

Adversarially Robust Kernel Smoothing

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March, 2022

Learning under distribution shift

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Empirical Risk Minimization

$$\min_{\theta} \frac{1}{N} \sum_{i=1}^N l(\theta, \xi_i), \quad \xi_i \sim \textcolor{magenta}{P}_0$$

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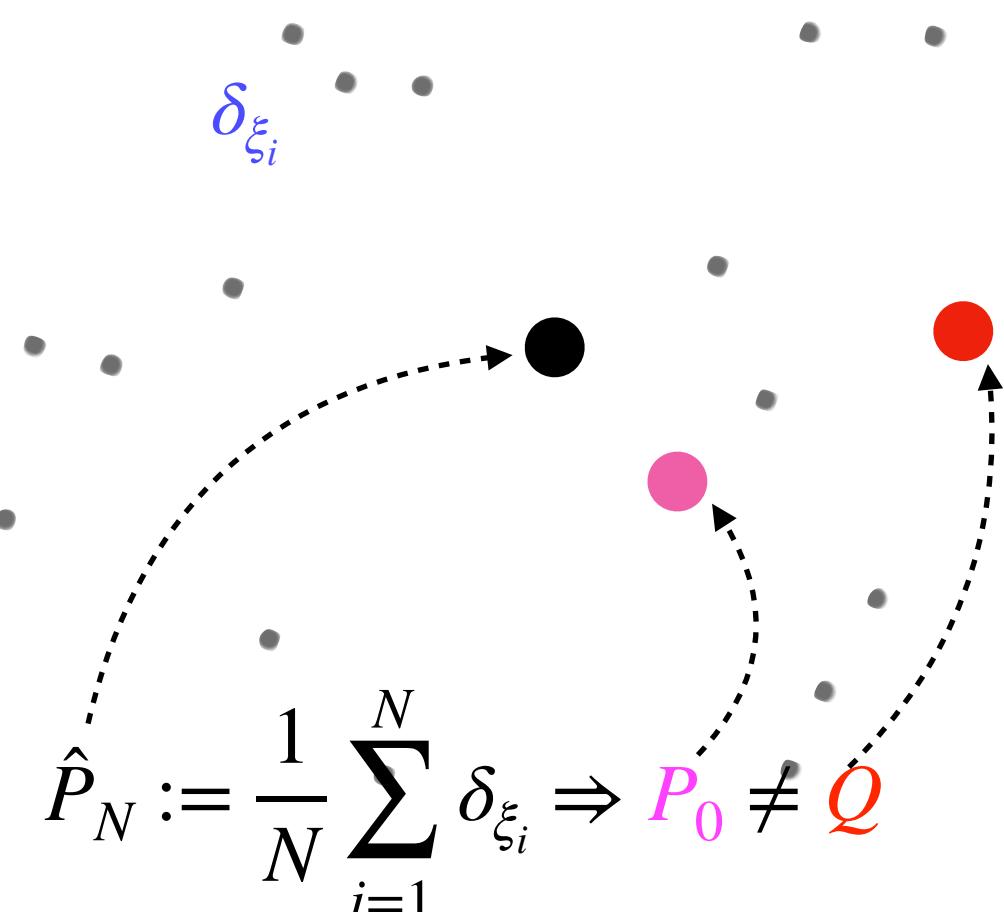
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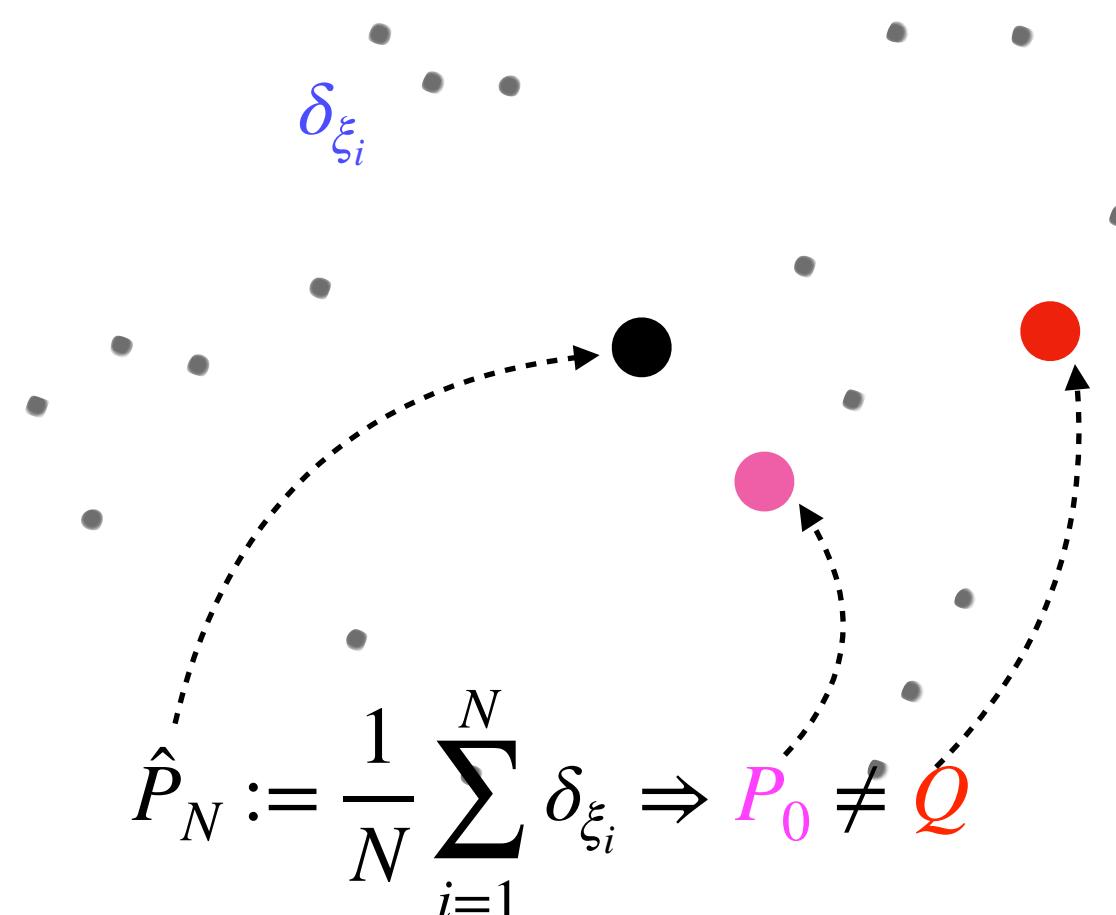
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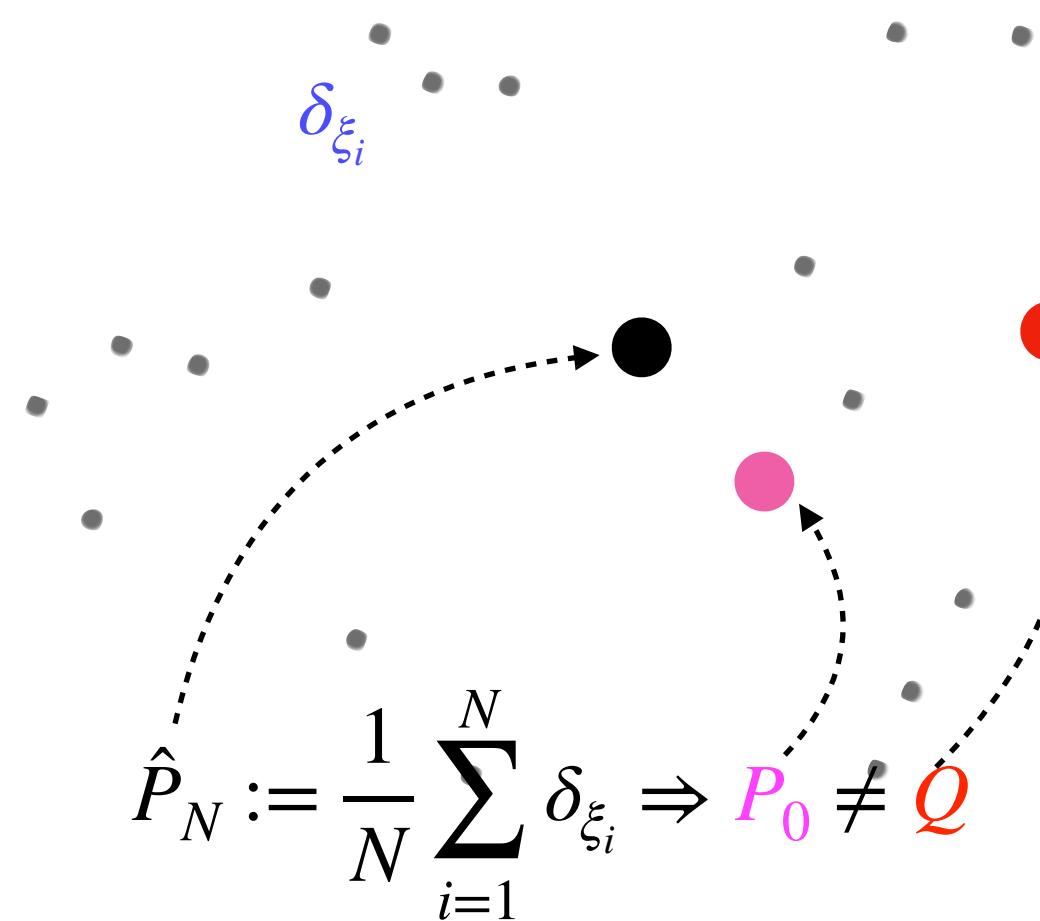


