Data Structures:
Trees: Introduction, Tree Traversal, Binary Trees, Binary Expression Trees

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(Lecture by Youngmin Oh)
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Trees

Real Tree

leaves

root

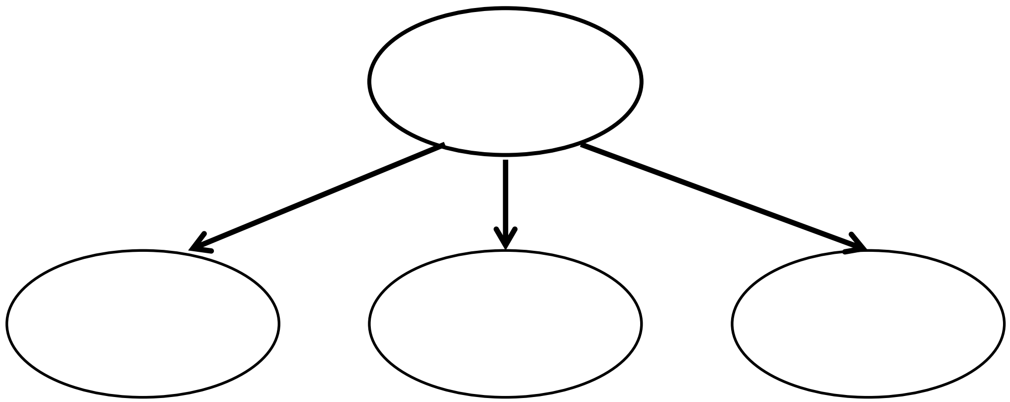
Definitions for Trees in Software Data Structures

Tree Data Structure

root

leaves

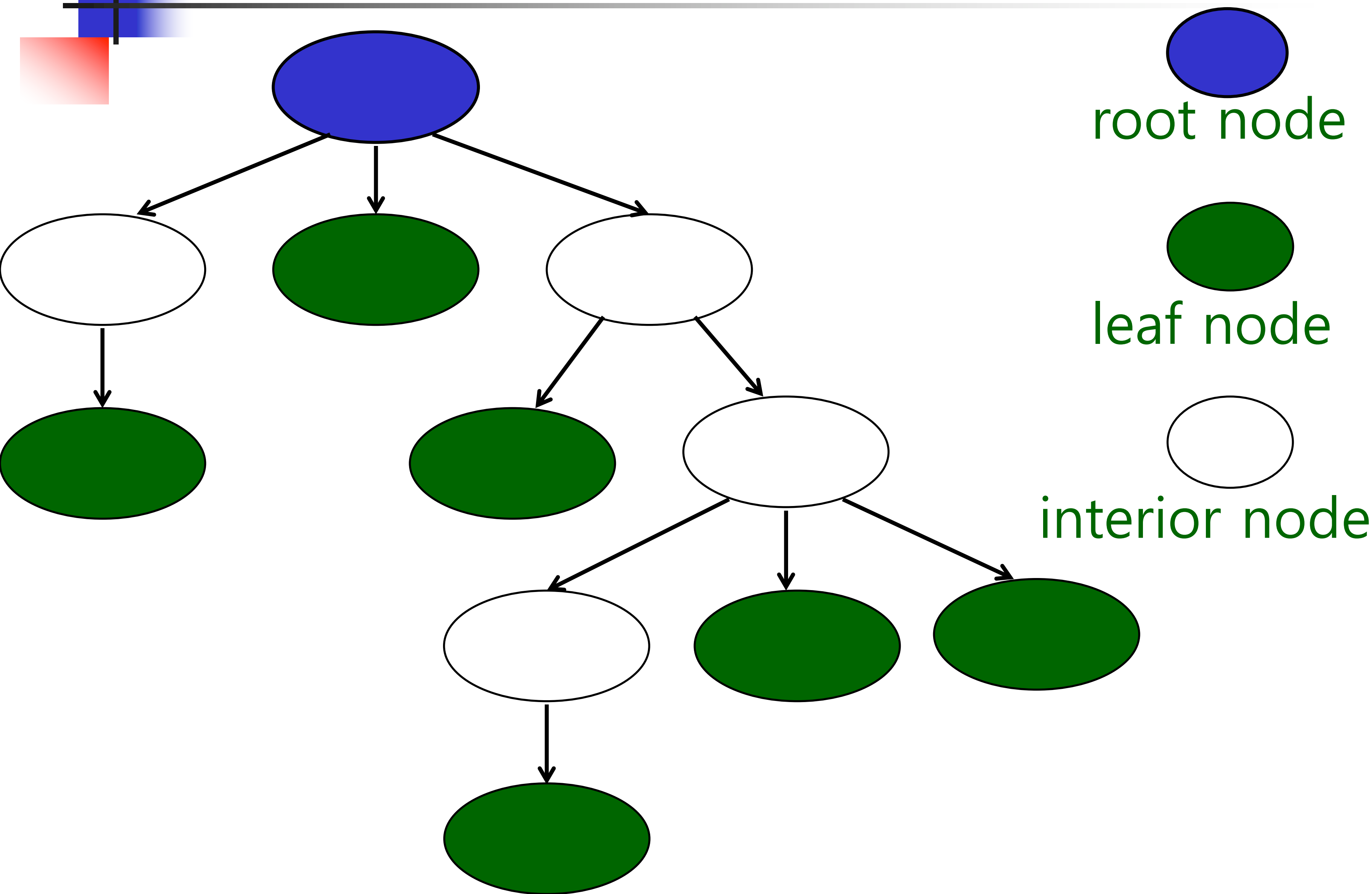
Nodes and Branches (Links)



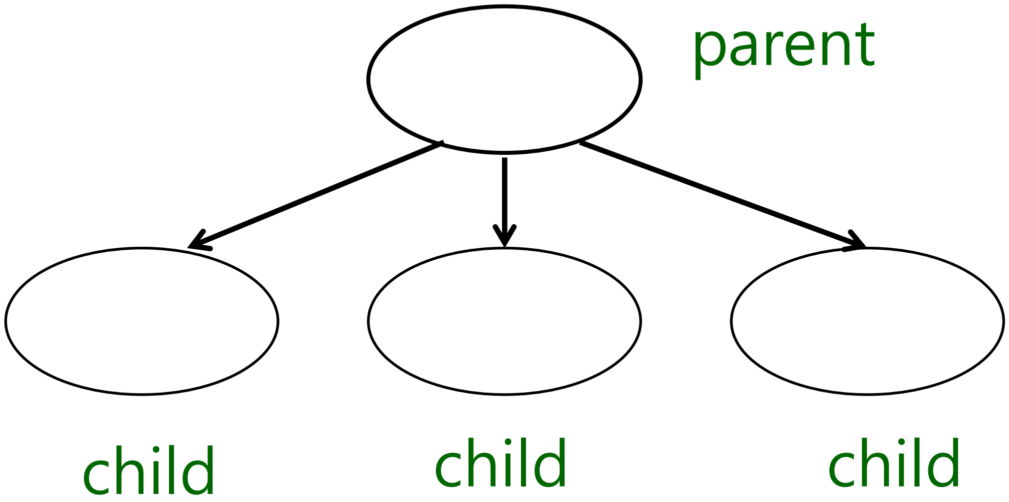
node (stores data)

branch (link)
(pointer stored in a node)

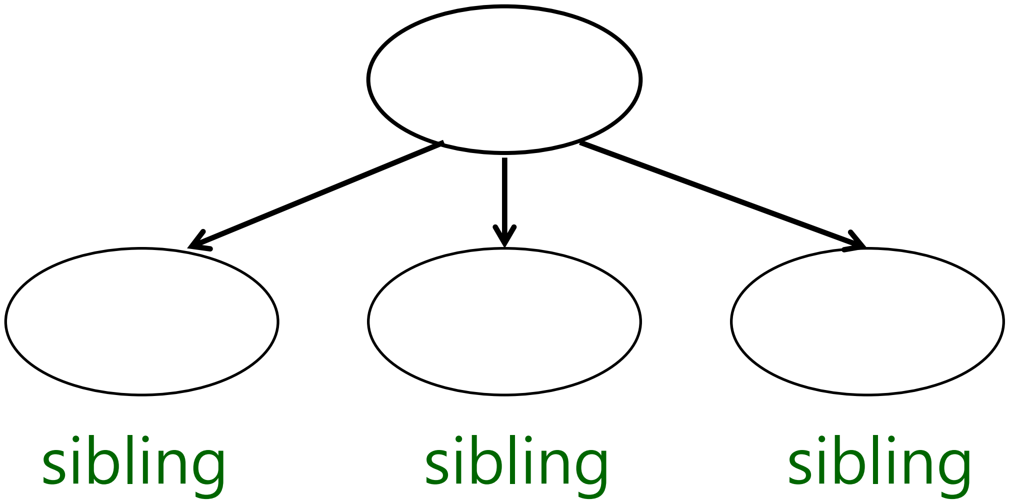
Root, Leaf, Interior (Non-Leaf) Nodes



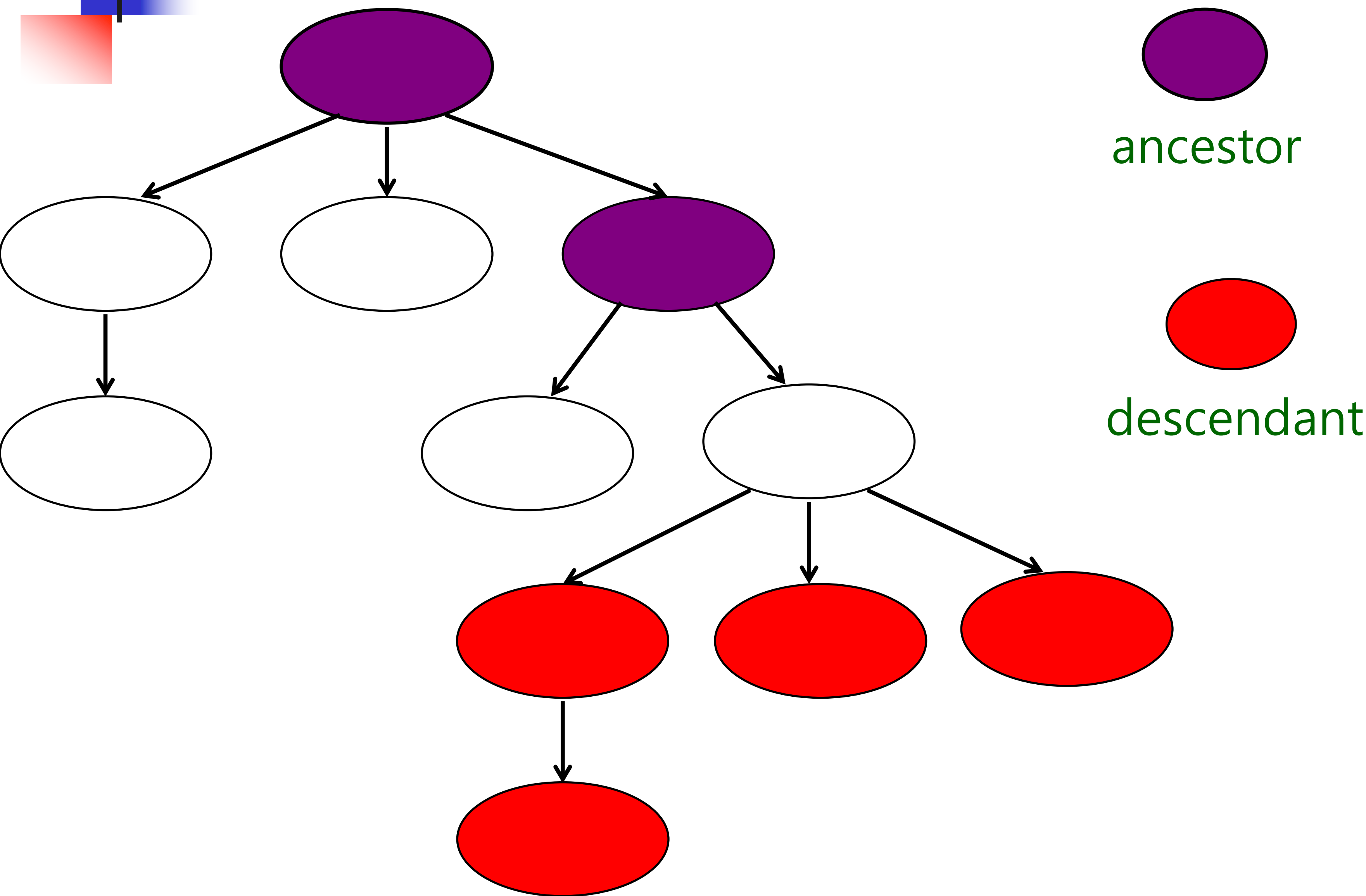
Parent and Child



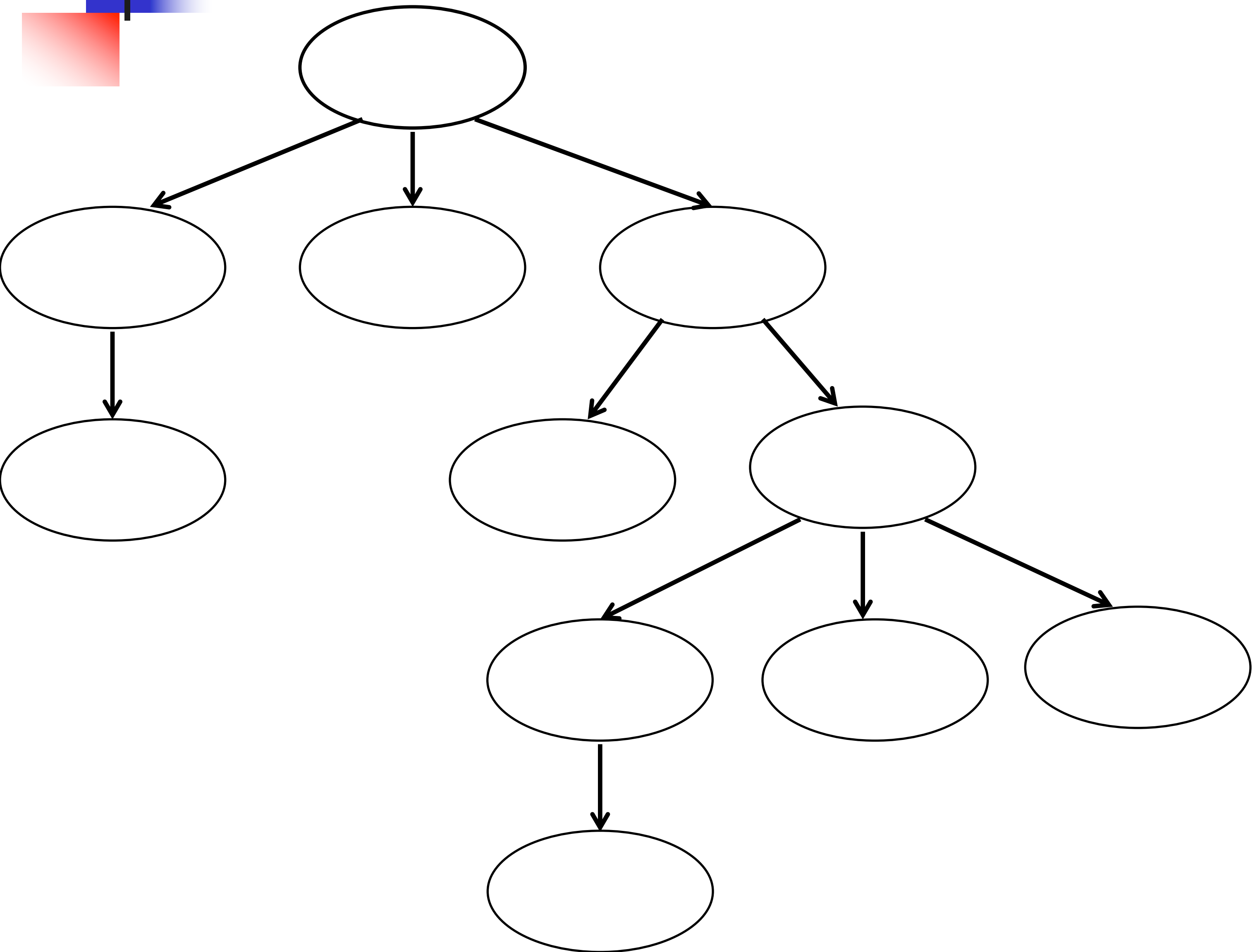
Siblings (brothers and sisters)



Ancestors and Descendants



Depth (Height, Level)



1 or 0

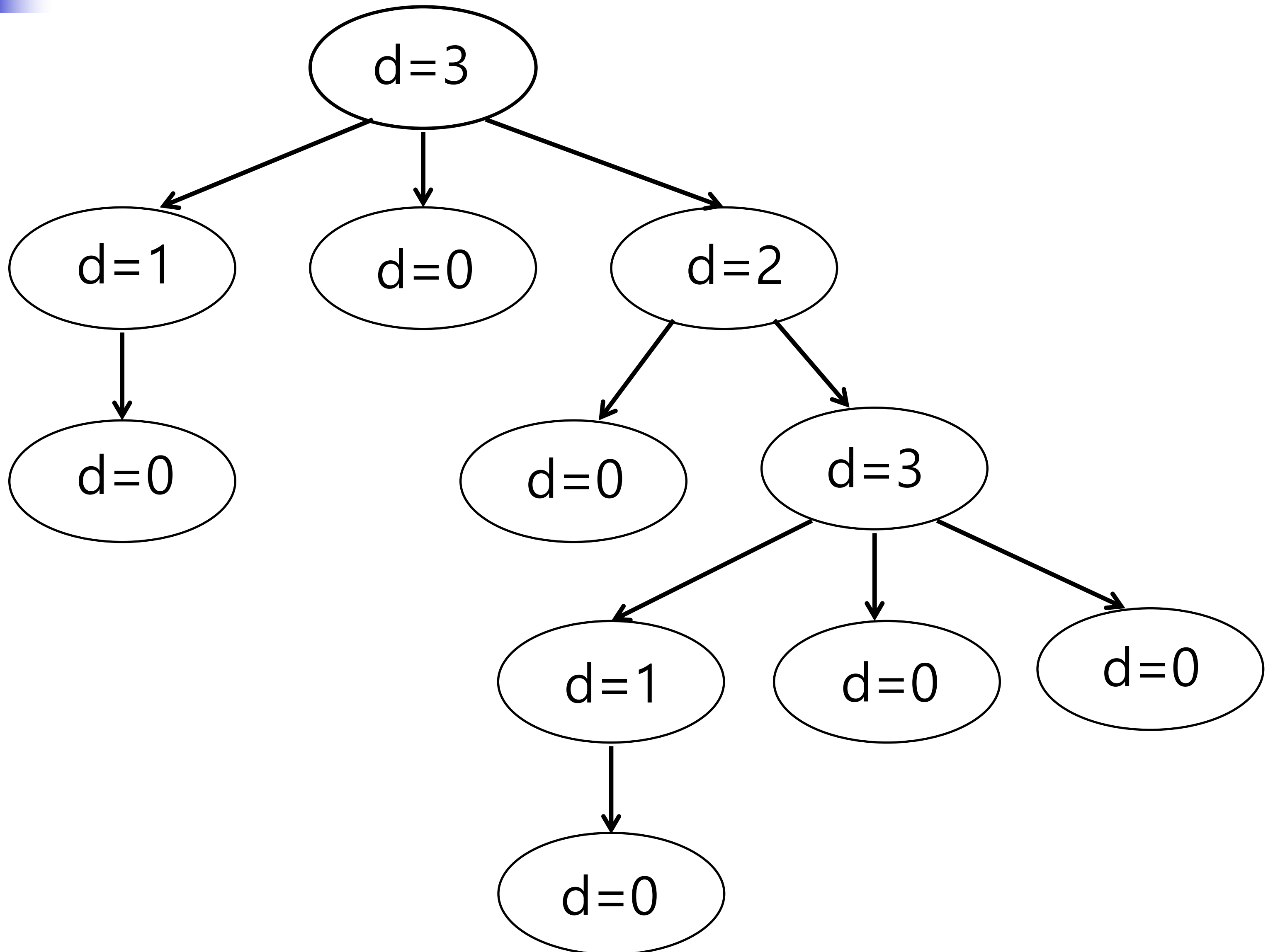
2 or 1

3 or 2

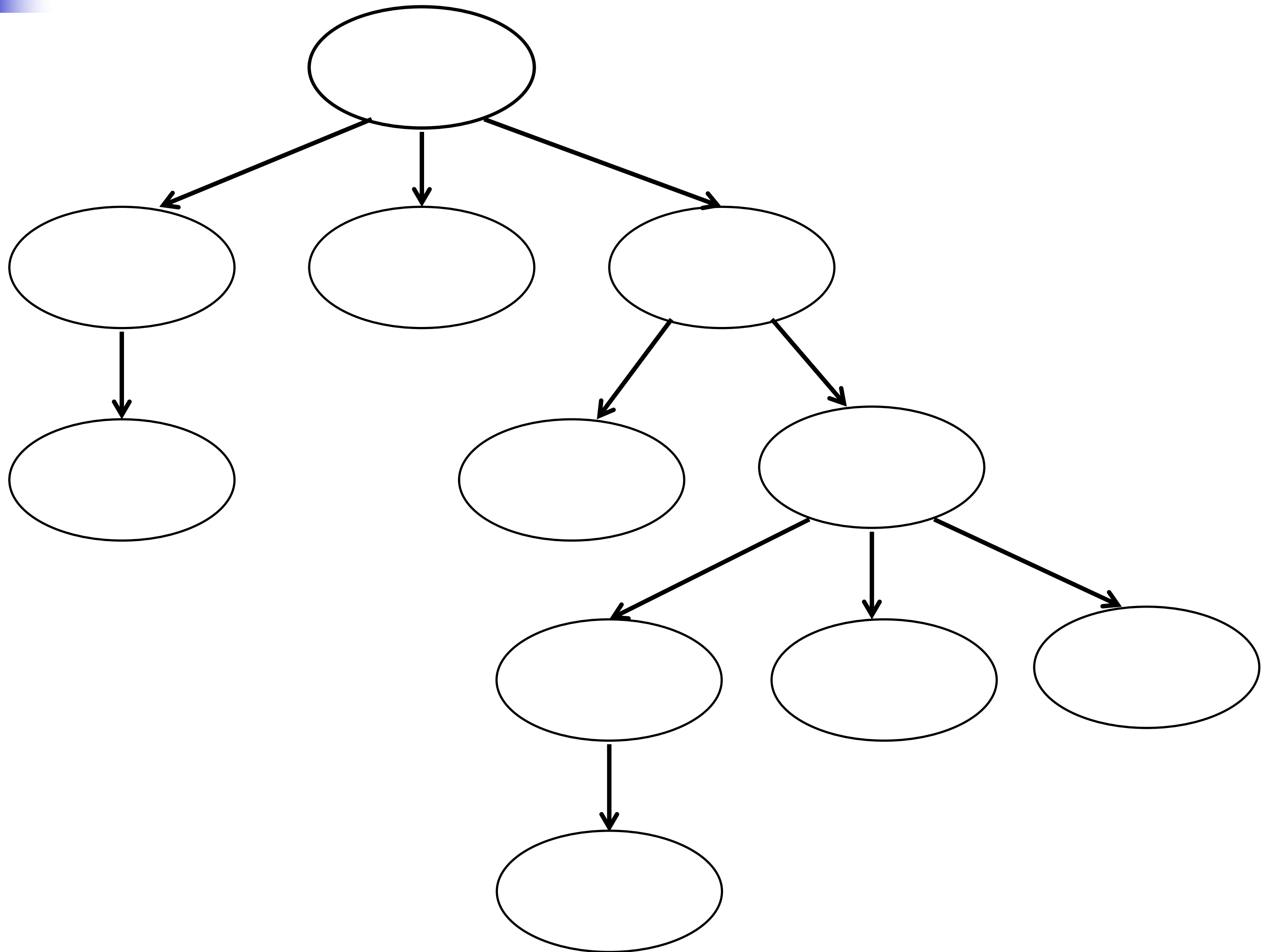
4 or 3

5 or 4

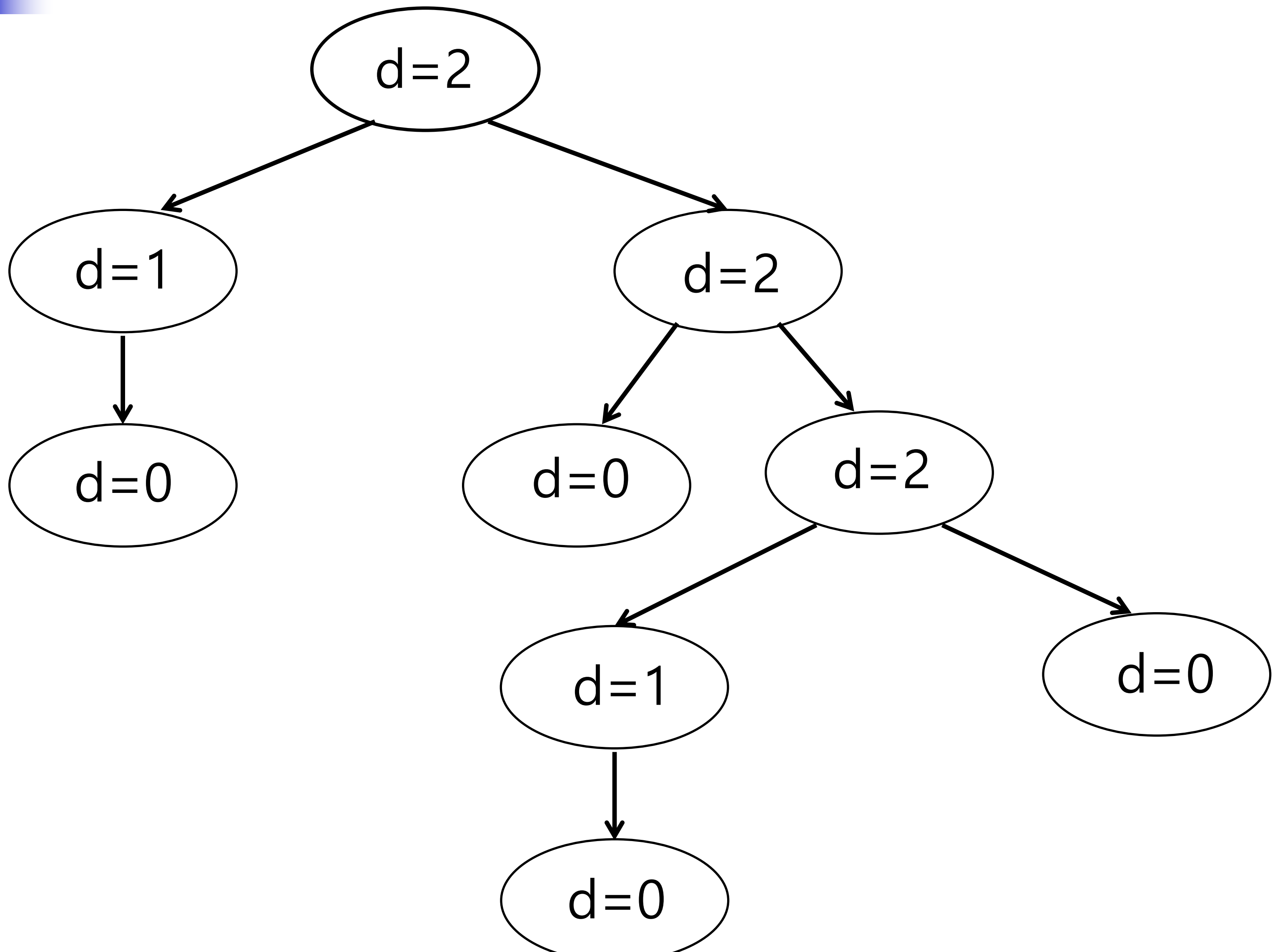
Degree (Fanout)



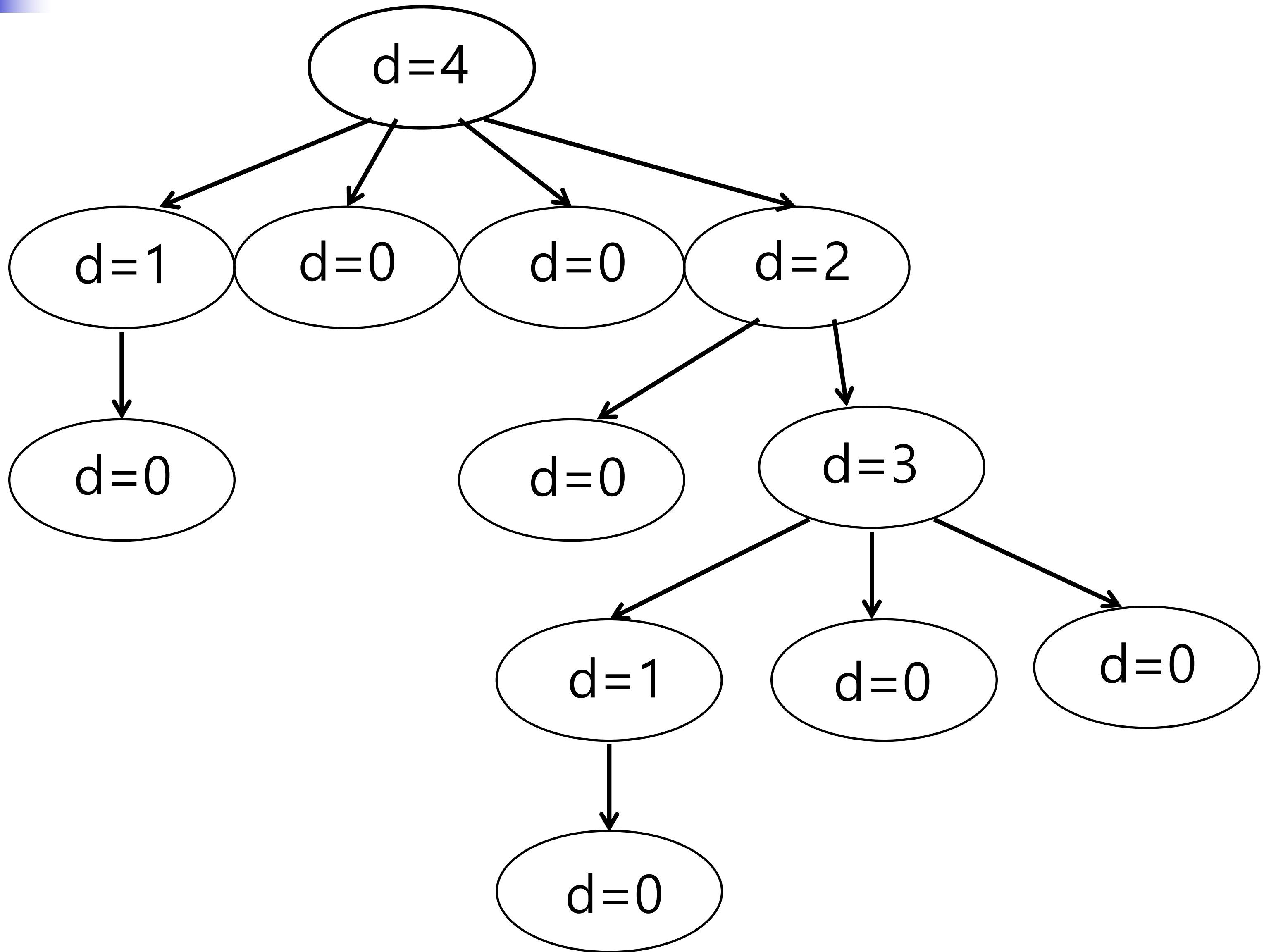
A General Tree



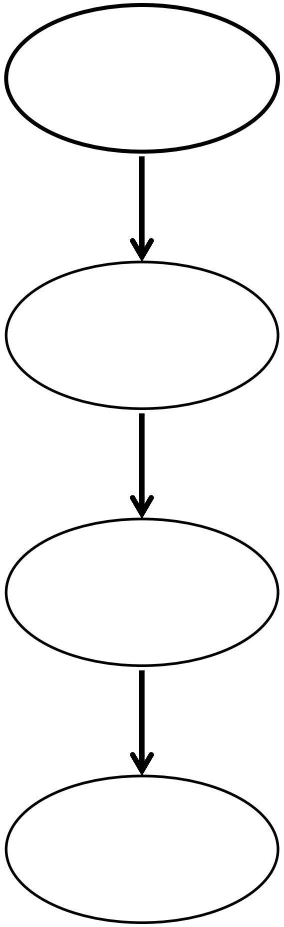
A Binary Tree (Degree <= 2)



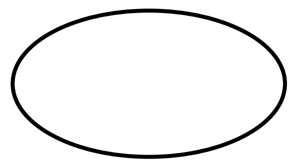
A Quad Tree (Degree =< 4)



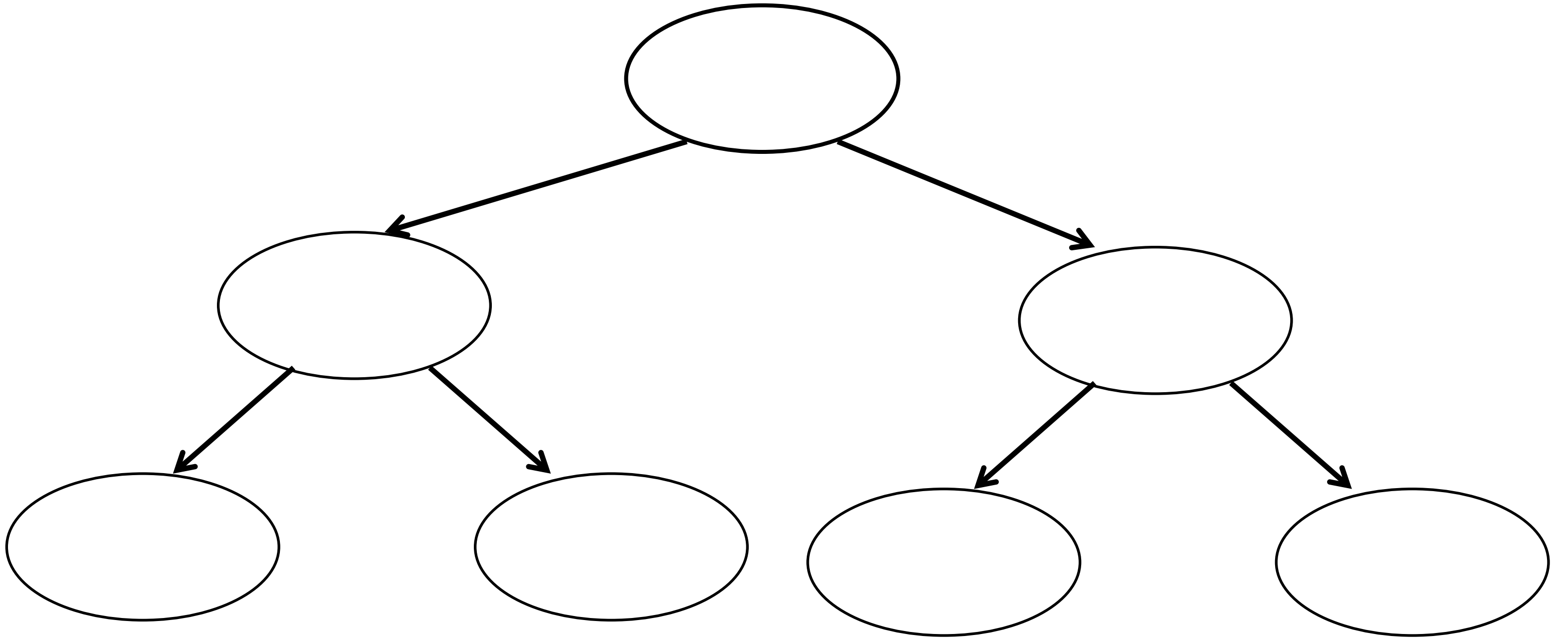
A Skewed Tree (Degenerate Tree)



(Theoretically A) Tree



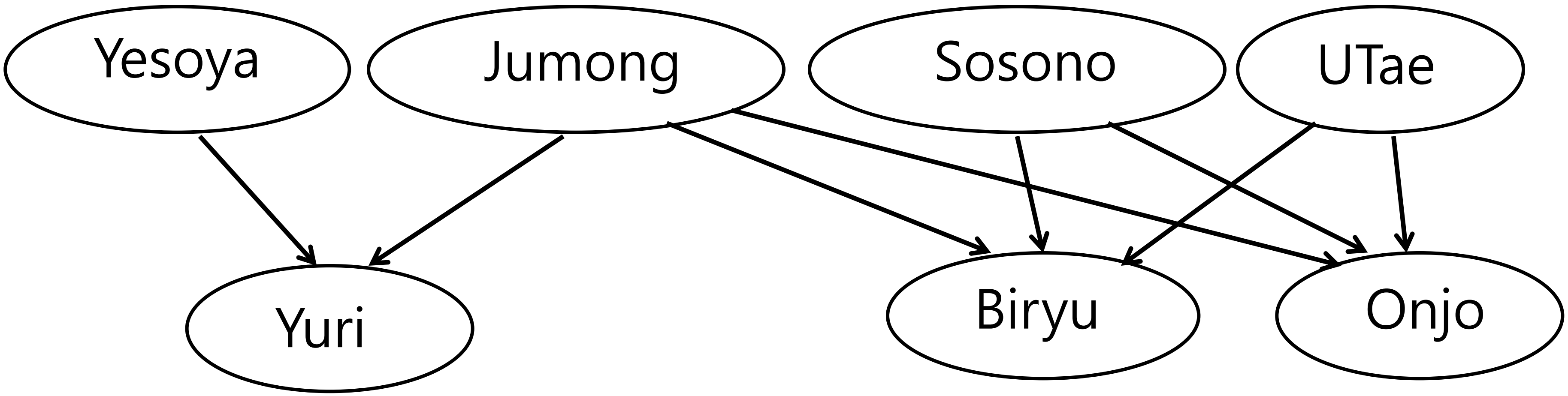
A Full (Perfect) Tree



Organization

- Each non-root node has only one parent node.
  - The root node has no parent node.
- Each non-leaf node has one or more child nodes.
  - Each leaf node has zero child node.
- A tree consists of
  - The root node, and j child nodes of the root node.
  - Each of the j child nodes is the root node of a tree.

