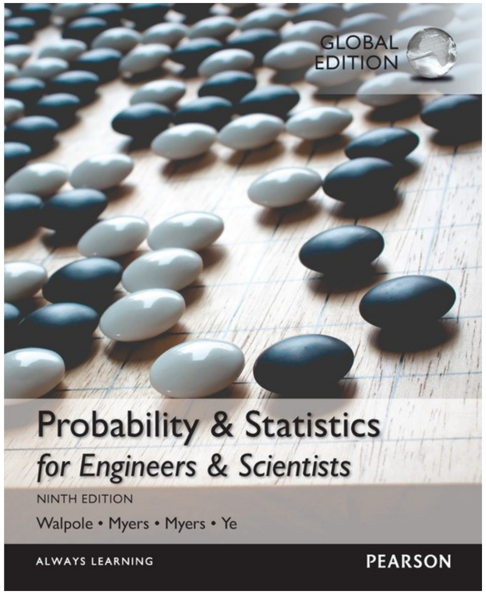
Chapter 3

Random Variables and Probability Distributions – part 2

School of Computing, Gachon Univ.
Joon Yoo



Wireless Intelligent Networking LAB

Outline

- Concept of a Random Variable
- Discrete Probability Distributions
- Continuous Probability Distributions
- Joint Probability Distribution

Wireless Intelligent Networking LAB

3.3 Continuous Probability Distributions

Discrete vs. Continuous

If a sample space contain a ? number of a possibilities

finite

infinite

Discrete Sample Space

Continuous Sample Space

r.v.

Discrete random variable (r.v.)

Continuous random variable (r.v.)

Probability distribution can be expressed via

Probability mass function (p.m.f.)

x	0	1	2
$f(x)$	$\frac{68}{95}$	$\frac{51}{190}$	$\frac{3}{190}$

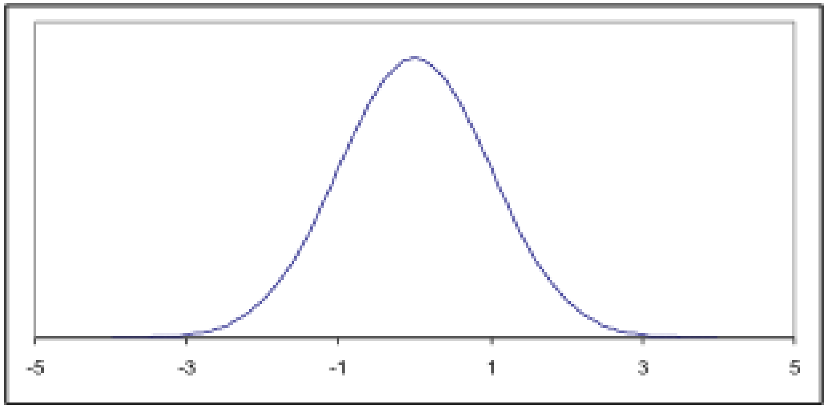
Probability density function (p.d.f.)

Wireless Intelligent Networking LAB

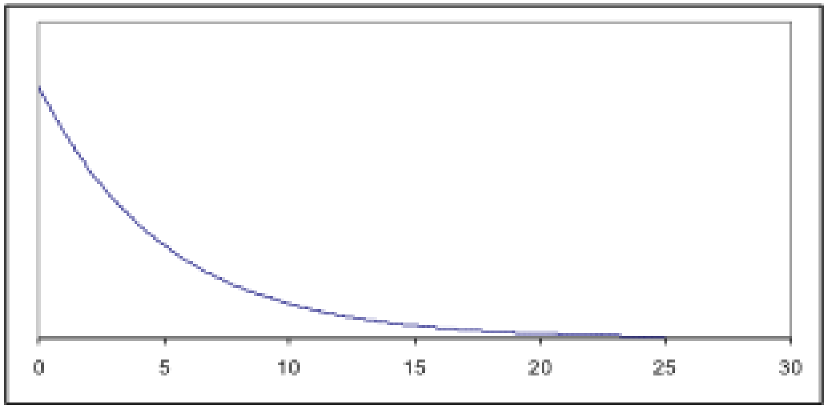
- In many cases, we shall concern ourselves with computing probabilities for various intervals of continuous random variables such as
P (**a** < **X** < **b**), **P** (**W** ≥ **c**), and so forth.