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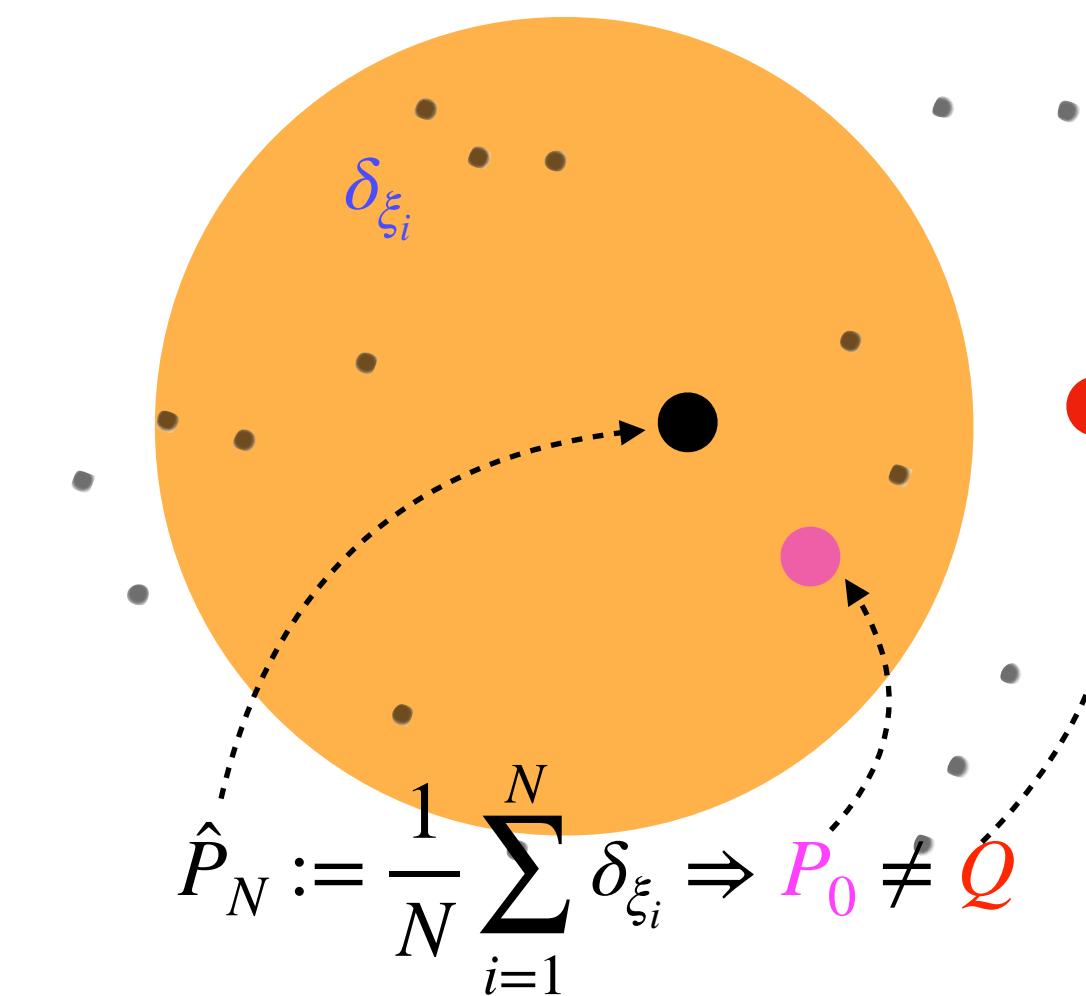
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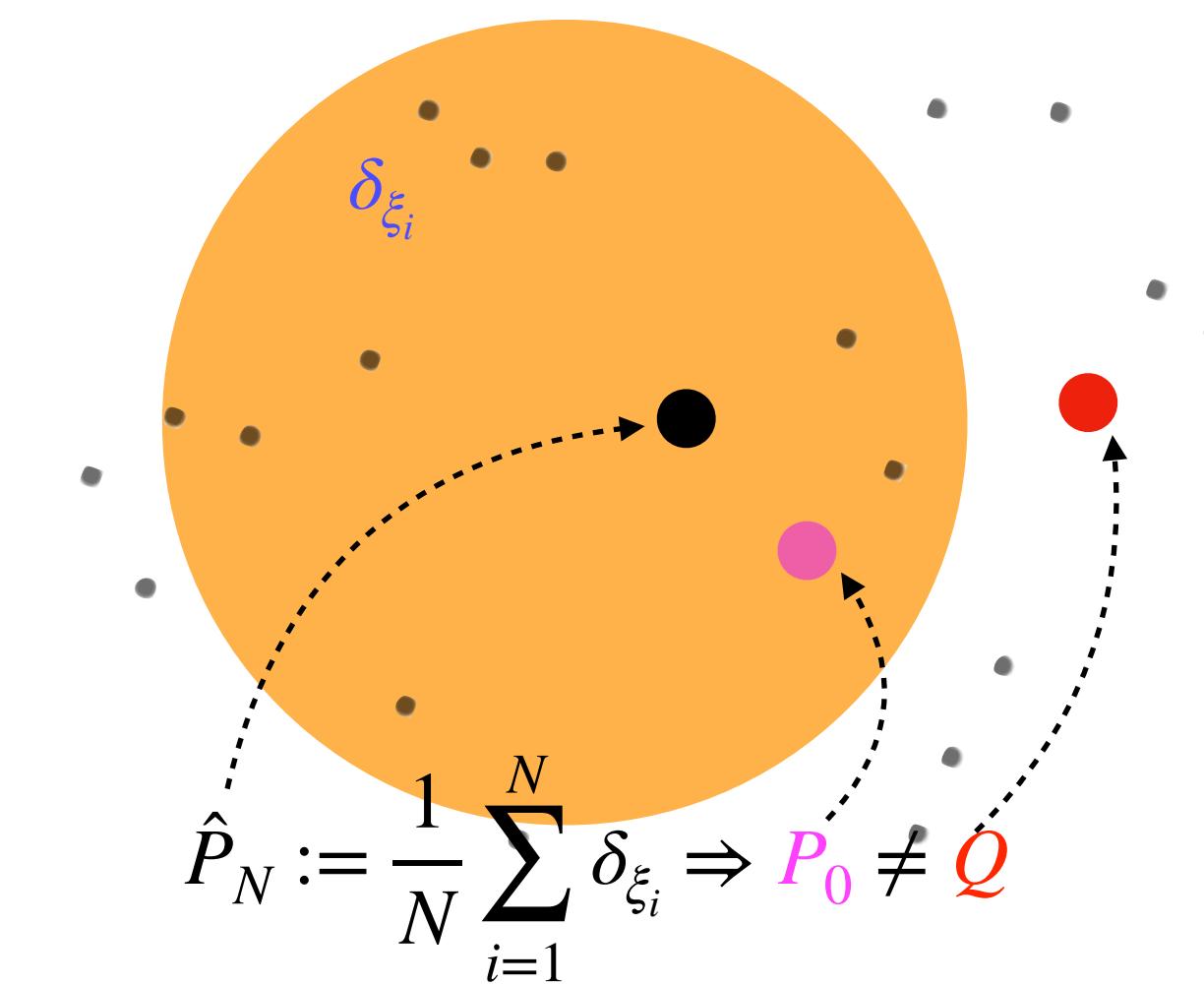
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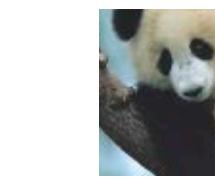


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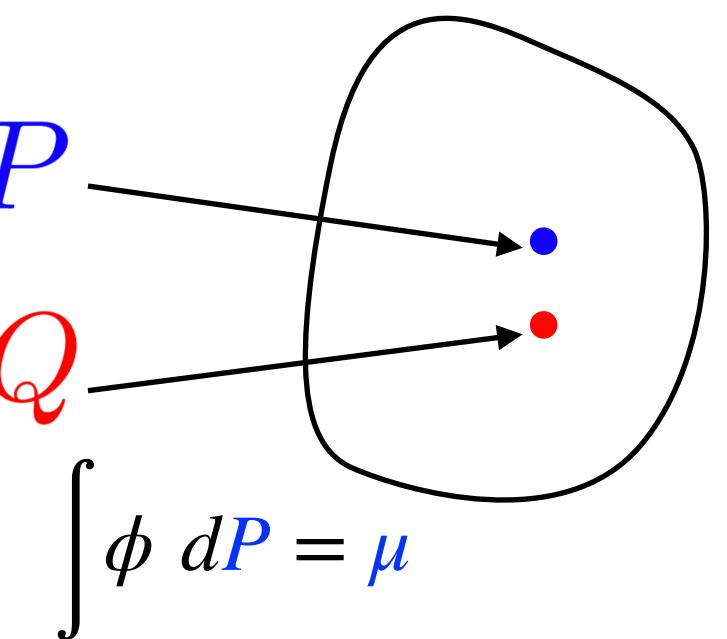
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$$\begin{aligned} \text{MMD}_{\mathcal{H}}(Q, P) &:= \sup_{\|f\|_{\mathcal{H}} \leq 1} \int f d(Q - P) \\ &= \mathbb{E}_{x, x' \sim Q} k(x, x') + \mathbb{E}_{y, y' \sim P} k(y, y') \\ &\quad - 2 \mathbb{E}_{x \sim Q, y \sim P} k(x, y). \end{aligned}$$



