = -L_1 L_2 \sin(\theta_2) \tag{4}$$

For the inverse kinematics problem, which determines the joint angles $\theta_1$ and $\theta_2$ that position the foot at a desired Cartesian location $(x, y)$, the distance from the hip to the desired foot position is:

$$r = \sqrt{x^2 + y^2} \tag{5}$$

The angle of the knee $\theta_2$ can be determined using the law of cosines applied to the triangle formed by the hip, knee, and foot, after which $\theta_1$ can be calculated.

$$\theta_2 = \arccos\left(\frac{r^2 - L_1^2 - L_2^2}{2L_1L_2}\right) \tag{6}$$

$$\theta_1 = \arctan 2(x, y) - \arctan 2[L_2 \sin(\theta_2),\ L_1 + L_2 \cos(\theta_2)] \tag{7}$$

Trajectory generation is implemented onboard and in Cartesian space, where each foot's position is parameterized by (x,y) coordinates relative to the body frame. These coordinates are then mapped to joint-space angles through inverse kinematics, ensuring that each leg maintains the desired parabolic motion throughout the gait cycle. The motion timing is defined by Buehler clock parameters, which allows the time duty cycle between stance and swing to be adjusted independently.

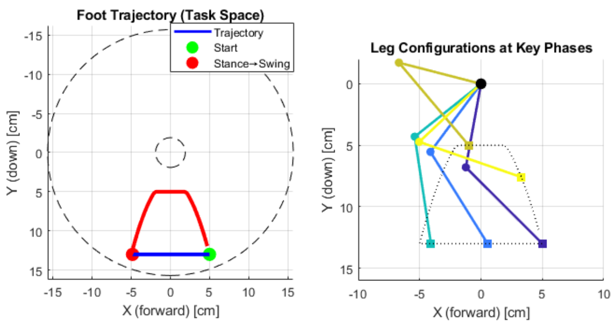


Fig. 4: The gait parabolic gait trajectory with max y clamped and feasible workspace (left). Joint and segment configuration over gait cycle (right).

### B. Experiment setup and parameter variation

To evaluate the effect of bilateral asymmetric gait parameters on climbing performance, a series of physical experiments will be conducted using the quadruped robot on a controlled sloped environment with discrete footholds. The objective is to compare lateral drift between symmetric and asymmetric trotting gaits at different slope inclines:

$\beta \in \{5°, 10°\}$

In the first phase, the robot will traverse the environment (Figure 5) using a baseline symmetric gait, where all legs operate with phase relationships standard to the trotting gait, as defined by the standard Buehler clock parameters.

In the second phase, asymmety will be introduced between contralateral leg pairs. These deviations will be systematically varied in magnitude, allowing observation of their influence on the robot's stability and trajectory correction. The same testing conditions (all other parameters kept constant aside from altering $\Delta\Phi_\alpha$) will be maintained to ensure fair comparison between gait modes.

By analyzing the robot's kinematic performance and orientation feedback under both gait configurations, the experiment aims to determine whether introducing controlled asymmetry can reduce mean lateral drift.

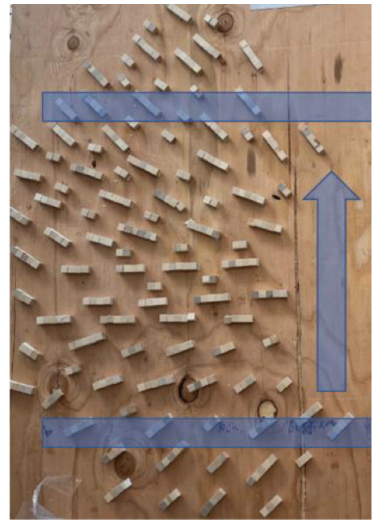

Fig. 5: Experiment Setup

For the trotting gait, we define contra-lateral asymmetry in terms of phase deltas for the actuation between each diagonal pair:

$$\Delta\Phi_1 = \Phi_{RR} - \Phi_{LF} \neq 0, \quad \Delta\Phi_2 = \Phi_{RF} - \Phi_{LR} \neq 0,$$

and the phase delta between both diagonal pairs:

$$\Delta\Phi_\alpha = \Phi_{LR} - \Phi_{RR} \neq \pi.$$

The first asymmetry parameter, $\Delta\Phi_1 = \Phi_{RR} - \Phi_{LF}$, quantifies phase asynchrony within the first diagonal pair. A value of $\Delta\Phi_1 = 0$ corresponds to the symmetric case where these diagonal partners move together, while $\Delta\Phi_1 > 0$ indicates that the right-rear leg leads the left-front leg in phase, initiating its stance phase earlier in the gait cycle.