Example

- The probability that the average daily temperature in L.A. during the month of August falls between 90 and 95 degrees is



- The probability that a given part will fail before 1000 hours of use is



probability density function*

- There is a function of the numerical values of the continuous random variable **X** and as such will be represented by the functional notation f(x).

- **DEFINITION:** The function **f**(**x**) is a **probability density function (p.d.f.)** for the continuous random variable **X**, defined over the set of real number **R**, if

1. $f(x) \geq 0$, for all $x \in R$.
2. $\int_{-\infty}^{\infty} f(x)\, dx = 1$.
3. $P(a < X < b) = \int_a^b f(x)\, dx$.

*확률밀도함수

$$P(a < X < b) = \int_a^b f(x)\, dx.$$

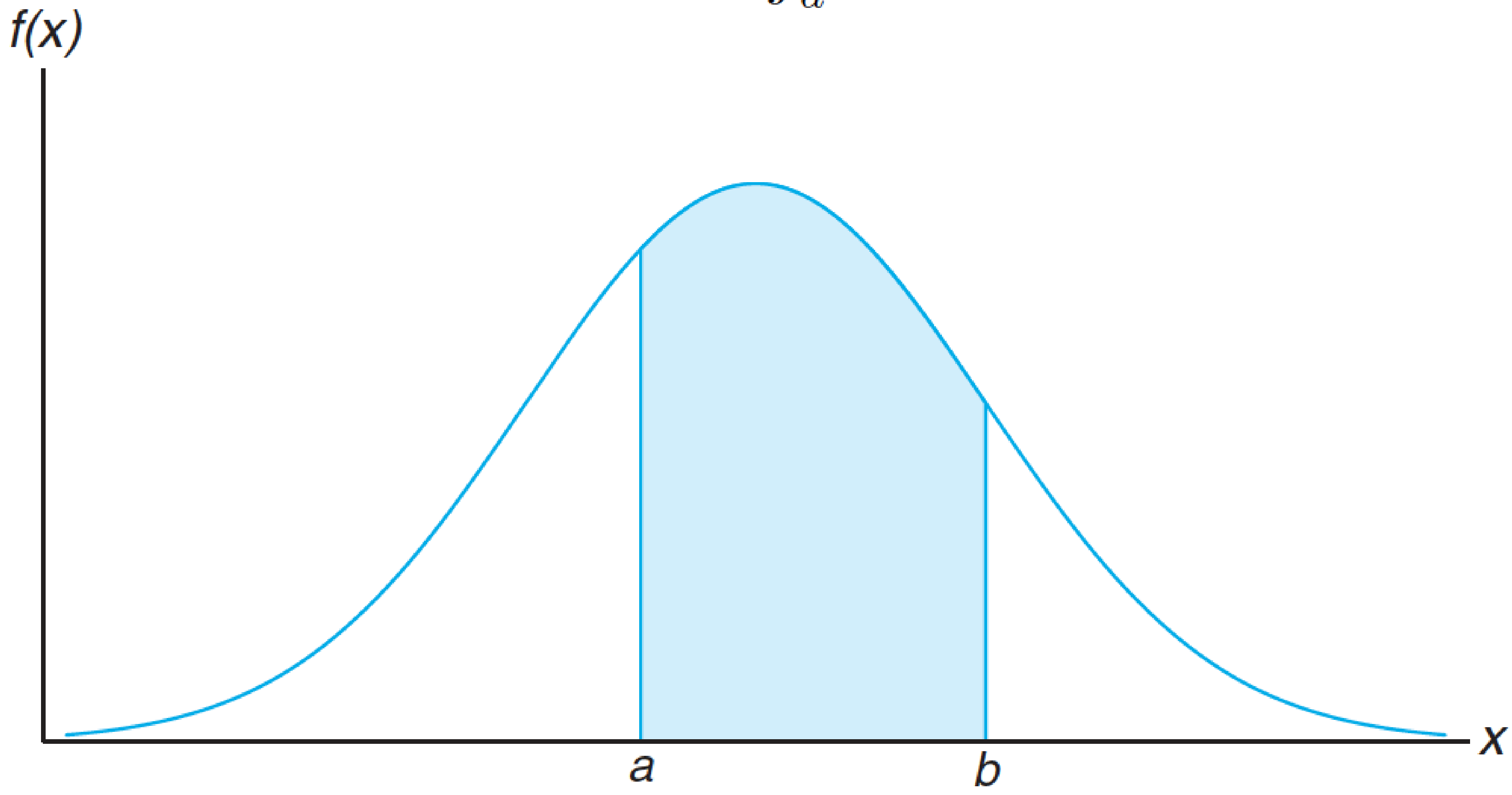


Figure 3.5: $P(a < X < b)$.