Not a Tree: General Genealogy



Many Types of Tree Data Structures
(* We will learn about the red highlighted trees in this course.)

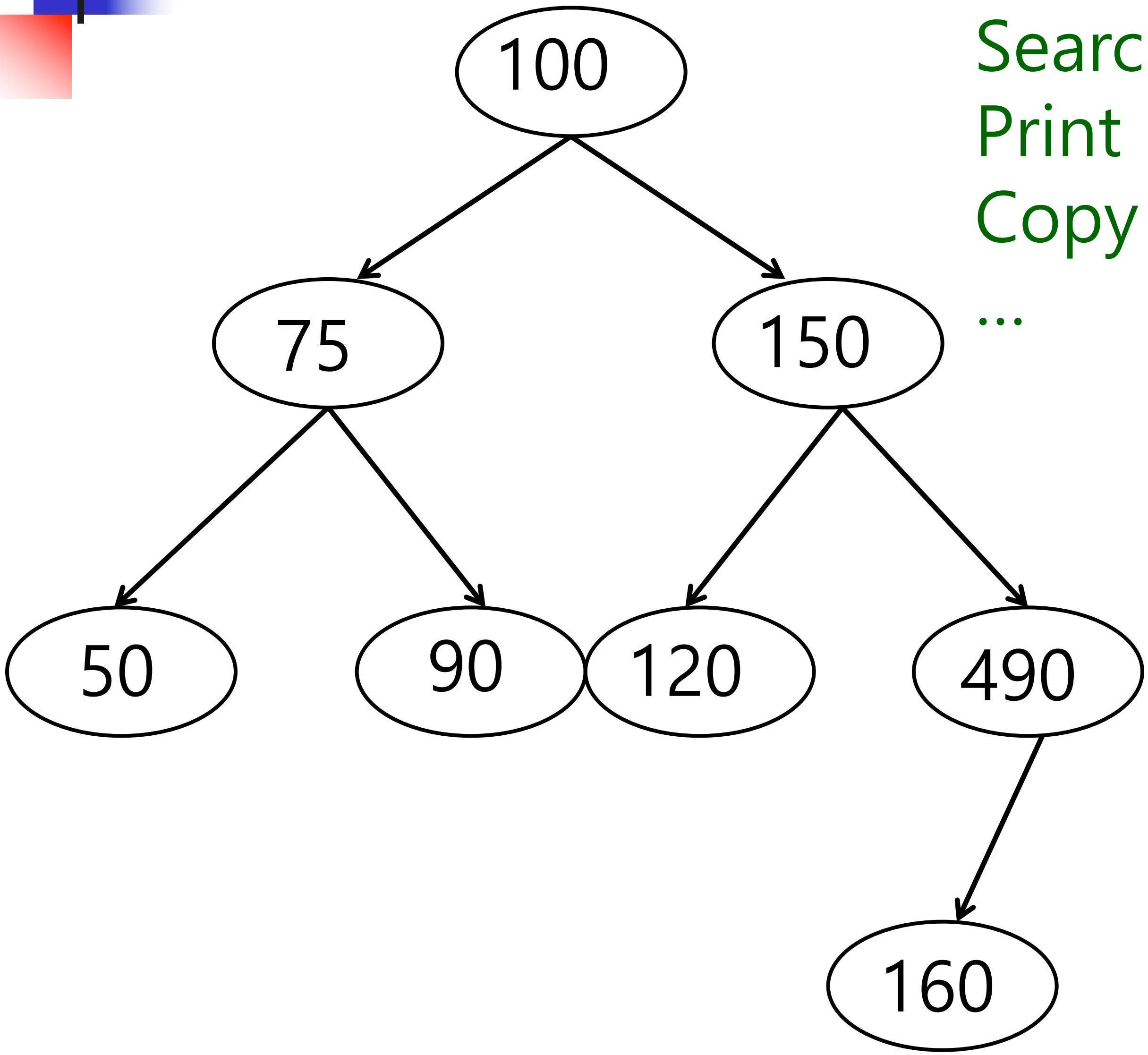
- Binary Tree
  - Binary Search Tree, Heap, Digital Search Tree, Trie
  - Red-Black Tree, AA Tree, Splay Tree,
- Height-Balanced Binary Trees
  - AVL Tree, T-Tree
- n-Way Tree
  - m-Way Trie
  - 2-3 Tree, 2-3-4 Tree
- Height-Balanced m-Way Tree
  - B-Tree, B+-Tree, K-d B-Tree
- Spatial Tree
  - Quad Tree, Oct Tree, K-d Tree, R-Tree, R* Tree

Summary Definition of a Tree

- A tree is a linked list of data, where a data item is linked to other data items by a certain relationship.
- A search for a data item on a tree proceeds from one data item to another data item that satisfies a certain relationship.
- The hierarchy of a tree is a convenient visualization of the relationships among the data items.

Tree Traversal

Tree Traversal (Tree Walk)



Search for a Key
Print Each Key
Copy Each Key
…

Two Basic Ways to Visit Each Node Once

- Visiting a Node: Taking Some Action on the Node
  - printing the data, pushing the data onto a stack, copying data into another tree,…
- Depth-First (Traversal / Search)
- Breadth-First (Traversal / Search)

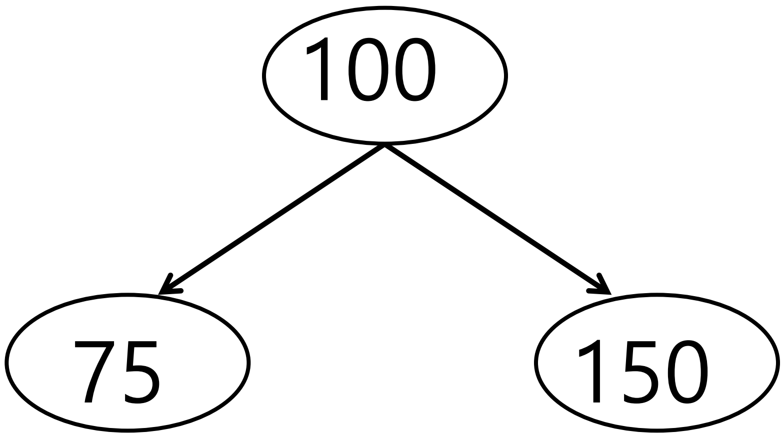
Breadth-First Traversal / Search

- **Level Order Traversal**
  - **Visit every node of a level, and move to the next level.**

Depth-First Traversal / Search

- Inorder Traversal
  - (Left Subtree, Visit the Root, Right Subtree)
- Postorder Traversal
  - (Left Subtree, Right Subtree, Visit the Root)
- Preorder Traversal
  - (Visit the Root, Left Subtree, Right Subtree)

Depth-First Traversal: Exercise



How to Memorize the 3 Types of Depth-First Traversal?

Think of a Mother with Two Children

Quiz: What are the 3 ways to share some cookies among the 3 people?

- Assumption: Mother likes the left child more.
- Your answers??

Answers

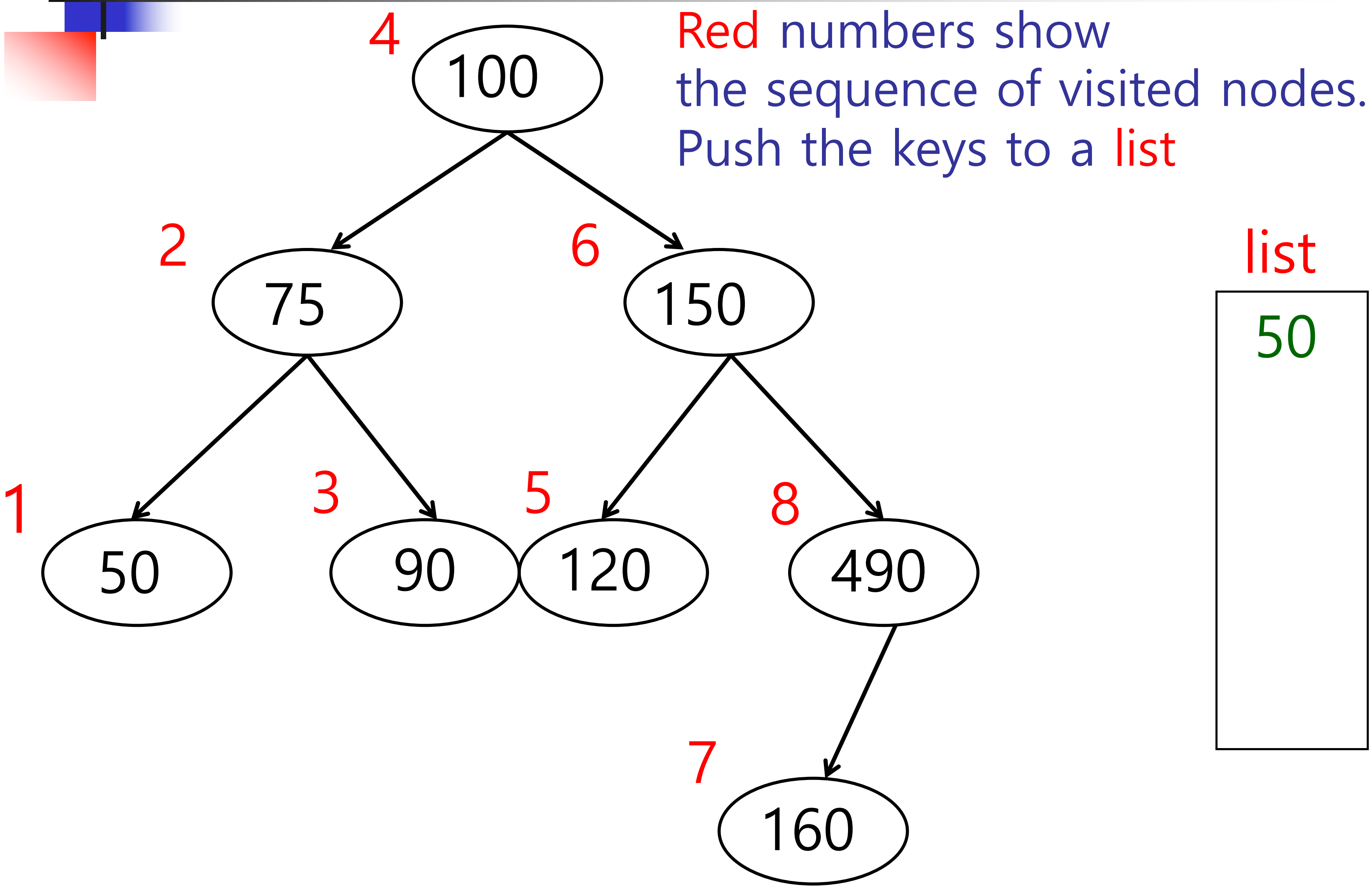
- left child first, mother next, right child last
  - (Inorder)
- left child first, right child next, mother last
  - Unselfish mother (postorder)
- mother first, left child next, right child last
  - Selfish mother, (preorder)

Inorder Traversal

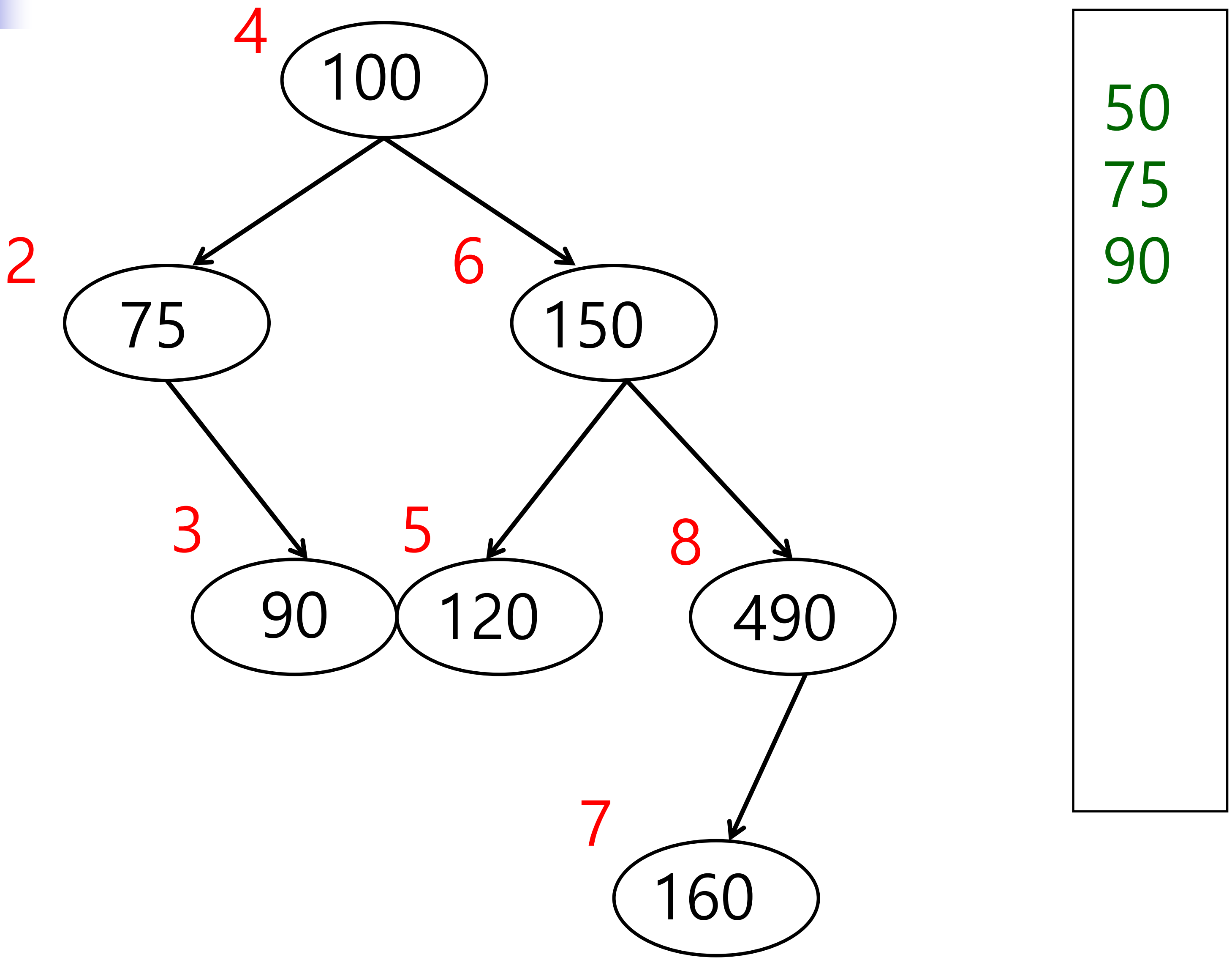
- First Child, Mother, Second Child
  - print or push the key of the visited node to a stack
- Applications
  - retrieval of a sorted sequence
- Makes Sense Only for a Binary Tree

Inorder Traversal: Example (1/6)

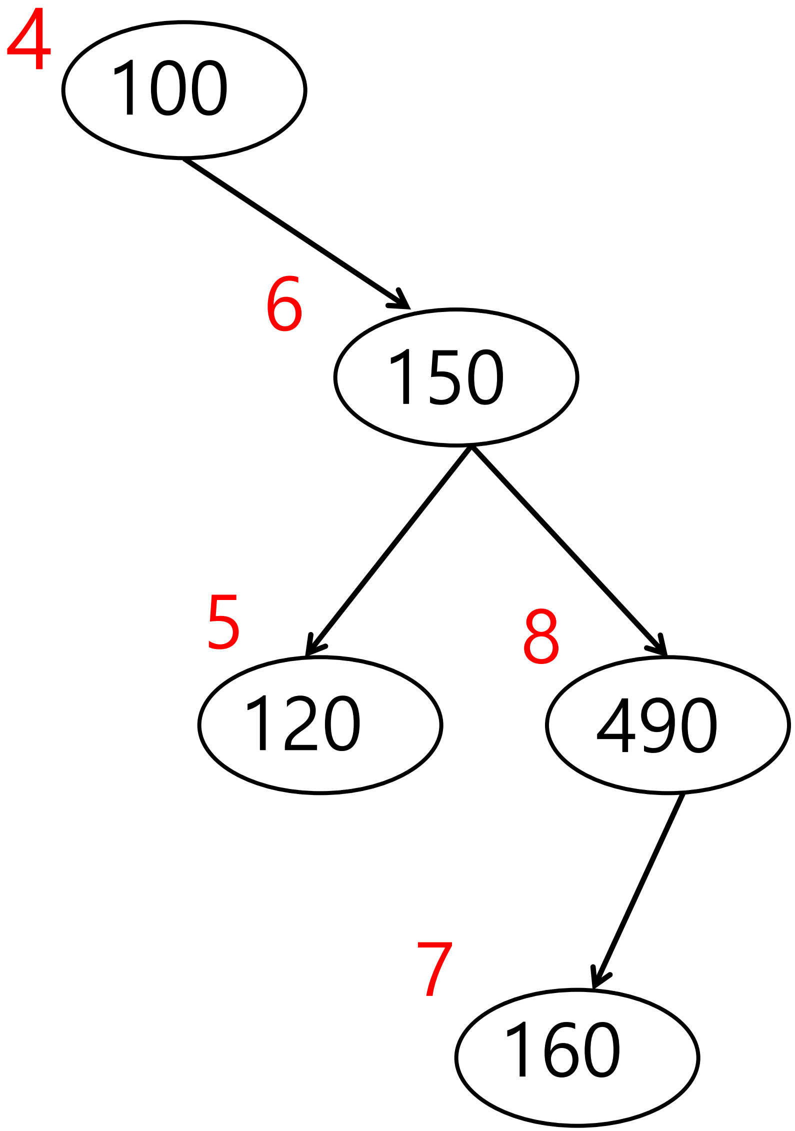
Red numbers show
the sequence of visited nodes.
Push the keys to a list

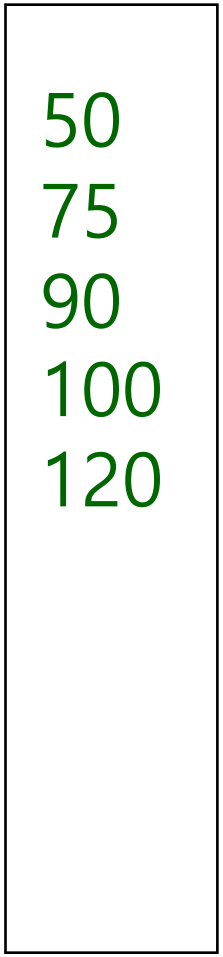


Inorder Traversal: Example (2/6)

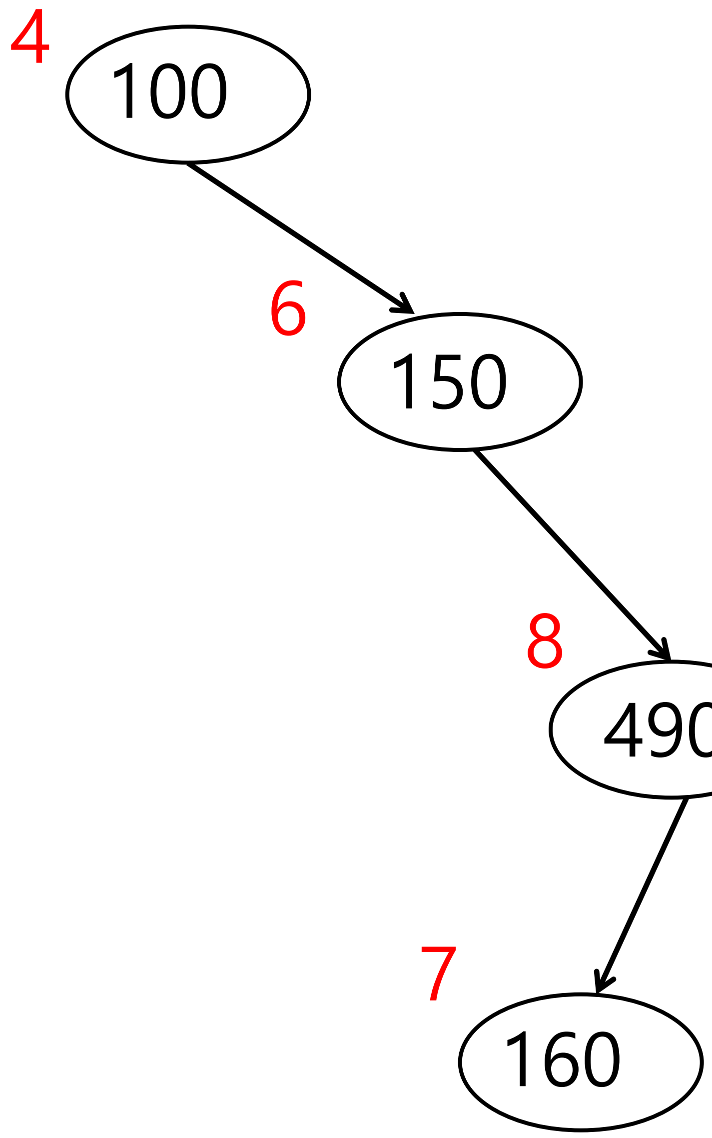


Inorder Traversal: Example (3/6)



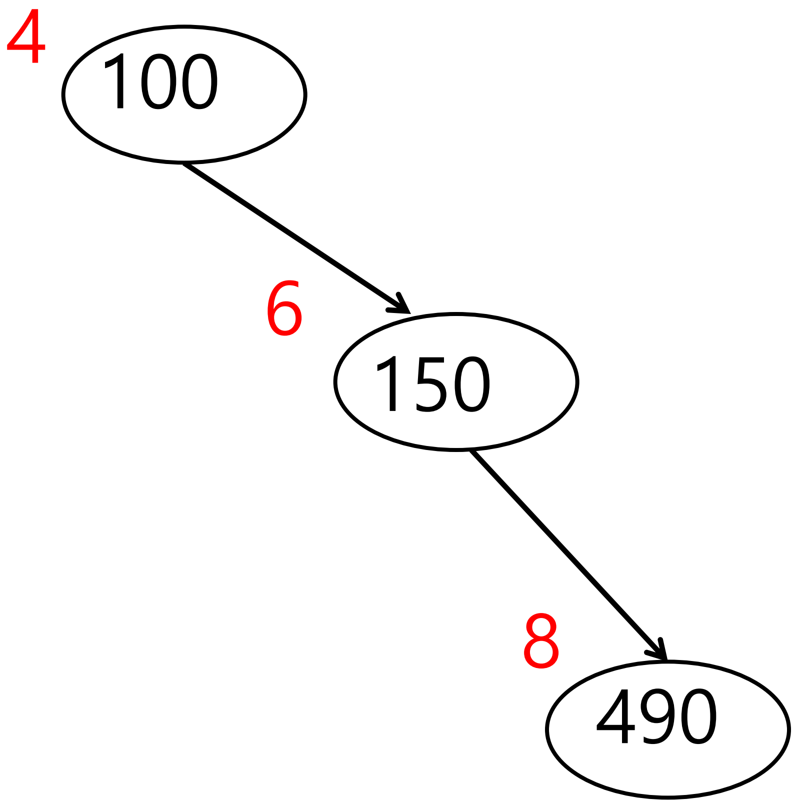


Inorder Traversal: Example (4/6)



50
75
90
100
120
150
160

Inorder Traversal: Example (5/6)



50
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120
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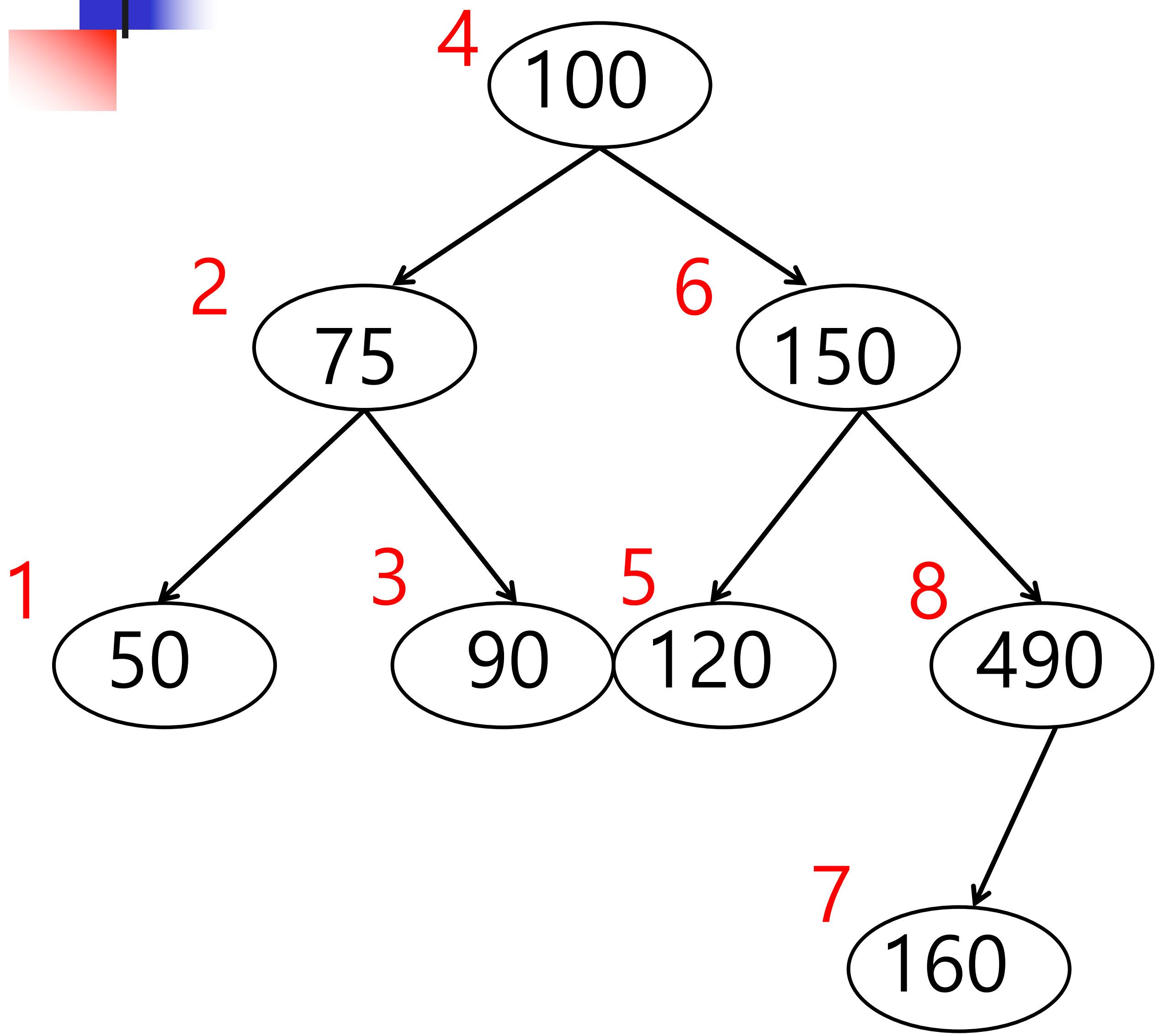
160
490

Inorder Traversal: Example (6/6)



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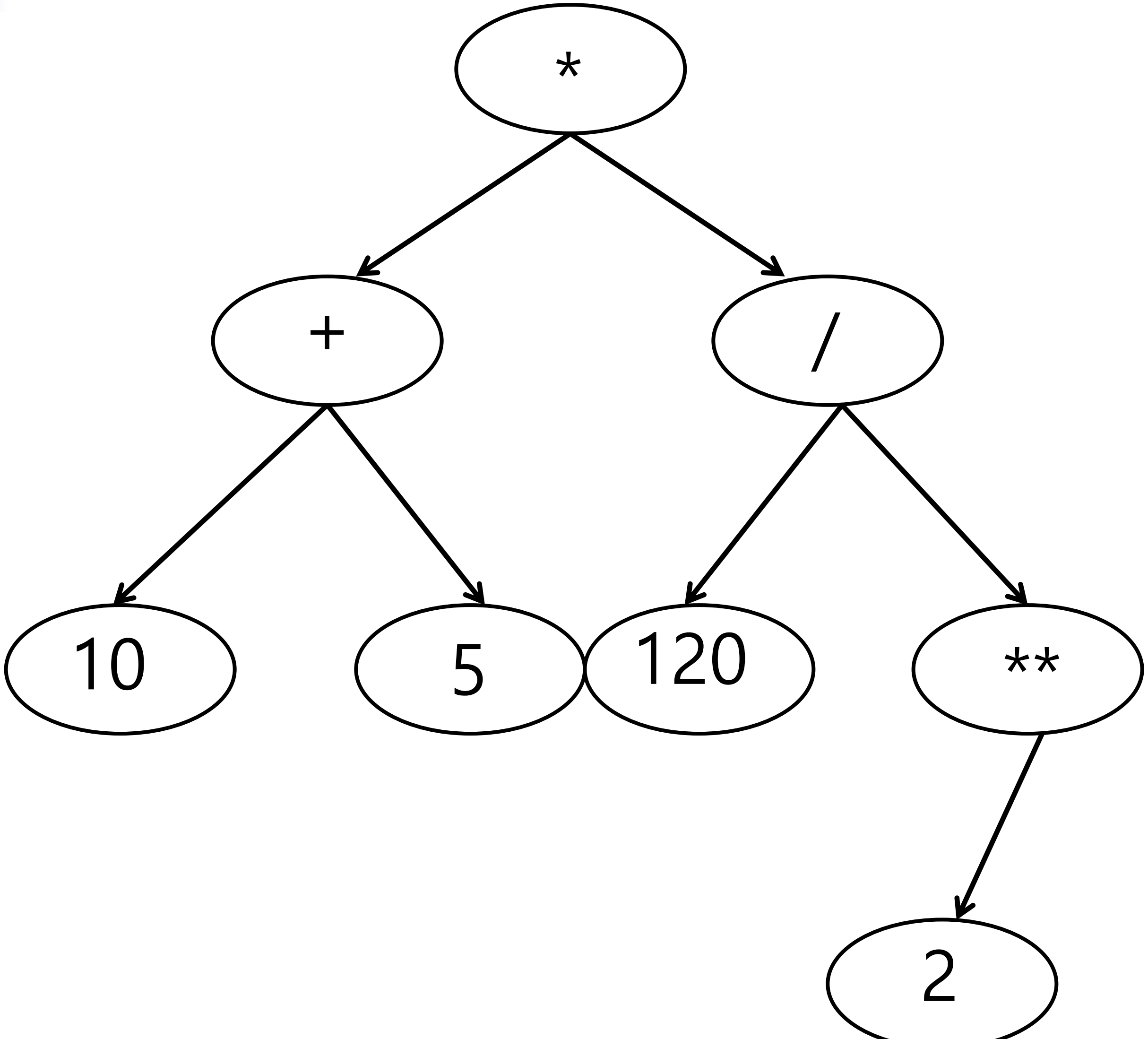
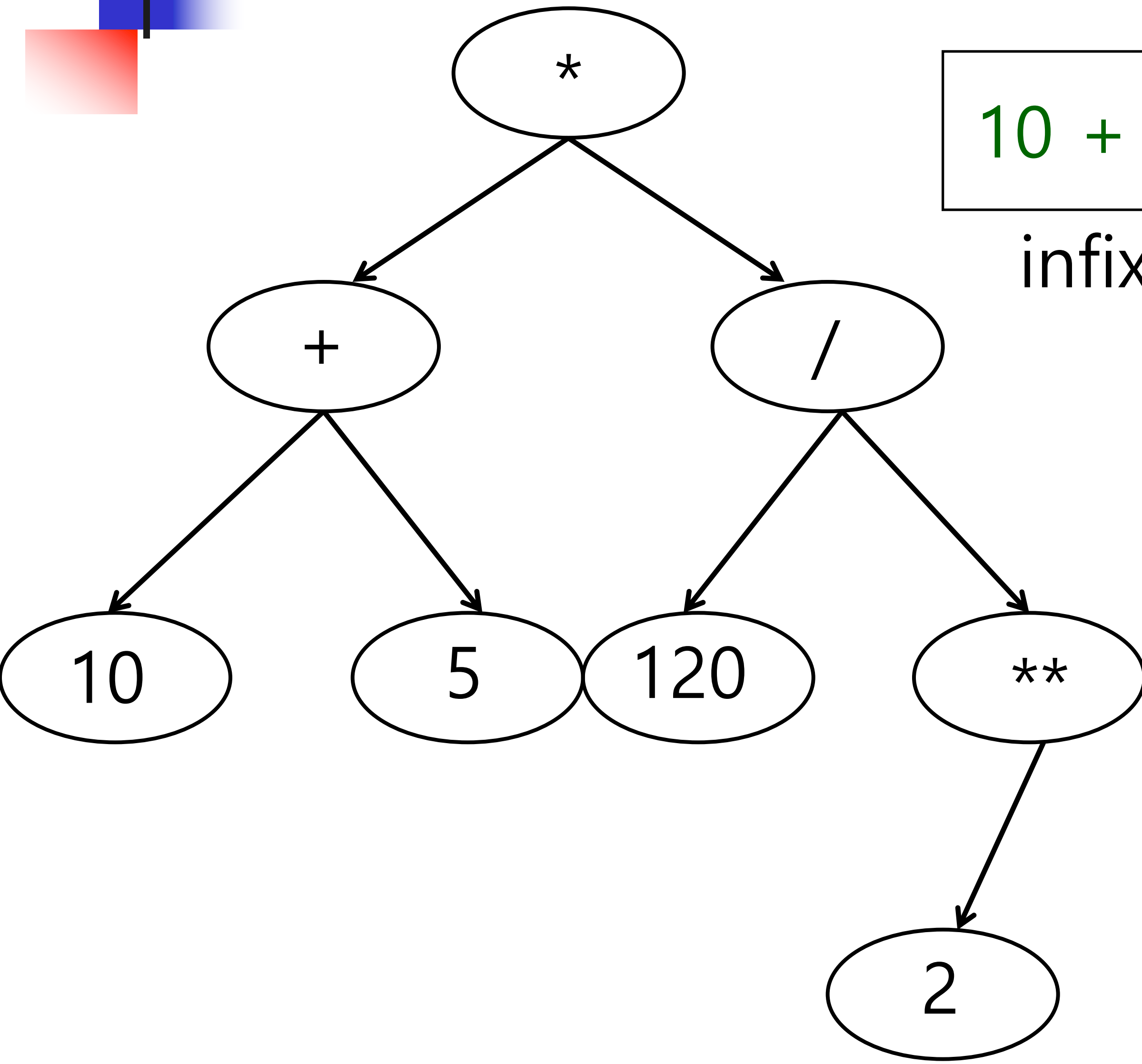
120
150
160
490