The second asymmetry parameter, $\Delta\Phi_2 = \Phi_{RF} - \Phi_{LR}$, similarly quantifies asynchrony within the second diagonal pair, with $\Delta\Phi_2 = 0$ representing the symmetric case and $\Delta\Phi_2 > 0$ indicating that the right-front leg leads the left-rear leg.

The third asymmetry parameter, $\Delta\Phi_\alpha = \Phi_{LR} - \Phi_{RR}$, characterizes the phase relationship between the two diagonal pairs, measuring the temporal offset between when the first diagonal pair (LF–RR) initiates stance and when the second diagonal pair (RF–LR) initiates stance. In a symmetric trot, this offset equals exactly $\pi$ radians.

We theorized that there are functional differences between the front and rear legs of a quadrupedal robot, suggesting that they contribute differently to propulsion and stabilization of its ascent. This asymmetry helps explain why adjustments to $\Delta\Phi_\alpha$ can act as an effective steering mechanism, enabling the robot to counter lateral drift.

When considering that each gait cycle begins with the LF-RR pair entering stance, deviations from $\Delta\Phi_\alpha = \pi$ cause:

- $\Delta\Phi_\alpha < \pi$ results in the LR-RF pair entering stance sooner
- $\Delta\Phi_\alpha > \pi$ results in the LR-RF pair entering stance later

The performance metric to be optimized is *lateral drift* (in centimeters per trial), defined by the robot successfully climbing to and from a discrete finishing and starting line spanning laterally on the footholds board, with a fixed distance of 65cm. We aim to determine the $\Delta\Phi_\alpha$ values at different slope inclinations ($\beta$) that achieve minimum lateral drift ($\Delta s$), and assess the asymmetric gait's performance using a standard symmetric trotting gait as the control:

$$\Delta\Phi_1, \Delta\Phi_2 = 0; \quad \Delta\Phi_\alpha = \pi.$$

For experimental feasibility, $\Delta\Phi_1$, $\Delta\Phi_2$, and the foothold slant angle ($\gamma$) will be kept constant:

$$\Delta\Phi_1 = 0, \quad \Delta\Phi_2 = 0,$$

Foothold slant angle ($\gamma$) will be kept constant in the form of using the same footholds board layout, with the footholds kept in fixed orientations.

The primary dependent variable, lateral drift, is defined as the horizontal distance the robot traveled from the starting point. Rightward drift is recorded as positive values and leftward drift as negative values, following the convention that the positive lateral direction points right when facing upslope. This metric directly quantifies the robot's ability to maintain a straight climbing trajectory despite bilateral asymmetries in ground contact mechanics induced by slanted footholds, and represents the cumulative effect of many small lateral perturbations over the course of several gait cycles required to traverse the $65cm$ distance.

This research systematically characterizes the relationship between $\Delta\Phi_\alpha$, terrain slope ($\beta$), and lateral drift $\Delta s$ through controlled experimentation on a fixed foothold configuration (constant slant angle $\gamma$), testing whether $\Delta\Phi_\alpha$ represents a useful control parameter for terrain adaptation and whether simple relationships (such as optimal $\Delta\Phi_\alpha$ proportional to slope angle $\beta$) emerge from the data. By holding $\Delta\Phi_1 = \Delta\Phi_2 = 0$, maintaining synchrony within each diagonal pair, and varying only the inter-diagonal relationship $\Delta\Phi_\alpha$, the experiment isolates the specific contribution of diagonal pair coordination to climbing performance, separating it from the more complex bilateral asymmetries that would arise from non-zero $\Delta\Phi_1$ and $\Delta\Phi_2$. This simplified parameter space makes the research feasible within project resource constraints while still addressing a genuine knowledge gap—no prior work has quantified how controlled deviations from the canonical trot phase relationship affect lateral stability on sloped terrain with discrete footholds.

