Probability and Statistics **Spring 2022**
Instructor: Joon Yoo (joon.yoo@gachon.ac.kr)
Homework 2 - Problem
Due date: Apr 4, 23:59, Cyber campus

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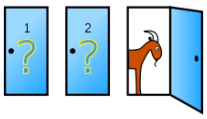
1. Of three cards, one is painted red on both sides; one is painted black on both sides; and one is painted red on one side and black on the other. A card is randomly chosen and placed on a table. If the side facing up is red, what is the probability that the other side is also red?

2. The following is the famous Monty Hall problem.

> Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?"



What would be your choice? Explain why. (Hint: link1, link2)

3. Prostate cancer is the most common type of cancer found in males. As an indicator of whether a male has prostate cancer, doctors often perform a test that measures the level of the PSA protein (prostate specific antigen) that is produced only by the prostate gland. Although higher PSA levels are indicative of cancer, the test is notoriously unreliable. Indeed, the probability that a noncancerous man will have an elevated PSA level is approximately .135, with this probability increasing to approximately .268 if the man does have cancer. If, based on other factors, a physician is 70 percent certain that a male has prostate cancer, what is the conditional probability that he has the cancer given that

(a) the test indicates an elevated PSA level;
(b) the test does not indicate an elevated PSA level?
(c) Repeat the preceding (a), this time assuming that the physician initially believes there is a 30 percent chance the man has prostate cancer.

4. Suppose that an insurance company classifies people into one of three classes — good risks, average risks, and bad risks. Their records indicate that the probabilities that good, average, and bad risk persons will be involved in an accident over a 1-year span are .05, .15, and .30, respectively. 20 percent of the population are "good risks," 50 percent are "average risks," and 30 percent are "bad risks". A policy holder Diana had ***no*** accidents in 1987.

(a) What is the probability that Diana is a good risk class?
(b) What is the probability that Diana is an average risk class?

5. An investment firm offers its customers municipal bonds that mature after varying numbers of years. Given that the cumulative distribution function of T, the number of years to maturity for a randomly selected bond, is

$$F(t) = \begin{cases} 0, & t < 1, \\ \frac{1}{4}, & 1 \le t < 3, \\ \frac{1}{2}, & 3 \le t < 5, \\ \frac{3}{4}, & 5 \le t < 7, \\ 1, & t \ge 7, \end{cases}$$

find

(a) $P(T = 5)$;

(b) $P(T > 3)$;

(c) $P(1.4 < T < 6)$;

(d) $P(T \leq 5 \mid T \geq 2)$.

6. If the density function of X equals

$$f(x) = \begin{cases} ce^{-2x} & \text{if } 0 < x < \infty, \\ 0 & \text{if } x < 0. \end{cases}$$

(1) Find c.

(2) What is $P(X > 2)$?

7. Suppose it is known from large amounts of historical data that X, the number of cars that arrive at a specific intersection during a 20-second time period, is characterized by the following discrete probability function:

$$f(x) = e^{-6}\frac{6^x}{x!}, \text{ for } x = 0, 1, 2, \ldots.$$

(a) Find the probability that in a specific 20-second time period, more than 8 cars arrive at the intersection.

(b) Find the probability that only 2 cars arrive.

8. The joint density of X and Y is given by

$$f(x, y) = \frac{6}{7}(x^2 + \frac{xy}{2}), \quad 0 < x < 1, \ 0 < y < 2$$

(a) Compute the marginal probability density of X, i.e., $f_X(x)$.

(b) Compute the marginal probability density of Y, i.e., $f_Y(y)$.

- Homework guidelines: (read carefully!)

1. Please write the answer and the solution process in detail.
2. Write in English.
3. Cheating is not accepted. (Do not copy any answer from the Internet, other students. All copied homework will result in ZERO points.)
4. Write down your answers in a single WORD file OR you may handwrite your answers and scan/copy&paste onto the word file.
5. The word file name MUST be "확률통계-학번-이름.docx".
6. You will get severely degraded if you do not follow the above five guidelines!!!
7. For your own sake do not submit at the last minute. Sometimes the cyber campus may fail to upload your file, and you may get ZERO points. Submit at least several hours before the deadline.