Inorder Traversal: Exercise



Inorder Traversal: Solution



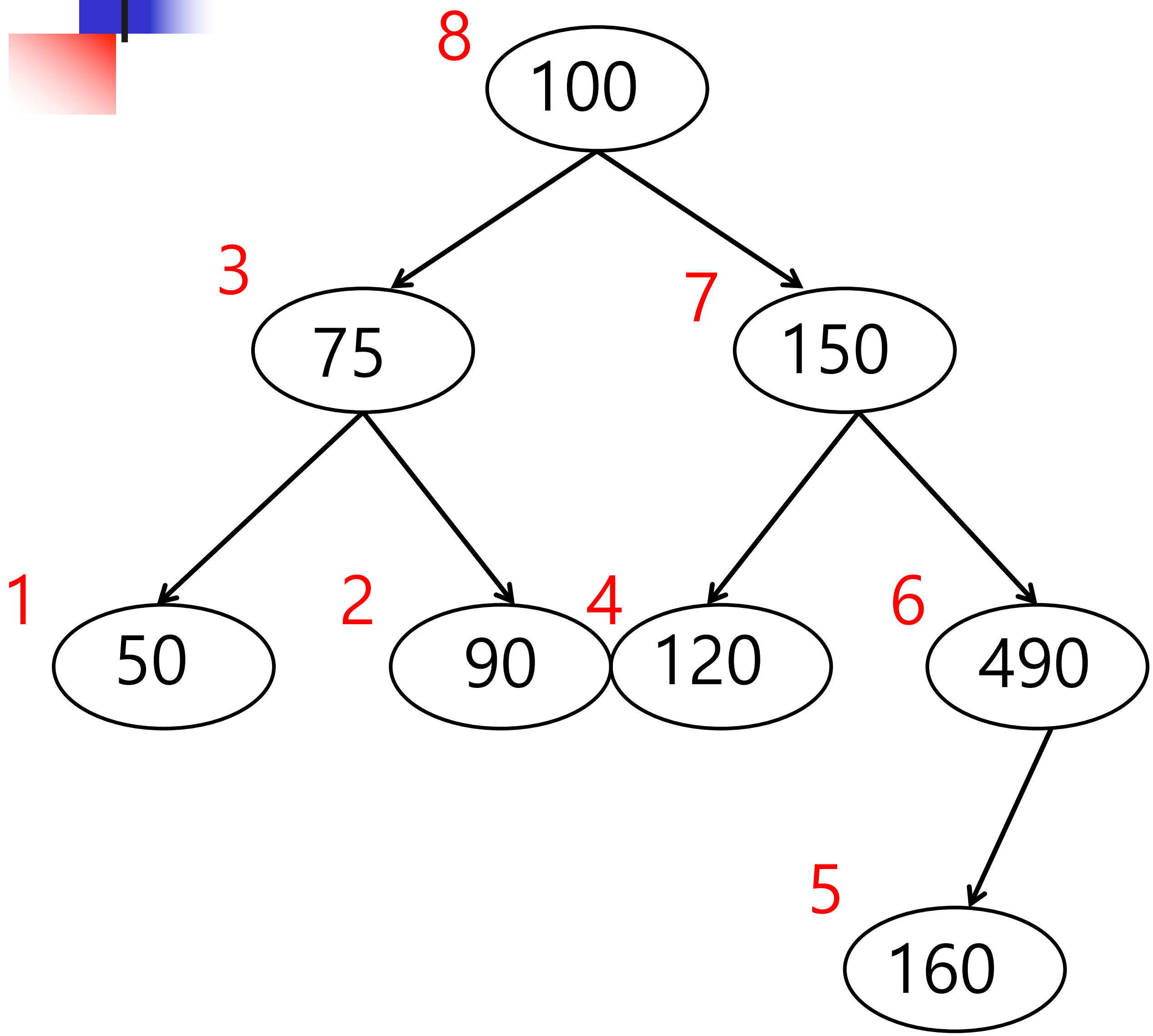
10 + 5 * 120 / 2 **

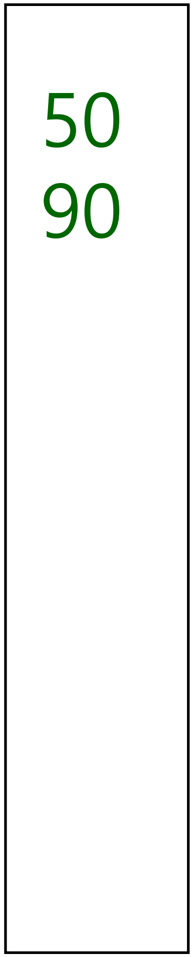
infix expression

Postorder Traversal

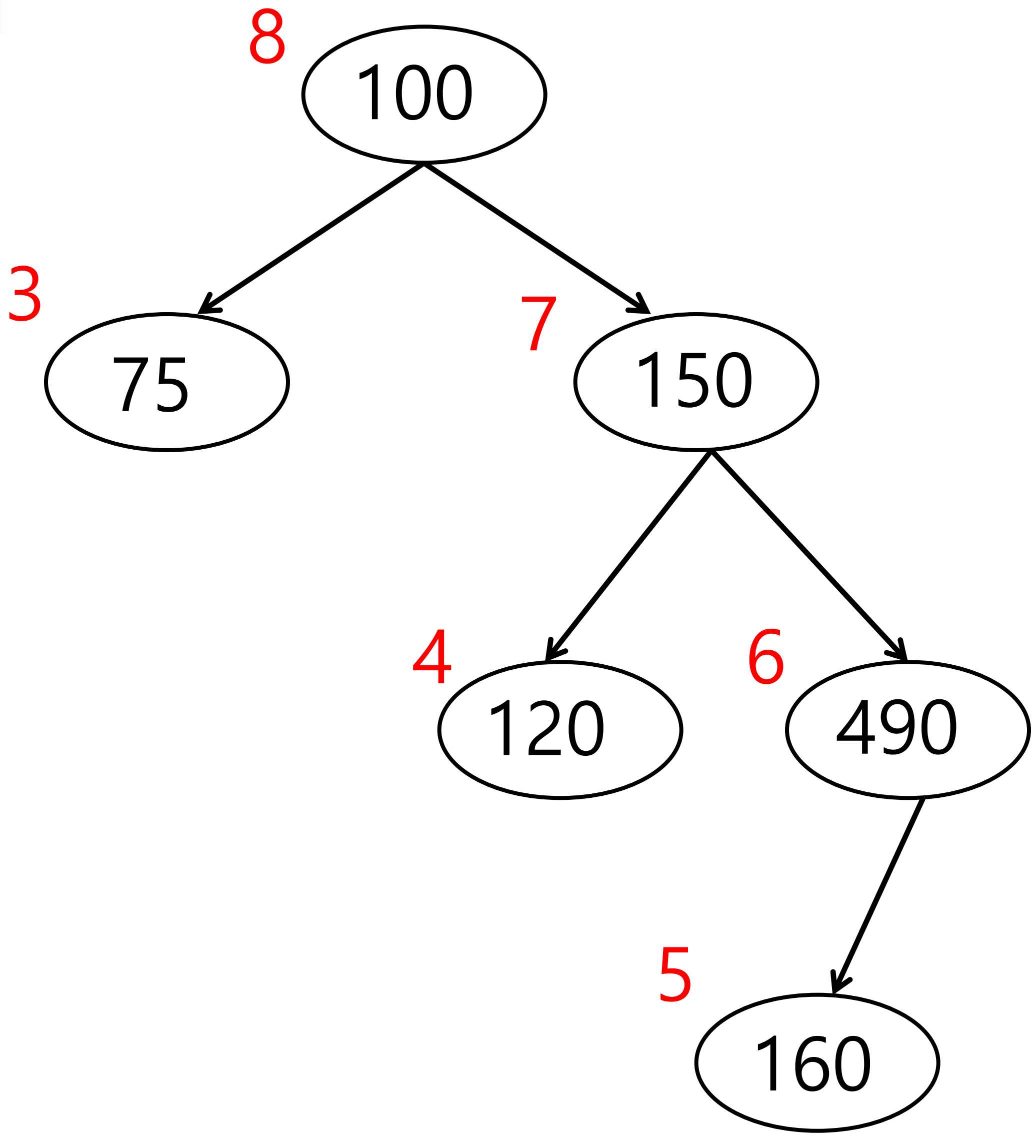
- First Child, Second Child, Mother
- Applications
  - (compiler) postfix expression evaluation
    - using a queue-like order, using a stack

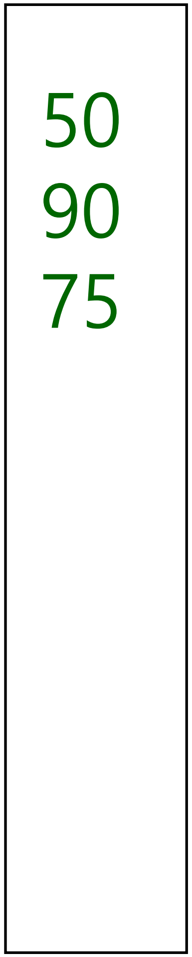
Postorder Traversal: Example (1/8)



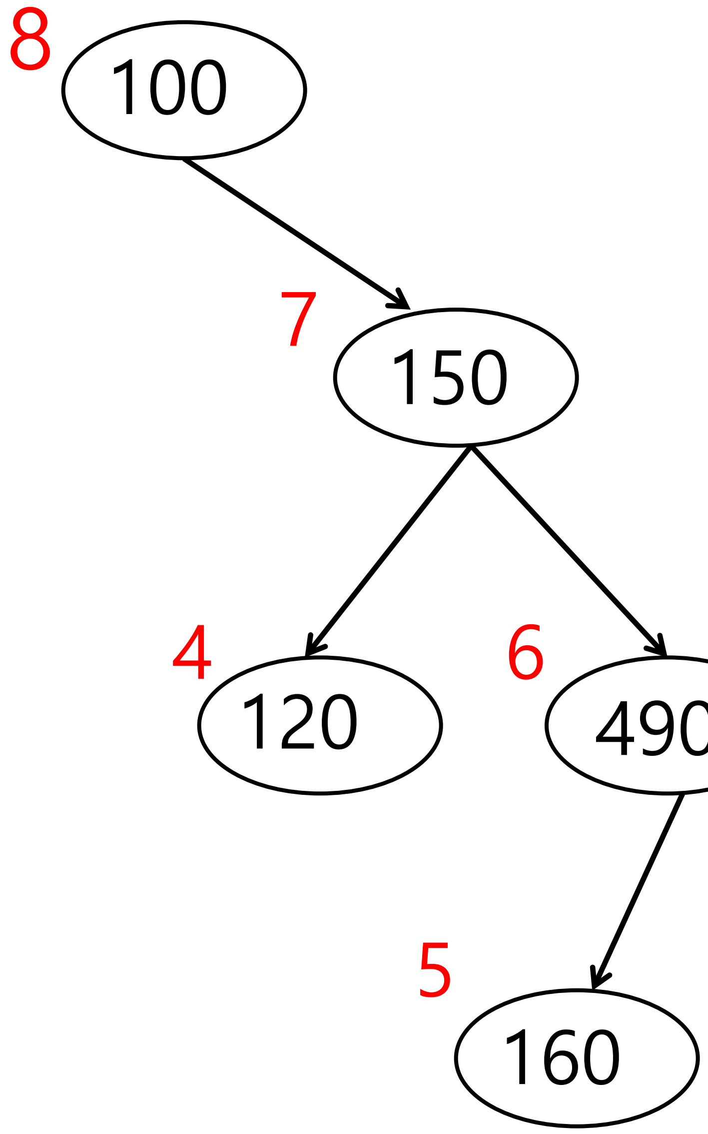


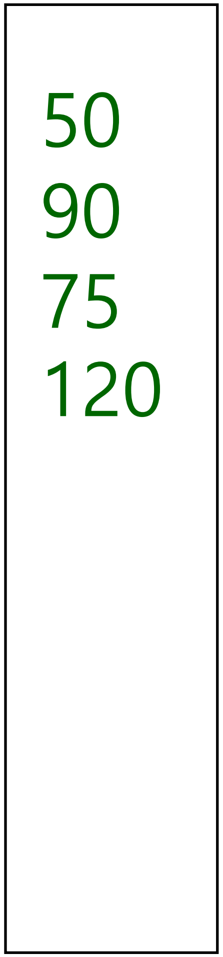
Postorder Traversal: Example (2/8)



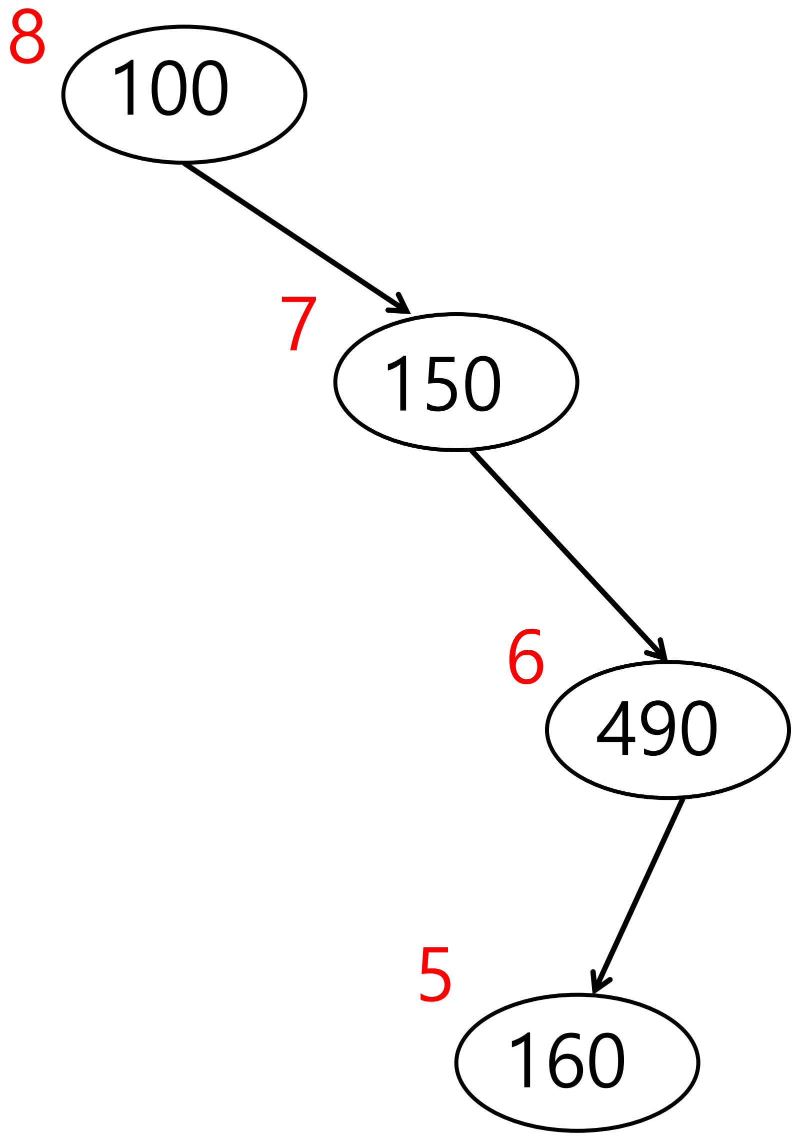


Postorder Traversal: Example (3/8)



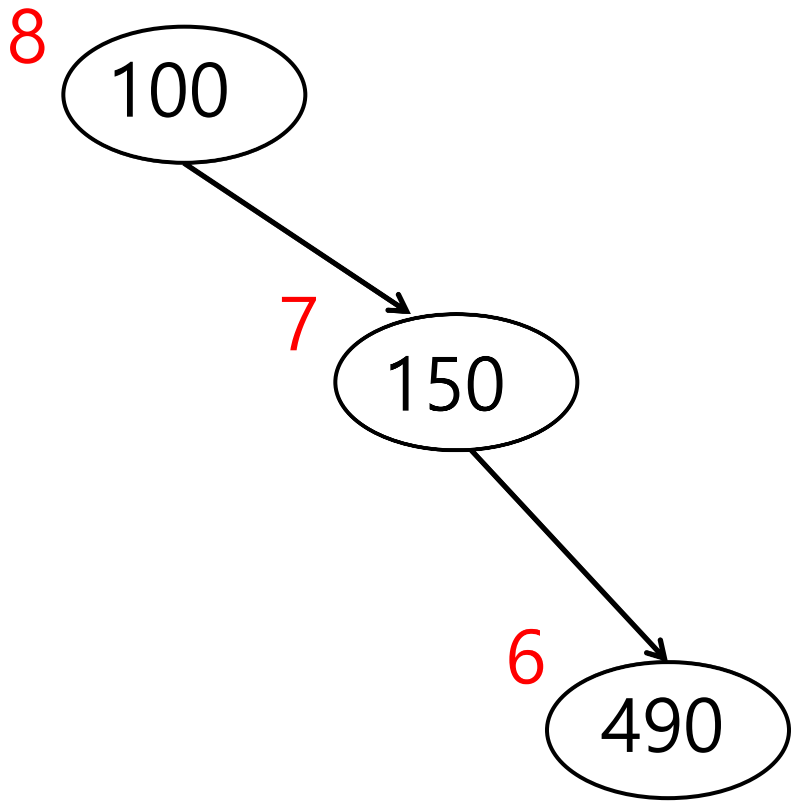


Postorder Traversal: Example (4/8)



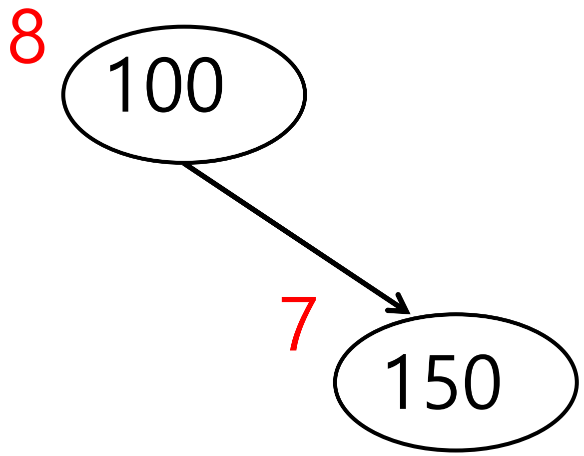
50
90
75
120
160

Postorder Traversal: Example (5/8)



50
90
75
120
160
490

Postorder Traversal: Example (6/8)



50
90
75
120
160
490
150

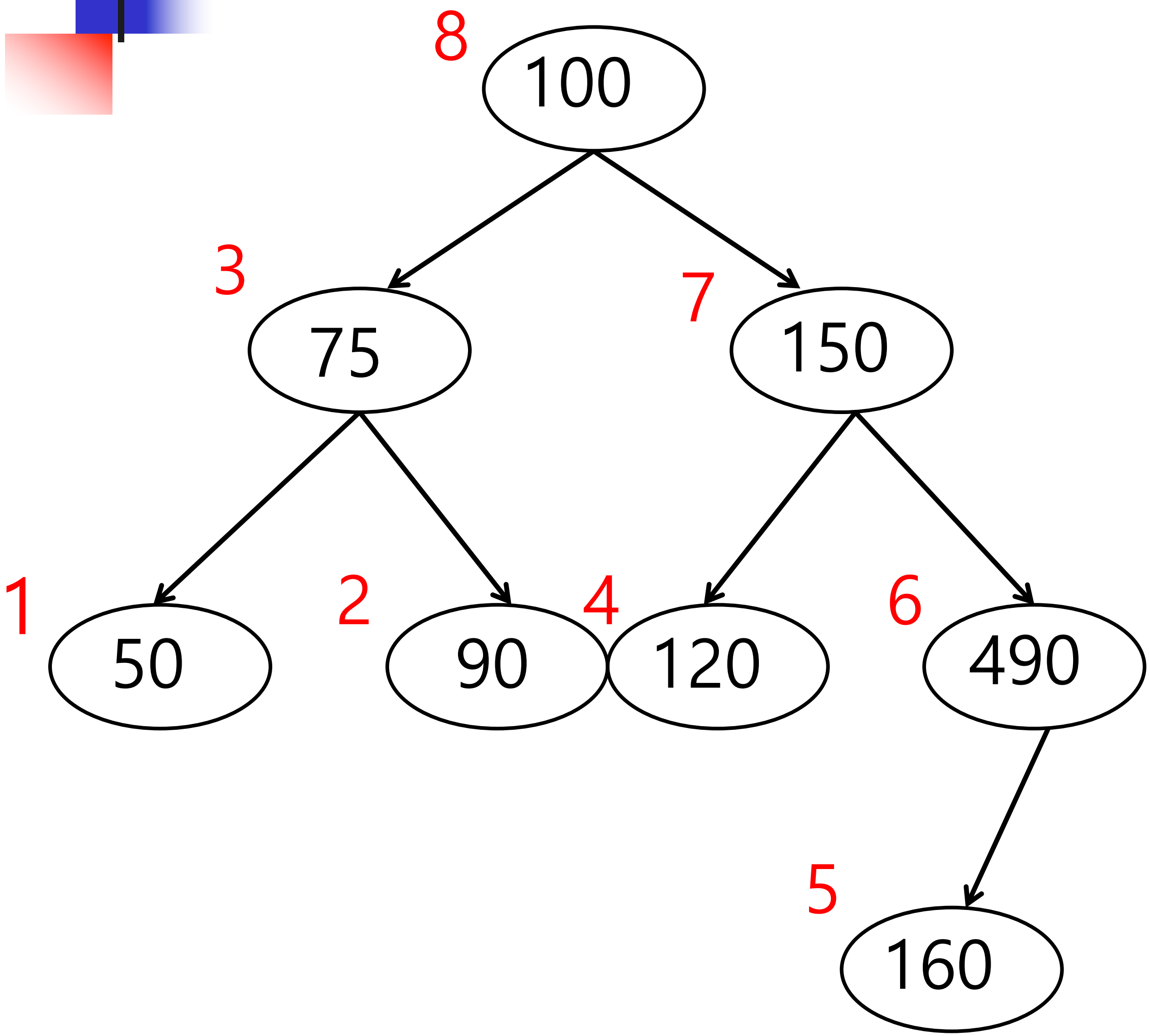
Postorder Traversal: Example (7/8)

8

100

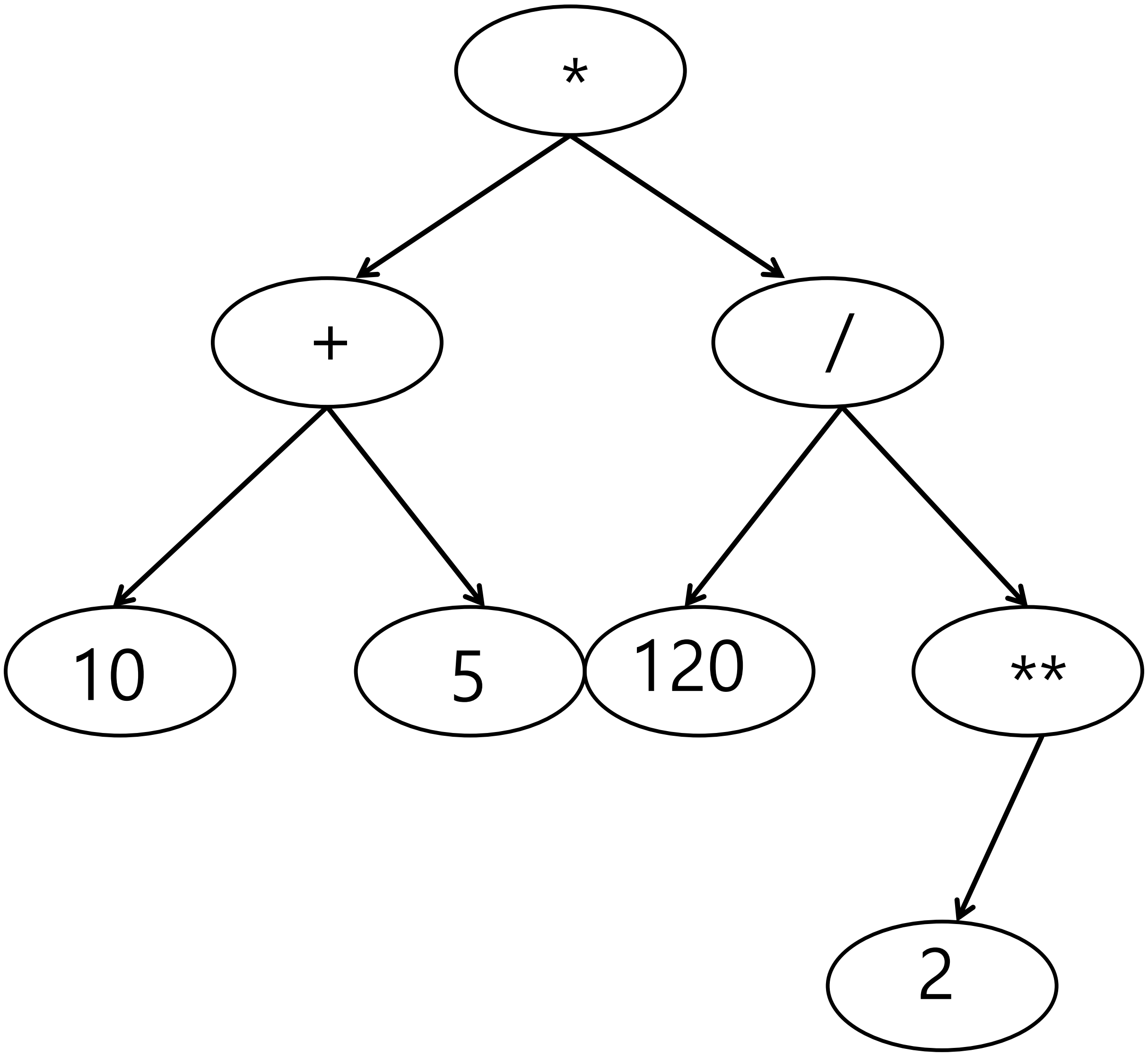
50
90
75
120
160
490
150
100

Postorder Traversal: Example (8/8)

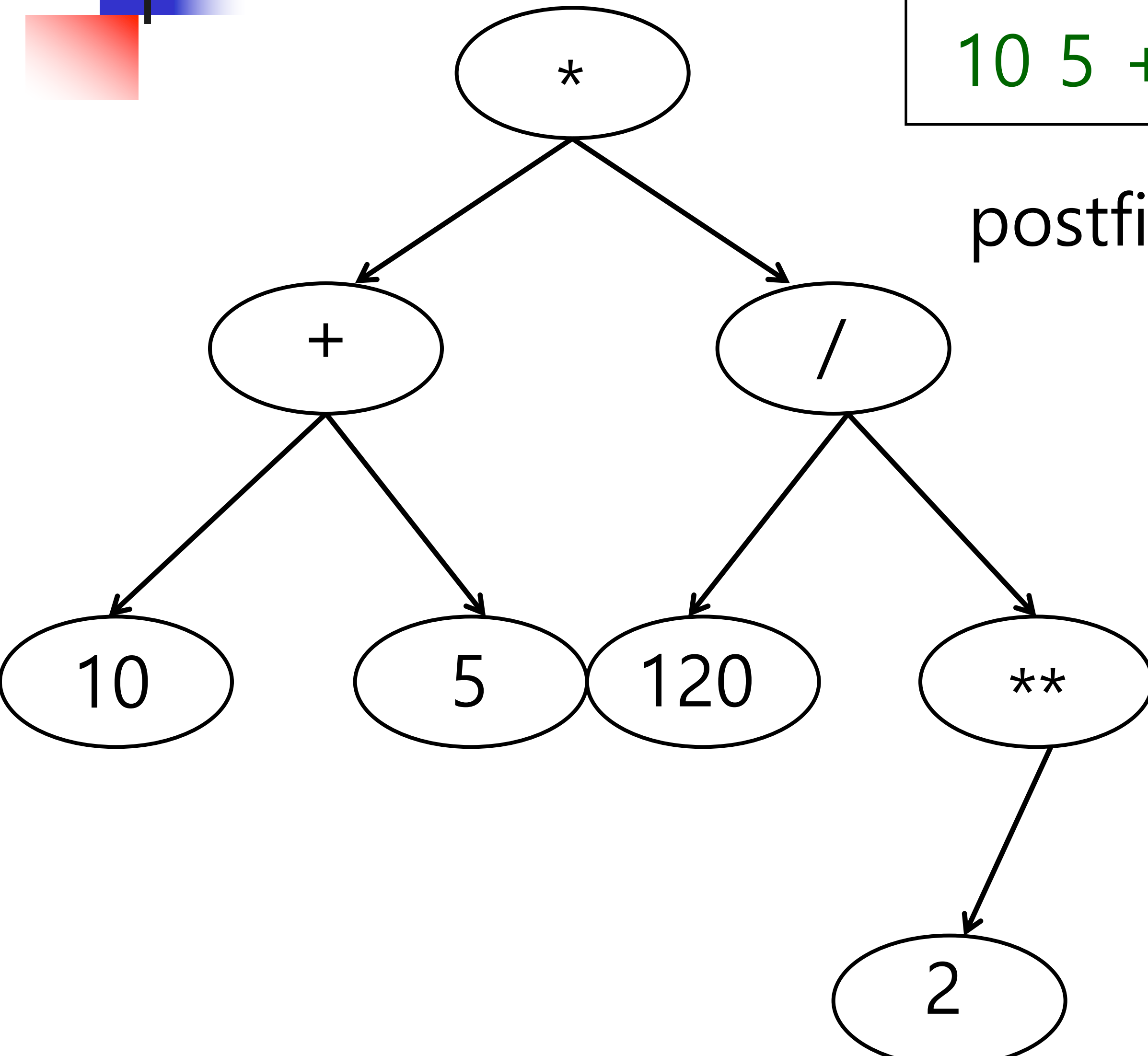


50
90
75
120
160
490
150
100

Postorder Traversal: Exercise



Postorder Traversal: Solution



10 5 + 120 2 ** / *

postfix expression

Evaluating a Postfix Expression Using a Stack (How compilers evaluate expressions) (1/5)