Question

$$P(a < X \le b) = P(a < X < b)\ ?$$

or

$$P(a < X \le b) \ne P(a < X < b)\ ?$$

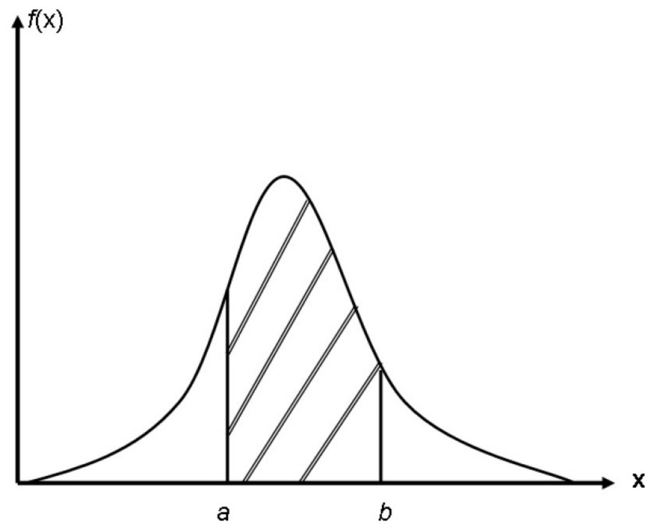
Wireless Intelligent Networking LAB

NOTE:

- A continuous random variable has a probability of zero of assuming exactly any of its values.
  - This is because $\int_a^a f(x)dx = 0$
- So, when X is continuous,

$$\boxed{P(a < X \leq b) = P(a < X < b) + P(X = b) = P(a < X < b).}$$

  - That is, it doesn't matter whether we include an endpoint of the interval or not. This is not true, though, when X is **discrete**.



Example 3.11

- Suppose that the error in the reaction temperature, for a controlled laboratory experiment is a continuous random variable X having the probability density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2; \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Verify that f(x) is a density function.

1. $f(x) \geq 0$, for all $x \in R$.
2. $\int_{-\infty}^{\infty} f(x)\, dx = 1$.
3. $P(a < X < b) = \int_a^b f(x)\, dx$.

We use Definition 3.6.

(a) Obviously, $f(x) \geq 0$. To verify condition 2 in Definition 3.6, we have

$$\int_{-\infty}^{\infty} f(x)\, dx = \int_{-1}^{2} \frac{x^2}{3} dx = \frac{x^3}{9}\Big|_{-1}^{2} = \frac{8}{9} + \frac{1}{9} = 1.$$

Wireless Intelligent Networking LAB

Solution

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2; \\ 0, & \text{elsewhere.} \end{cases}$$

1. $f(x) \geq 0$, for all $x \in R$.
2. $\int_{-\infty}^{\infty} f(x)\, dx = 1$.
3. $P(a < X < b) = \int_a^b f(x)\, dx$.

(b) Find P(0 < X ≤ 1).

- Using formula 3 in Definition 3.6, we obtain

$$P(0 < X \leq 1) = \int_0^1 \frac{x^2}{3} dx = \left. \frac{x^3}{9} \right|_0^1 = \frac{1}{9}.$$

Wireless Intelligent Networking LAB

Cumulative Distribution Function: Continuous

Definition 3.6:

3. $P(a < X < b) = \int_a^b f(x)\, dx.$

- Definition 3.7

The **cumulative distribution function** $F(x)$ of a continuous random variable X with density function $f(x)$ is

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\, dt, \quad \text{for } -\infty < x < \infty.$$

- As an immediate consequence,

$$\boxed{P(a < X < b) = F(b) - F(a) \text{ and } f(x) = \frac{dF(x)}{dx},}$$

if the derivation exists.

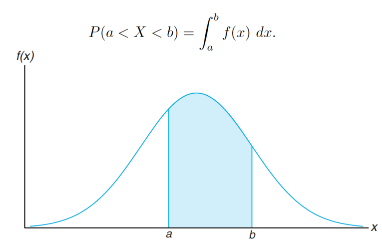


Figure 3.5: $P(a < X < b)$.

Wireless Intelligent Networking LAB

Example 3.11

- Suppose that the error in the reaction temperature, in °C, for a controlled laboratory experiment is a continuous random variable X having the probability density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2; \\ 0, & \text{elsewhere.} \end{cases}$$

- **(c)** find $F(x)$, and use it to evaluate $P(0 < X \leq 1)$.

