Probability and Statistics **Spring 2022**
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Homework 2 - Solutions
Due date: Apr 4, 23:59, Cyber campus

1.

$$P(RR|\text{red side up}) = \frac{P(RR, \text{red side up})}{P(\text{red side up})}$$
$$= \frac{P(RR)P(\text{red side up}|RR)}{P(\text{red side up})}$$
$$= \frac{(1/3)(1)}{1/2} = 2/3$$

2. We want to pick door 2 (switch choice). Refer to the links for further explanation.

3. (a) Define C: Have cancer, E: high PSA level
$P(E|C^C)=0.135$, $P(E|C)=0.268$
$P(C)=0.7$, $P(C|E)$ =?
$P(C|E) = P(E|C)P(C) / [P(E|C)P(C) + P(E|C^c)P(C^c)] = 0.268*0.7 / (0.268*0.7+0.135*0.3) = 0.8224$

(b) $P(E|C^C)=0.135$, $P(E|C)=0.268$ ➔ $P(E^C|C^C)=0.865$, $P(E^c|C)=0.732$
$P(C)=0.7$, $P(C|E^c)$ =?
$P(C|E^c) = P(E^c|C)P(C) / [P(E^c|C)P(C) + P(E^c|C^c)P(C^c)] = 0.732*0.7/ (0.732*0.7+0.865*0.3) = 0.6638$

(c) $P(C)=0.3$

$P(C|E) = P(E|C)P(C) / [P(E|C)P(C) + P(E|C^c)P(C^c)] = 0.268*0.3 / (0.268*0.3+0.135*0.7) = 0.4596$

4.
$$P\{\text{good}|O\} = P\{\text{good}, O\}/P\{O\}$$
$$= .2P\{O|\text{good}\}/[P\{O|\text{good}\}.2 + P\{O|\text{average}\}.5 + P\{O|\text{bad}\}.3]$$
$$= .2 \times .95/[.95 \times .2 + .85 \times .5 + .7 \times .3] = 190/825$$

5.

(a) $P(T = 5) = F(5) - F(4) = 3/4 - 1/2 = 1/4.$

(b) $P(T > 3) = 1 - F(3) = 1 - 1/2 = 1/2.$

(c) $P(1.4 < T < 6) = F(6) - F(1.4) = 3/4 - 1/4 = 1/2.$

(d) $P(T \leq 5|T \geq 2) = \frac{P(2 \leq T \leq 5)}{P(T \geq 2)} = \frac{3/4-1/4}{1-1/4} = \frac{2}{3}.$

6. (1) $\int_0^\infty ce^{-2x}\,dx = -\frac{c}{2}[e^{-2x}]_0^\infty = -\frac{c}{2}[0-1] = \frac{c}{2} = 1$, thus c = 2.

(2) $P(X > 2) = \int_2^\infty 2e^{-2x}\,dx = -[e^{-2x}]_2^\infty = -[0 - e^{-4}] = e^{-4}$ $(or\ \frac{1}{e^4})$

7.

(a) $P(X > 8) = 1 - P(X \le 8) = \sum_{x=0}^{\circ} e^{-6}\frac{6^x}{x!} = e^{-6}\left(\frac{6^0}{0!} + \frac{6^1}{1!} + \cdots + \frac{6^8}{8!}\right) = 0.1528.$

(b) $P(X = 2) = e^{-6}\frac{6^2}{2!} = 0.0446.$

8. (a) $\int_0^2 f(x,y)dy = 12x^2/7 + 6x/7$

(b) $\int_0^1 \int_0^x f(x,y)dy\,dx = \int_0^1 (6x^3/7 + 3x^3/14)dx = 15/56.$