## III. Results and Discussion

To evaluate our hypothesis that varying the phase delta between the start of the swing phase for both diagonal pairs of legs from the cannonical value ($\Delta\phi_\alpha$ = $\phi_{LR}$ - $\phi_{RR} \neq \pi$) can mitigate accumulated lateral drift, experiments were conducted across two slopes (5° and 10°) while only varying the bilateral phase deviation parameter $\Delta\phi_\alpha$. All gait and climbing parameters, as well as the physical robot design, were tuned for each tested incline angle ($\beta$ = 5°, 10°), after which they remained constant for all trials for each $\beta$. A value of $\Delta\phi_\alpha$ = 180° corresponds to the symmetric trotting gait, while values above or below 180° represent controlled phase asymmetries. For successful climbs, lateral drift and forward distance gained were recorded at the finish line. For unsuccessful trials (before the robot was about to fall or cross the vertical line) the final lateral drift (x) and vertical progress (y) were logged at the robot's stopping point. Three trials were conducted for each of our selected $\Delta\phi_\alpha$ values. For the 5° incline, we tested $\Delta\phi_\alpha$ = 140°,160°, 180°, 200°, 220° (refer to video 1 to 5). For the 10° incline, we tested $\Delta\phi_\alpha$ = 160°,170°, 180°, 190°, 200° (refer to video 6 to 10). Each lateral drift and vertical progress value was measured with a rounding error of 0.1 cm, taken from the starting point to the center of the Arduino board. The robot was always placed on the same, marked starting position, and the timer started, when the LF-RR leg pair entered stance phase.

When $\Delta\phi_\alpha < 180°$, we expected the robot to drift leftward because the left front leg touches down earlier than the right front leg in each gait cycle. Likewise, when $\Delta\phi_\alpha > 180°$, we expected the robot to drift to the rightward because the right front leg touches down earlier than the left front leg. We hoped to find whether there was a significant relationship between lateral drift and varying $\Delta\phi_\alpha$ and whether there were values of $\Delta\phi_\alpha$ that resulted in lower, absolute lateral drift.

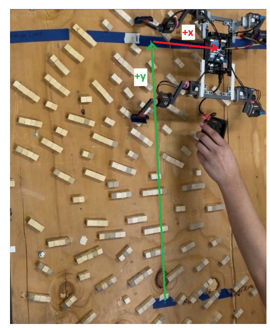


Fig. 6: The starting position and ending position measurements with x (lateral drift) and y (vertical distance climbed).

At the 5° slope, there was an apparent trend of turning more right when $\Delta\phi_\alpha > 180°$ and more left when $\Delta\phi_\alpha < 180°$ (Fig 7b). As expected, the 140° and 160° gaits resulted in significant leftward drift (averaging 19.9 cm and 18.8 cm, respectively), leading to failure as the robot exited the left side of the board. Conversely, the 220° gait caused a rapid rightward drift (+27.9 cm), leading to failure on the right side. The symmetric control gait (180°) was stable and successfully completed all trials, though it exhibited a natural rightward bias with an average drift of +13.1 cm.

Interestingly, a 200° phase offset on the 5° slope, resulted in a near-zero average drift of 2.7 cm, seemingly canceling out the natural rightward bias seen in the symmetric control gait. However, this correction seemed to produce high variance in lateral drift, as the 200° gait exhibited the highest standard deviation in lateral drift (±19.6 cm) of all tested configurations. In individual trials, the robot drifted significantly left in two

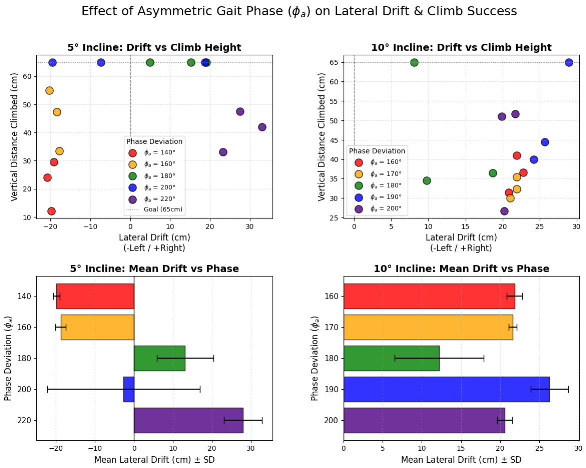


Fig. 7: Effect of Asymmetric Gait Phase ($\Delta\phi_\alpha$) on Lateral Drift and Climb Success. (Fig. 7a Top left) lateral drift vs climb height for the 5° incline; (Fig. 7b Bottom left) mean drift vs phase deviation for the 5° incline, with standard deviation; (Fig. 7c Top right) lateral drift vs climb height for the 10° incline; (Fig. 7d Bottom right) mean drift vs phase deviation for the 10° incline, with standard deviation.