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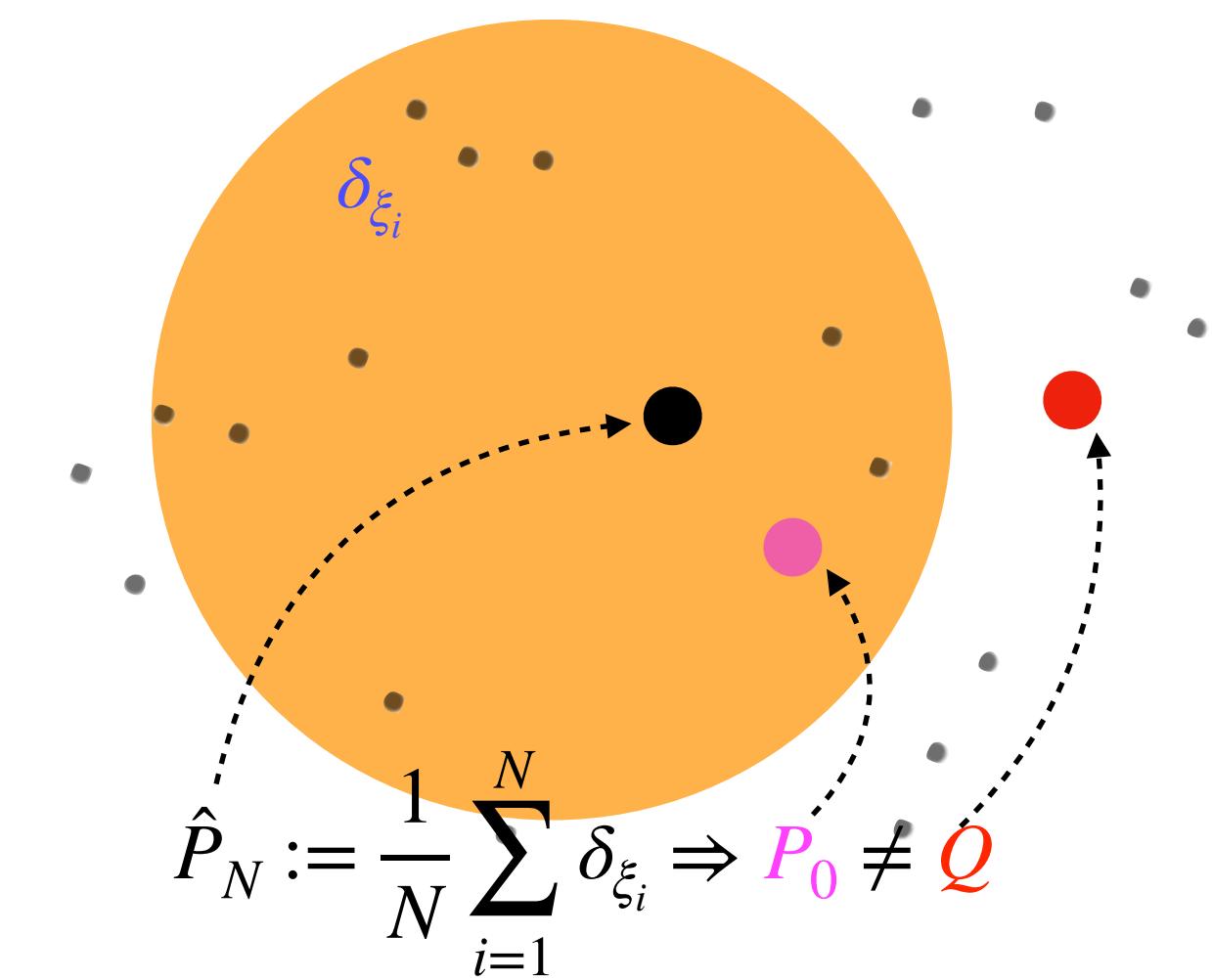


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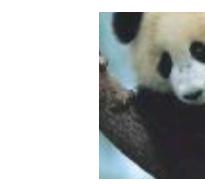


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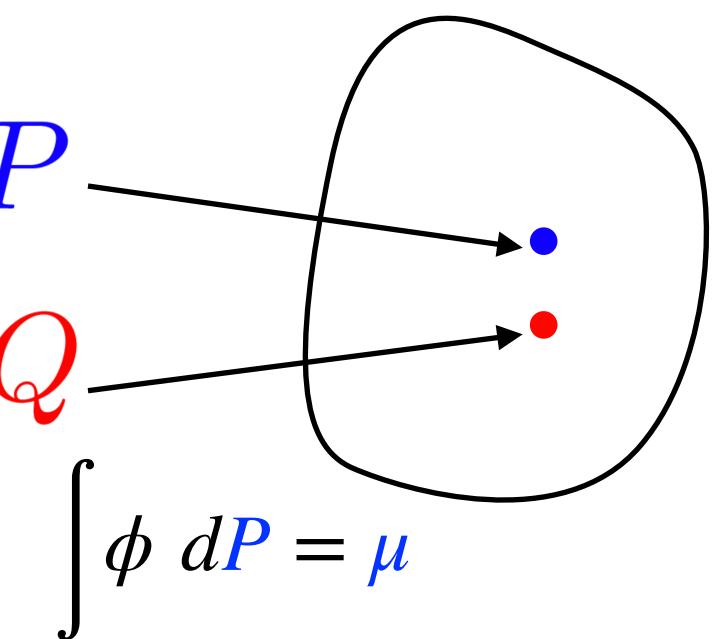
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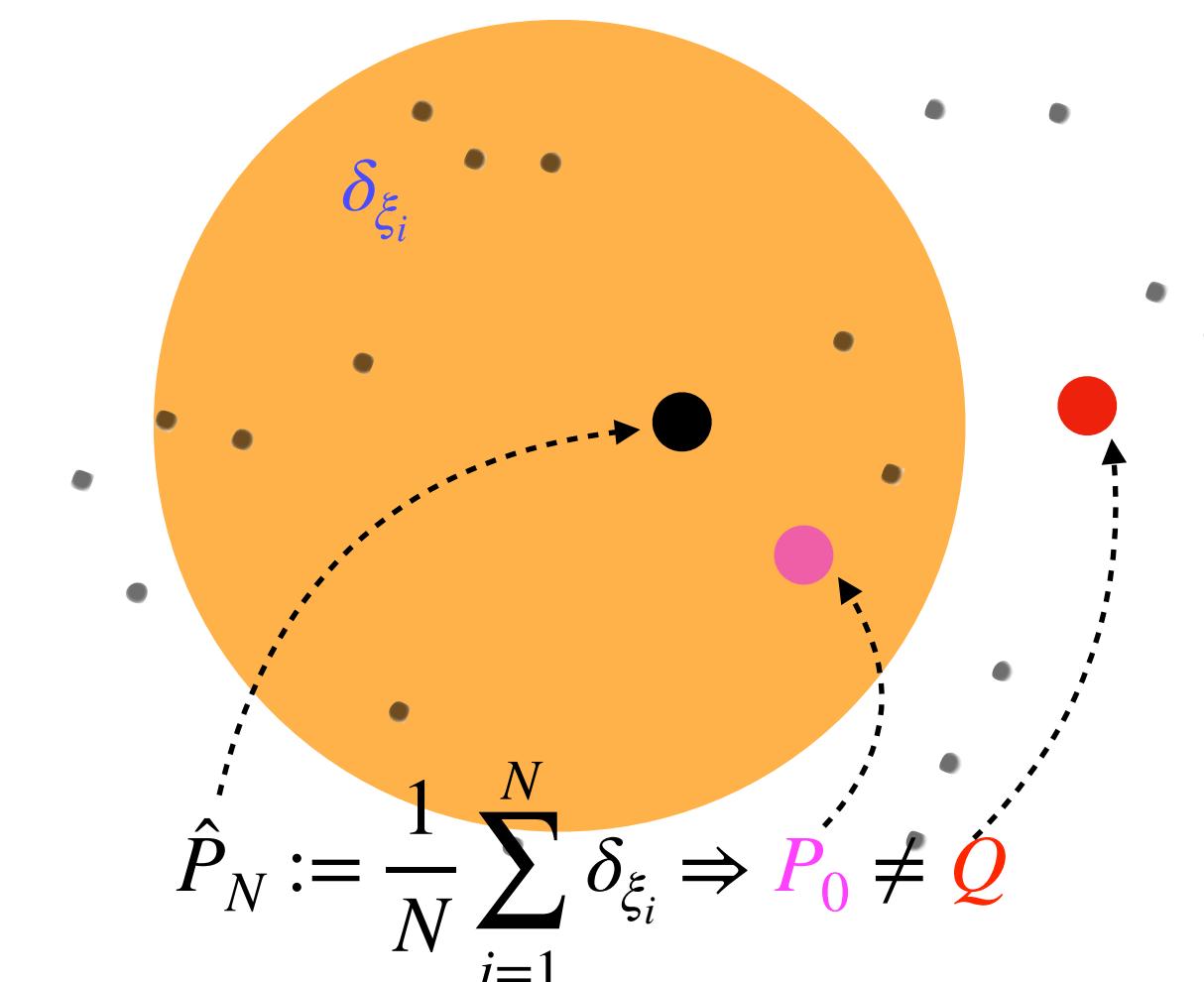
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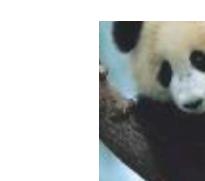


