CS7646 Fall 2019
Project 1: Martingale
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1. In Experiment 1, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning thoroughly.
    - **Approximately 100%**. Figure 2 shows that when the simple simulator is run 1000 times, on average, by approximately the 200th bet, every simulation has reached $80. Further proof is that the standard deviation is 0 after that as well given every simulation reached $80 by approximately the 200th bet.

2. In Experiment 1, what is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning thoroughly. Go here to learn about expected value: https://en.wikipedia.org/wiki/Expected_value
    - According to the Wikipedia article, according to the law of large numbers, the "arithmetic mean of the values almost surely converges to the expected value as the number of repetition approaches infinity". In our case, given the value converges to $80 before the 1000th sequential bet, **the expected value of our winnings after 1000 sequential bets is $80.**

3. In Experiment 1, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not) thoroughly.
    - The standard deviations **do reach maximum value** as the episode_winnings start to increase in variance at a 2-folder pace per bet where the results are losses. This is especially true in simulations where there were multiple losses prior to reaching $80.
    - Once maximum value is reached, the standard deviations **quickly converge back to the mean** as more and more simulations reach $80, leaving no room for variance among the samples.

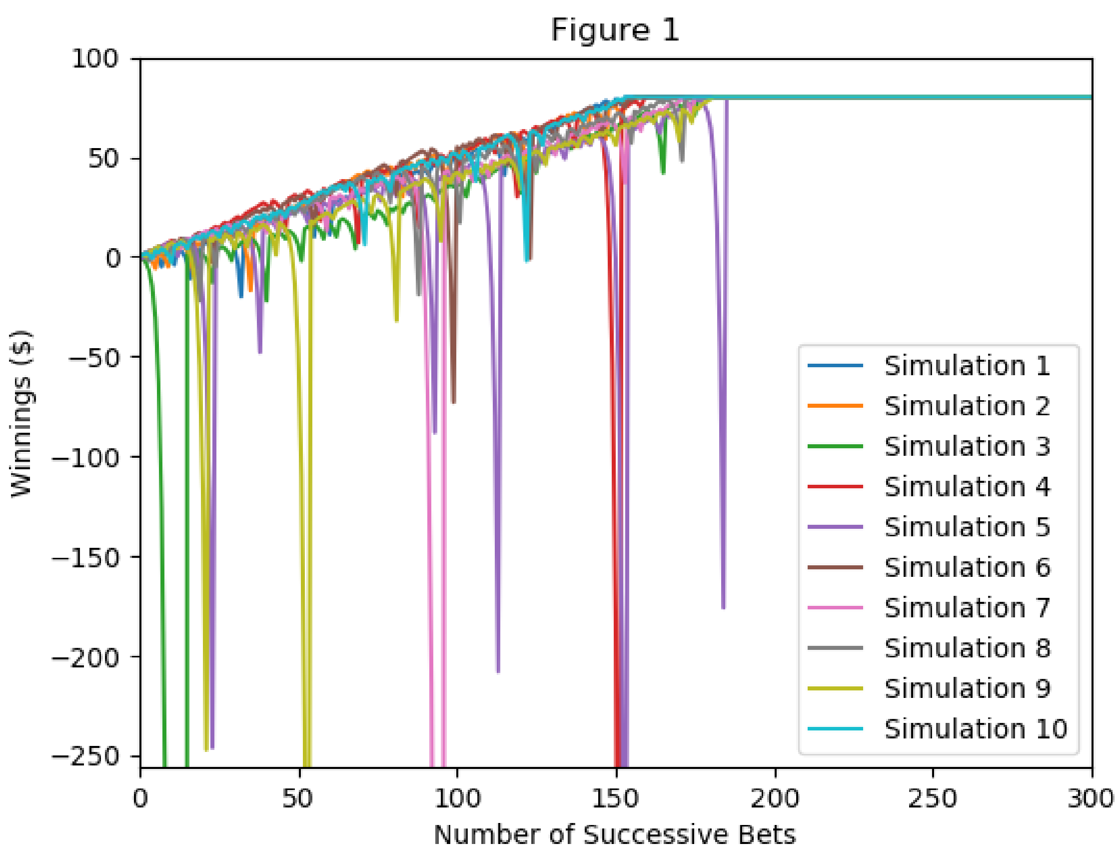
4. In Experiment 2, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning using the experiment thoroughly. (not based on plots)
    - The probably of winning $80 within 1000 sequential bets according to my experiment was **approximately 66%.** After printing out the final, 1000th bet on each of the 1000 simulations, I found 660 instances out of 1000 had a value of $80.