instances and right in one, suggesting that maybe the robot became highly sensitive to initial conditions and foothold irregularities, rather than robustly holding a straight line, though it is difficult to claim with certainty this is the case, due to sampling only 3 trials per $\Delta\phi_\alpha$

Further analysis was performed on the steeper 10° incline (Fig. 7d) to determine if the steering effects observed at 5° could scale to more challenging terrain. However, the 10° slope did not show the same consistent drift trends. Instead, two specific footholds repeatedly caused the robot's left rear foot to get stuck for multiple gait cycles, triggering an unintended rightward rotation and overshadowing the effects of $\Delta\phi_\alpha$. We expect that footholds spaced farther apart would have avoided this issue, as the parabolic gait trajectory with high steepness and clamped maximum y would allow the leg to clear the foothold without getting stuck, while footholds spaced closer together would have similarly avoided the issue because they would be too close together for the robot's foot to even fit between them. This issue is unique to a margin of foothold spacing that is nearly perfect for the shape of the foot to become securely wedged. Since this terrain interaction dominated the robot's behavior, the relationship between phase asymmetry and drift could not be conclusively evaluated at this slope, and regardless of the phase deviation (ranging from 160° to 200°), the robot consistently drifted to the right (+x direction) in every single trial, with no gait successfully generating a leftward counter-steer. The symmetric 180° gait performed best in terms of minimizing drift (+12.2 cm) and achieved the only partial success (reaching the top in one trial), while the asymmetric deviations that noticeably influenced steer on the lower slope only worsened the rightward drift and hastened failure (any leg of the robot reaching past +38cm in lateral drift from the starting point). The value of +38 cm was chosen because the left side of the board ends at -38 cm in lateral drift from the starting point, resulting in the robot falling. This deviation from expected behavior does not necessarily contradict the hypothesis; rather, it clearly shows that asymmetry significantly affected lateral drift, albeit negatively. It indicates that foothold-leg interaction effects can more easily exceed gait-level adjustments at higher inclines, and that further examination may need to be performed on a tighter spread of $\Delta\phi_\alpha$ values in order to find an optimum for reducing lateral drift.

To better understand the raw measurements collected during the climbing trials, we performed additional analysis on the robot's lateral drift, progress along the slope, and trajectory curvature across the different phase-offset conditions, in videos demo, which show the lateral drift trajectories of the third trial for all $\Delta\phi_\alpha$ at the 5° incline. The videos clearly highlight the nonlinear drift patterns observed in the asymmetric trials where some $\Delta\phi_\alpha$ values generated increasingly curved or parabolic paths, while the symmetric gait produced a curved, but much straighter climb. Similarly, videos demo show the lateral drift trajectories of the third trial for all $\Delta\phi_\alpha$ at the 10° incline. The 140° asymmetric gait displayed strong instability: the robot failed to reach even half of the board and consistently drifted to the left (–x direction) by 20.8 cm, producing a pronounced left-turning parabolic trajectory. The 160° asymmetric gait also could not complete the climb and drifted left by 17.8 cm, with the left-front leg eventually slipping off the board, indicating insufficient uphill stability. The 180° symmetric gait served as the control and reached the end goal in 2 min and 1 sec. It initially drifted slightly right (+x) on the lower section of the slope but maintained a nearly straight path on the upper half, finishing with a modest right drift of 7 cm. The 200° asymmetric gait successfully completed the climb in 2 min and 4 sec; however, it showed a more complex pattern—drifting slightly left during the lower half of the climb, then gradually shifting right near the top as foothold perturbations accumulated, ending with a small net leftward drift of 5.4 cm. Finally, the 220° asymmetric gait could not reach the upper half of the slope and drifted consistently to the right (+x) by 23.3 cm, eventually causing the right-front leg to pass our threshold for failure.

When compared with our original hypothesis, the videos partially align with expectations. We predicted that gaits with $< 180°$ would drift leftward, while gaits with $> 180°$ would drift rightward. The experiments broadly matched these trends: 140° and 160° (both <180°) indeed drifted left and were unable to complete the climb, while 220° drifted far to the right, and 200° performed marginally better, actually having drifted to the left with a low absolute lateral drift, though the standard deviation between the 3 trials was the greatest. The symmetric condition drifted moderately toward the right. Overall, the additional trajectory analysis reinforces the hypothesis that $\Delta\phi_\alpha$ asymmetry can influence lateral drift as a steering mechanism.