Pop stack-1.
If it is a number,
    push it to stack-2
If it is an operator,
    pop stack-2 and compute,
Push the result to stack-2

reverse stack

10 5 + 120 2 ** / *

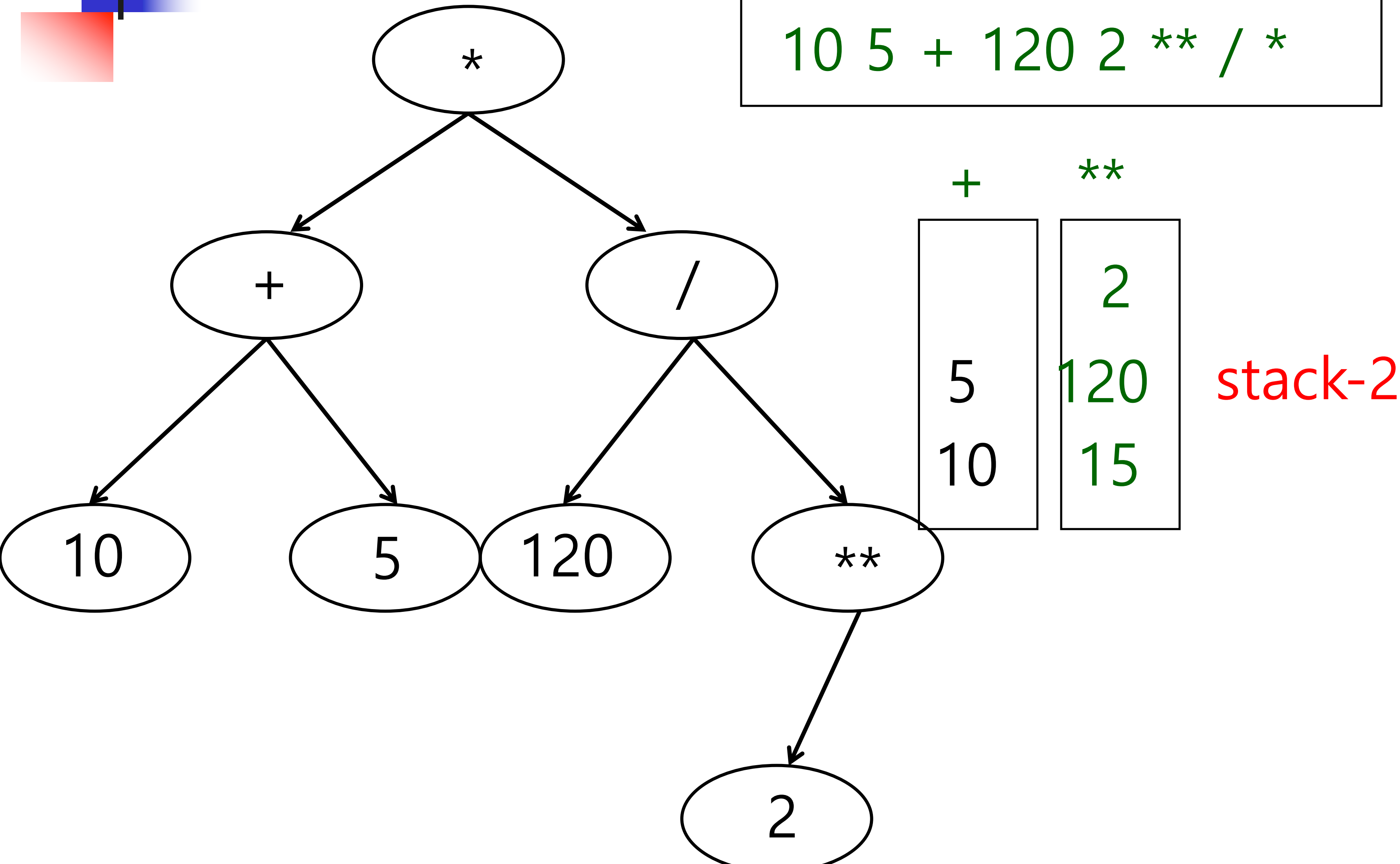
+

5
10

stack-2

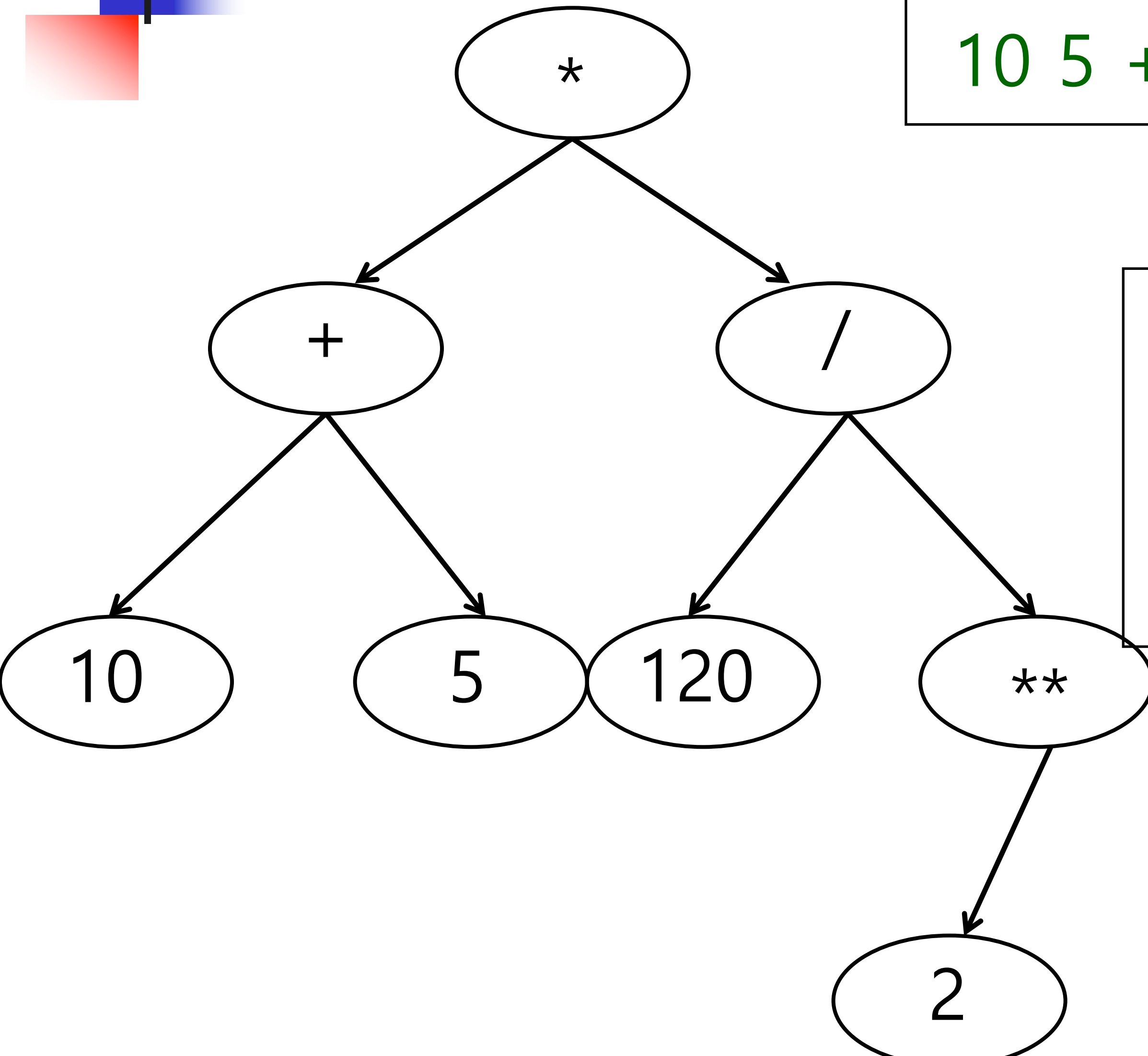
(2/5)

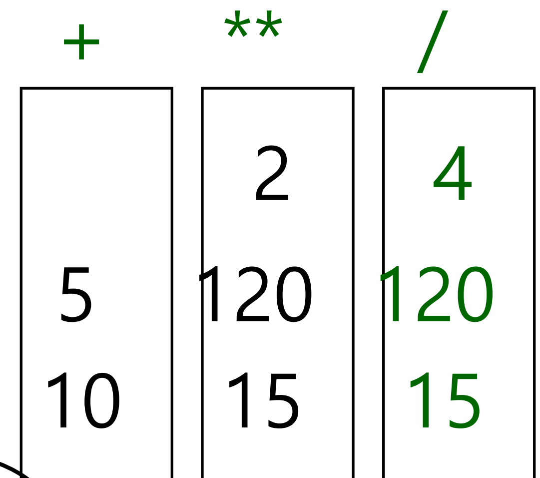
10 5 + 120 2 ** / *



(3/5)

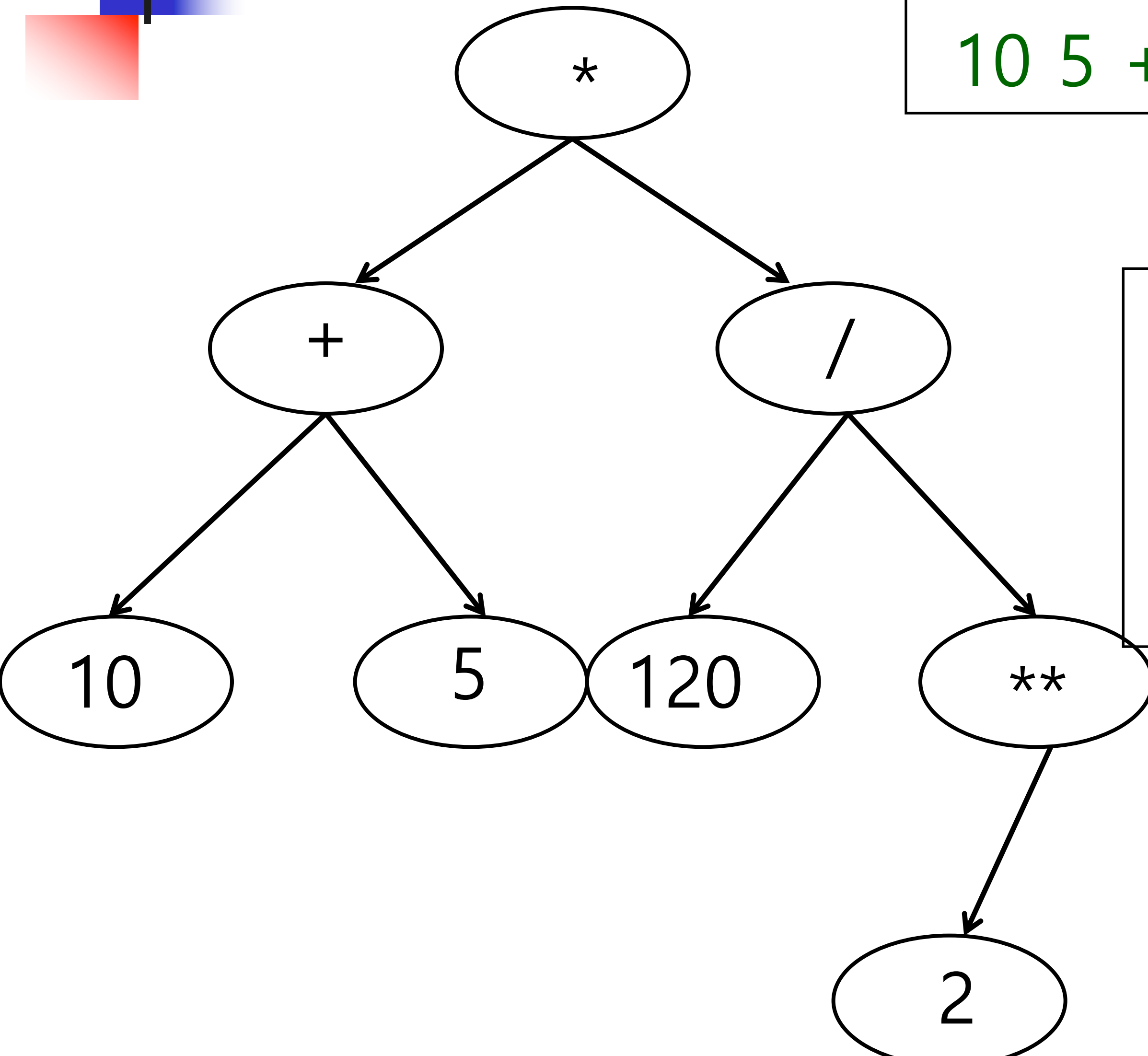
10 5 + 120 2 ** / *

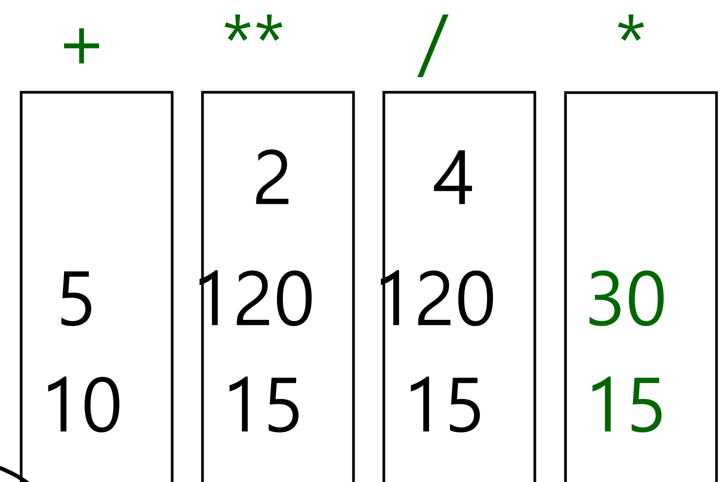




(4/5)

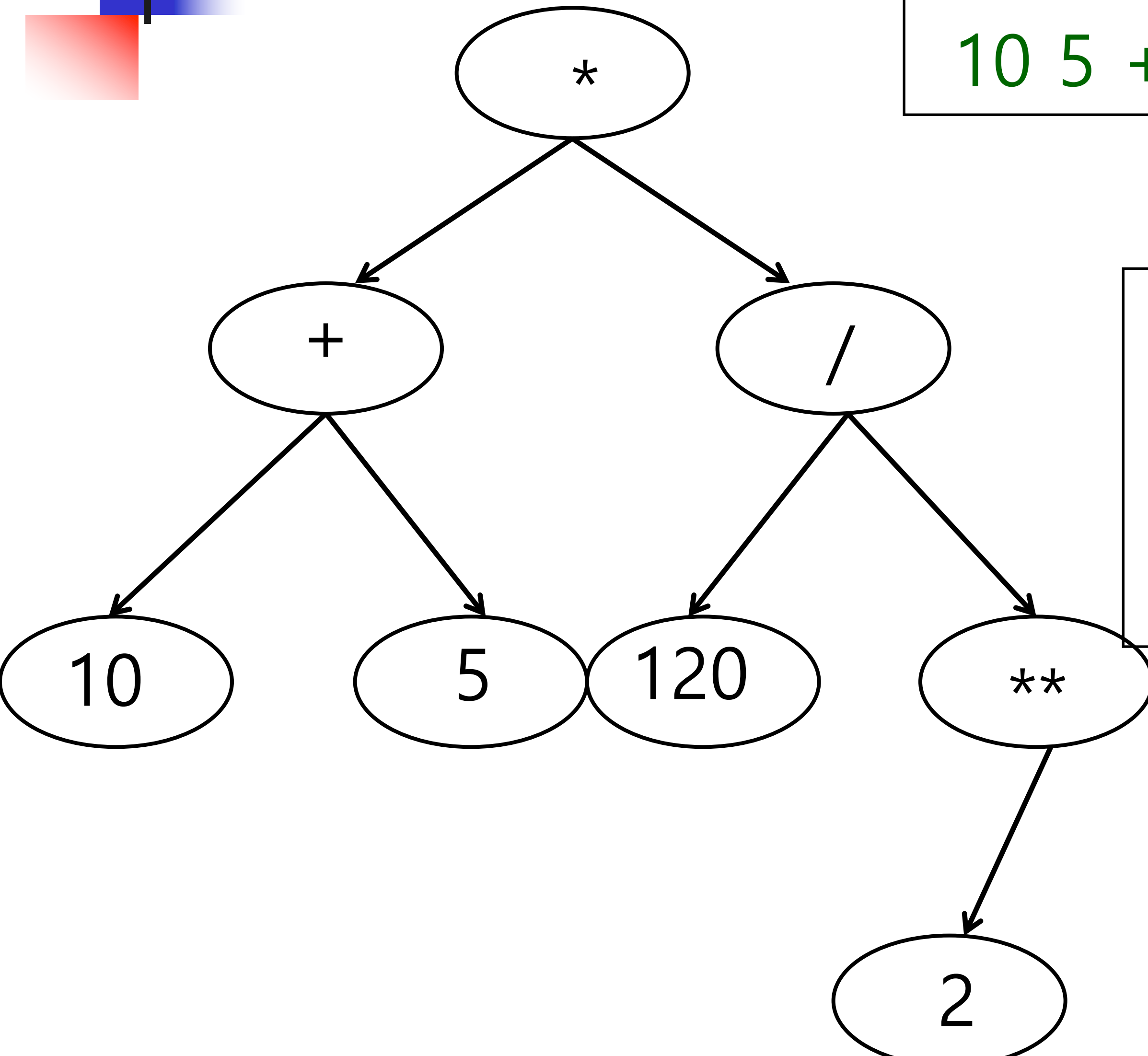
10 5 + 120 2 ** / *

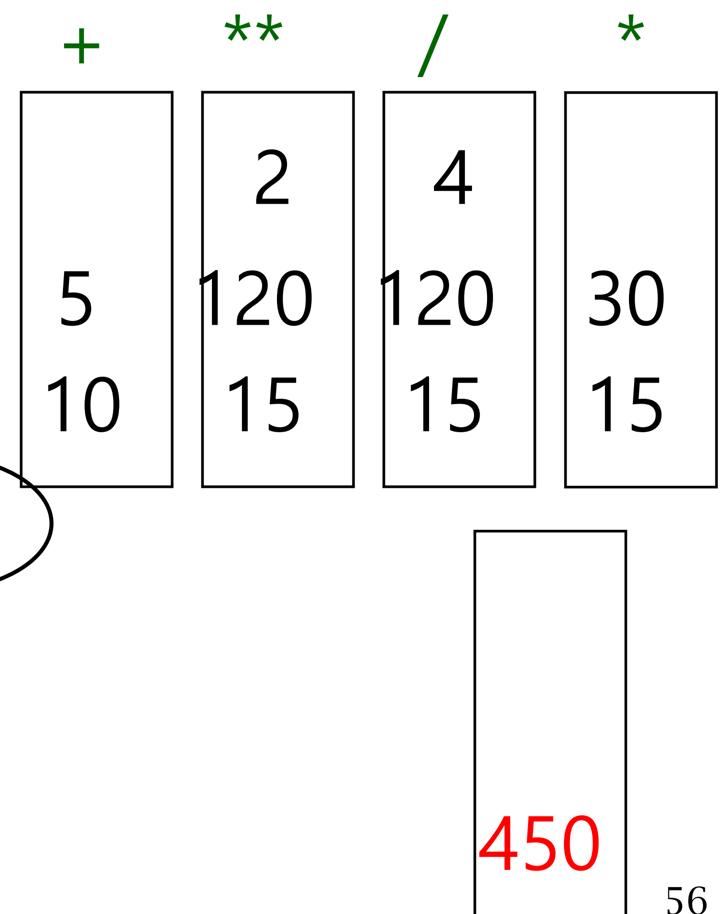




(5/5)

10 5 + 120 2 ** / *

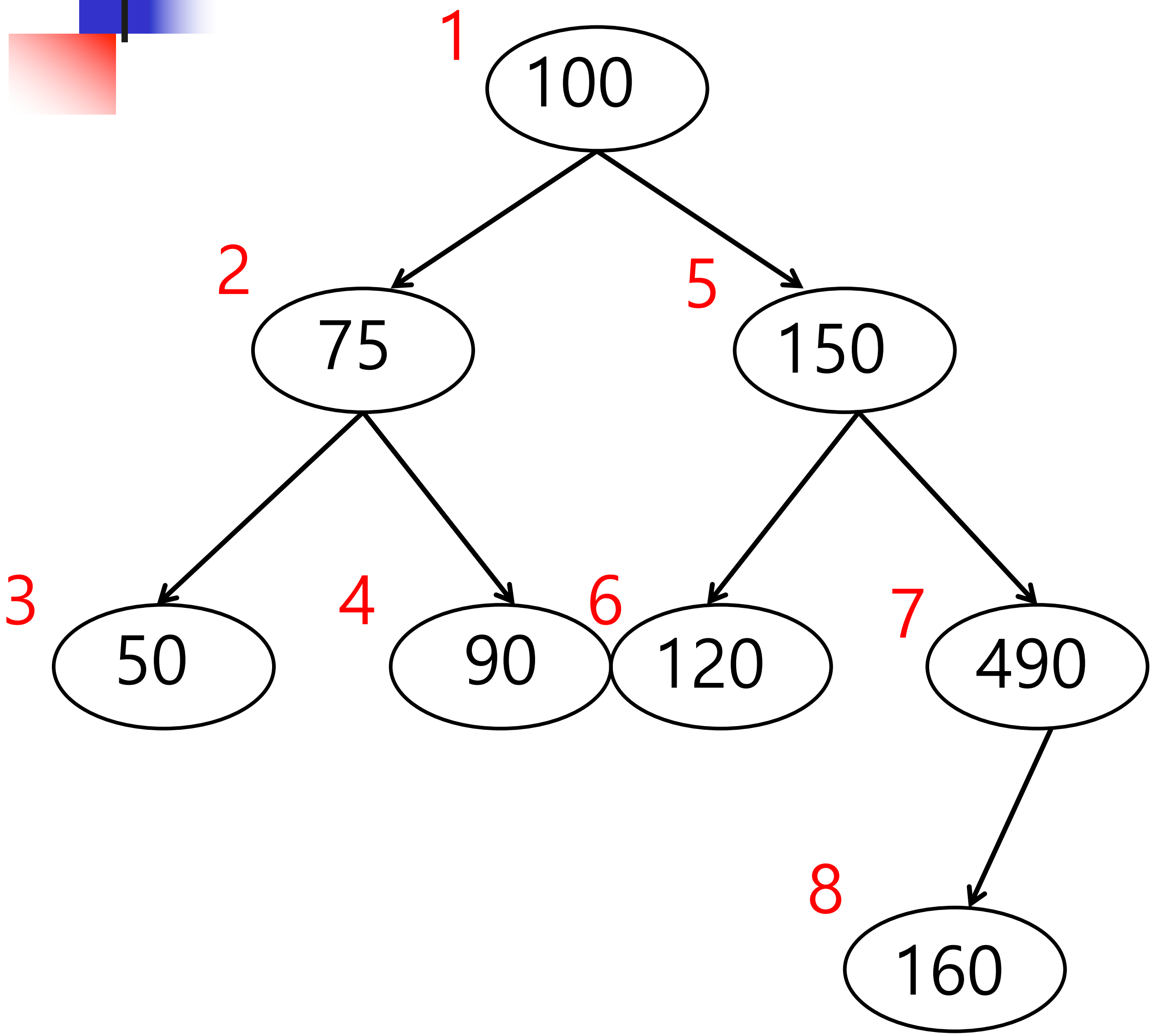




Preorder Traversal

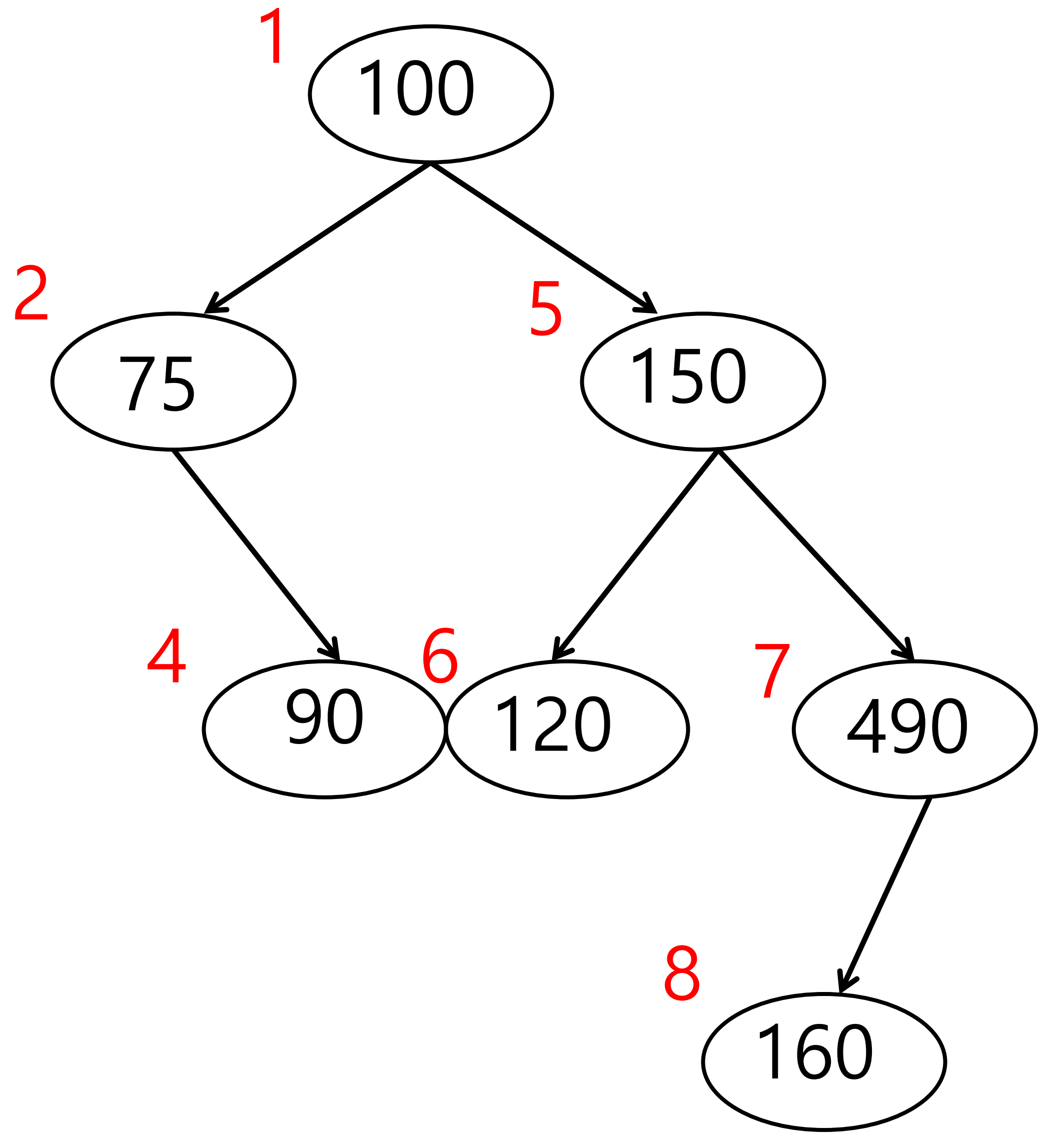
- Mother, First Child, Second Child
- Applications
  - Make a complete copy of a tree
  - (compiler) prefix expression evaluation
    - In stack-like (reverse) order, using a stack

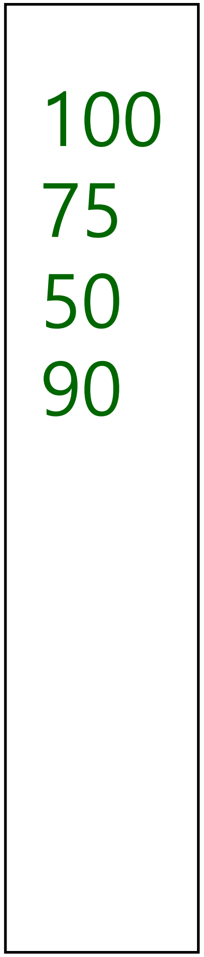
Preorder Traversal: Example (1/6)



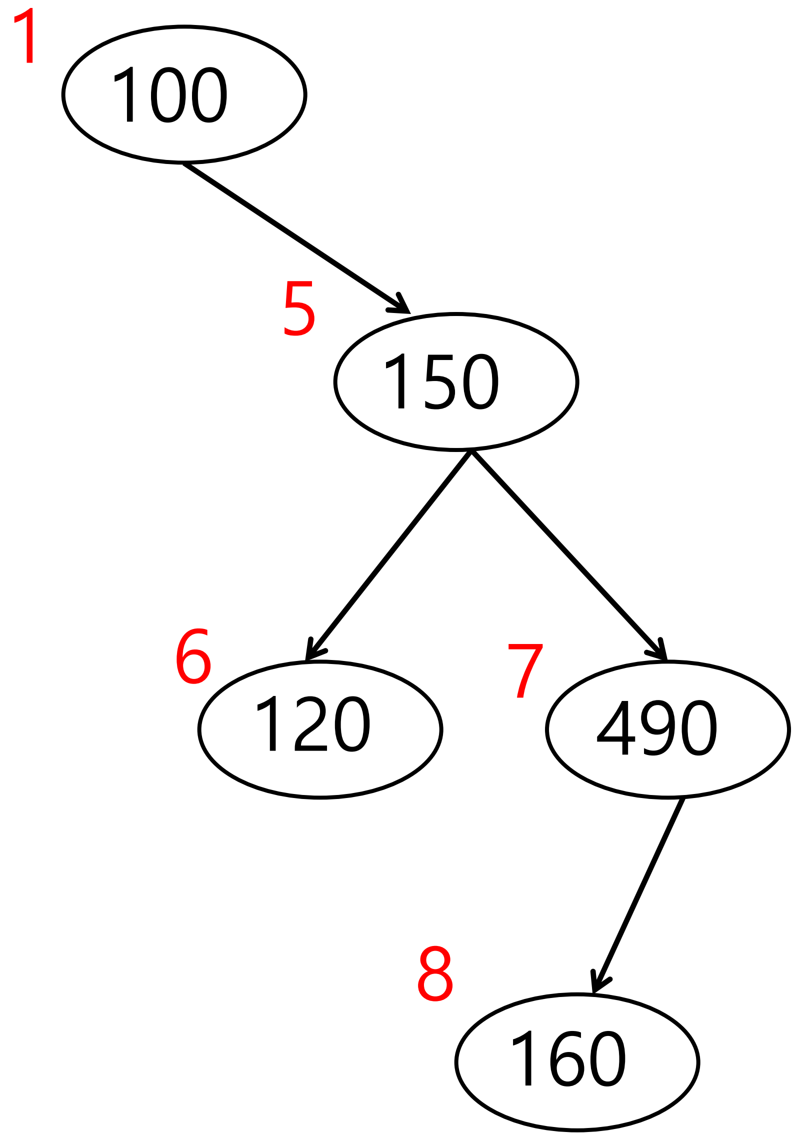
100
75
50

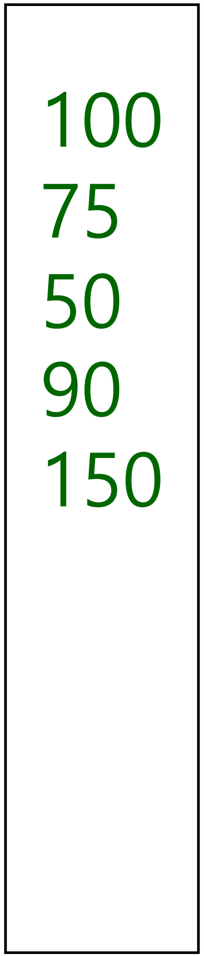
Preorder Traversal: Example (2/6)



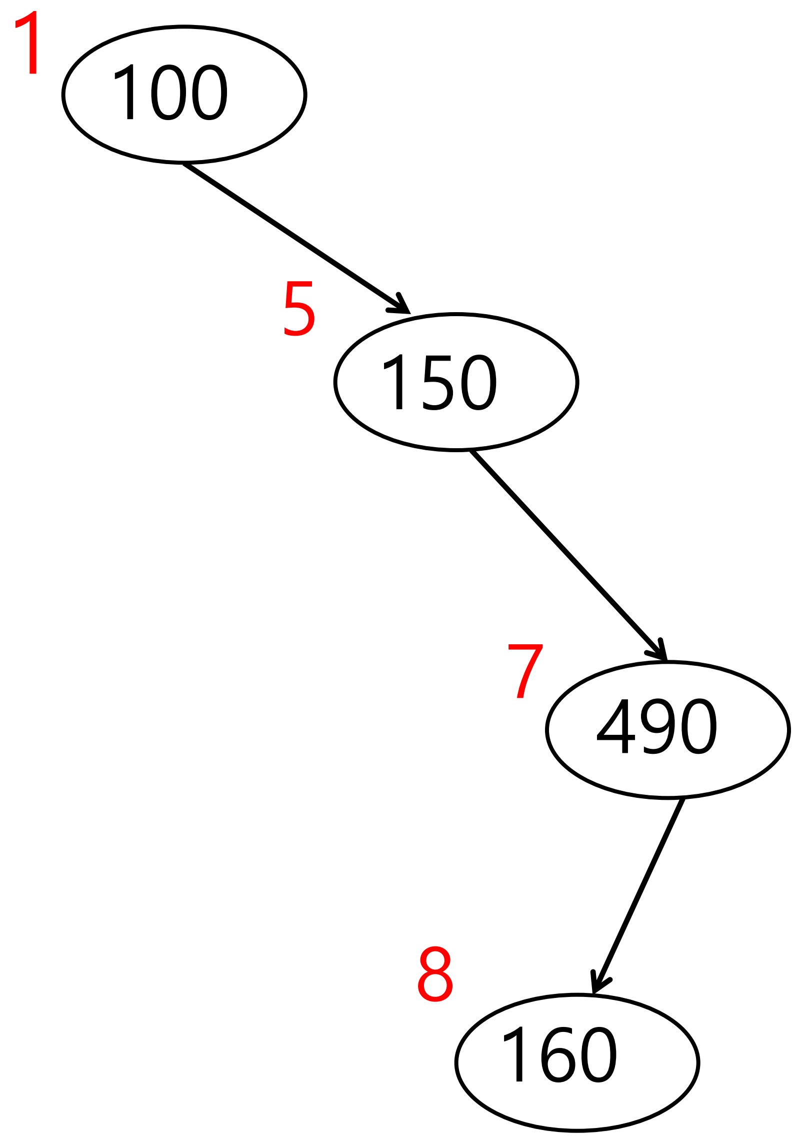


Preorder Traversal: Example (3/6)





Preorder Traversal: Example (4/6)



100
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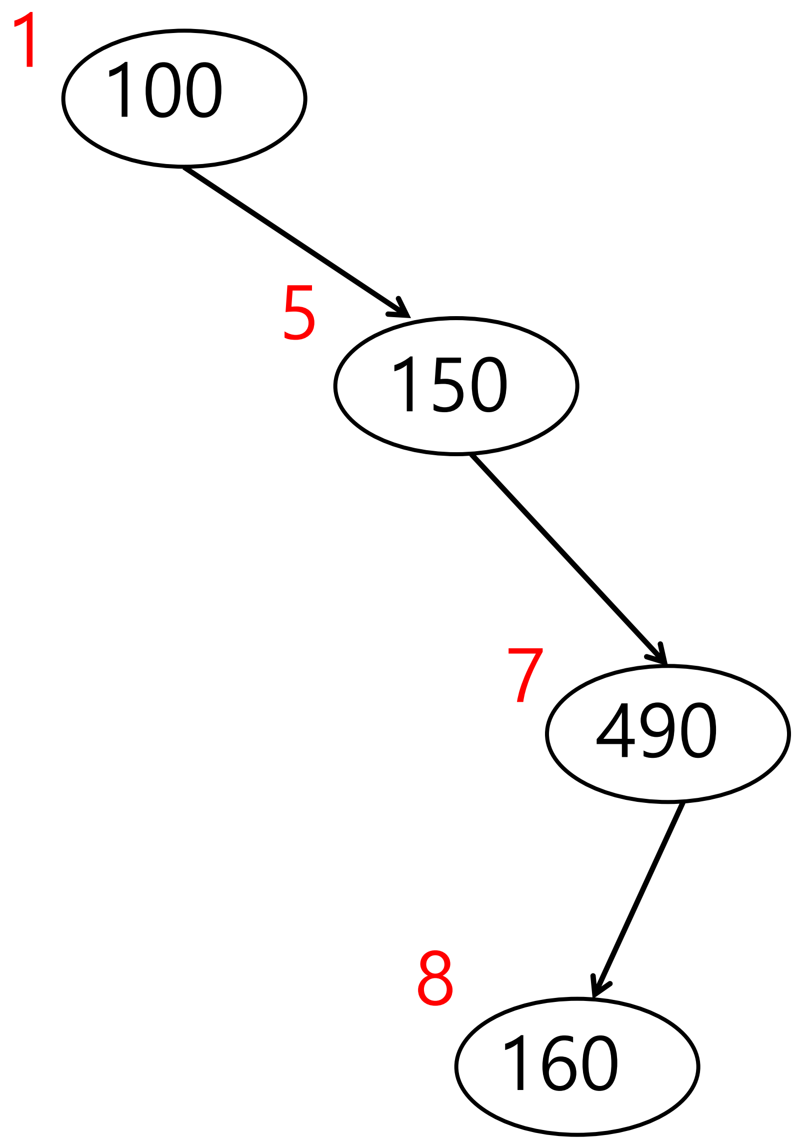
150
120