5. In Experiment 2, what is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning thoroughly. (not based on plots)

- By printing out the average episode winnings per each sequential bet, I was able to find that by the end of the 1000 sequential bets, average episode winning was -$33.6. This is an indication that the **estimated expected value of our winnings after 1000 sequential bets is -$33.6**

6. In Experiment 2, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not) thoroughly.
    - As the number of sequential bets increase, the standard deviations **reach a maximum value, then stabilize.**
    - The standard deviation reaches the maximum value as simulations start to reach the maximum threshold of winnings at either -$256 or $80.
    - The standard deviation then stabilizes as there is a finite amount of bankroll that the simulator can use. Those that have already reached the maximum threshold of winnings at either -$256 or $80 will stay that way. Others which have not yet reached it will soon reach it, and then stay that way, resulting in the stabilization.

7. Include figures 1 through 5.



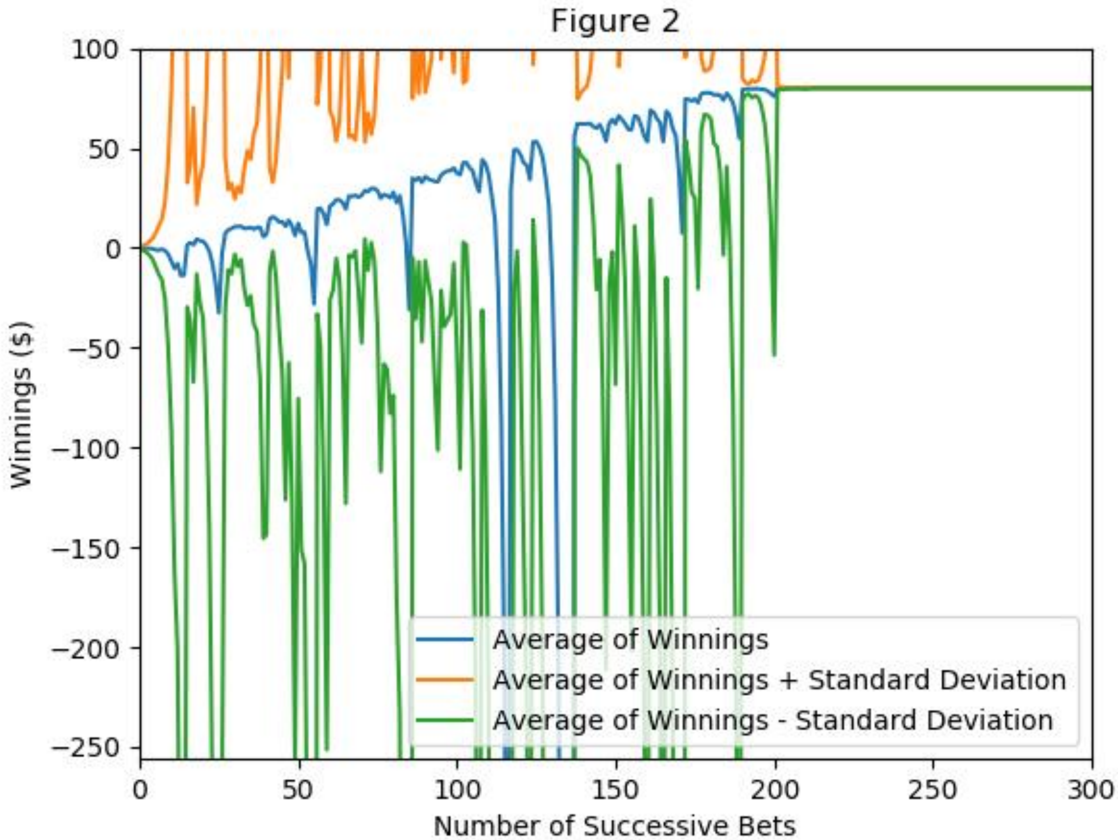


Figure 2
Winnings ($)
Number of Successive Bets
Average of Winnings
Average of Winnings + Standard Deviation
Average of Winnings - Standard Deviation
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300