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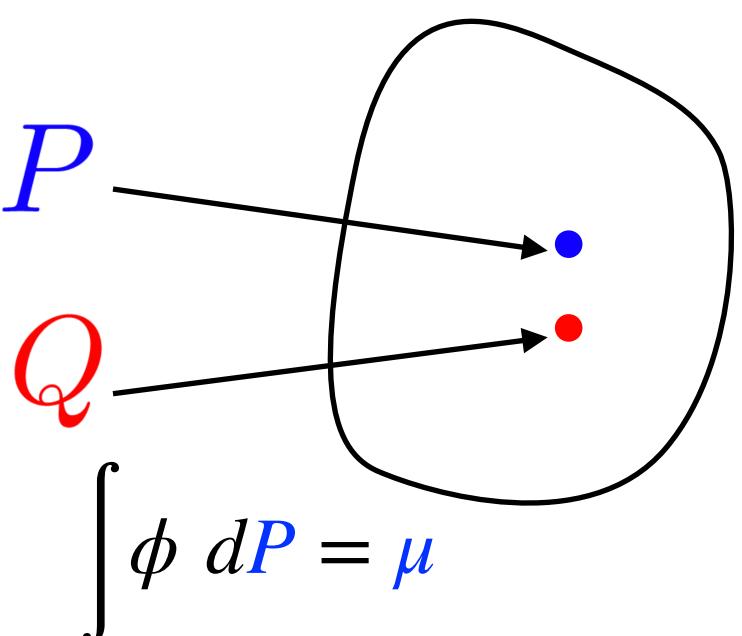
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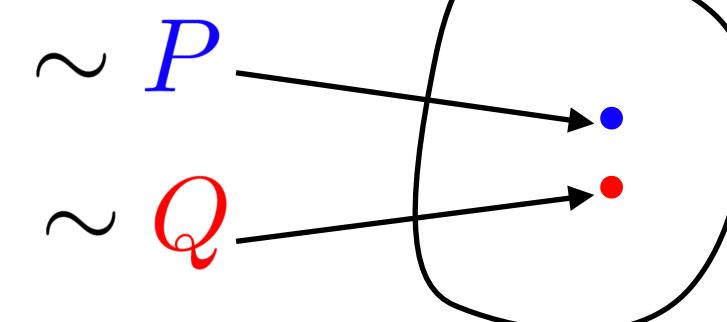
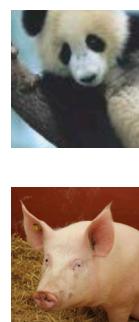
- We can bound performance under Q ($\neq P_0$) beyond statistical fluctuation (classical learning theory)
- **Question:** how do we actually solve an MMD-constrained optimization problem? **(Non-trivial!)**

Kernel distributionally robust optimization

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Primal DRO (not solvable as it is)

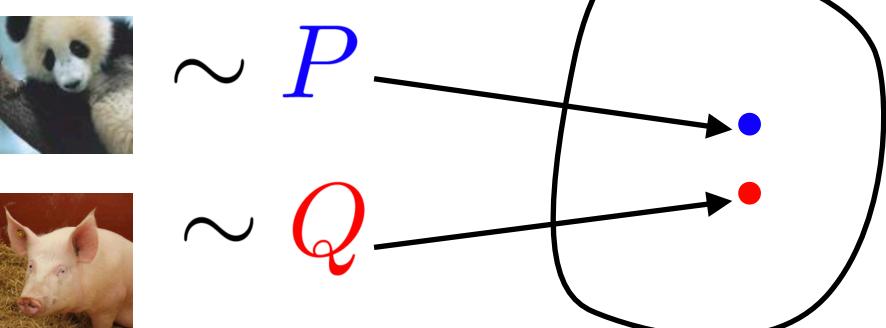
$$(\text{DRO}) \quad \min_{\theta} \sup_{\substack{\mathbb{E}_{\mathcal{Q}} l(\theta, \xi) \\ \text{MMD}(\mathcal{Q}, \hat{P}) \leq \epsilon}} \mathbb{E}_{\mathcal{Q}} l(\theta, \xi)$$



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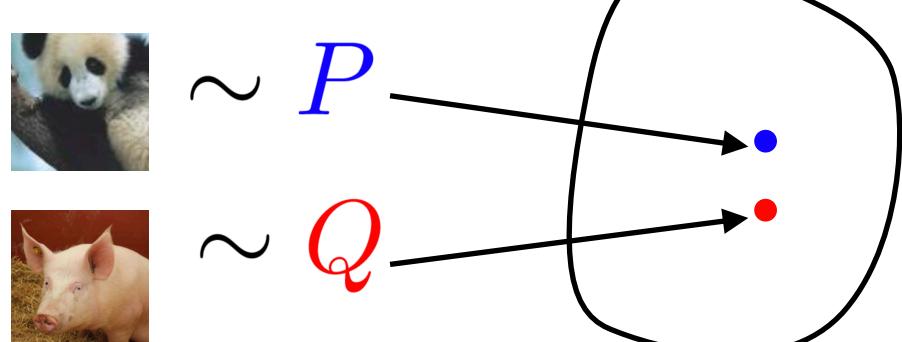
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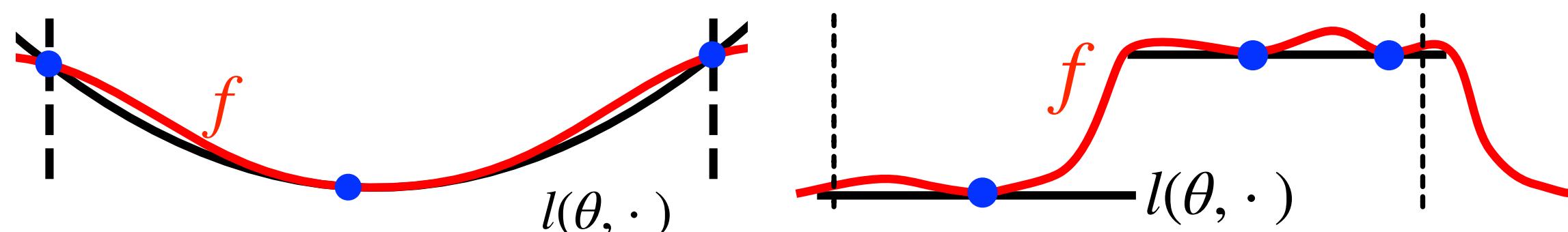
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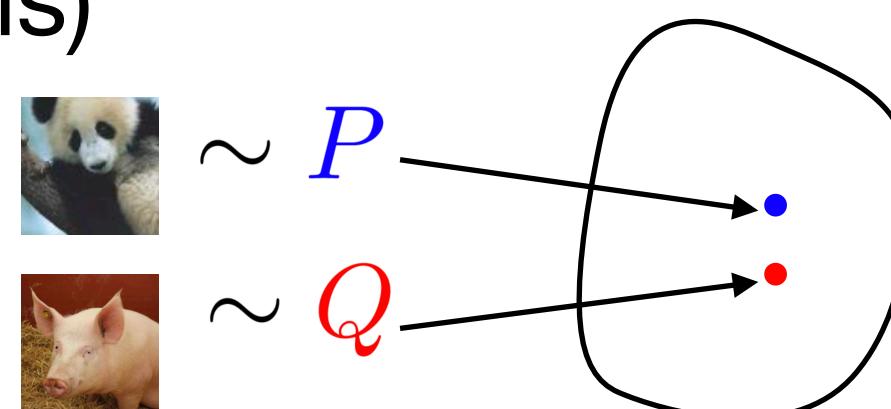
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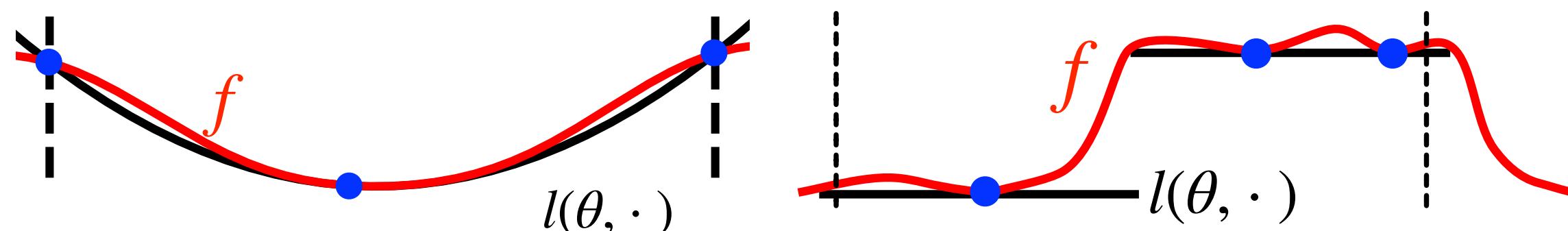