Preorder Traversal: Example (5/6)



100
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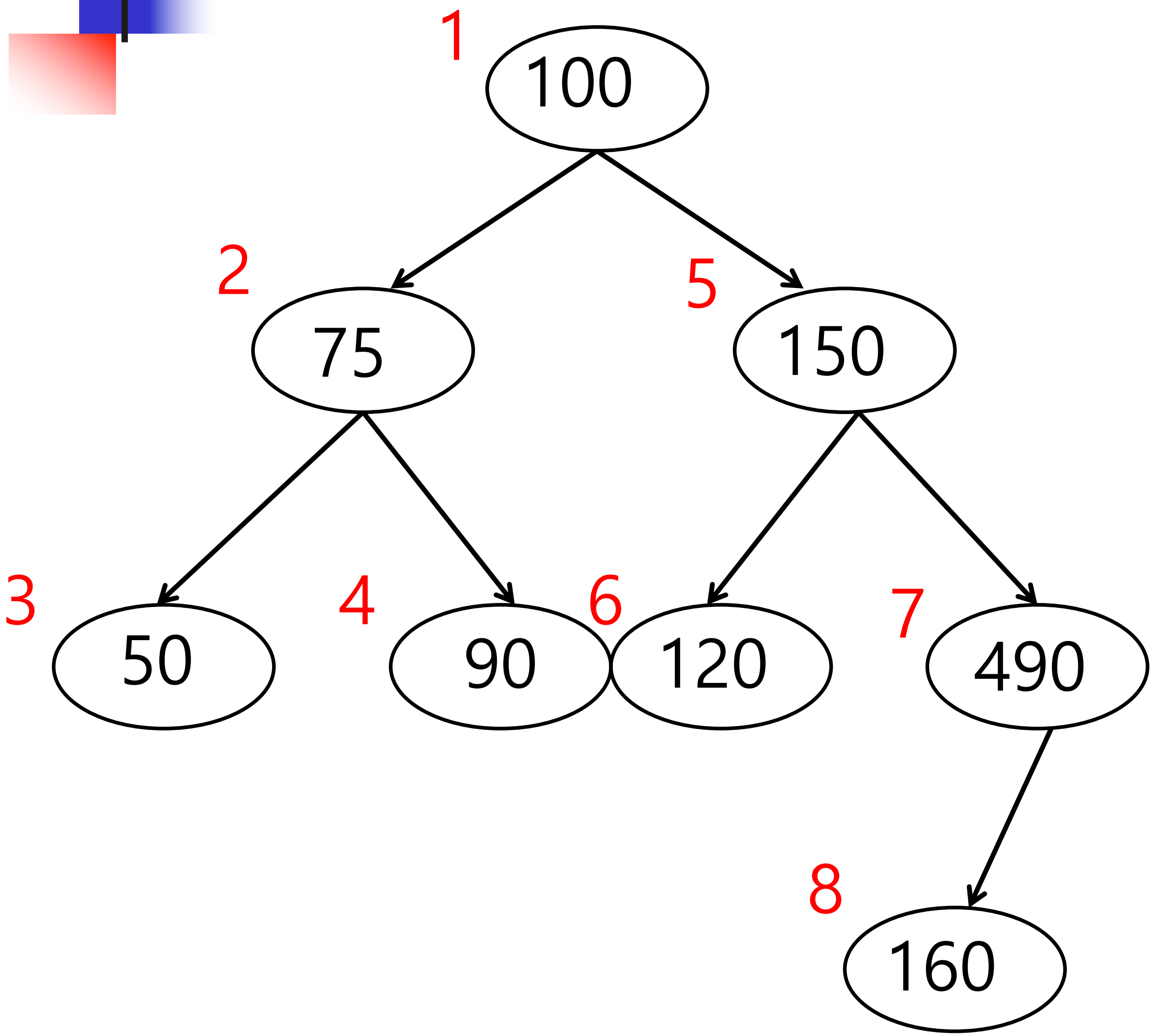
50
90
150
120
490
160

Preorder Traversal: Example (6/6)



100
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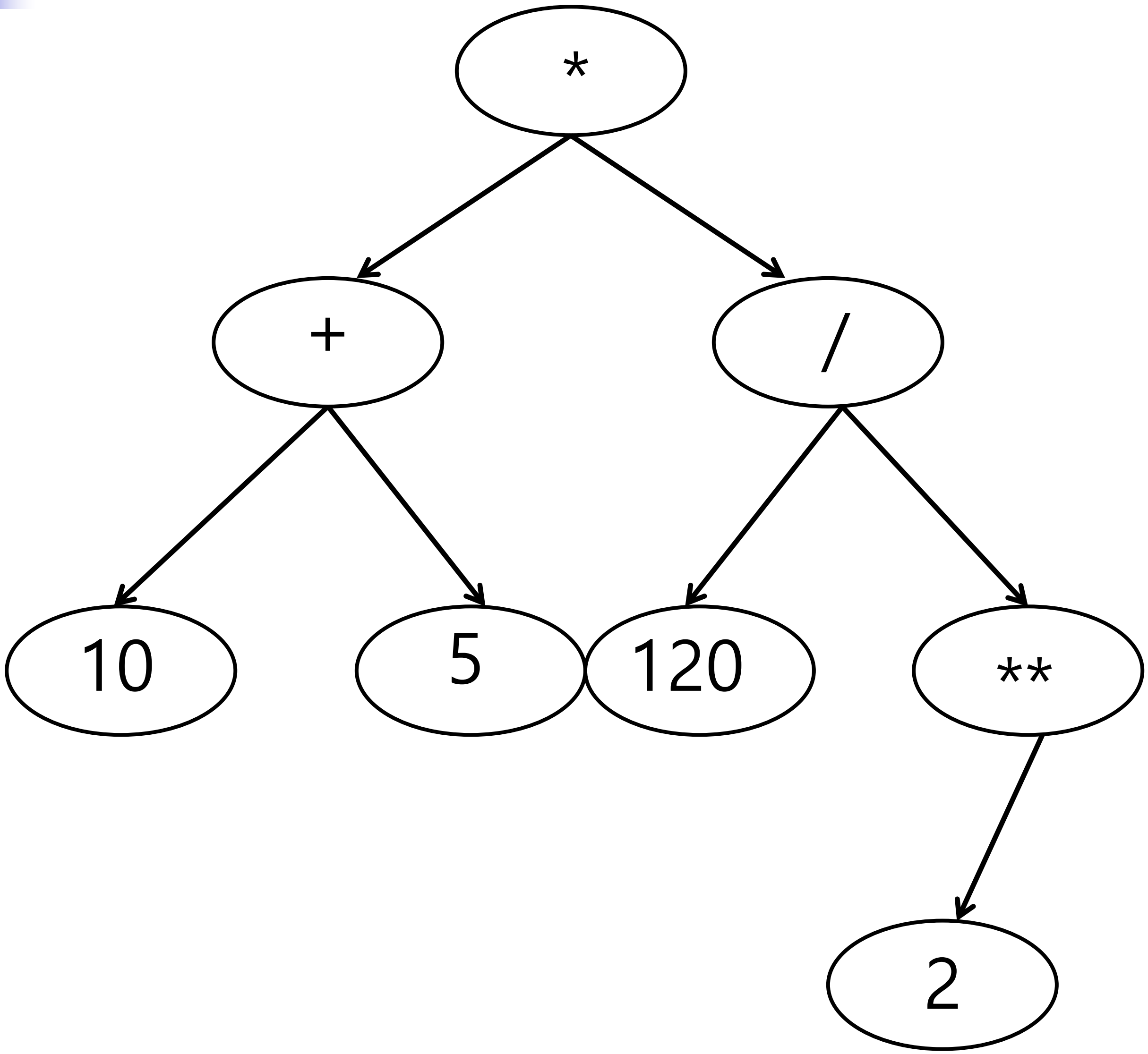
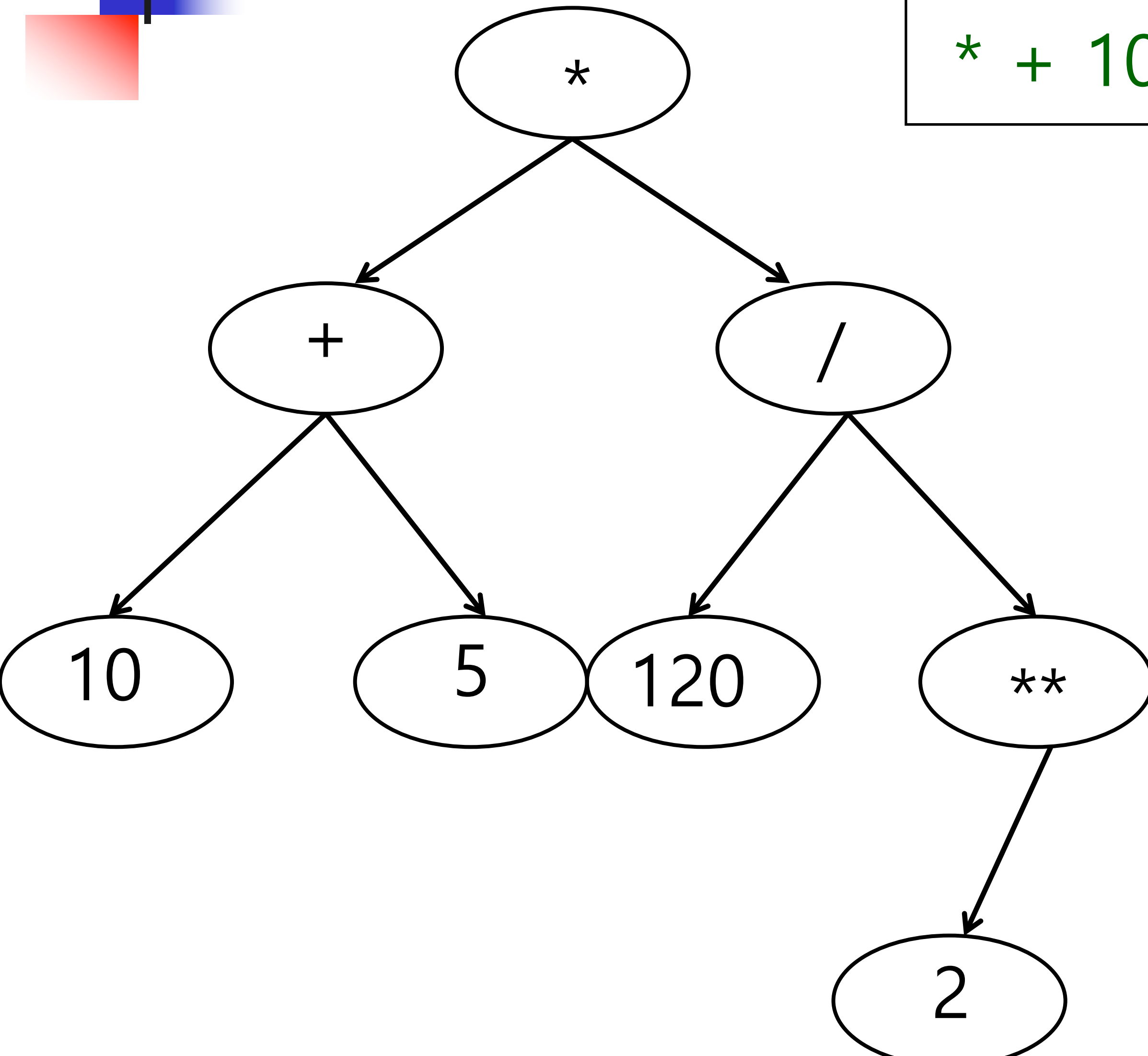
50
90
150
120
490
160

Preorder Traversal: Exercise

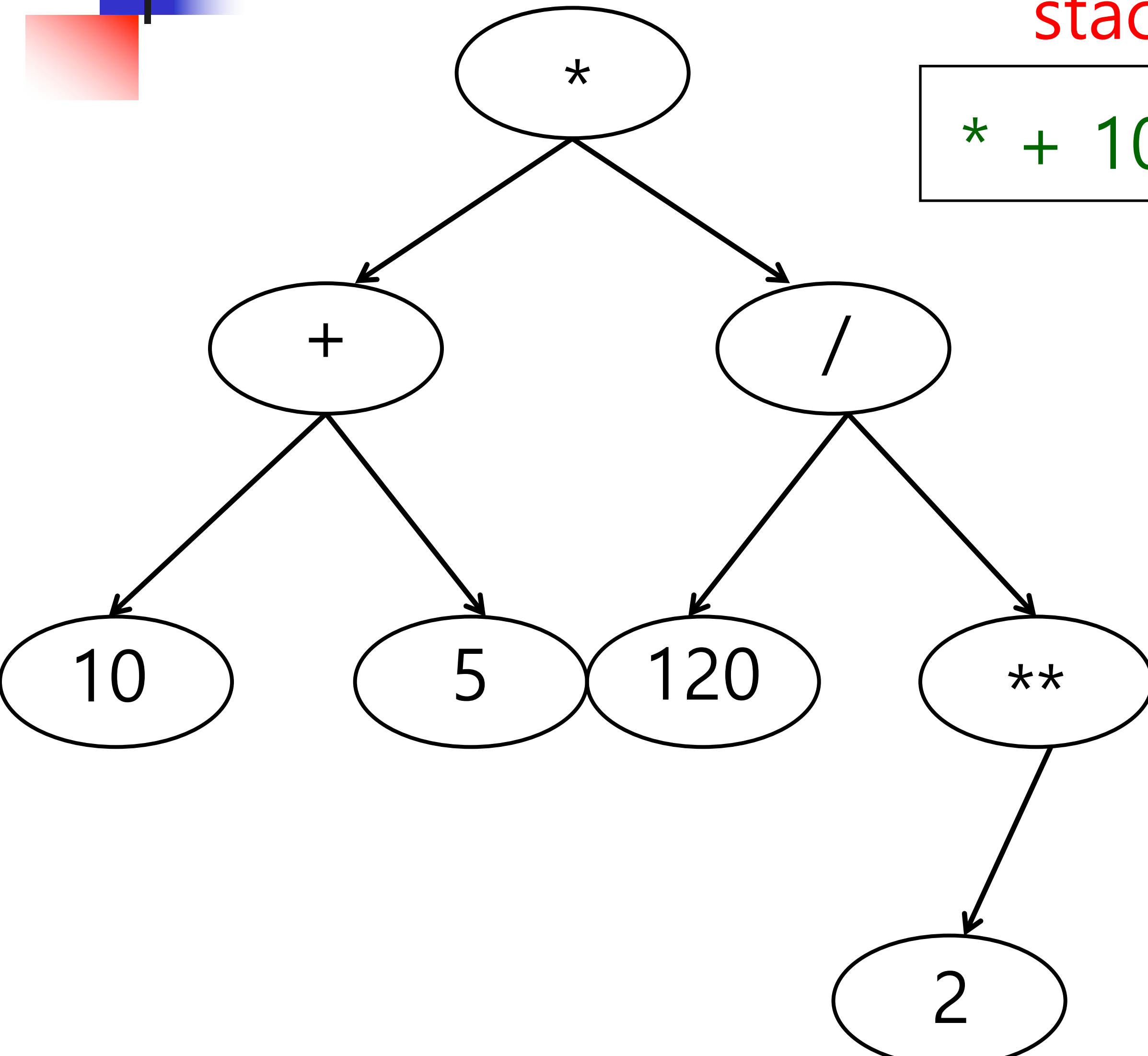


Preorder Traversal: Solution



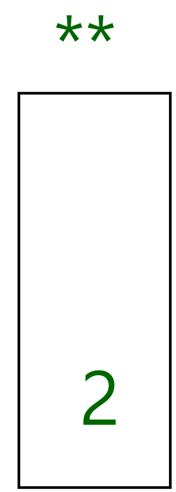
* + 10 5 / 120 ** 2

Evaluating a Prefix Expression Using a Stack (1/5)

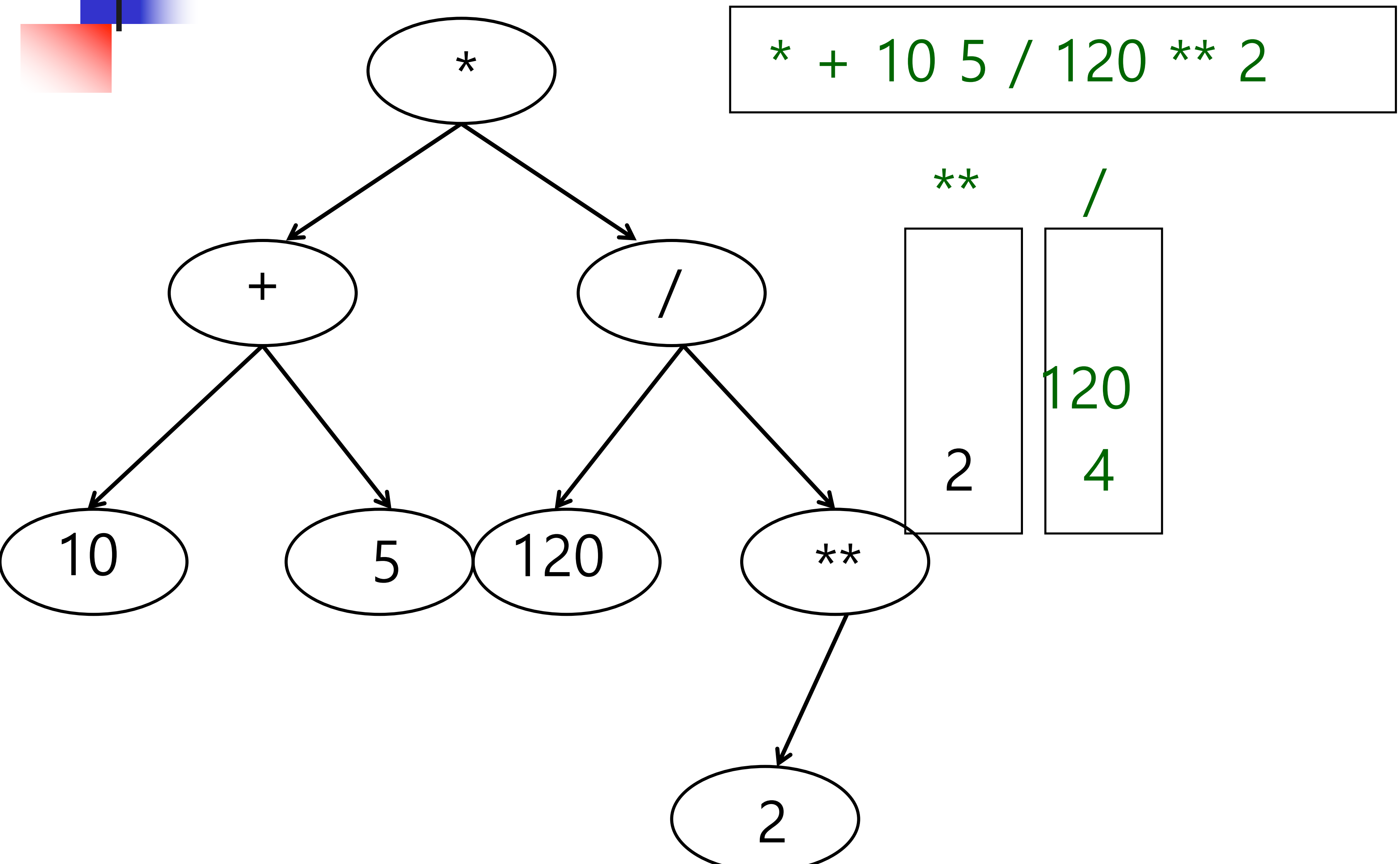


stack-1

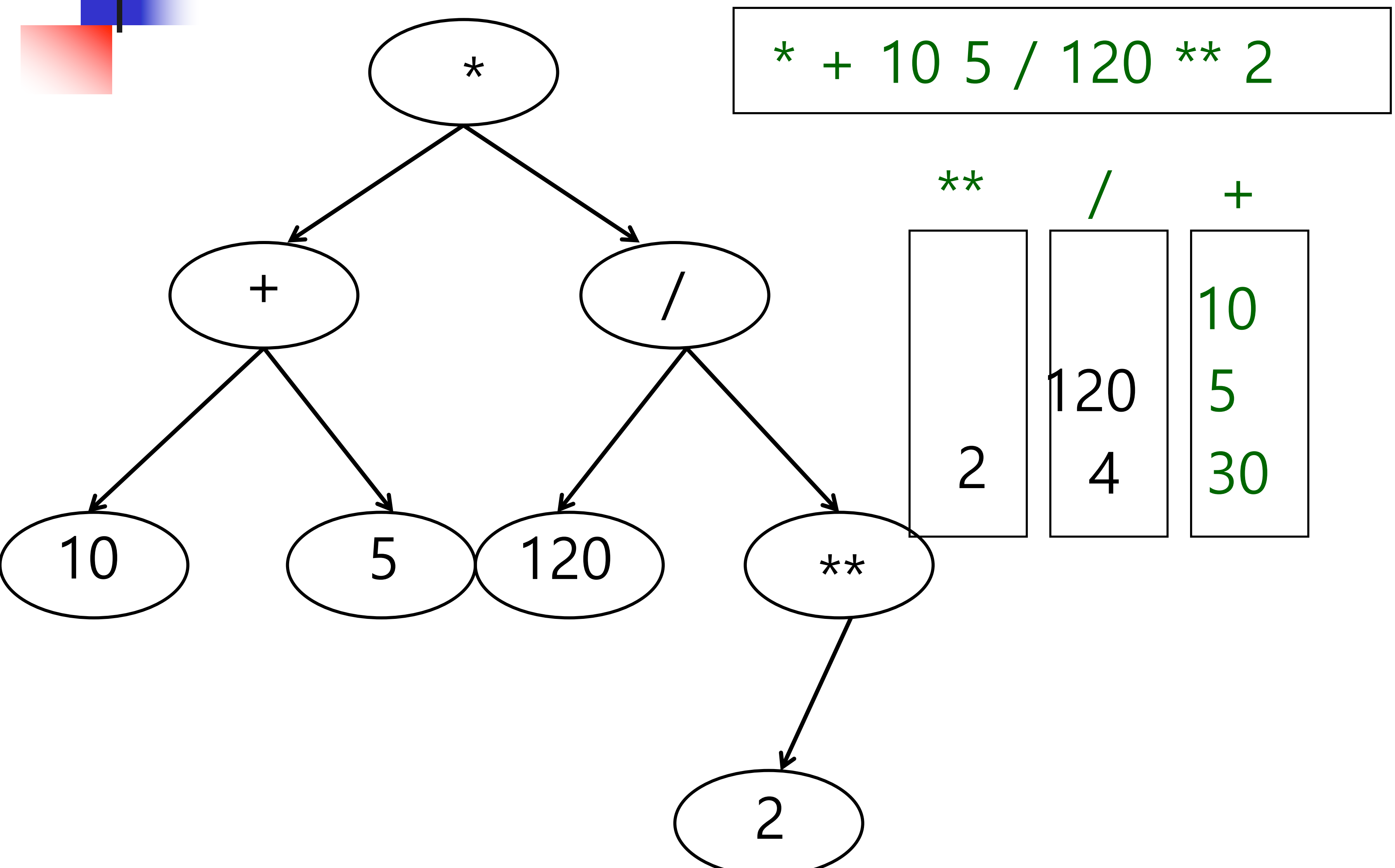
* + 10 5 / 120 ** 2



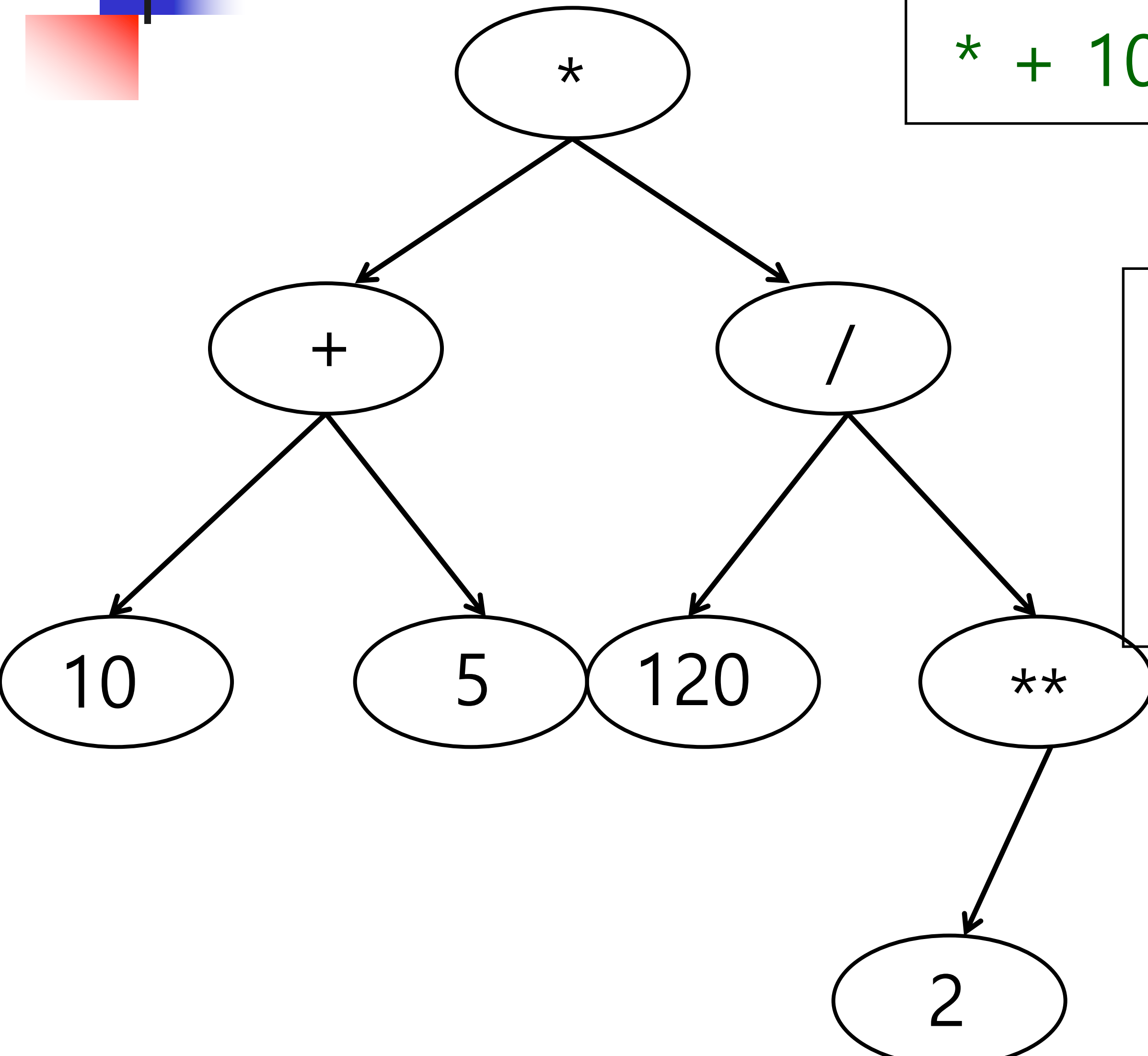
(2/5)



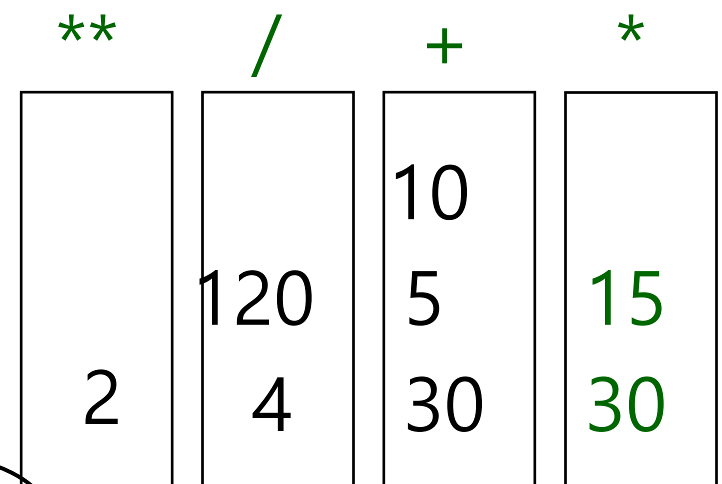
(3/5)



(4/5)

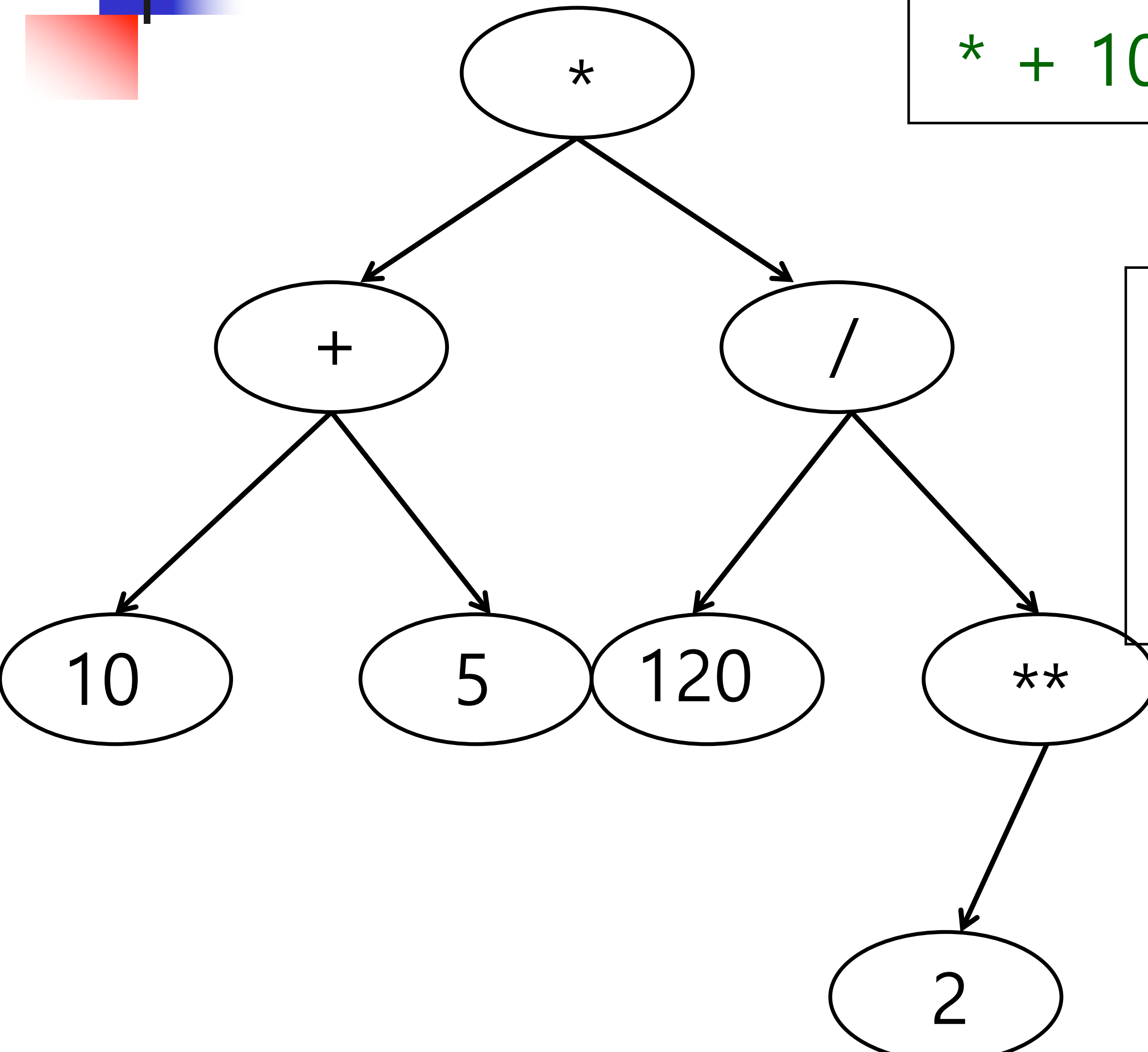


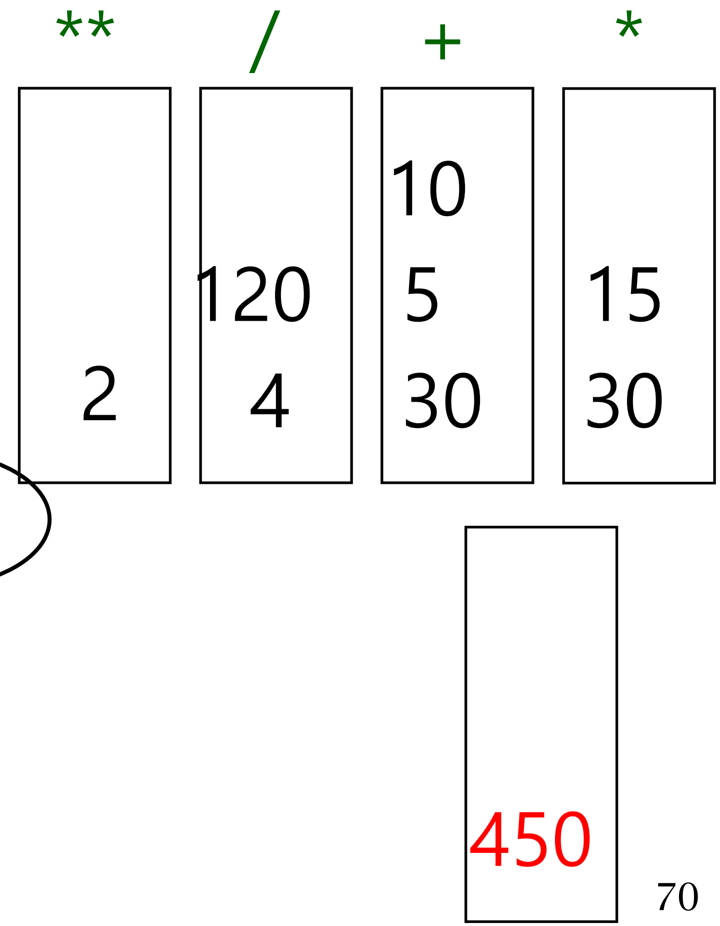
* + 10 5 / 120 ** 2



(5/5)