- Solution

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2; \\ 0, & \text{elsewhere.} \end{cases}$$

For $-1 < x < 2$,

$$F(x) = \int_{-\infty}^{x} f(t)\, dt = \int_{-1}^{x} \frac{t^2}{3} dt = \left.\frac{t^3}{9}\right|_{-1}^{x} = \frac{x^3+1}{9}.$$

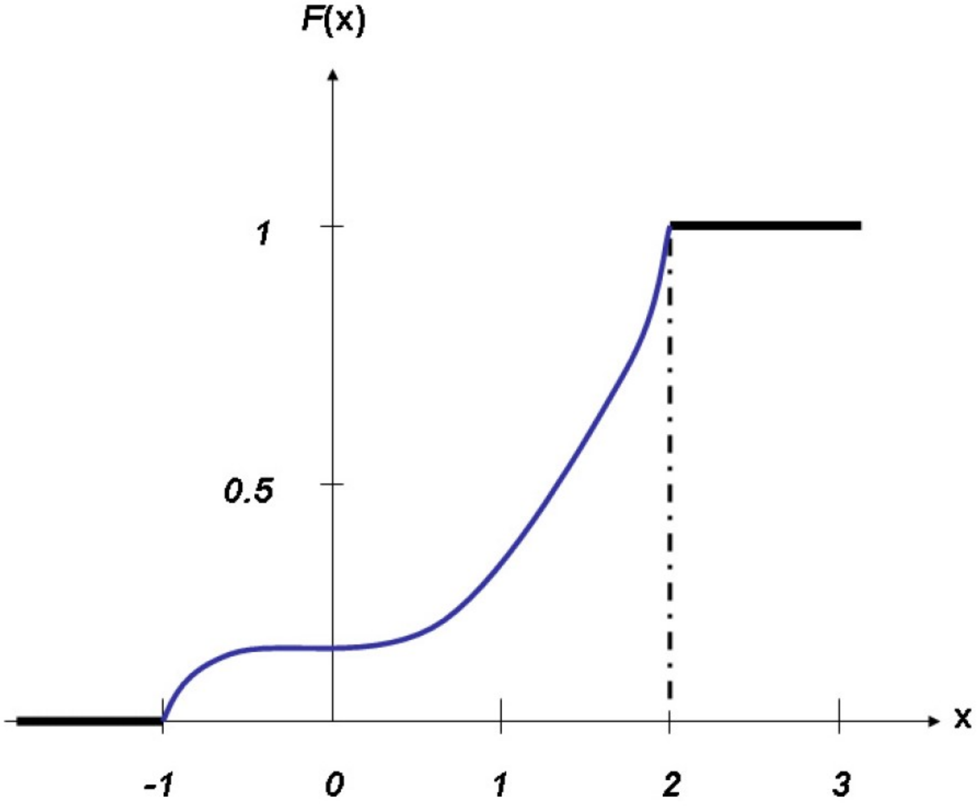
Therefore,

$$F(x) = \begin{cases} 0, & x < -1, \\ \frac{x^3+1}{9}, & -1 \le x < 2, \\ 1, & x \ge 2. \end{cases}$$

- Thus,

$$P(0 < X \le 1) = F(1) - F(0) = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}$$

GWINL Wireless Intelligent Networking LAB



$$F(x) = \begin{cases} 0, & x < -1; \\ \frac{x^3+1}{9}, & -1 \le x < 2; \\ 1, & x \ge 2. \end{cases}$$

Example 3.13

- The Department of Energy (DOE) puts projects out on bid and generally estimates what a reasonable bid should be. Call the estimate **b**. The DOE has determined that the density function of the winning (low) bid is

$$f(y) = \begin{cases} \frac{5}{8b}, & \frac{2}{5}b \le y \le 2b, \\ 0, & \text{elsewhere.} \end{cases}$$

- Find **F**(**y**) and use it to determine the probability that the winning bid is less than the DOE's preliminary estimate **b** .