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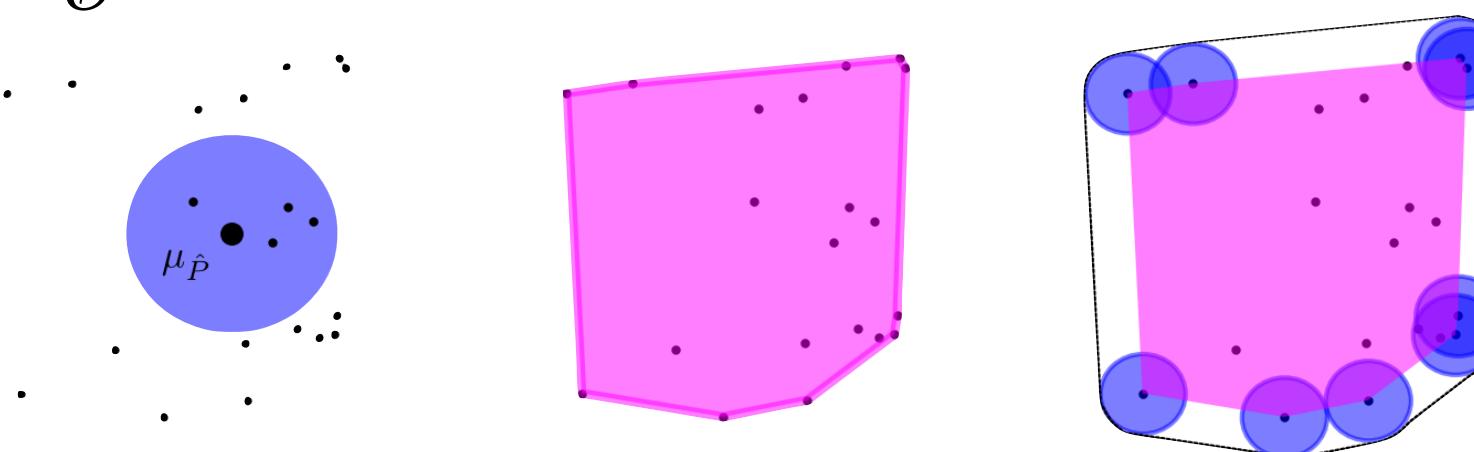
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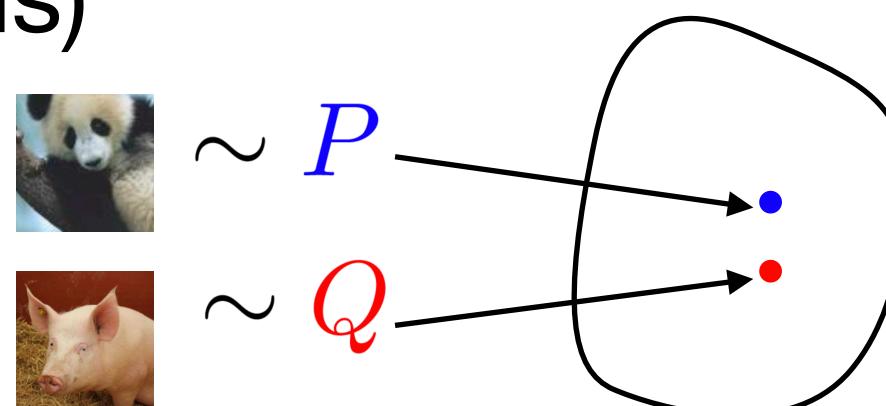


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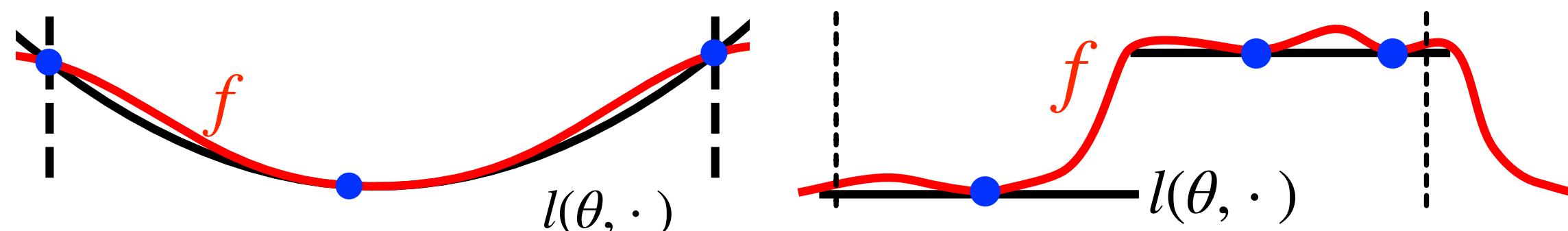