* + 10 5 / 120 ** 2





Level Order Traversal

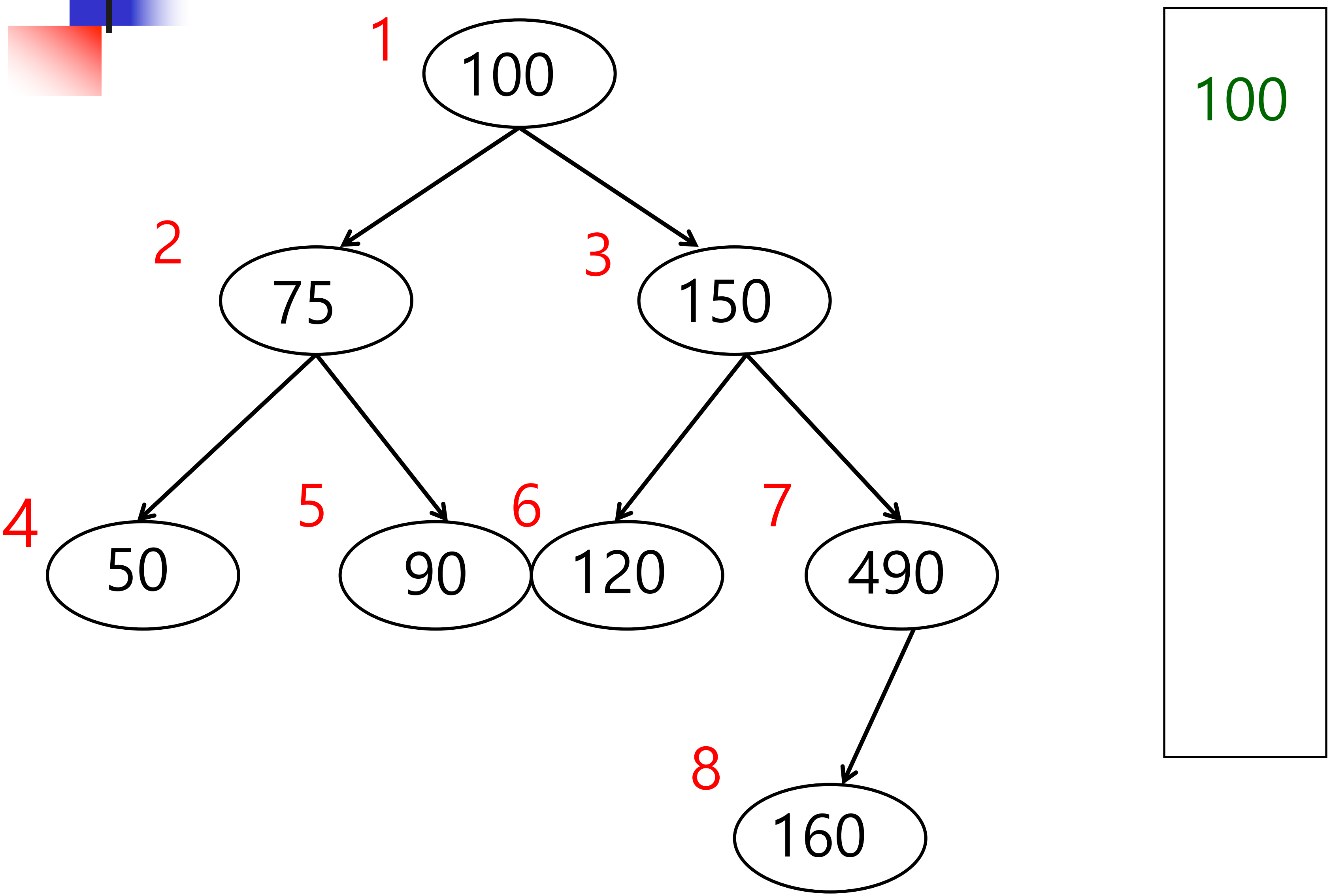
- Range (Level) Retrieval
  - People of the same rank on an organizational chart
  - Groups of the same rank on an organization chart
  - Possible next moves in a chess/go game

Playing Chess

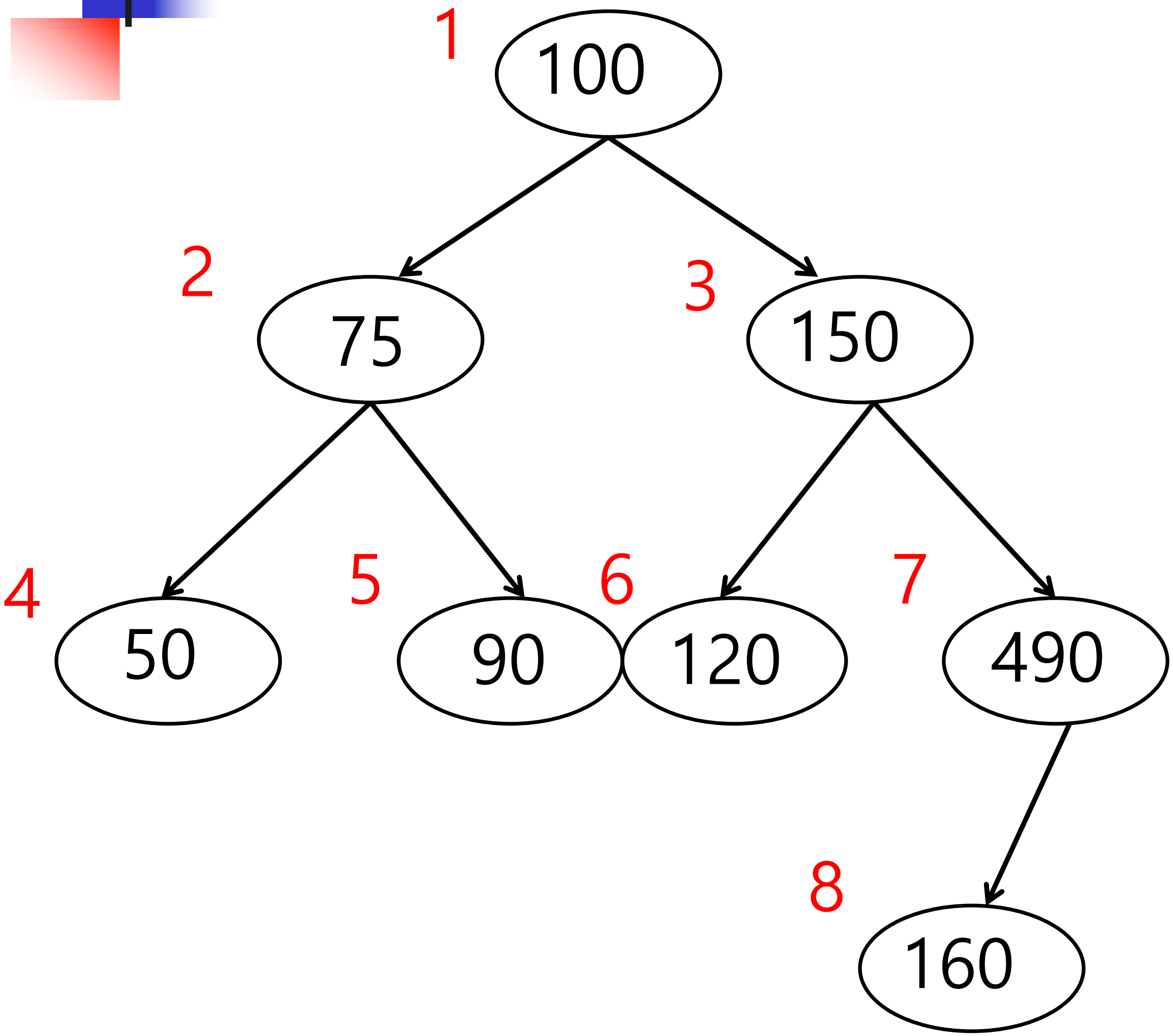
IBM Deep Blue – Chess Playing Computer

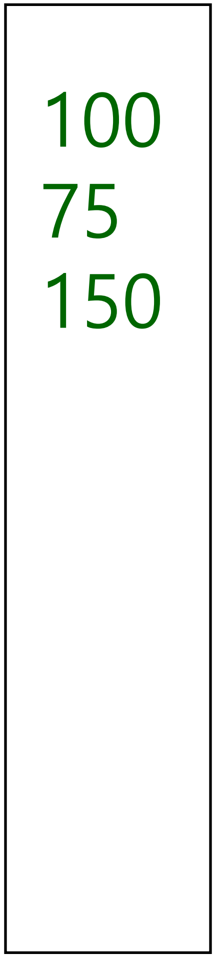
Playing Go

Level Order Traversal: Example (1/4)

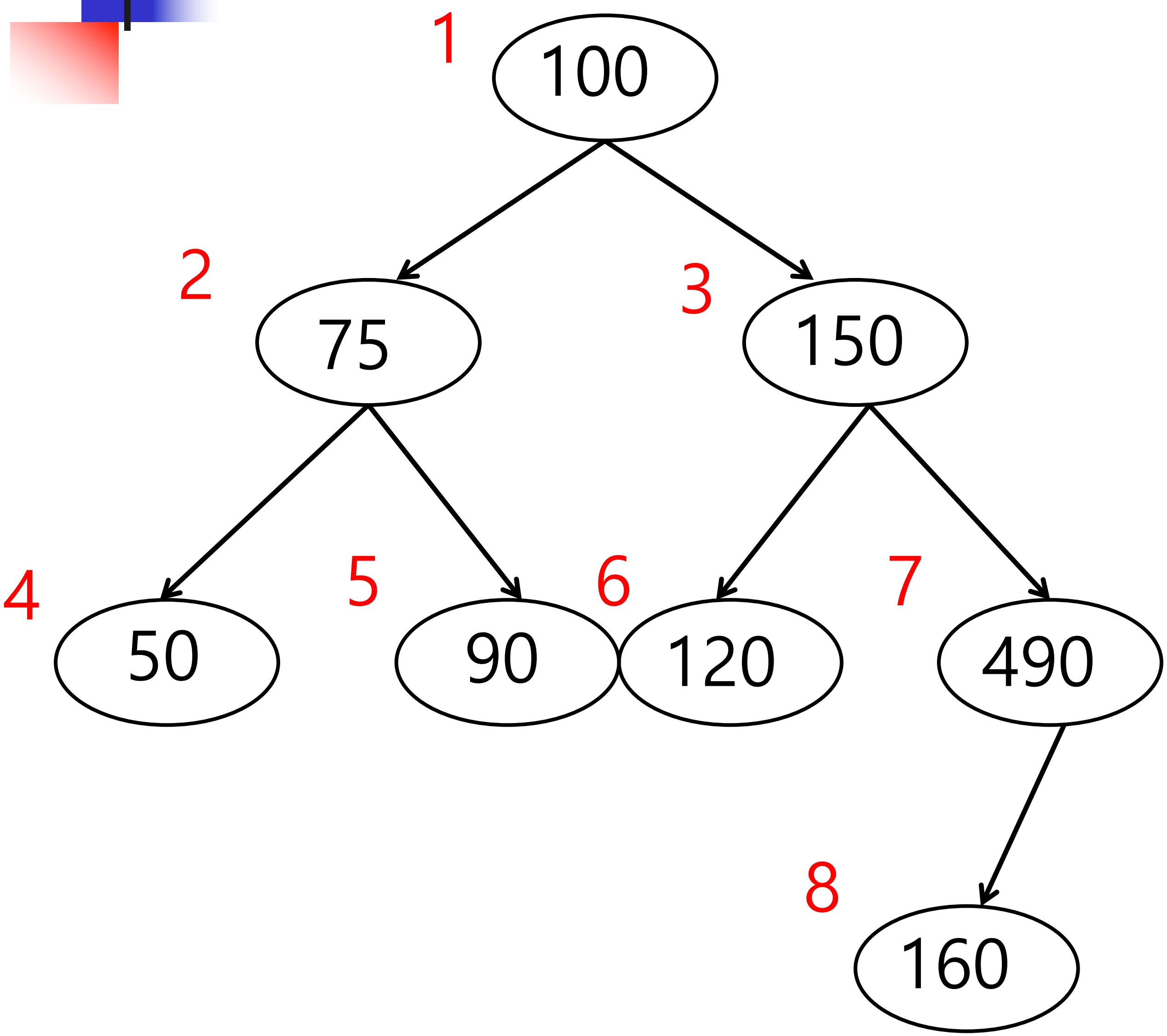


Level Order Traversal: Example (2/4)



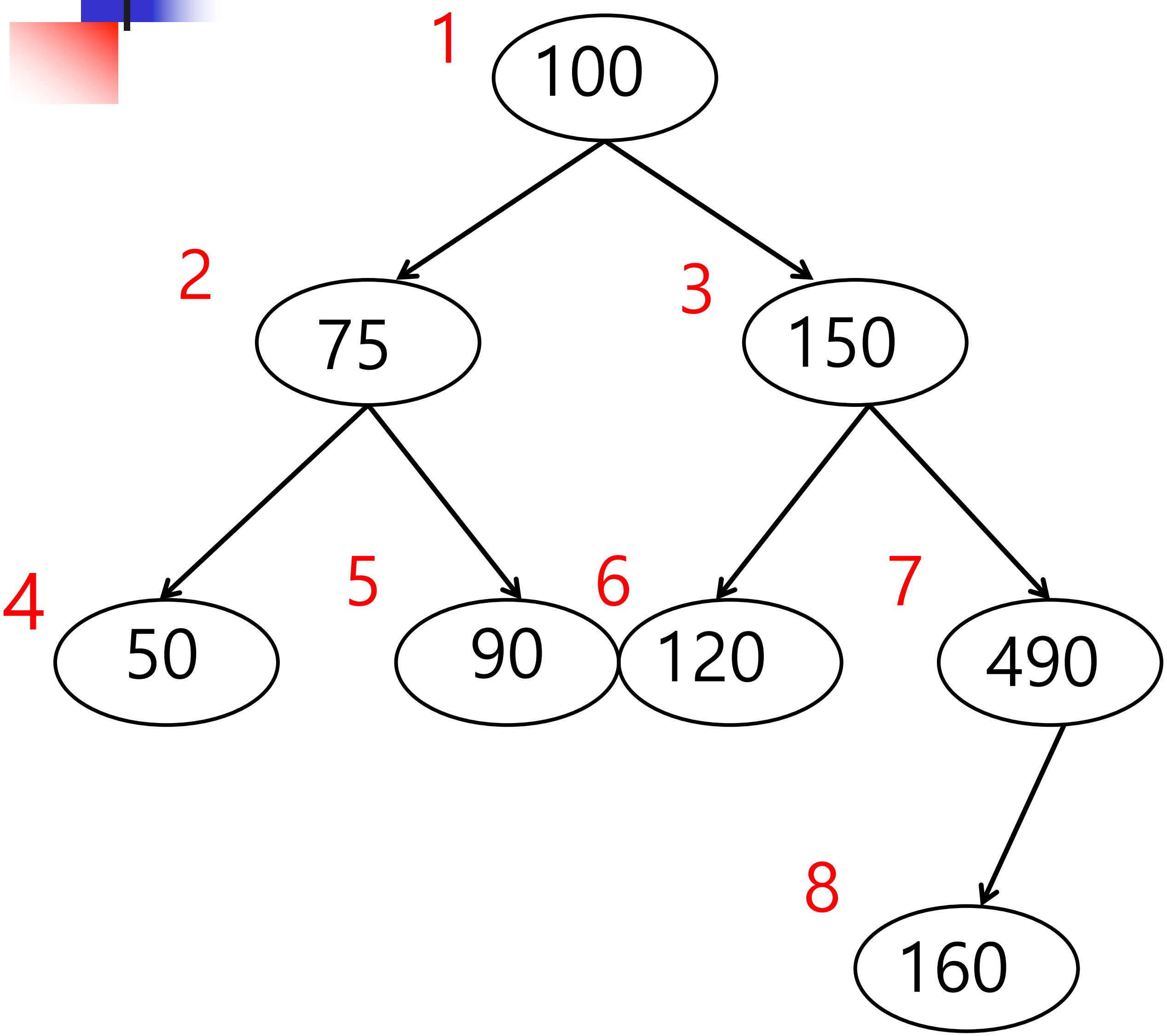


Level Order Traversal: Example (3/4)



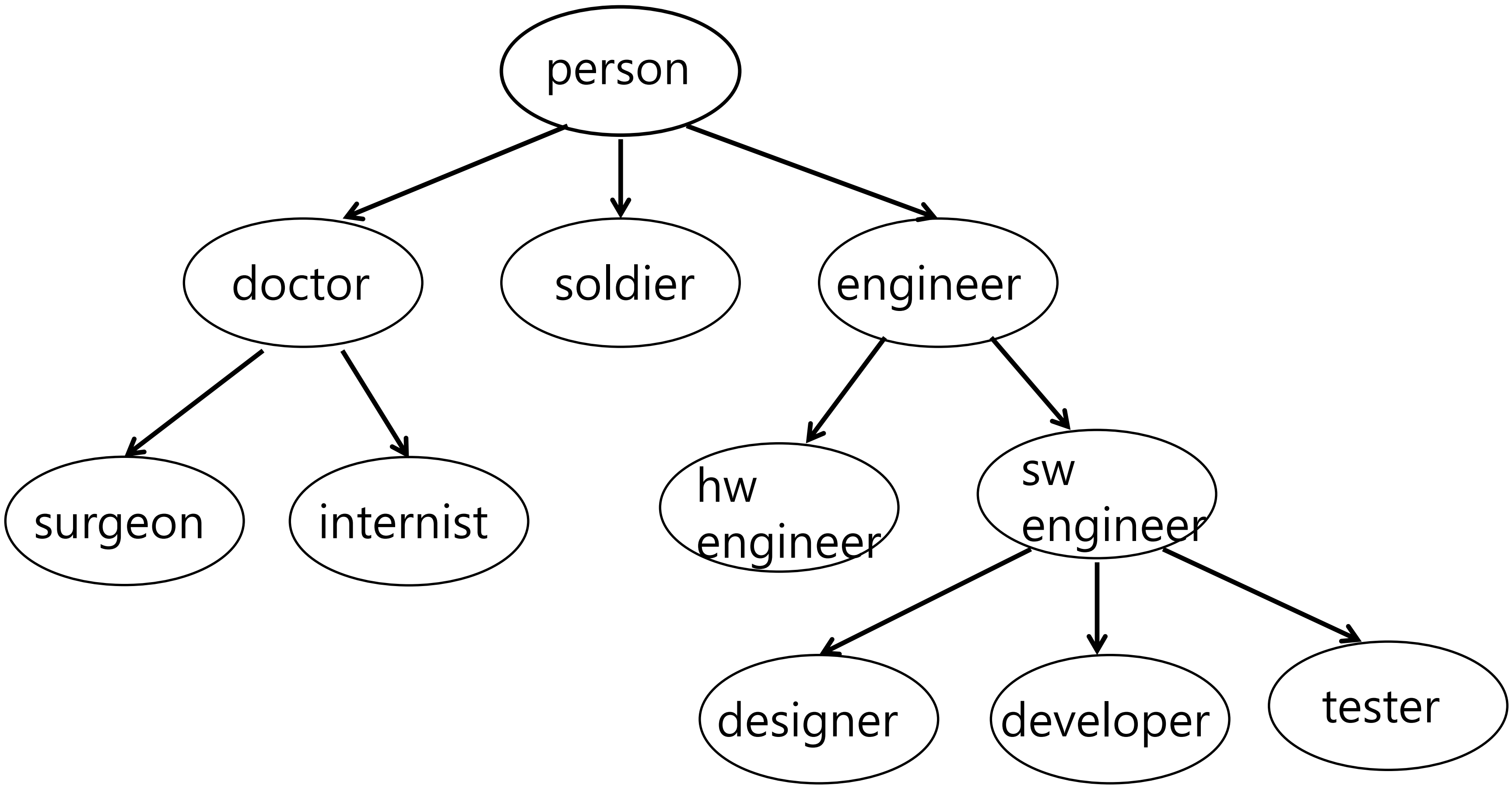
100
75
150
50
90
120
490

Level Order Traversal: Example (4/4)



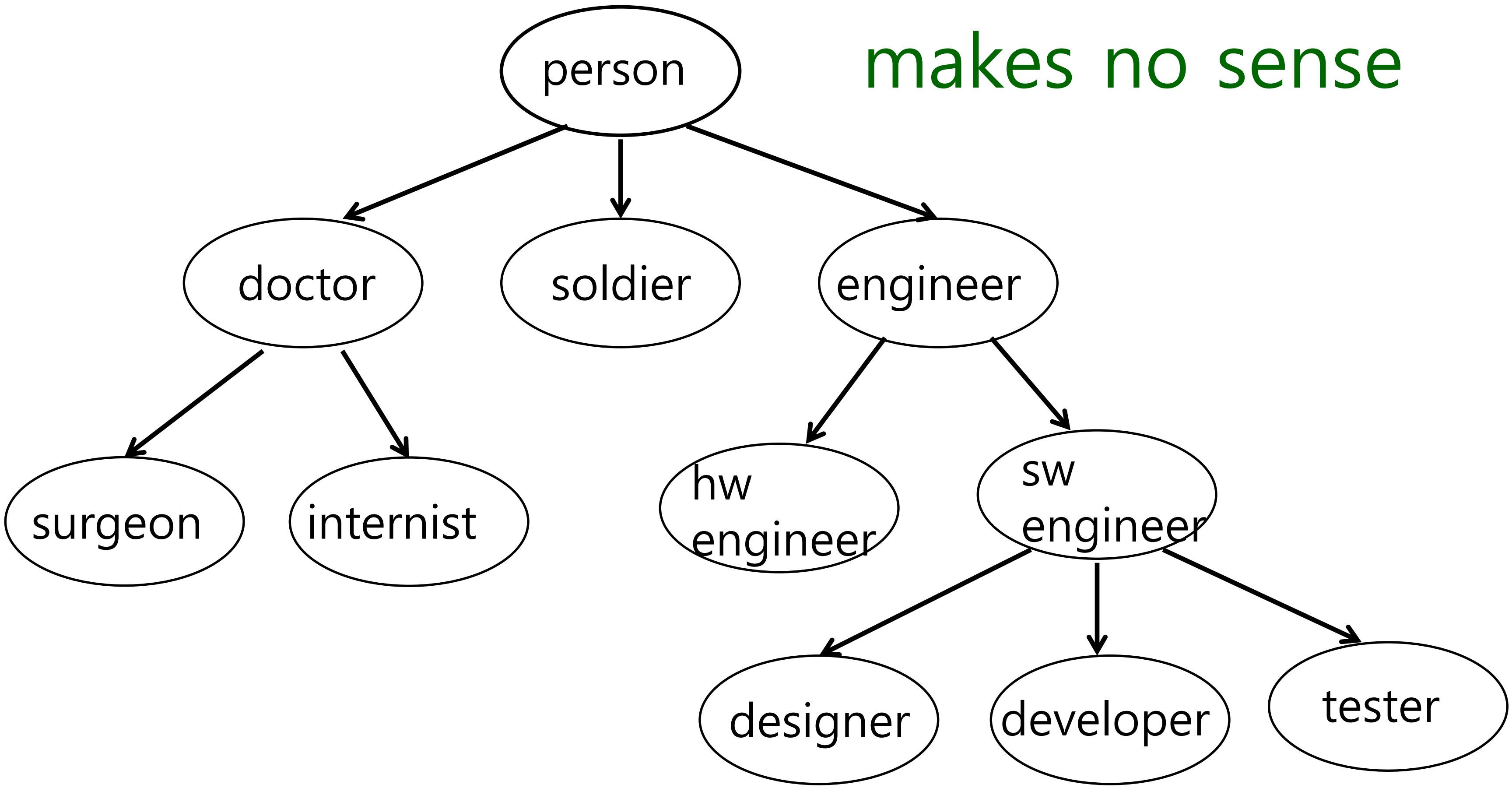
100
75
150
50
90
120
490
160

Inorder Traversal: Exercise

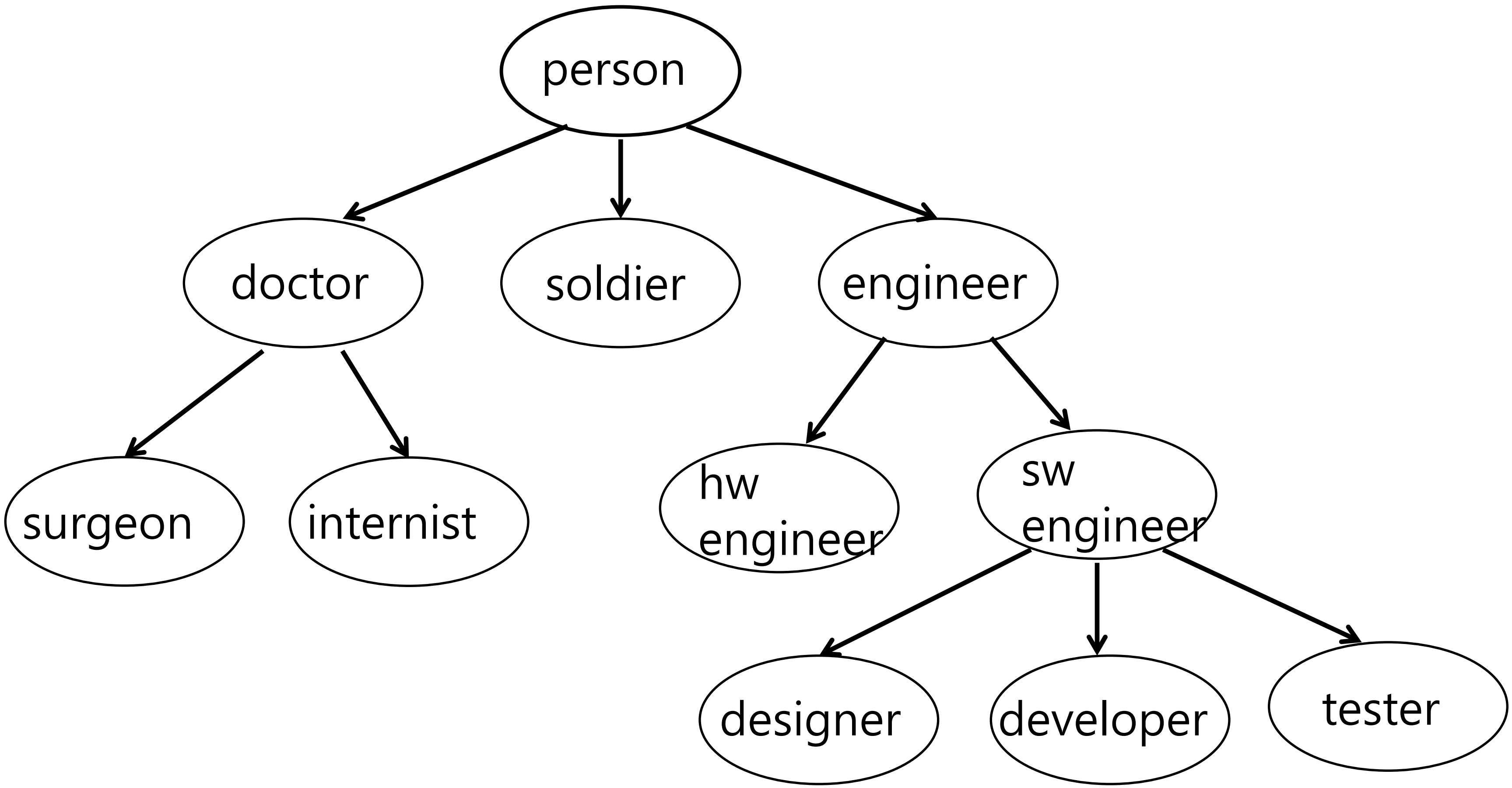


Inorder Traversal: Solution

makes no sense

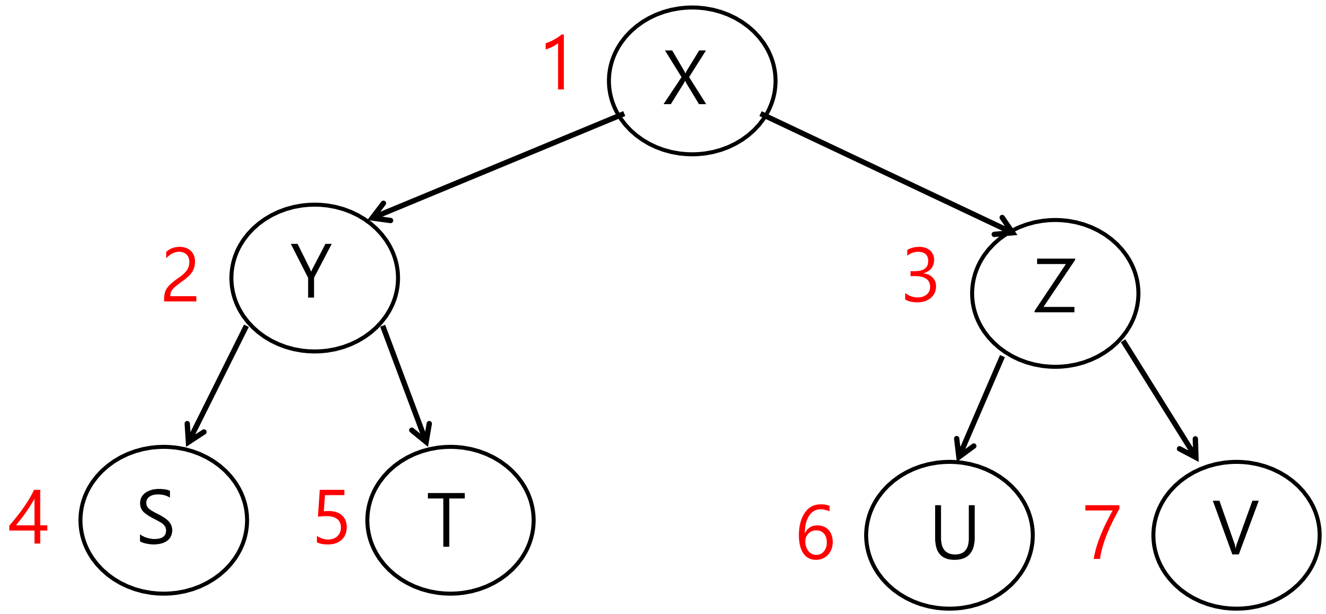


Exercise: What Are the Results of #1 level order, #2 Postorder, #3 Preorder Traversal?