Solution

$$f(y) = \begin{cases} \frac{5}{8b}, & \frac{2}{5}b \le y \le 2b, \\ 0, & \text{elsewhere.} \end{cases}$$

For $\frac{2b}{5} < y < 2b$

$$F(y) = \int_{2b/5}^{y} \frac{5}{8b} dt = \frac{5t}{8b} \Big|_{2b/5}^{y} = \frac{5y}{8b} - \frac{1}{4}.$$

Thus

$$F(y) = \begin{cases} 0, & \text{if } y < \frac{2b}{5}; \\ \frac{5y}{8b} - \frac{1}{4}, & \text{if } \frac{2b}{5} \le y < 2b; \\ 1, & \text{if } y \ge 2b. \end{cases}$$

To determine the probability that the winning bid is less than the preliminary bid estimate b, we have

$$P(Y \le b) = F(b) = \frac{5}{8} - \frac{1}{4} = \frac{3}{8}.$$

3.4 Joint Probability Distributions*

*결합확률분포

Joint Probability Distribution

- If X and Y are two random variables, the probability distribution for their simultaneous occurrence can be represented by a function with values $f(x, y)$ for any pair of values (x, y). We refer to this function as **the joint probability distribution** of X and Y .

Joint Probability Distribution*

- For **discrete** cases,
  - **f(x, y) = P(X = x, Y = y);**
  - **that is, the values f(x, y) give the probability that outcomes x and y occur at the same time.**
- **DEFINITION 3.8**

The function $f(x, y)$ is a **joint probability distribution** or **probability mass function** of the discrete random variables X and Y if

1. $f(x, y) \geq 0$ for all (x, y),
2. $\sum_x \sum_y f(x, y) = 1$,
3. $P(X = x, Y = y) = f(x, y)$.

For any region A in the xy plane, $P[(X, Y) \in A] = \sum\sum_A f(x, y)$.

*결합확률질량함수

- Example 3.14
  - **Two** ballpoint pens are selected at random from a box that contains 3 blue pens, 2 red pens, and 3 green pens. If X is the number of blue pens selected and Y is the number of red pens selected, find
  - (a) the joint probability function f(x, y),

- # Solution (a)

- 3 blue pens, 2 red pens, and 3 green pens
- the joint probability function f(x, y)
  - blue pens
  - red pens

The possible pairs of values (x, y) are $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$, $(0,2)$, and $(2,0)$.

Now, $f(0,1)$, for example, represents the probability that a red and a green pens are selected. The total number of equally likely ways of selecting any 2 pens from the 8 is $\binom{8}{2} = 28$. The number of ways of selecting 1 red from 2 red pens and 1 green from 3 green pens is $\binom{2}{1}\binom{3}{1} = 6$. Hence, $f(0,1) = 6/28 = 3/14$.

$f(x, y)$		$x=0$	$x=1$	$x=2$	Row Totals
y	0	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$	$\frac{15}{28}$
	1	$\frac{3}{14}$	$\boxed{\frac{3}{14}}$	0	$\frac{3}{7}$
	2	$\frac{1}{28}$	0	0	$\frac{1}{28}$
Column Totals		$\frac{5}{14}$	$\frac{15}{28}$	$\frac{3}{28}$	1

- 3 blue pens, 2 red pens, and 3 green pens

- The joint probability distribution of (X; Y) can be represented:

- blue pens
- red pens

$$f(x, y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{2-x-y}}{\binom{8}{2}},$$

for $x = 0, 1, 2;\ y = 0, 1, 2;$ and $0 \le x + y \le 2$.

Wireless Intelligent Networking LAB

- (b) P[(X,Y) ∈ A], where A is the region { (x,y)| x+y≤1}.

- Solution

$$P[(X,Y) \in A] = P(X+Y \leq 1) = f(0,0)+f(1,0)+f(0,1) = \frac{9}{14}$$

Definition 3.8:

For any region A in the xy plane, $P[(X,Y) \in A] = \sum\sum_{A} f(x,y)$.

Joint Density Function*

- **When X and Y are continuous random variables,** the joint density function f(x, y) is a surface lying above the xy plane, and P [(X,Y) ∈ A], where A is any region in the xy plane.
- **DEFINITION 3.9**

The function $f(x, y)$ is a **joint density function** of the continuous random variables X and Y if

1. $f(x, y) \geq 0$, for all (x, y),
2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1$,
3. $P[(X, Y) \in A] = \int \int_A f(x, y)\, dx\, dy$, for any region A in the xy plane.

*결합확률밀도함수

Wireless Intelligent Networking LAB

Example

- A privately owned business operates both a drive-in facility and a walk-in facility.
- On a randomly selected day, let X and Y, respectively, be the **proportions of the time** that the drive-in and the walk-in facilities are in use, and suppose that the joint density function of these random variables is

$$f(x,y) = \begin{cases} \frac{2}{5}(2x+3y), & 0 \le x \le 1, 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Verify $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\ dx\ dy = 1$,
- (b) Find $P[(X,Y) \in A]$, where $A = \{(x,y) \mid 0 < x < \frac{1}{2}, \frac{1}{4} < y < \frac{1}{2}\}$.