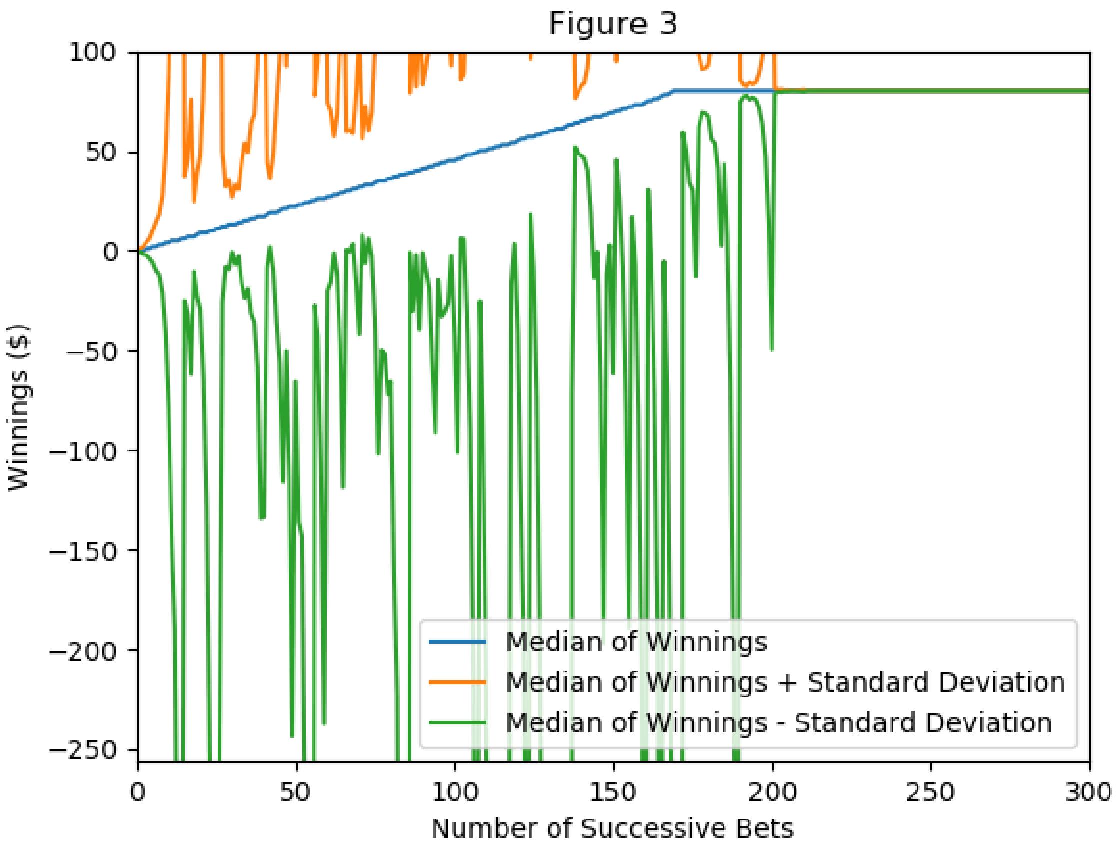


Figure 3
Winnings ($)
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300
Number of Successive Bets
Median of Winnings
Median of Winnings + Standard Deviation
Median of Winnings - Standard Deviation