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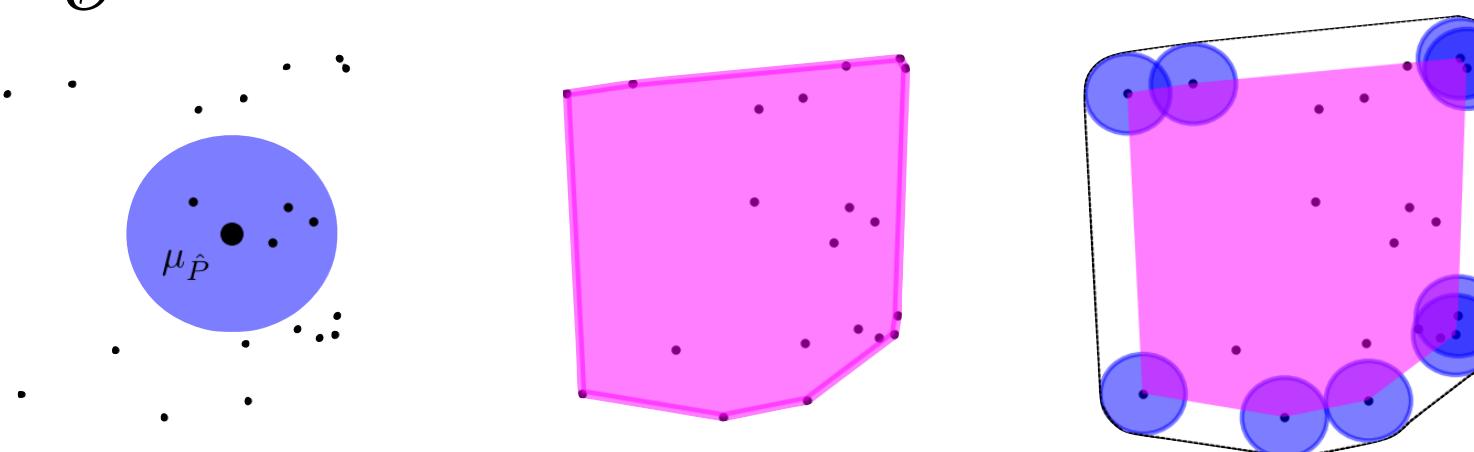
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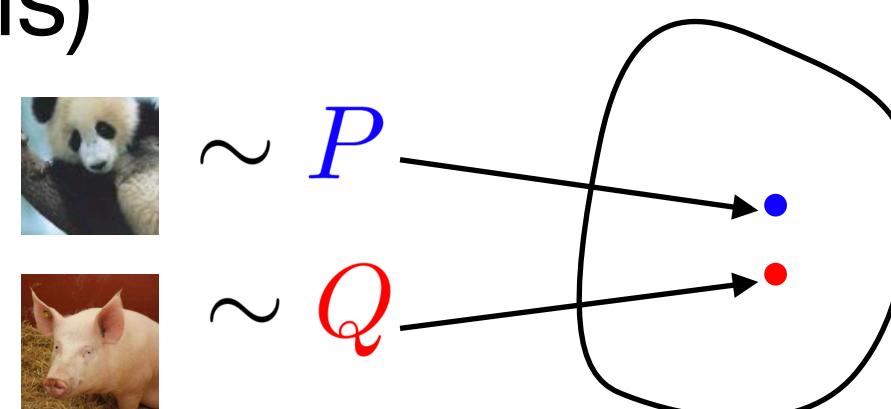
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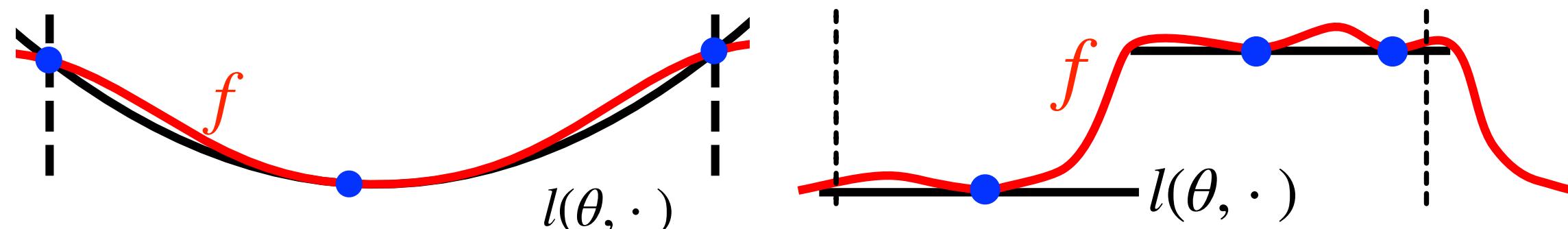
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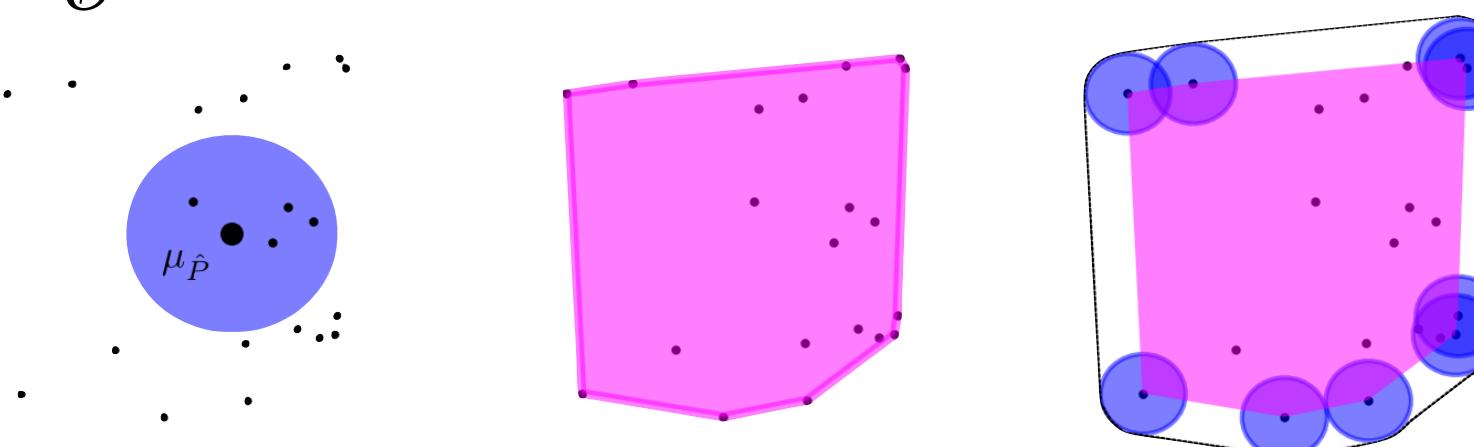
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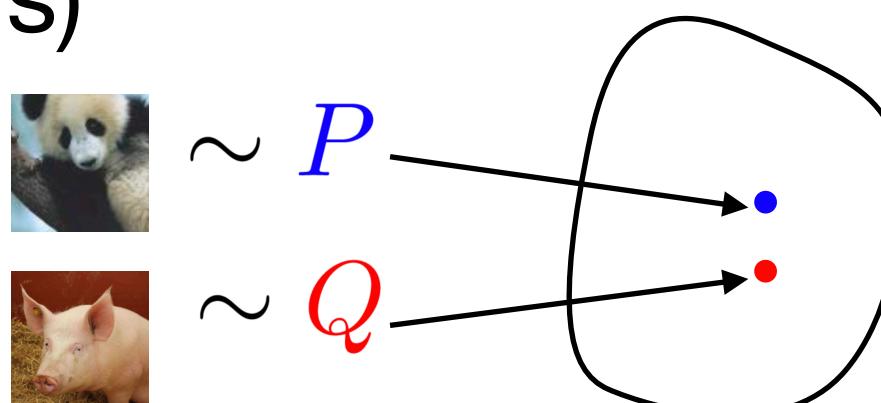
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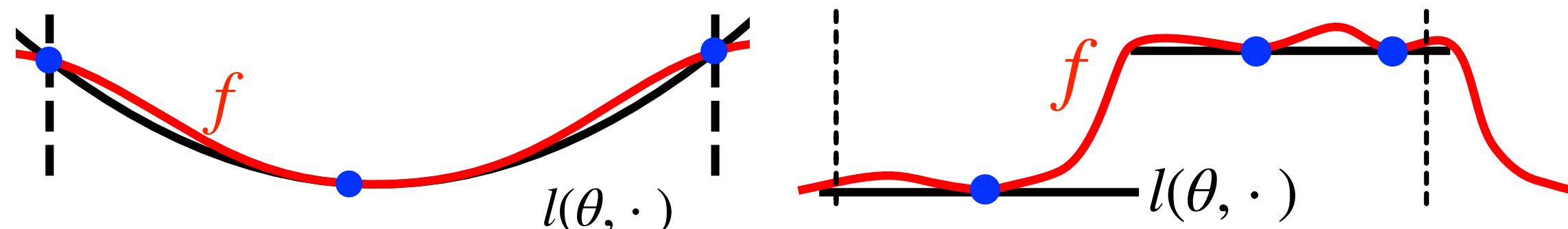
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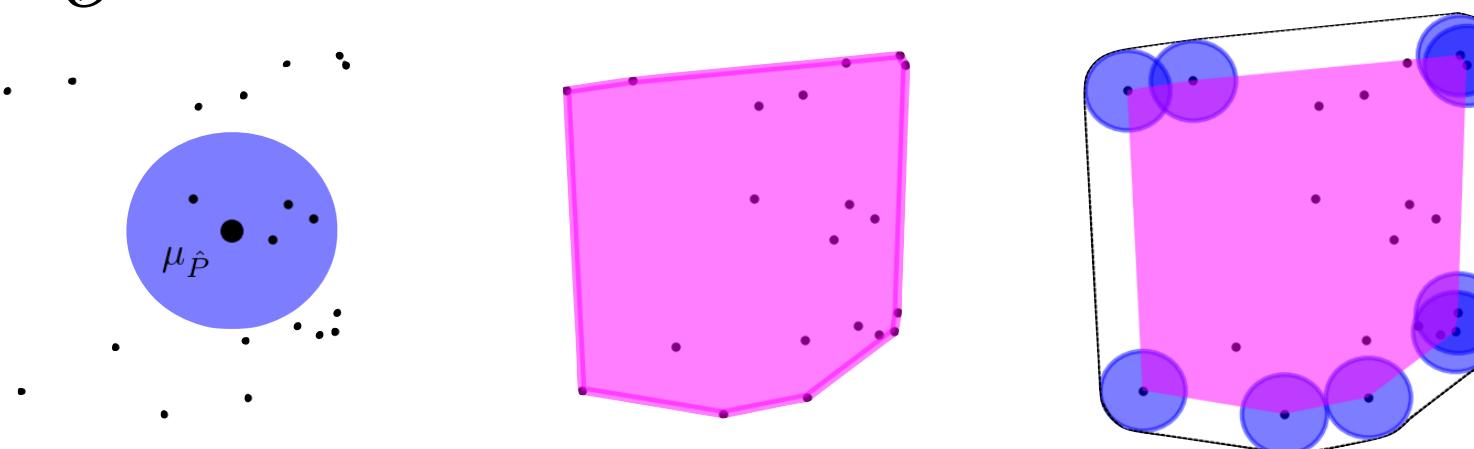
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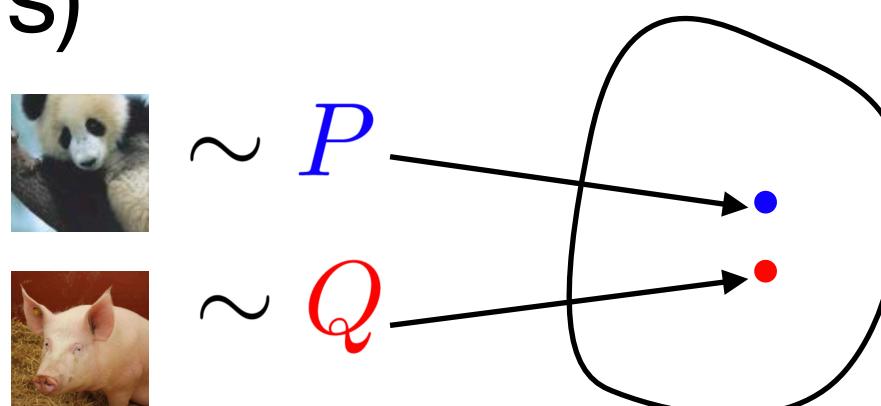
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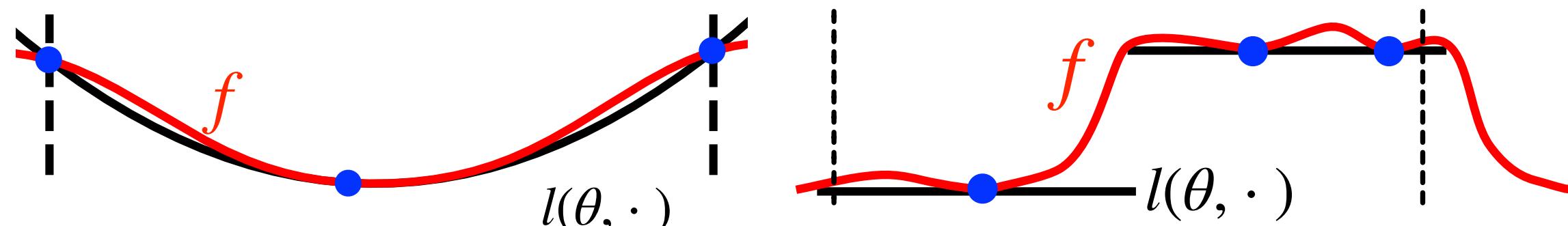
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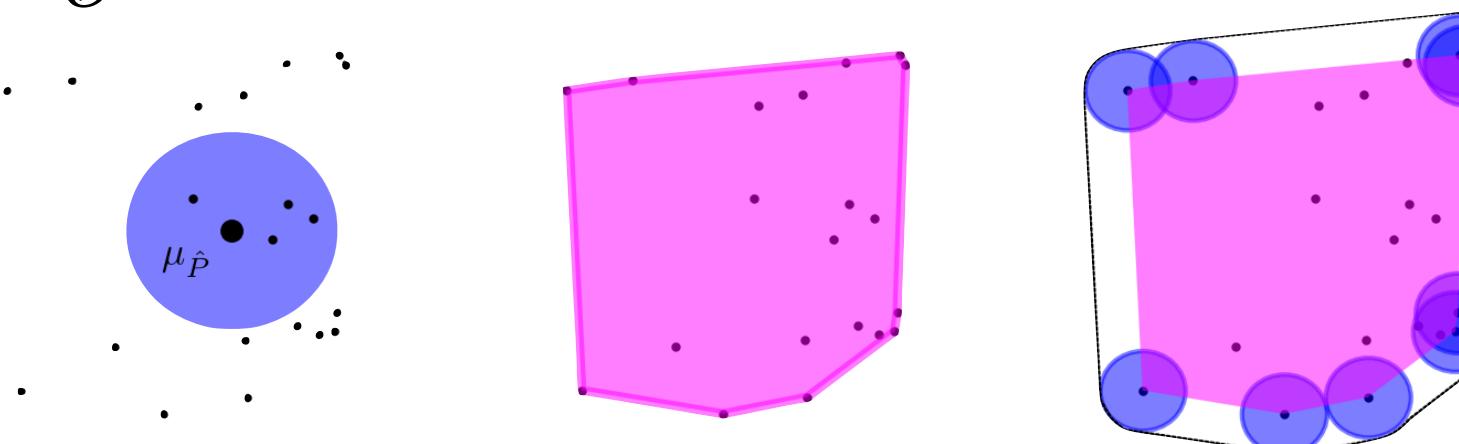
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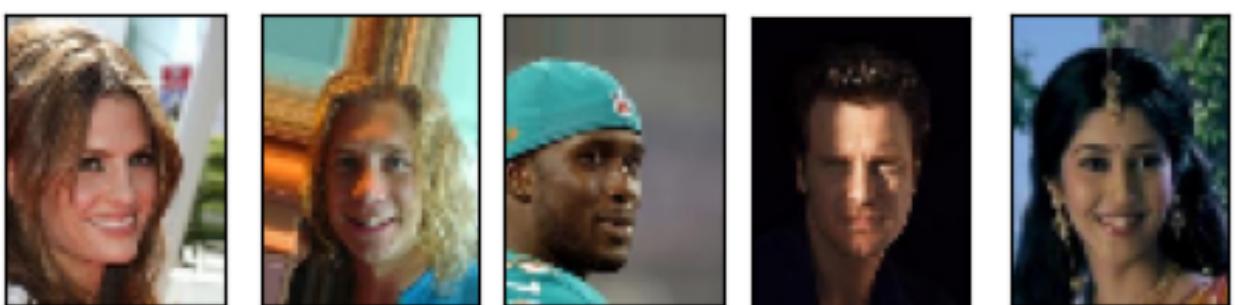
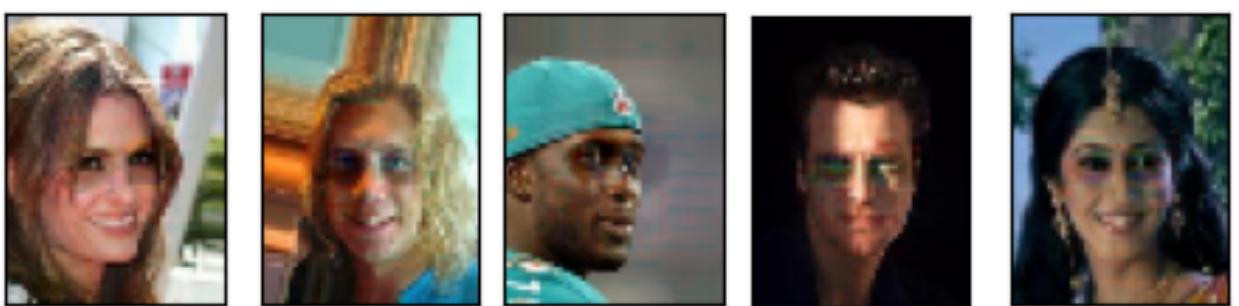
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- Comparison with Wasserstein DRO:
 - MMD enjoys closed-form estimator for fast computation and favorable convergence rate
 - For general ML loss $l(\theta, \cdot)$ with nonlinear models, there exists no exact reformulation of Wasserstein DRO. Kernel DRO can be applied in such cases thanks to the universality of RKHSs.