WINL Wireless Intelligent Networking LAB

$$f(x,y) = \begin{cases} \frac{2}{5}(2x+3y), & 0 \le x \le 1, 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Verify $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\ dx\ dy = 1,$

- Solution

The integration of $f(x,y)$ over the whole region is

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\ dx\ dy = \int_0^1\int_0^1 \frac{2}{5}(2x+3y)\ dx\ dy$$
$$= \int_0^1 \left(\frac{2x^2}{5} + \frac{6xy}{5}\right)\Bigg|_{x=0}^{x=1} dy$$
$$= \int_0^1 \left(\frac{2}{5} + \frac{6y}{5}\right) dy = \left(\frac{2y}{5} + \frac{3y^2}{5}\right)\Bigg|_0^1 = \frac{2}{5} + \frac{3}{5} = 1.$$

GWINL Wireless Intelligent Networking LAB

$$f(x,y) = \begin{cases} \frac{2}{5}(2x+3y), & 0 \le x \le 1, 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (b) Find $P[(X,Y) \in A]$, where $A = \{(x,y) \mid 0 < x < \frac{1}{2}, \frac{1}{4} < y < \frac{1}{2}\}$.

$$\begin{aligned} P[(X,Y) \in A] &= P\left(0 < X < \frac{1}{2}, \frac{1}{4} < Y < \frac{1}{2}\right) \\ &= \int_{1/4}^{1/2} \int_{0}^{1/2} \frac{2}{5}(2x+3y)\ dx\ dy \\ &= \int_{1/4}^{1/2} \left(\frac{2x^2}{5} + \frac{6xy}{5}\right)\Bigg|_{x=0}^{x=1/2} dy = \int_{1/4}^{1/2} \left(\frac{1}{10} + \frac{3y}{5}\right) dy \\ &= \left(\frac{y}{10} + \frac{3y^2}{10}\right)\Bigg|_{1/4}^{1/2} \\ &= \frac{1}{10}\left[\left(\frac{1}{2} + \frac{3}{4}\right) - \left(\frac{1}{4} + \frac{3}{16}\right)\right] = \frac{13}{160}. \end{aligned}$$

Wireless Intelligent Networking LAB

NEXT..

From the given joint probability distribution $f(x, y)$ of the discrete random variables X and Y,

How can you derive $P(a < X < b)$ **?**

$$\boxed{P(a < X < b) = P(a < X < b, -\infty < Y < \infty)}$$

$$= \int_a^b \int_{-\infty}^{\infty} f(x, y)\ dy\ dx = \int_a^b g(x)\ dx.$$

Wireless Intelligent Networking LAB

Marginal Probability Distribution*

- Given two jointly distributed random variables X and Y, the marginal distribution of X is simply the probability distribution of X ignoring information about Y (= summing $f(x,y)$ over all the values of Y).
- DEFINITION 3.10.
  - For the discrete case,

$$g(x) = \sum_y f(x,y) \quad \text{and} \quad h(y) = \sum_x f(x,y)$$

  - For continuous case,

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\, dy \quad \text{and} \quad h(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$$

*주변확률분포, 한계확률분포

		x			Row Totals
	$f(x,y)$	0	1	2	
y	0	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$	$\frac{15}{28}$
	1	$\frac{3}{14}$	$\frac{3}{14}$	0	$\frac{3}{7}$
	2	$\frac{1}{28}$	0	0	$\frac{1}{28}$
Column Totals		$\frac{5}{14}$	$\frac{15}{28}$	$\frac{3}{28}$	1

- Example 3.14 (AGAIN)
  - Give the marginal distribution of X alone and of Y alone in example 3.14.
- Sol
  - For r.v. X,

$$g(0) = \sum_y f(0,y) = f(0,0) + f(0,1) + f(0,2) = \frac{5}{14},$$

$$g(1) = \sum_y f(1,y) = f(1,0) + f(1,1) + f(1,2) = \frac{15}{28},$$

$$g(2) = \sum_y f(2,y) = f(2,0) + f(2,1) + f(2,2) = \frac{3}{28}.$$

x	0	1	2
$g(x)$	$\frac{5}{14}$	$\frac{15}{28}$	$\frac{3}{28}$

y	0	1	2
$h(y)$	$\frac{15}{28}$	$\frac{3}{7}$	$\frac{1}{28}$

$$f(x,y) = \begin{cases} \frac{2}{5}(2x+3y), & 0 \le x \le 1, 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Example 3.17: Find $g(x)$ and $h(y)$ for the joint density function of Example 3.15.