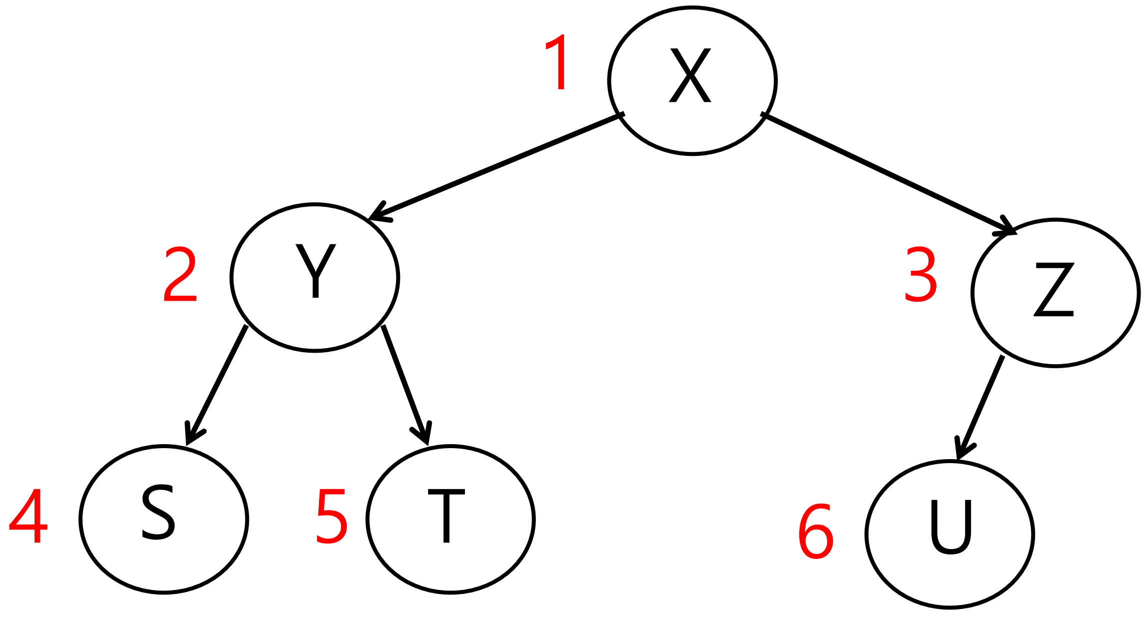


Binary Trees

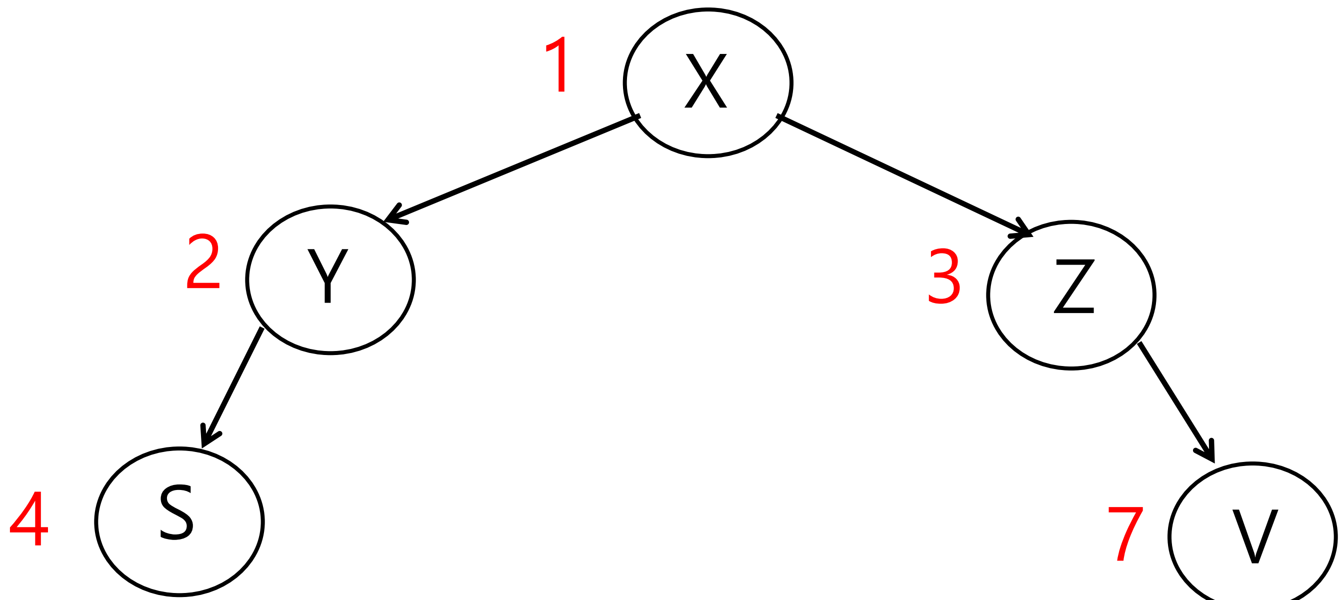
A Full Binary Tree



A Complete Binary Tree

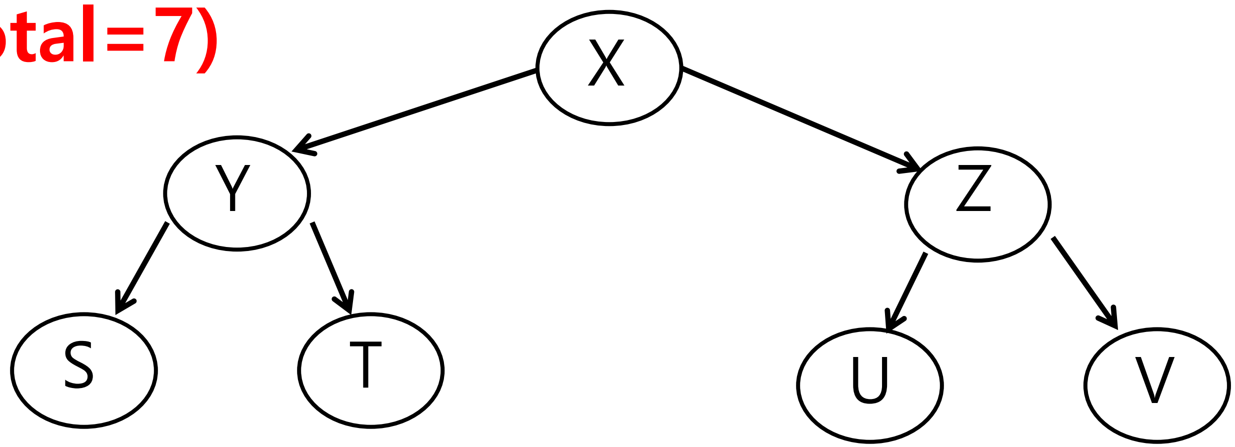


Not a Complete Binary Tree (* middle teeth missing ^ ^ *)

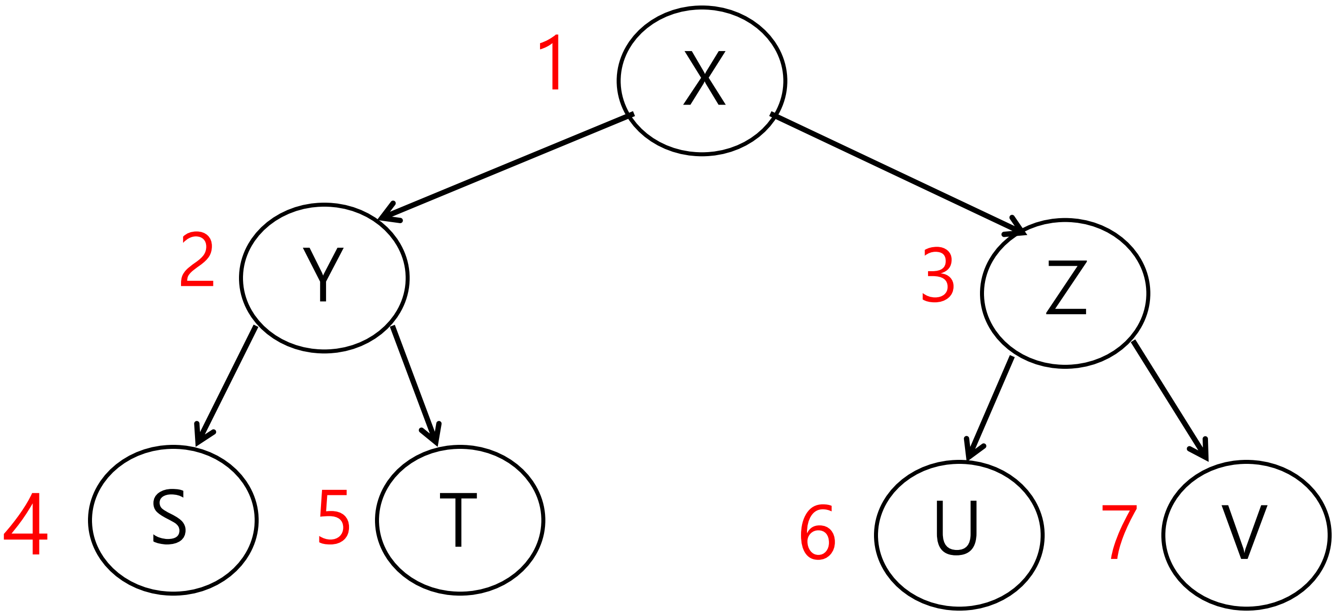


Binary Tree: Properties

- The degree of each non-leaf node is one or two.
- The maximum number of nodes on the i-th level of a tree = 2^{i-1} **(i=3, max num= 4)**
- The maximum total number of nodes in a tree of height h = $2^h - 1$ **(h=3, max total=7)**



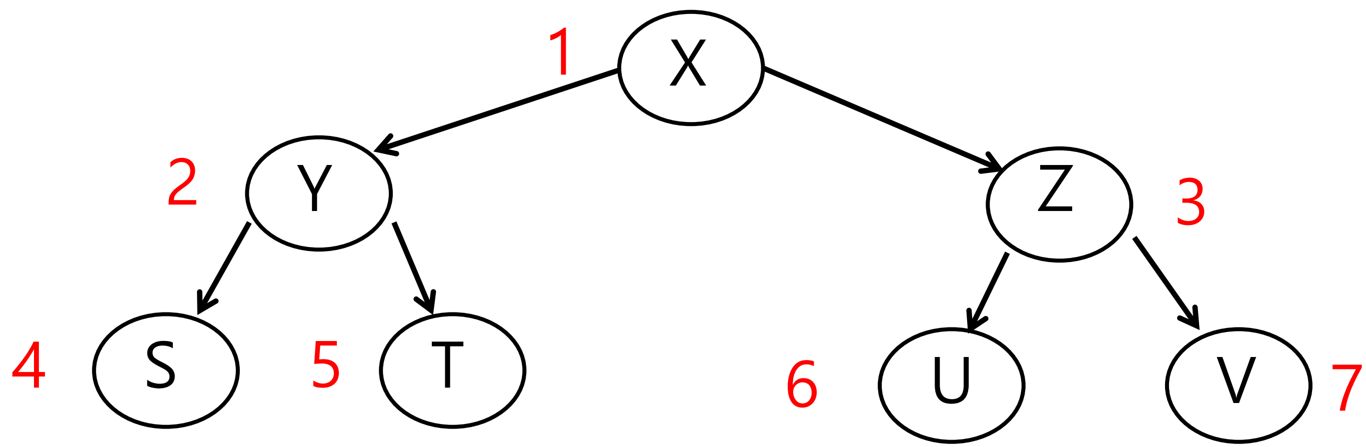
Implementing a Binary Tree Using an Array



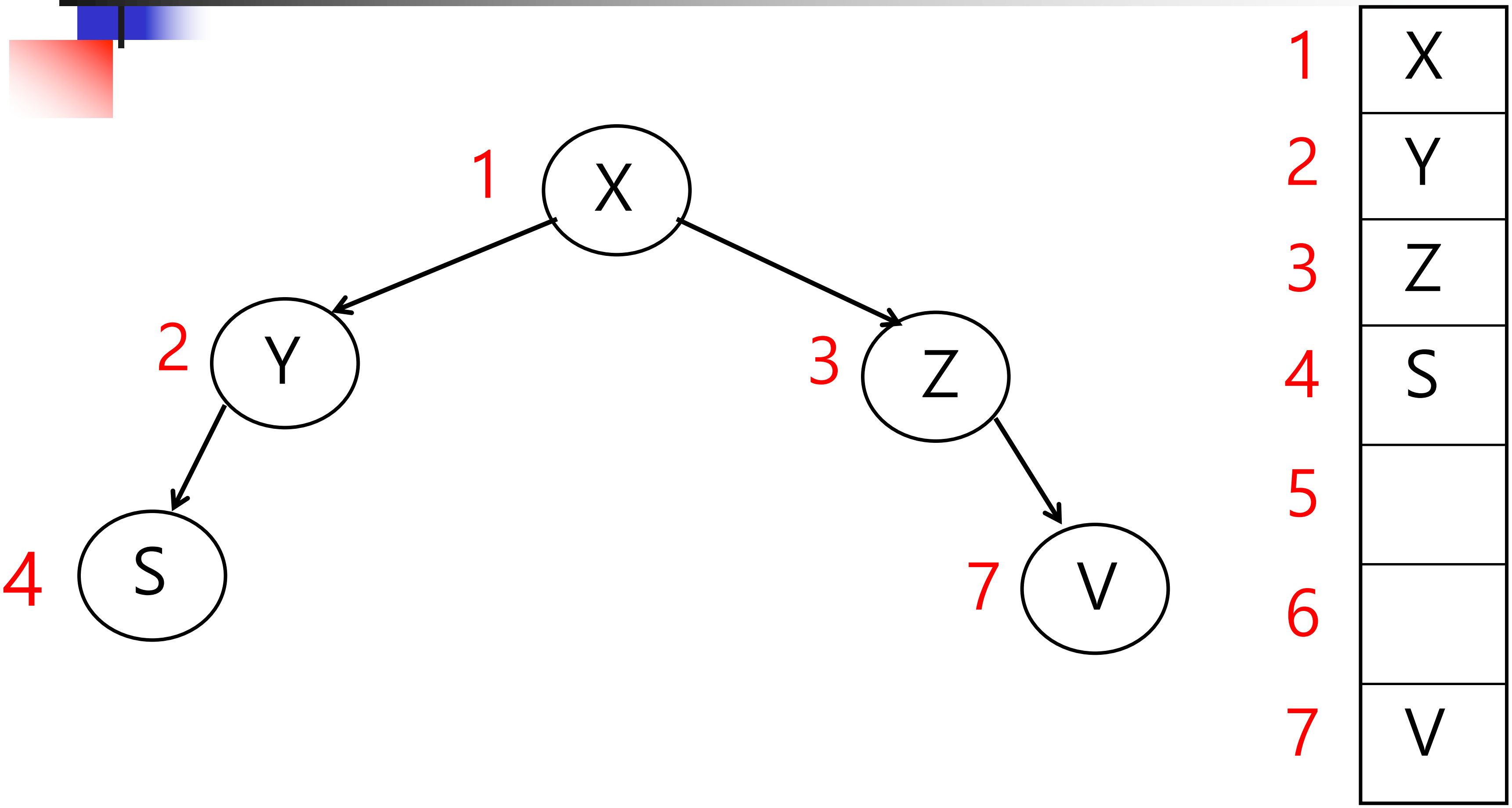
1	X
2	Y
3	Z
4	S
5	T
6	U
7	V

Computing the Locations of Nodes

- For a node with array index i on a full binary tree with n nodes
  - parent of i $= \lceil i/2 \rceil$
  - left child of i = 2i
  - right child of i = 2i + 1



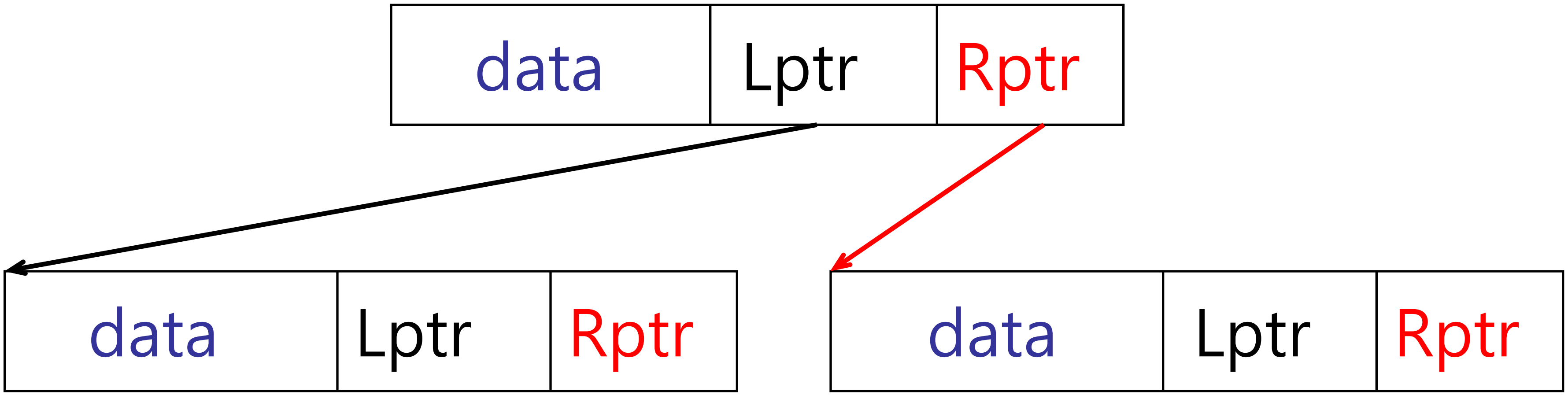
Implementing a Binary Tree Using an Array



Problems

- Insertion or deletion of nodes in the middle of an array is expensive.
- Memory is wasted.
  - When the tree is not full/complete

Implementation of a Binary Tree Using a Singly Linked List With 2 Pointers



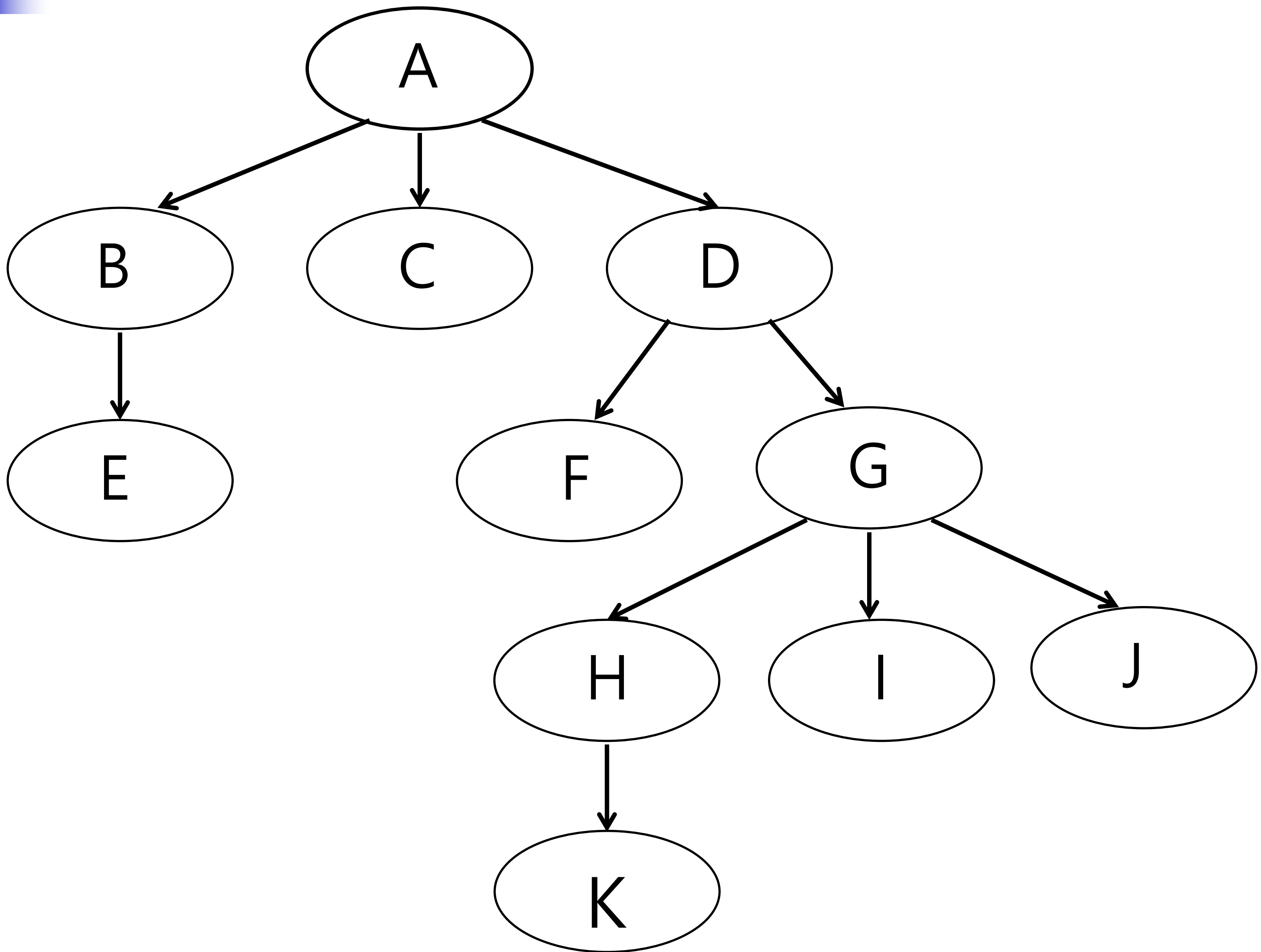
Coding in C

- **Each Node as a Structure**
  - **(data, left-child ptr, right-child ptr)**

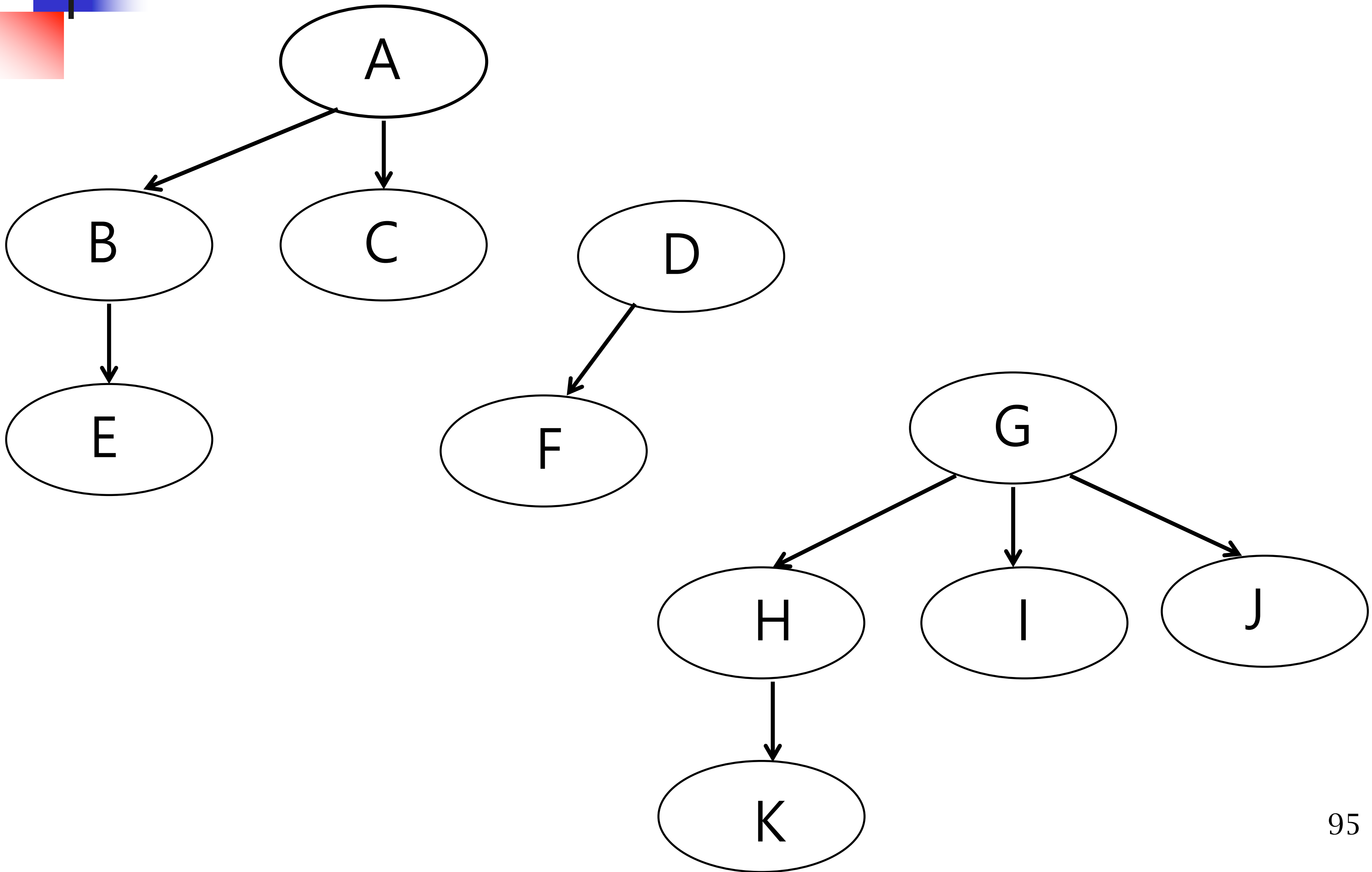
```
typedef  struct  node *tree_ptr;
typedef  struct  node  {
                (data_type    data);
                tree_ptr  left_child, right_child;
                };
```

Representations of Trees

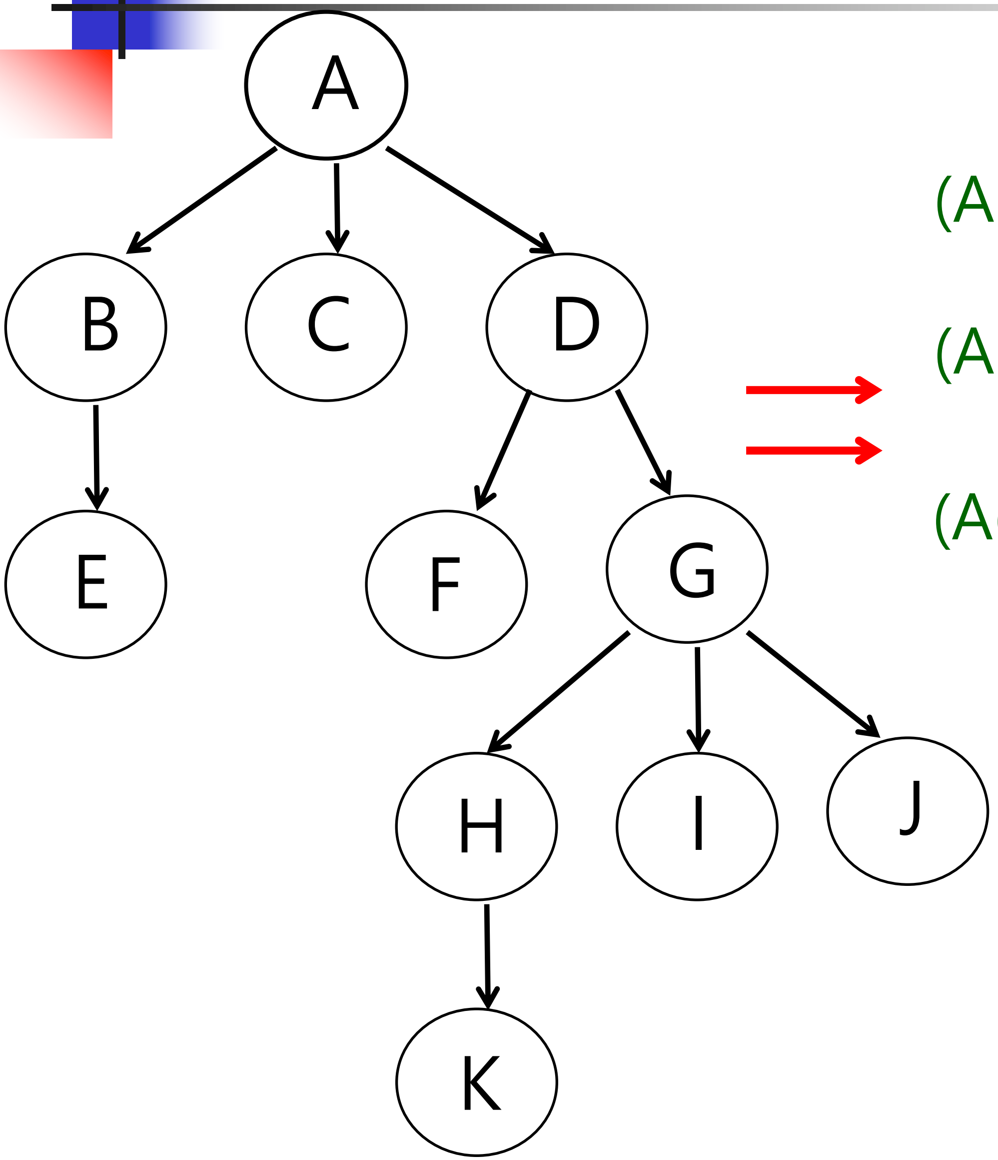
General Tree



Forest (of 3 Trees)



List Representation of a Tree



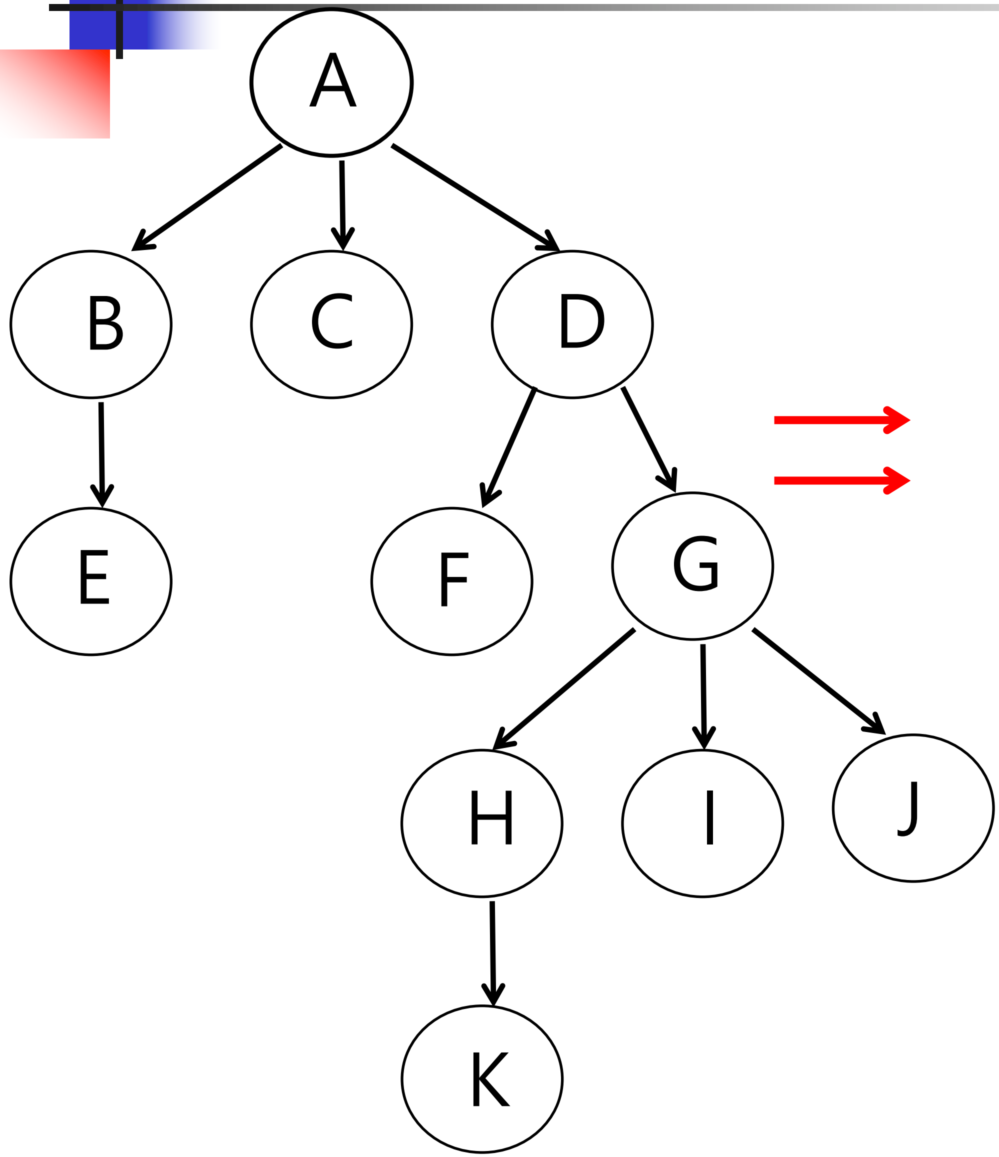
(A(B,C,D))

(A(B(E),C,D(F,G)))

(A(B(E),C,D(F,G(H,I,J))))

(A(B(E),C,D(F,
G(H(K),I.J))))

Linked List Representation of a Tree (In Memory) (* pB, etc. is pointer to B, etc.)



A	pB	pC	pD	
B	pE			
C				
D	pF	pG		
E				
F				
G	pH	pI	pJ	

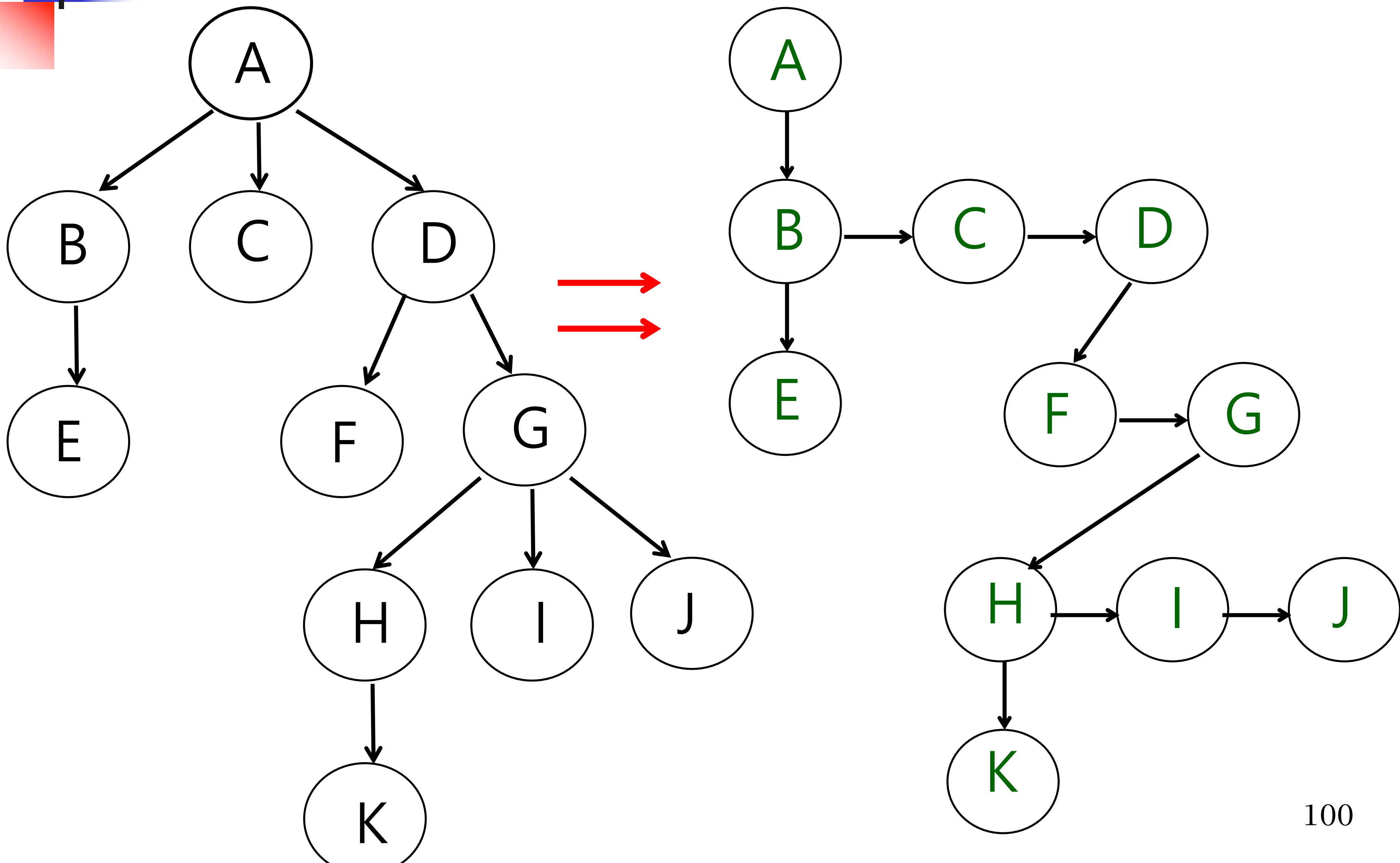
Converting a General Tree into a Binary Tree

- The varying degree of the node in a general tree makes it difficult to read, insert, and delete nodes.
- It is best to transform a general tree into a binary tree.
- We can use the "Leftmost Child-Right Siblings" Representation

Leftmost Child-Right Siblings Representation

- Step 1
  - Connect all siblings of a parent, and delete all links from the parent to its children (except for the one to the leftmost child).
- Step 2
  - Make the right sibling the right child.