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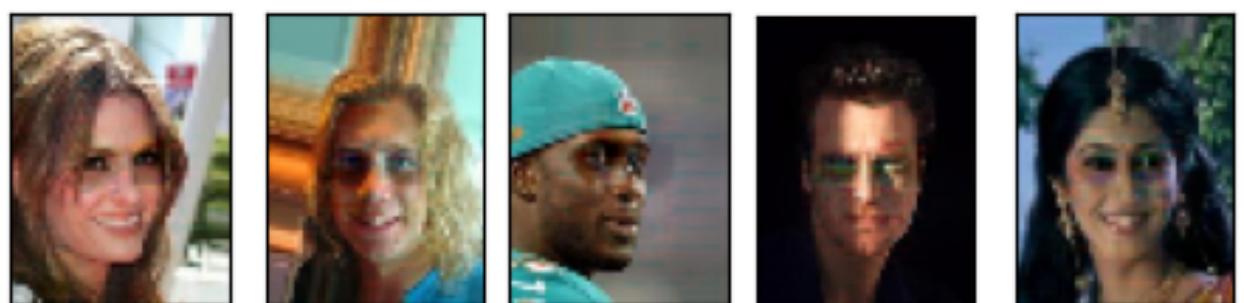
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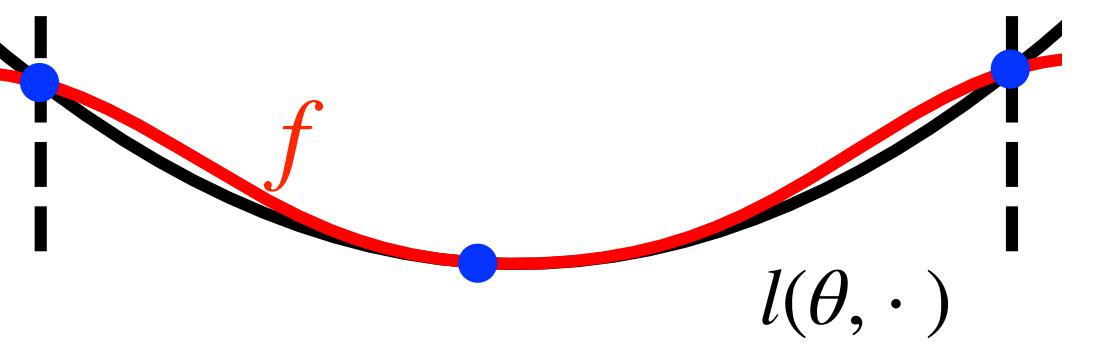


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We construct a solution:

$$f(x) = \sup_{z} \{l(\theta, z)k(z, x)\}$$



•kernel choice: $k(x, x') := e^{-c(x,x')/\sigma}$ (OT in log-scale)

c : transport cost in OT, $\sigma > 0$: bandwidth

Adversarially Robust Kernel Smoothing (ARKS)

Example. Certified
adversarially robust
learning
(Classify the presence of
glasses)

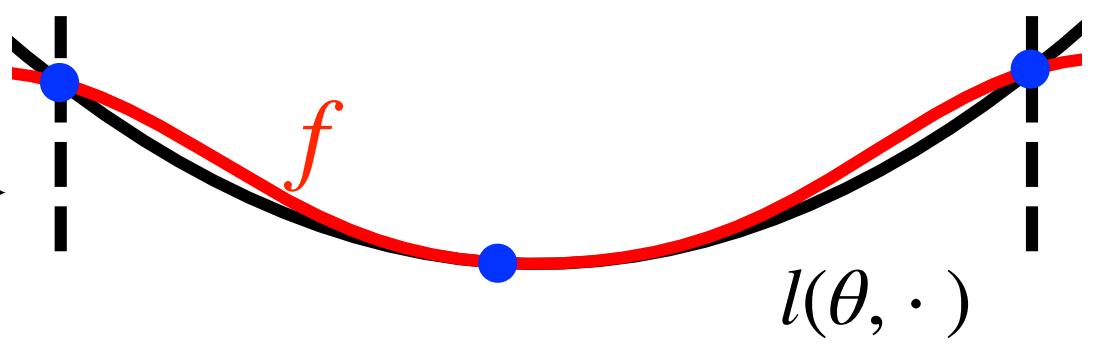


Recall: **Kernel DRO Theorem:**

$$(K) \min_{\theta, f \in \mathcal{H}} \frac{1}{N} \sum_{i=1}^N f(\xi_i) + \epsilon \|f\|_{\mathcal{H}} \quad \text{subject to } l(\theta, \cdot) \leq f$$