***Solution*:** By definition,

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\ dy = \int_0^1 \frac{2}{5}(2x+3y)\ dy = \left(\frac{4xy}{5} + \frac{6y^2}{10}\right)\bigg|_{y=0}^{y=1} = \frac{4x+3}{5},$$

for $0 \le x \le 1$, and $g(x) = 0$ elsewhere. Similarly,

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\ dx = \int_0^1 \frac{2}{5}(2x+3y)\ dx = \frac{2(1+3y)}{5},$$

for $0 \le y \le 1$, and $h(y) = 0$ elsewhere.

NOTE!

$$\boxed{P(a < X < b) = P(a < X < b, -\infty < Y < \infty)}$$

$$= \int_a^b \int_{-\infty}^{\infty} f(x, y)\ dy\ dx = \int_a^b g(x)\ dx.$$

- The marginal distributions *g*(*x*) and *h*(*y*) are indeed the probability distributions of the individual variables *X* and *Y* alone.

GWINL Wireless Intelligent Networking LAB

Conditional Distribution

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \text{ provided } P(A) > 0,$$

where A and B are now the events defined by $X = x$ and $Y = y$, respectively, then

$$P(Y = y \mid X = x) = \frac{P(X = x, Y = y)}{P(X = x)} = \frac{f(x, y)}{g(x)}, \text{ provided } g(x) > 0,$$

where X and Y are discrete random variables.

This is also true when $f(x, y)$ and $g(x)$ are the joint density and marginal distribution, respectively, of continuous random variables.

This distribution is called **conditional probability distribution**.

Conditional Distribution

- **Definition 3.11:**

Let X and Y be two random variables, discrete or continuous. The **conditional distribution** of the random variable Y given that $X = x$ is

$$f(y|x) = \frac{f(x,y)}{g(x)}, \text{ provided } g(x) > 0.$$

Similarly, the conditional distribution of X given that $Y = y$ is

$$f(x|y) = \frac{f(x,y)}{h(y)}, \text{ provided } h(y) > 0.$$

If we wish to find the probability that the discrete random variable X falls between a and b when it is known that the discrete variable $Y = y$, we evaluate

$$P(a < X < b \mid Y = y) = \sum_{a<x<b} f(x|y),$$

where the summation extends over all values of X between a and b. When X and Y are continuous, we evaluate

$$P(a < X < b \mid Y = y) = \int_a^b f(x|y)\, dx.$$

$f(x,y)$		x = 0	1	2	Row Totals
y	0	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$	$\frac{15}{28}$
	1	$\frac{3}{14}$	$\frac{3}{14}$	0	$\frac{3}{7}$
	2	$\frac{1}{28}$	0	0	$\frac{1}{28}$
Column Totals		$\frac{5}{14}$	$\frac{15}{28}$	$\frac{3}{28}$	1

- Example 3.18
  - Referring to Example 3.14,

  find the conditional distribution of **X**, given that **Y** = 1, and use it to determine **P**(**X** = 0**,Y** = 1).

  - Solution

Definition 3.11:

$$\boxed{f(x|y) = \frac{f(x,y)}{h(y)}}$$

We need to find $f(x|y)$, where $y = 1$. First, we find that

$$h(1) = \sum_{x=0}^{2} f(x,1) = \frac{3}{14} + \frac{3}{14} + 0 = \frac{6}{14}.$$

$$f(x|1) = \frac{f(x,y)}{h(1)} = \frac{7}{3} f(x,1), \quad \text{for} \quad x = 0,1,2.$$

Wireless Intelligent Networking LAB

$$f(x|1) = \frac{f(x,1)}{h(1)} = \left(\frac{7}{3}\right) f(x,1), \quad x = 0, 1, 2.$$

Therefore,

$$f(0|1) = \left(\frac{7}{3}\right) f(0,1) = \left(\frac{7}{3}\right)\left(\frac{3}{14}\right) = \frac{1}{2}, \; f(1|1) = \left(\frac{7}{3}\right) f(1,1) = \left(\frac{7}{3}\right)\left(\frac{3}{14}\right) = \frac{1}{2},$$
$$f(2|1) = \left(\frac{7}{3}\right) f(2,1) = \left(\frac{7}{3}\right)(0) = 0,$$

and the conditional distribution of X, given that $Y = 1$, is

x	0	1	2
$f(x\|1)$	$\frac{1}{2}$	$\frac{1}{2}$	0

Finally,

$$P(X = 0 \mid Y = 1) = f(0|1) = \frac{1}{2}.$$

Therefore, if it is known that 1 of the 2 pen refills selected is red, we have a probability equal to $1/2$ that the other refill is not blue.