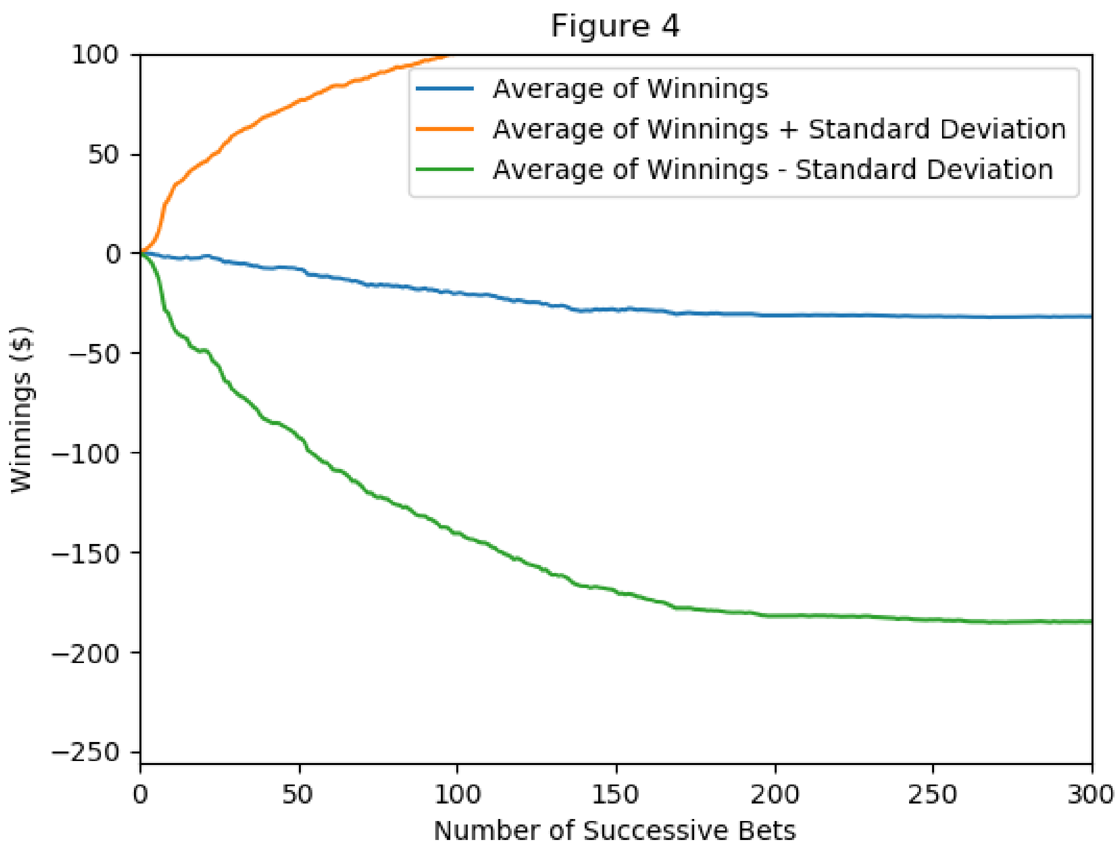


Figure 4
100
50
0
−50
−100
−150
−200
−250
Winnings ($)
0
50
100
150
200
250
300
Number of Successive Bets
Average of Winnings
Average of Winnings + Standard Deviation
Average of Winnings - Standard Deviation

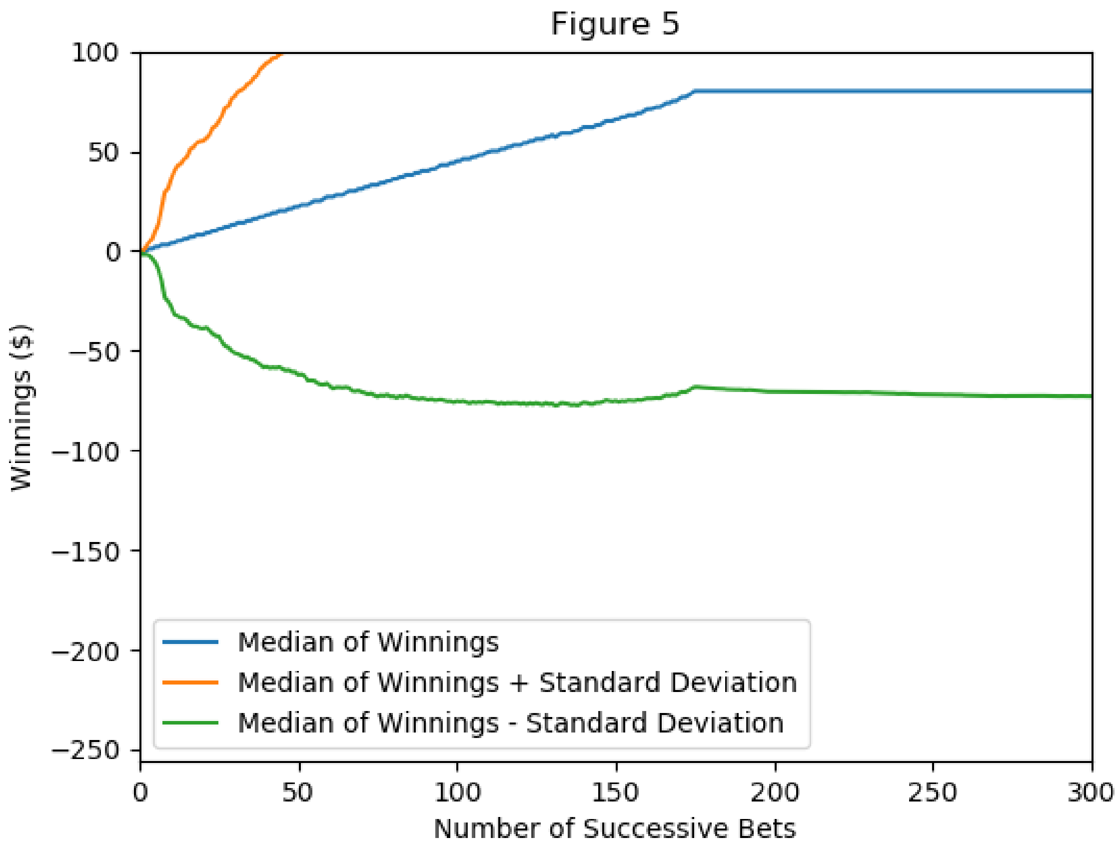


Figure 5
Winnings ($)
Number of Successive Bets
Median of Winnings
Median of Winnings + Standard Deviation
Median of Winnings - Standard Deviation
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300