We construct a solution:

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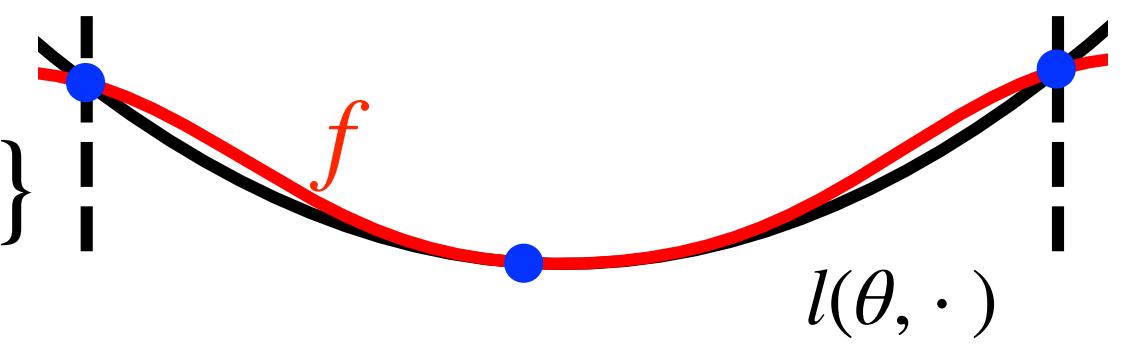


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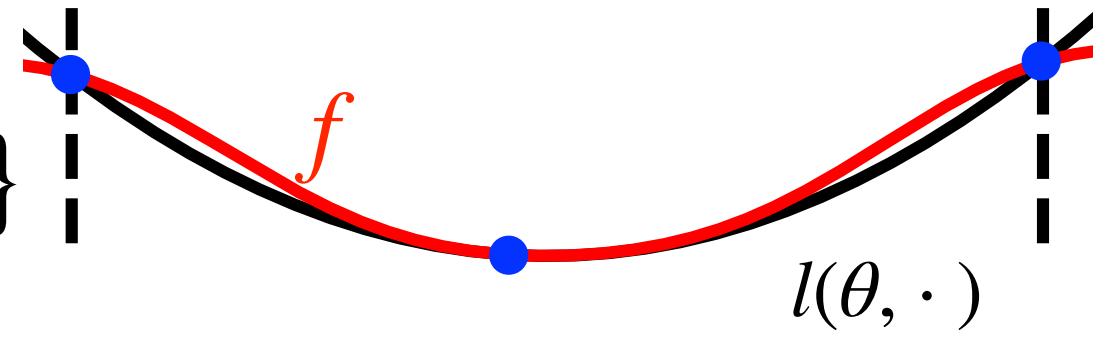


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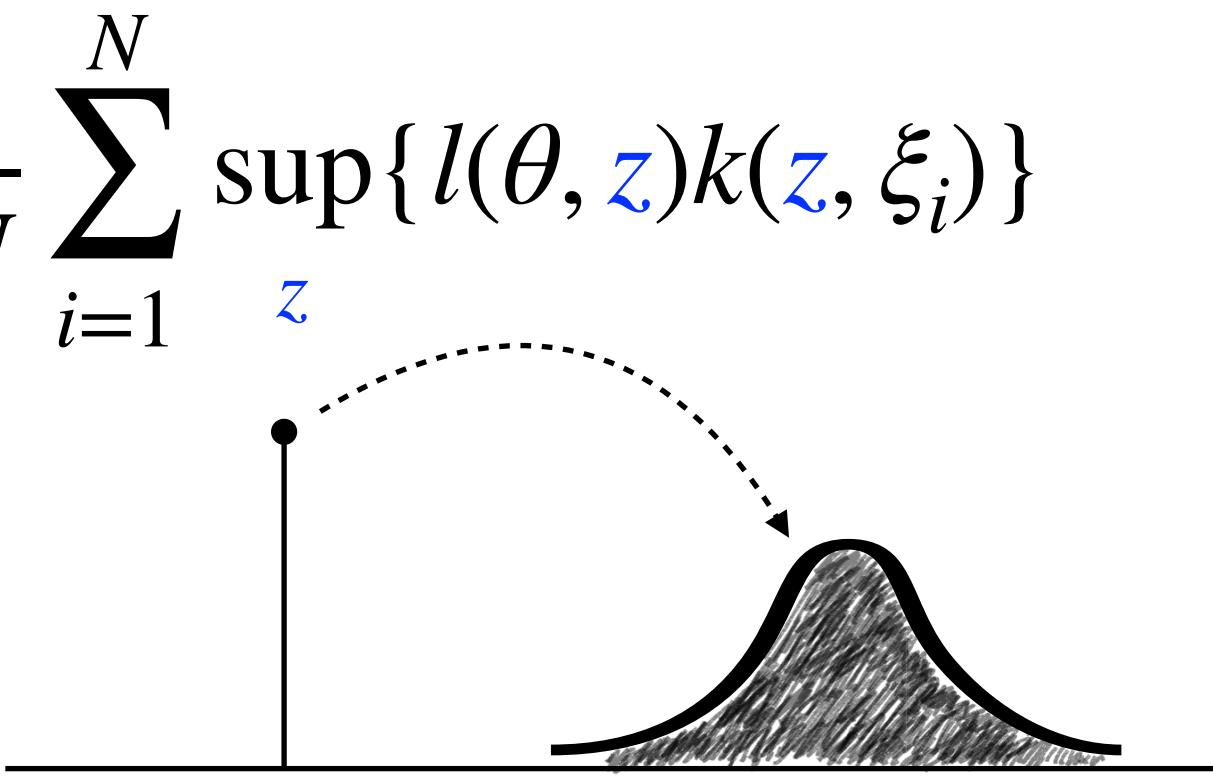
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Intuition: modeling adversarial perturbation using density $k(z, \xi_i)$.



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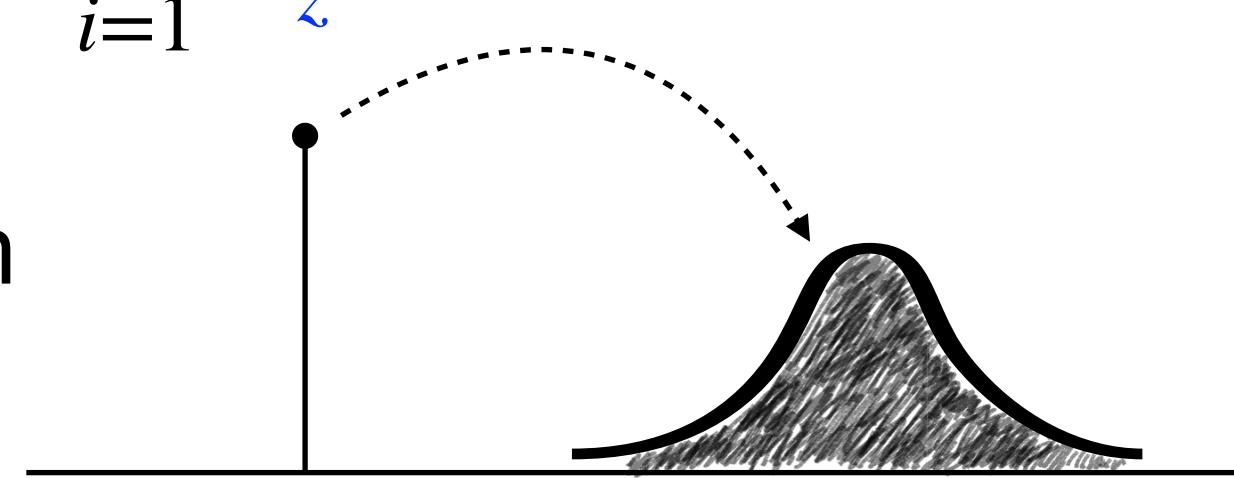
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Distributional robustness certificate.

$\mathcal{W}_c(\cdot, \cdot)$: OT metric with transport cost c

$\epsilon_N \rightarrow 0$, computable robustness certificate:

$$\begin{aligned} & \sup_{\mathcal{W}_c(Q, P_0) \leq \rho} \mathbb{E}_Q \ln l(\hat{\theta}, \xi) \\ & \leq \underbrace{\ln \left\{ \frac{1}{N} \sum_{i=1}^N \sup_z \{l(\hat{\theta}, z)k(z, \xi_i)\} \right\} + \frac{\rho}{\sigma} + \epsilon_N}_{\text{ARKS objective}} \end{aligned}$$

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Interesting future directions

- Design specific kernels for robustness beyond norm-ball perturbation
- Physics, information geometry, and general dynamic OT
- Causal inference via distributional robustness

References

- **Zhu, J.-J., Jitkrittum, W., Diehl, M. & Schölkopf, B.** Kernel Distributionally Robust Optimization. AISTATS 2021
- **Zhu, J.-J., Kouridi, C., Nemmour, Y. & Schölkopf, B.** Adversarially Robust Kernel Smoothing. AISTATS 2022

Code

- KDRO: <https://github.com/jj-zhu/kdro>
- ARKS: <https://github.com/christinakouridi/arks>