To further evaluate robot performance on the steeper 10° incline, we analyzed the lateral drift profiles, climb progress, and failure modes across five gait phase-offset conditions ranging from 160° to 200°. At 160° $\Delta\phi_\alpha$ the robot was able to climb approximately 95% of the board before exceeding our right-side threshold for failure. During the lower half of the climb, it drifted slightly to the right (+x direction), and this rightward drift increased in the upper half as foothold perturbations accumulated, eventually causing the right-front leg to slip off the board. The 170° configuration performed worse, reaching only half of the board before failing; it drifted to the right by 21.9 cm, again ending with the right-front limb stepping past the failure threshold. The third trial of the symmetric condition, which previously demonstrated a slight rightward bias on the 5° slope, was unable to complete the climb at 10°. It reached only half of the board and consistently drifted to the right, with the right-front leg once more slipping past the threshold. Similarly, the 190° condition climbed up to roughly 95% of the board but exhibited rightward drift throughout the ascent. In the lower half of the board the drift began gradually, then increased in the upper half where the robot slipped on the unblocked downhill (right) side, ultimately leading to failure when the right-front leg reached the threshold. Finally, at 200° $\Delta\phi_\alpha$ the robot reached about 95% of the board. It showed minimal lateral movement during the lower half of the climb but began drifting rightward in the upper half, where slip again occurred due to the lack of lateral support on the downhill side, causing the right-front leg to again reach the failure threshold. Across all conditions at 10°, every video-recorded $\Delta\phi_\alpha$ trial experienced significant rightward drift and ultimately failed before reaching the goal, with consistent failure through the downhill (right) front leg exceeding our failure threshold of +38 cm of lateral drift from the starting point. This pattern highlights the increased challenge posed by the steeper incline and indicates that none of the tested phase offsets provided adequate lateral stability under these conditions.

A notable performance difference emerged between the 5° and 10° incline trials. Additional testing revealed that reducing the step length improved locomotion at the steeper 10° slope. This effect is partly due to the increasing density of footholds as the incline rises. At both very shallow and very steep angles, the robot's effectiveness becomes strongly dependent on the geometry and design of its feet, highlighting the importance of foot size and shape in discrete-foothold environments.

These findings provide a nuanced answer to our research question. The results at 5° confirm that contralateral, inter-leg phase asymmetry is indeed a functional mechanism for steering and drift correction, producing a predictable, monotonic response that aligns with our hypothesis. However, the 10° results, which show that the performance of all asymmetric offsets for the 10° incline performed worse than the 180° control, reveal a critical boundary condition: the influence of steering via altering the $\Delta\phi_\alpha$ of the gait is finite. When the slope angle increases, the foothold-terrain interactions and lack of downhill support dominate the robot's dynamics, rendering the phase adjustments insufficient to counteract the physical forces pulling the robot downhill. Perhaps more testing with a tighter array of $\Delta\phi_\alpha$ values for higher inclines is necessary to find an optimum for reducing lateral drift (e.g. $\Delta\phi_\alpha$ = 176, 178, 180, 182, 184). But from the data we do have, we cannot reach any conclusion for the 10° incline, other than that our tested $\Delta\phi_\alpha$ offsets all resulted in an increase in lateral drift.

Our results and analysis reveal a novel finding: phase asymmetry between concurrently actuating diagonal leg pairs provides a controllable and monotonic "fine-tuning" method of steering correction at low inclines ($\beta$<=5°) but cannot compensate for the dominating environmental conditions introduced by steeper terrain, as factors such as foothold geometry can overpower gait-based corrections and more drastically impact the robot's trajectory. This may be useful for designing legged locomotion controllers on discrete foothold terrain. The findings imply that while asymmetric gaits can be integrated as a low-level steering tool for minor course corrections, they must be part of a hybrid control strategy. For steeper slopes, gait asymmetry alone is insufficient, and must likely be coupled with active foothold selection or posture adaptation to maintain mobility.