GWINL Wireless Intelligent Networking LAB

- Example 3.20
  - Given the joint density function

$$f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4}, & 0 < x < 2, 0 < y < 1; \\ 0, & \text{elsewhere.} \end{cases}$$

(a) find g(x), h(y), f(x|y), and

(b) evaluate $P\left(\frac{1}{4} < X < \frac{1}{2} \middle| Y = \frac{1}{3}\right)$.

- Solution

  (a) find g(x), h(y), f(x|y), and

$$f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4}, & 0 < x < 2,\ 0 < y < 1, \\ 0, & \text{elsewhere}, \end{cases}$$

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\ dy = \int_0^1 \frac{x(1+3y^2)}{4} dy$$
$$= \left(\frac{xy}{4} + \frac{xy^3}{4} \right) \Bigg|_{y=0}^{y=1} = \frac{x}{2},$$

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\ dx = \int_0^2 \frac{x(1+3y^2)}{4} dx$$
$$= \left(\frac{x^2}{8} + \frac{3x^2y^2}{8} \right) \Bigg|_{x=0}^{x=2} = \frac{1+3y^2}{2}.$$

Therefore, using the conditional density definition, for $0 < x < 2$,

$$f(x|y) = \frac{f(x,y)}{h(y)} = \frac{x(1+3y^2)/4}{(1+3y^2)/2} = \frac{x}{2},$$

(b) evaluate $P\left(\frac{1}{4} < X < \frac{1}{2} \middle| Y = \frac{1}{3}\right)$.

$$P\left(\frac{1}{4} < X < \frac{1}{2} \ \middle|\ Y = \frac{1}{3} \right) = \int_{1/4}^{1/2} \frac{x}{2} dx = \frac{3}{64}.$$

GWINL Wireless Intelligent Networking LAB

Statistical Independent

- Let X and Y be two random variables with joint probability distribution f(x; y) and marginal distributions g(x) and h(y), respectively.

Definition 3.12:

- The random variables X and Y are said to be statistically independent if and only if

$$f(x,y) = g(x)h(y)$$

for all (x, y) within their range.

Definition 3.11: $\boxed{f(x|y) = \frac{f(x,y)}{h(y)}}$

- Example 3.19

The joint density for the random variables (X, Y), where X is the unit temperature change and Y is the proportion of spectrum shift that a certain atomic particle produces, is

$$f(x,y) = \begin{cases} 10xy^2, & 0 < x < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the marginal densities $g(x)$, $h(y)$, and the conditional density $f(y|x)$.

(b) Find the probability that the spectrum shifts more than half of the total observations, given that the temperature is increased by 0.25 unit.

(c) determine if X and Y are independent.

- # Solution

$$f(x,y)=\begin{cases}10xy^2, & 0<x<y<1,\\ 0, & \text{elsewhere.}\end{cases}$$

  - (a) By definition,

$$g(x)=\int_{-\infty}^{\infty} f(x,y)\,dy=\int_x^1 10xy^2\,dy$$

$$=\frac{10}{3}xy^3\Big|_{y=x}^{y=1}=\frac{10}{3}x(1-x^3),\ 0<x<1,$$

$$h(y)=\int_{-\infty}^{\infty} f(x,y)\,dx=\int_0^y 10xy^2\,dx=5x^2y^2\Big|_{x=0}^{x=y}=5y^4,\ 0<y<1.$$

$$\boxed{f(y|x)=\frac{f(x,y)}{g(x)}=\frac{10xy^2}{\frac{10}{3}x(1-x^3)}=\frac{3y^2}{1-x^3},\ 0<x<y<1.}$$

  - (b) $P\left(Y>\frac{1}{2}\,\Big|\,X=0.25\right)=\int_{1/2}^1 f(y\mid x=0.25)\,dy=\int_{1/2}^1 \frac{3y^2}{1-0.25^3}\,dy=\frac{8}{9}.$

  - (c) NO!

End of chapter

https://www.psycom.net/bipolar-questions-answers

Wireless Intelligent Networking LAB