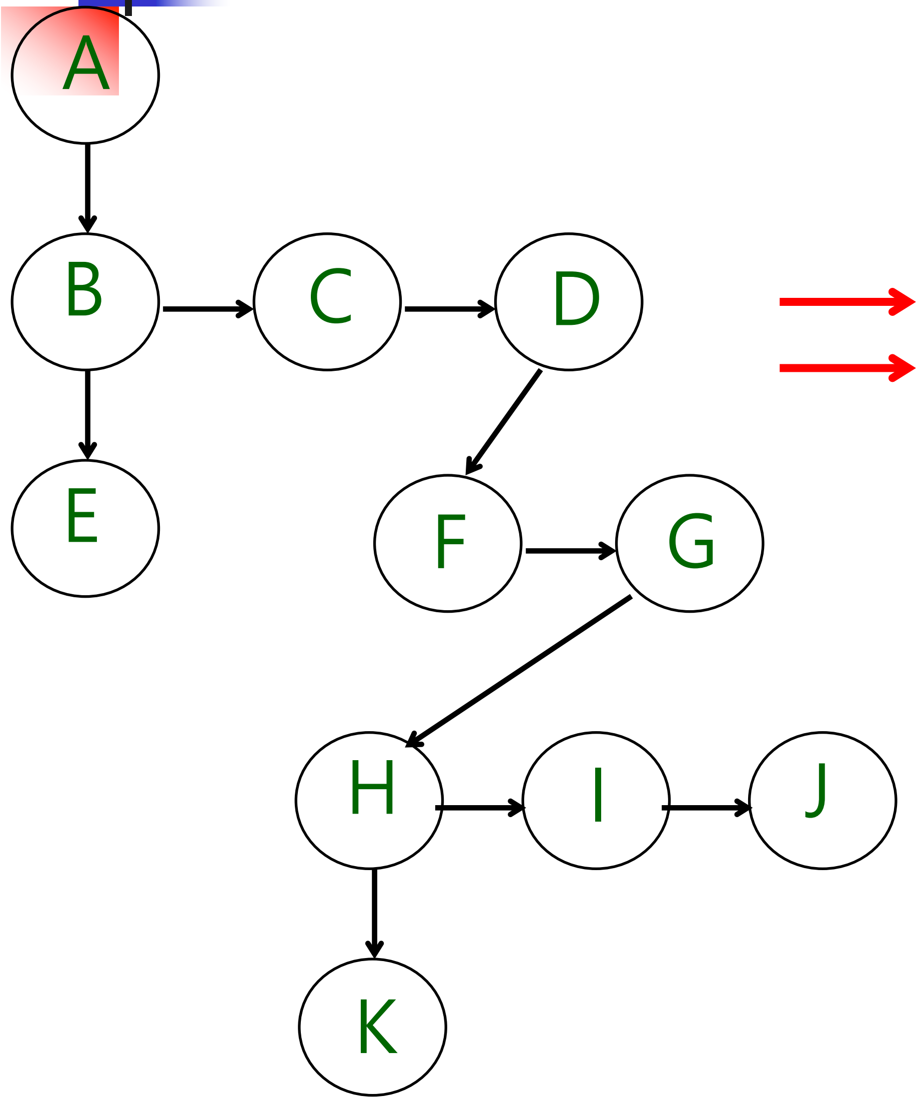
Illustration of Step 1

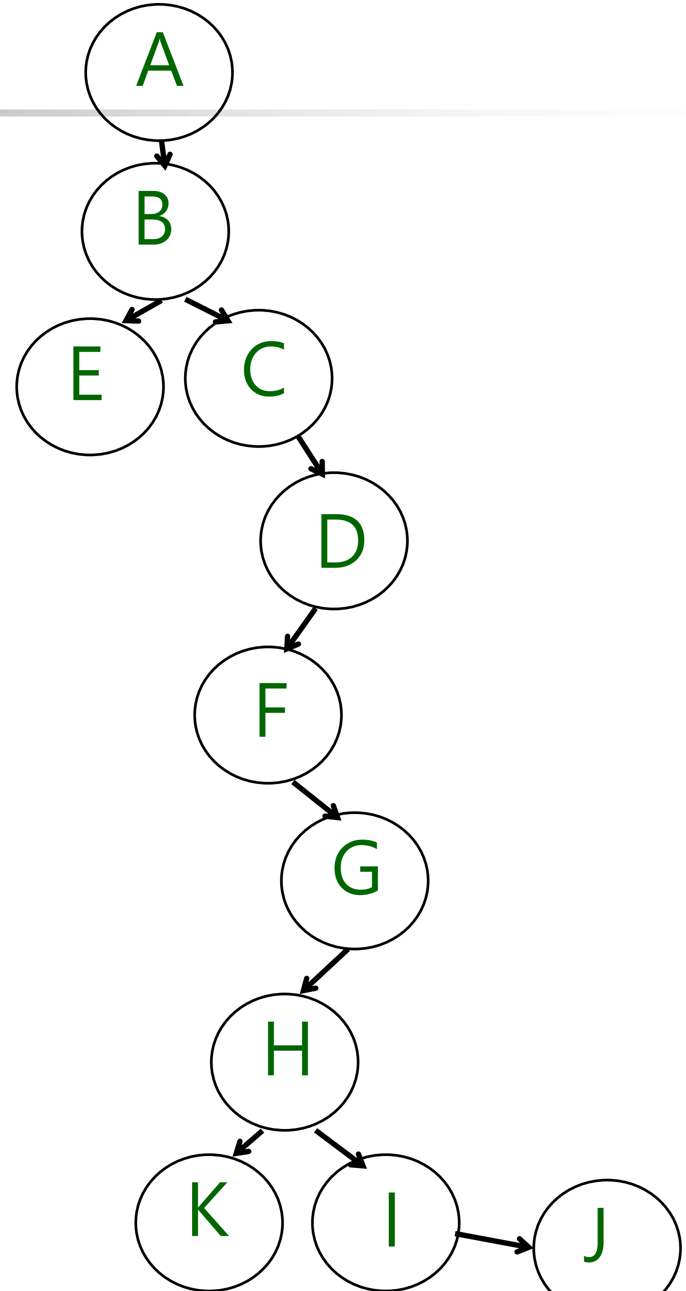


Each Node Has…

data	left child ptr	right sibling ptr

Illustration of Step 2





Each Node Now Has...

data	left child ptr	right child ptr

Algorithm (1/2)

- The root of the general tree is the root of the binary tree.
- Traverse the general tree in a depth-first order from the root, and make the leftmost child of the node the left child of the corresponding node in the binary tree.
- If the node has no left child, make the right sibling the right child of the node in the binary tree.
- Repeat the above process for the entire general tree.

Algorithm (2/2)

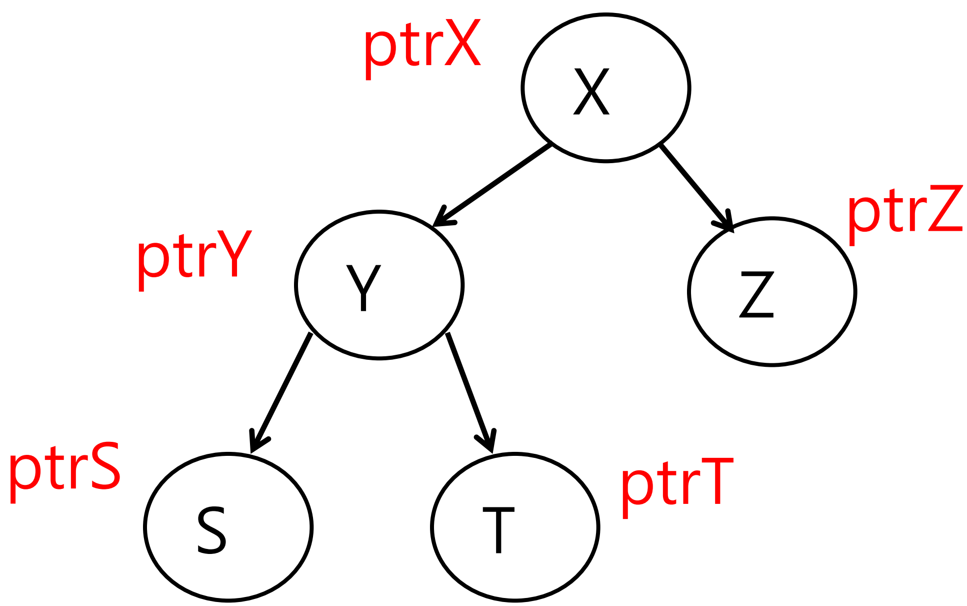
- In the Transformation
  - All nodes of the original general tree are included.
  - Node relative levels are preserved.
  - Parent-descendant relationship is preserved.

Implementing Inorder Traversal in C

```
void inorder (tree_ptr  ptr)
{
    if  (ptr)  {
        inorder (ptr->left_child);
        (visit node);
        inorder (ptr->right_child);
    }
}
```

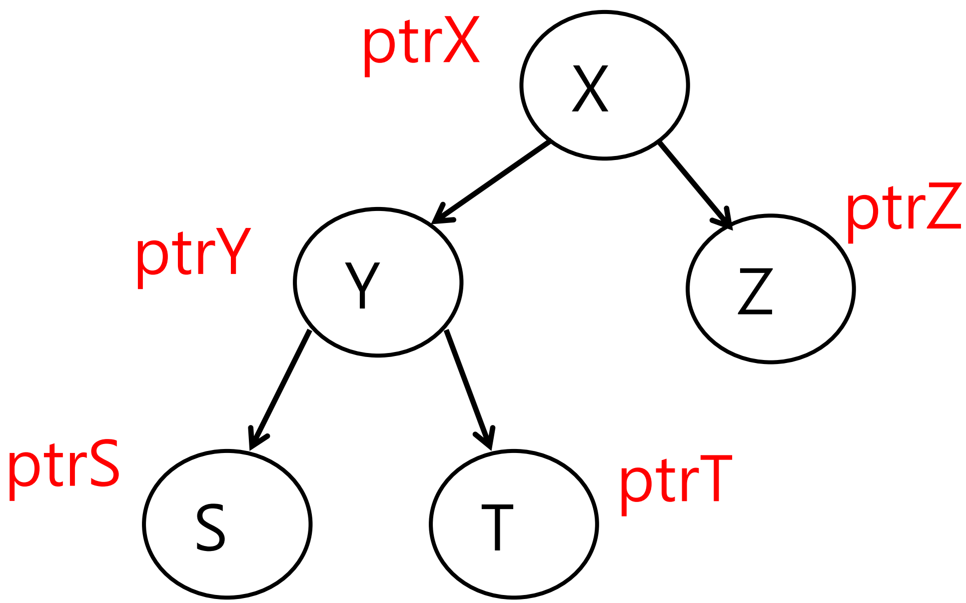
How Recursion Works: (Inorder traversal of a sample tree)

```
void inorder (tree_ptr  ptr)
{ if  (ptr)  {
        inorder (ptr->left_child);
        (visit node);
        inorder (ptr->right_child); }
}
```



How Recursion Works (execution sequence)

```
void inorder (tree_ptr  ptr)
{ if  (ptr)  {
        inorder (ptr->left_child);
        (visit node);
        inorder (ptr->right_child); }
}
```

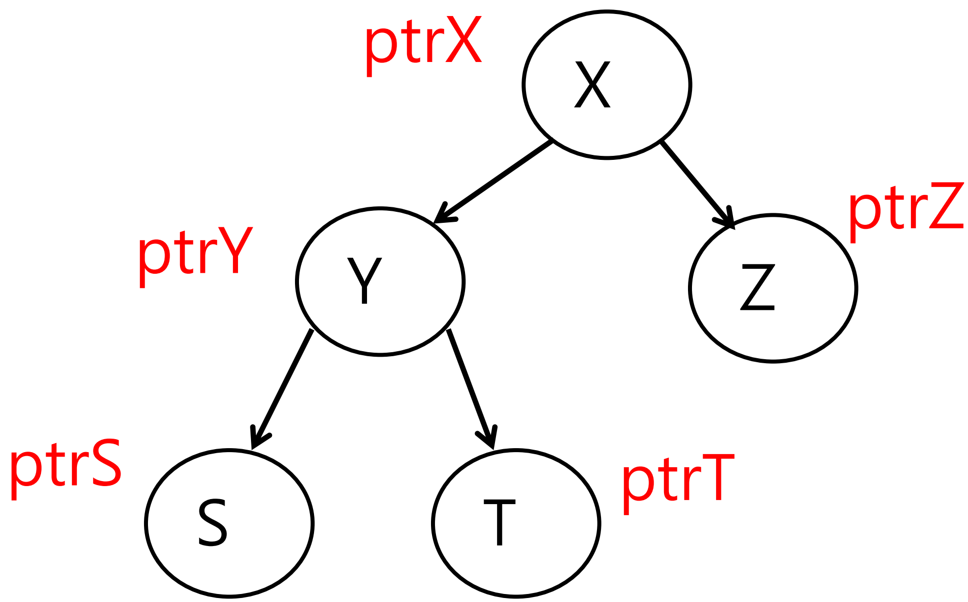


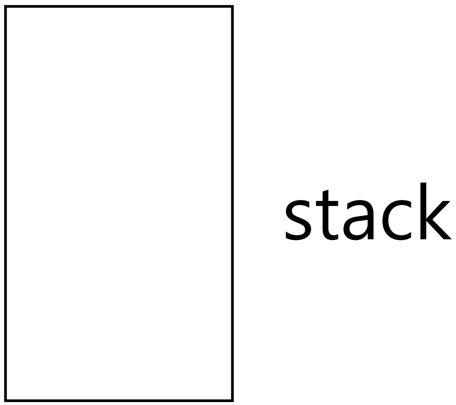

```
inorder (ptrX)
  inorder (ptrY)
    inorder (ptrS)
      inorder (null)
      visit S
      inorder (null)
    visit Y
    inorder (ptrT)
      inorder (null)
      visit T
      inorder (null)
  visit X
  inorder (ptrZ)
    inorder (null)
    visit Z
    inorder (null)
```

Stack Is Used to Save the Activation Record (* simplified illustration 1/7 *)

```
void inorder (tree_ptr  ptr)
{ if  (ptr)  {
        inorder (ptr->left_child);
        (visit node);
        inorder (ptr->right_child); }
}
```

inorder (ptrX)



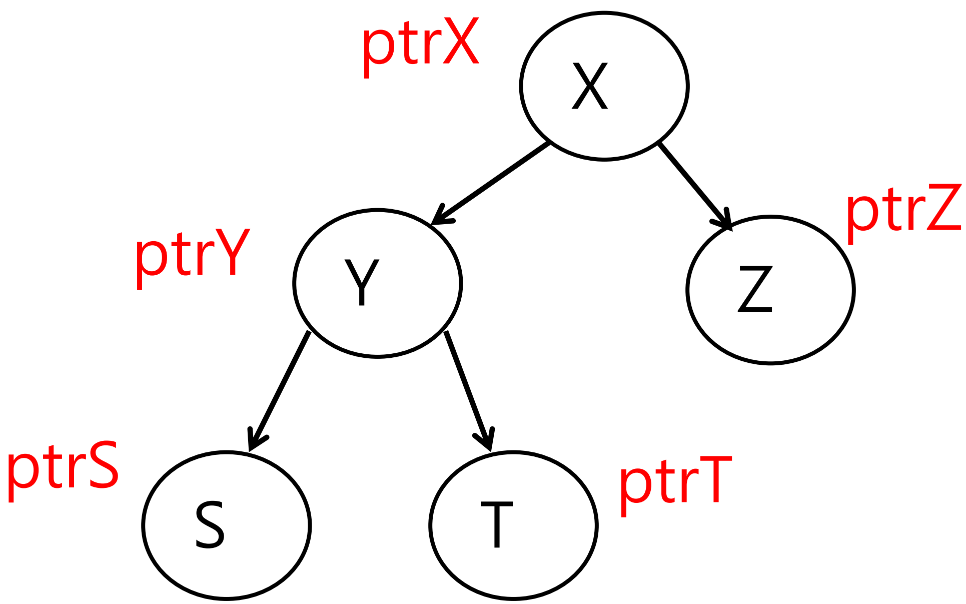


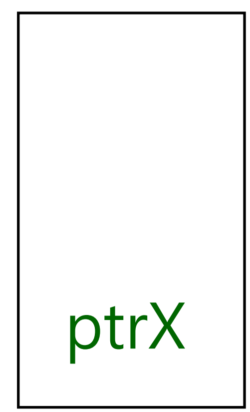
(2/7)

```
void inorder (tree_ptr  ptr)
{ if  (ptr)  {
        inorder (ptr->left_child);
        (visit node);
        inorder (ptr->right_child); }
}
```

inorder (ptrX)

inorder (ptrY)





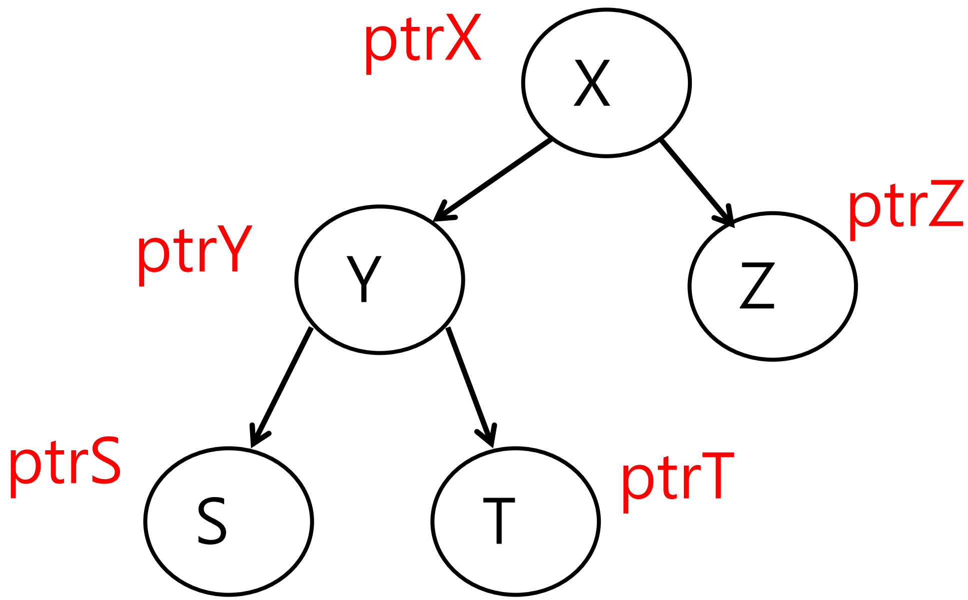
(3/7)

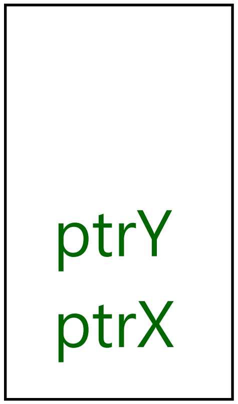
```
void inorder (tree_ptr  ptr)
{ if  (ptr)  {
        inorder (ptr->left_child);
        (visit node);
        inorder (ptr->right_child); }
}
```

inorder (ptrX)

inorder (ptrY)

inorder (ptrS)





(4/7)

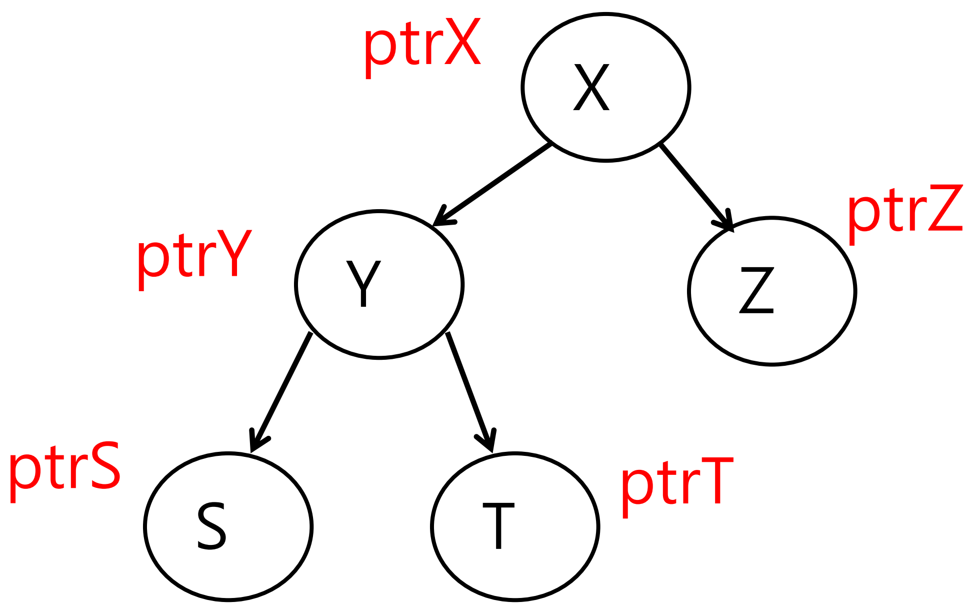
```
void inorder (tree_ptr  ptr)
{ if  (ptr)  {
        inorder (ptr->left_child);
        (visit node);
        inorder (ptr->right_child); }
}
```

inorder (ptrX)

inorder (ptrY)

inorder (ptrS)

inorder (Lnull)



ptrS
ptrY
ptrX

(5/7)

```
void inorder (tree_ptr  ptr)
{ if  (ptr)  {
          inorder (ptr->left_child);
          (visit node);
          inorder (ptr->right_child); }
}
```

inorder (ptrX)

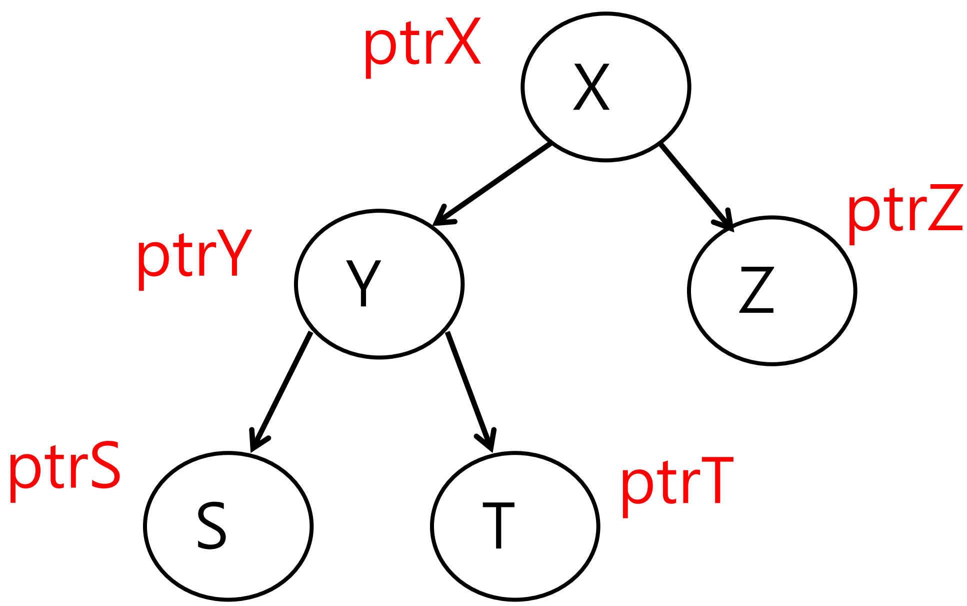
inorder (ptrY)

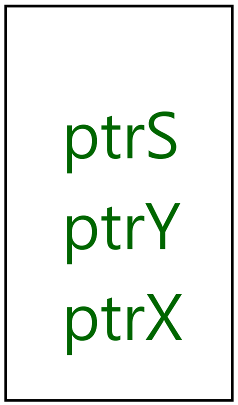
inorder (ptrS)

inorder (Lnull)

visit S

inorder (Rnull)



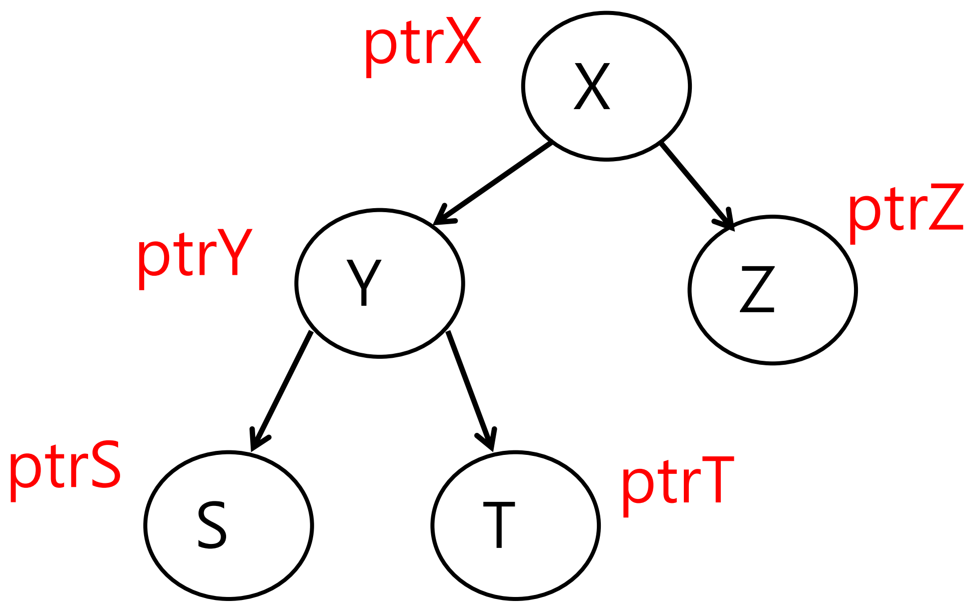


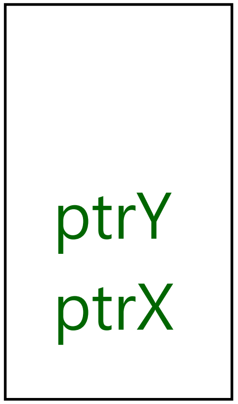
(6/7)

```
void inorder (tree_ptr  ptr)
{ if  (ptr)  {
        inorder (ptr->left_child);
        (visit node);
        inorder (ptr->right_child); }
}
```

inorder (ptrX)

inorder (ptrY)





(7/7)

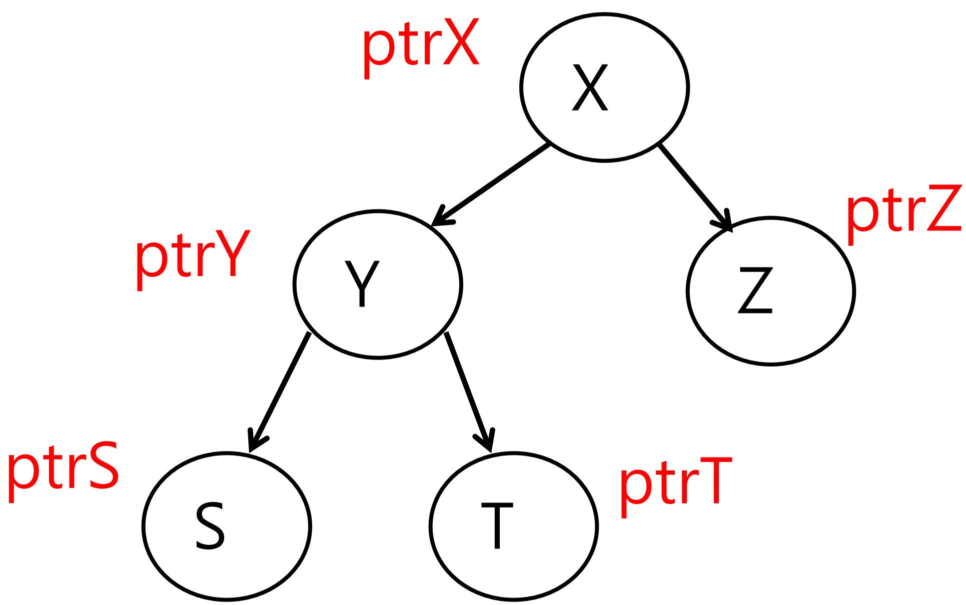
```
void inorder (tree_ptr  ptr)
{ if  (ptr)  {
        inorder (ptr->left_child);
        (visit node);
        inorder (ptr->right_child); }
}
```

inorder (ptrX)

inorder (ptrY)

visit Y

inorder (ptrT)



ptrT
ptrY
ptrX

** Activation Record (or Stack Frame)

- An activation record is a block of memory used for managing the information needed by a single execution of a program (that includes nested function calls and returns).
- In this lecture, for simplicity, only the function call actual parameter is shown in the stack.
- An activation record actually includes a lot more information.
  - temporary variables, local variables
  - saved machine registers
  - control link, access link
  - (function call) actual parameters, return values

Implementing Postorder Traversal in C

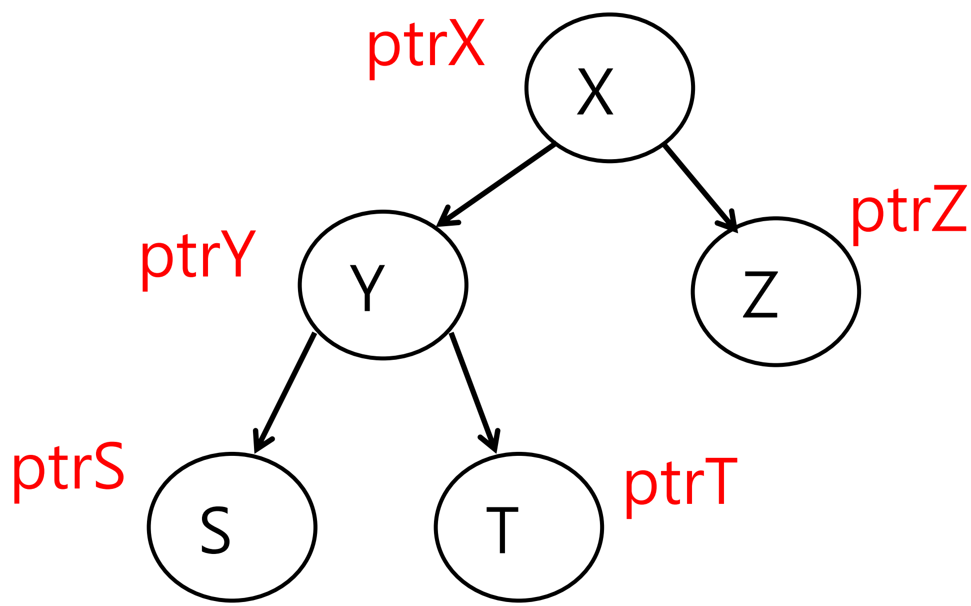
```
void postorder (tree_ptr  ptr)
{
    if  (ptr)  {
        postorder (ptr->left_child);
        postorder (ptr->right_child);
        (visit node);
     }
}
```

Implementing Preorder Traversal in C

```
void preorder (tree_ptr  ptr)
{
    if  (ptr)  {
        (visit node);
        preorder (ptr->left_child);
        preorder (ptr->right_child);
     }
}
```

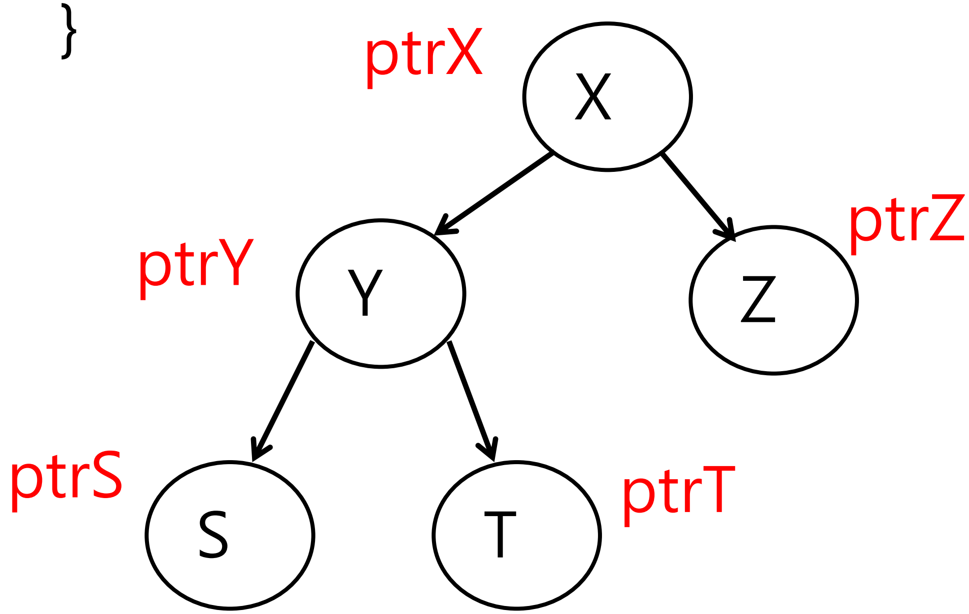
WHW 1-1: Draw the Recursion Execution Sequence and Stack State Sequence (10 points)

```
void postorder (tree_ptr  ptr)
{ if  (ptr)  {
        postorder (ptr->left_child);
        postorder (ptr->right_child);
        (visit node);}
}
```



WHW 1-2. (5 points) Draw the Recursion Execution Sequence

```
void preorder (tree_ptr  ptr)
{ if  (ptr)  {
        (visit node);
        preorder (ptr->left_child);
        preorder (ptr->right_child);
        }
}
```



Programming Homework (PHW) 2

- Two problems
- Specifications in a separate WORD file.

End of Lecture

Review of
Programming Homework 1

Review: malloc and free functions

```
int *nums, i;

nums = (int *) malloc (10*sizeof(int));
if (nums == (int *) NULL) {
    printf (“malloc failed”);
    exit(1);
    }
for (i=0; i<10; i++) {
    printf (“₩n type an integer”)
    scanf (“%d”, &nums[i]);
    printf (“%d”, nums[i]);
    }
free(nums);
```

Review: malloc and free functions

```
int *nums, i;

nums = (int *) malloc (10*sizeof(int));
if (nums == (int *) NULL) {
    printf ("malloc failed");
    exit(1);
    }
for (i=0; i<10; i++) {
    printf ("₩n type an integer")
    scanf ("%d", &nums[i]);
    printf ("%d", nums[i]);
    }
free(nums);
```

Review: Programming Homework 1

- Implementing a stack or queue using a singly linked list
- (WRONG) It makes no sense to test for stack_full, queue_full condition
  - A linked list has infinite memory. All you need to do is call malloc and request additional memory.
- If this is the case, why is there malloc in the first place?

Why Is Memory Not Infinite?

- Computer main memory is taken up by the operating system and other programs and the buffer space they need.
- Your program is given limited space (by the operating system).
- On top of that, there are memory leaks.

Memory Leak

- Imperfect programming causes memory leaks – many large programs have them.
- A **memory leak** occurs when a computer program does not free memory that is no needed.
  - no 'free' after 'malloc'
- A **memory leak** also occurs when an object stored in **memory** cannot be accessed
  - (e.g.) node A has a pointer to node B; the pointer is deleted.

stack_full, queue_full testing

- defensive coding by checking for malloc failure.

```
int *nums, i;

nums = (int *) malloc (10*sizeof(int));
if (nums == (int *) NULL) {
    (malloc failed);
  }
```

Damages Done by Careless Memory Management

- Not checking for malloc failure
- Not freeing memory no longer needed
- Lead to system crashes for large important programs
- A lot of time, people and money spent to debug and re-release the programs