In conclusion, we were able to answer our original research question with some limitations, which especially include the very small sample size of data and possible inconsistencies in data collection due to human error–which could have resulted in minor variances in starting conditions and consequently, foothold dynamics for each climb, despite our best efforts to always initiate the climb at $\phi$=0–and the restrictive scope and timeline of the project. Our data for the 5° incline seems to support our hypothesis, while the data for the 10° incline is inconclusive.

## IV. Conclusion

This work set out to determine whether contralateral asymmetric gait parameters—implemented through deviations in the inter-diagonal phase offset $\Delta\Phi_\alpha$ — could reduce lateral drift and improve climbing performance for quadrupedal robots navigating sloped terrain with discrete, slanted footholds. Experiments on a 5° incline confirmed that controlled phase asymmetry produced predictable, monotonic steering behavior: values of $\Delta\Phi_\alpha < 180°$ consistently steered the robot left, while values $> 180°$ steered it right. In particular, the 200° configuration substantially reduced the inherent rightward drift observed in the symmetric gait, demonstrating that phase asymmetry can function as a fine-tuning mechanism to counteract terrain-induced lateral forces. These results support the hypothesis that mild bilateral asymmetry can improve directional stability on 5° incline slopes.

However, on the steeper 10° incline, robot behavior was dominated by foothold–leg interaction effects—especially sticking and slipping of the left-rear leg—which overwhelmed the steering influence of phase asymmetry and caused the right side of the robot to repeatedly slip, thus steering the robot rightward. Consequently, all tested $\Delta\Phi_\alpha$ values resulted in rightward drift and mainly occurred as failure, indicating that

TABLE I: 5 degree Inclined Trials

| $\phi_a$ | Trial 1 | | | Trial 2 | | | Trial 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | x(cm) | y(cm) | Time(Minutes:Seconds) | x(cm) | y(cm) | Time(Minutes:Seconds) | x(cm) | y(cm) | Time(Minutes:Seconds) |
| 140 | -19.1 | +29.5 | 1:43 | -19.7 | +12.0 | 1:31 | -20.8 | +24.0 | 1:43 |
| 160 | -18.4 | +47.3 | 2:38 | -20.3 | +55.0 | 3:12 | -17.8 | +33.5 | 1:52 |
| 180 | +19.1 | +65.0 | 1:38 | +15.3 | +65.0 | 1:32 | +5.0 | +65.0 | 2:01 |
| 200 | +18.8 | +65.0 | 2:05 | -19.5 | +65.0 | 1:35 | -7.4 | +65.0 | 2:04 |
| 220 | +33.0 | +42.0 | 1:00 | +27.5 | +47.5 | 1:08 | +23.3 | +33.0 | 0:52 |

TABLE II: 10 degree Inclined Trials

| $\phi_a$ | Trial 1 | | | Trial 2 | | | Trial 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | x(cm) | y(cm) | Time(Minutes:Seconds) | x(cm) | y(cm) | Time(Minutes:Seconds) | x(cm) | y(cm) | Time(Minutes:Seconds) |
| 160 | +22.8 | +36.6 | 2:05 | +20.8 | +31.4 | 1:25 | +21.9 | +41.0 | 3:58 |
| 170 | +21.0 | +30.0 | 1:40 | +21.9 | +35.4 | 2:17 | +21.9 | +32.3 | 1:32 |
| 180 | +9.8 | +34.5 | 1:37 | +8.1 | +65.0 | 3:26 | +18.7 | +36.4 | 2:05 |
| 190 | +25.7 | +44.5 | 1:40 | +24.2 | +40.0 | 1:38 | +28.9 | +65.0 | 2:06 |
| 200 | +20.2 | +26.6 | 1:01 | +21.7 | +51.7 | 1:58 | +19.9 | +51.0 | 1:59 |

on more challenging terrain, gait-level phase modulation alone is insufficient to counteract asymmetric loading and reduced lateral support. Thus, while the hypothesis is supported at mild slopes, the results at higher slopes remain inconclusive.

Future work should explore (1) a finer resolution of $\Delta\Phi_\alpha$ values near performance-critical regions, (2) integration of active foothold selection or posture adaptation strategies, (3) closed-loop modulation of phase asymmetry using force-sensing resistor (FSR) feedback to better characterize foot–terrain contact dynamics, and (4) exploring a combination of control methods with phase asymmetry as a hybrid control strategy. Together, these directions may enable hybrid locomotion controllers that preserve the advantages of asymmetric gaits while overcoming their limitations on steeper or more irregular terrain.

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