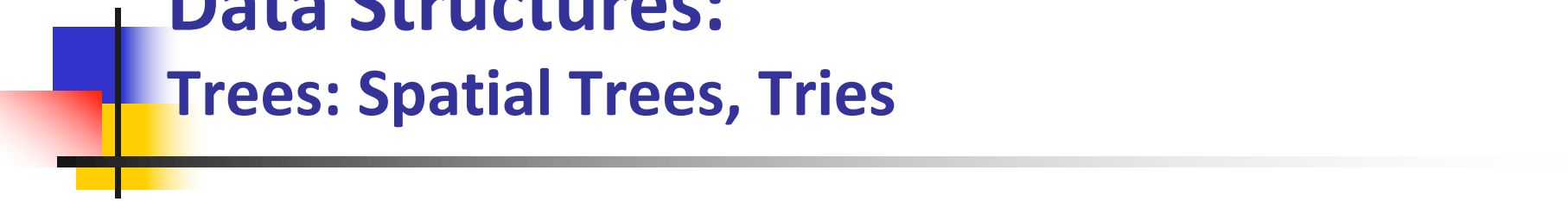


Data Structures:
Trees: Spatial Trees, Tries

Won Kim
(Lecture by Youngmin Oh)
Spring 2022

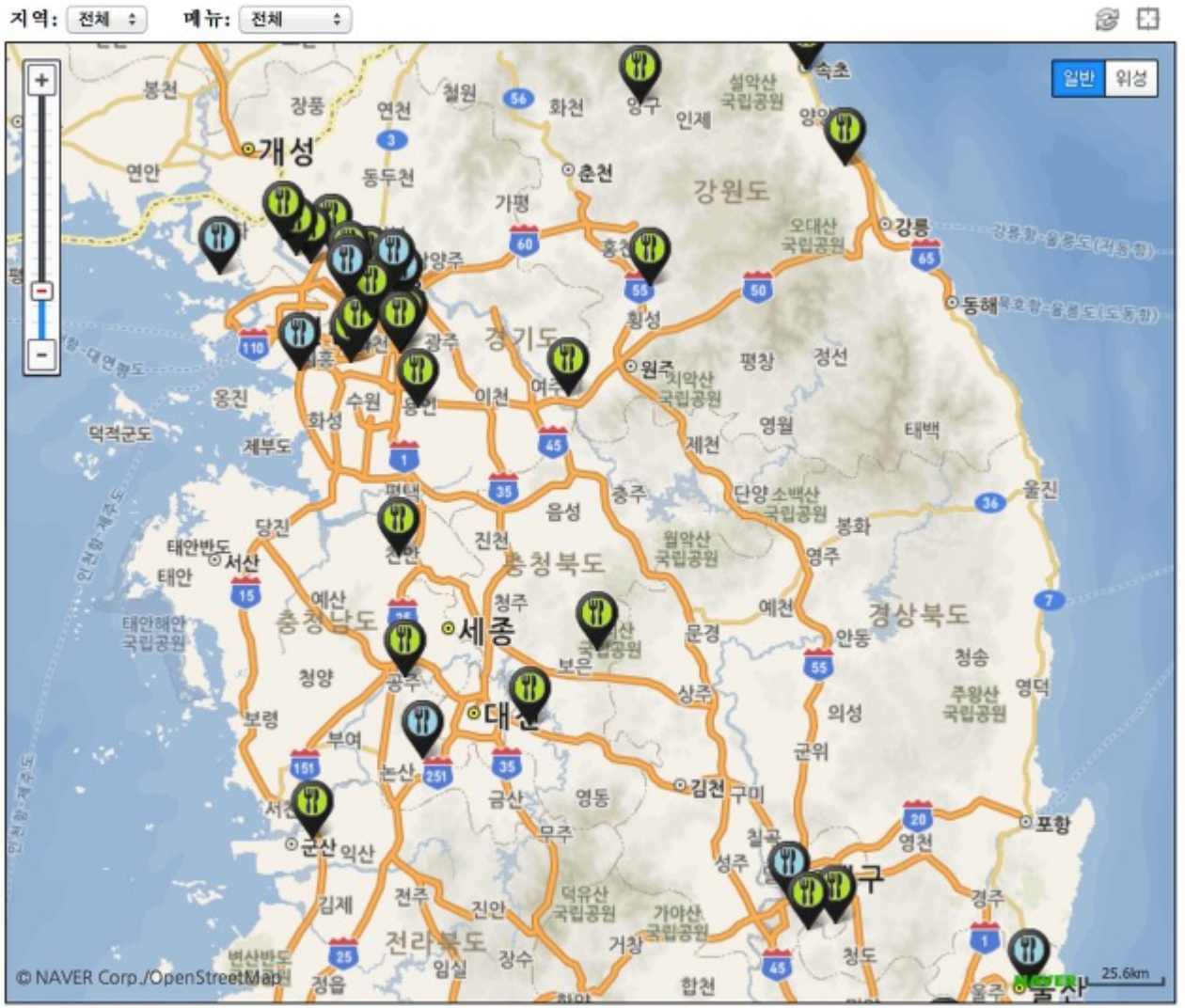
Characterizing Trees

Tree Properties

- Degrees of Trees
  - unary tree
    - linked list
  - binary tree
    - binary search tree
  - m-way tree (m>2)
- Height Balanced
  - unbalanced
  - balanced (but not perfectly)
    - AVL tree, T-tree
  - perfectly balanced
    - 2-3 tree
- Dimension
  - one dimension
    - AVL tree, T-tree
  - n-dimension (n >=2)
    - quad tree, kd tree
- Interior node has no value
  - trie, region quadtree

Spatial Data Structures

Storing and Searching for Popular Restaurants on a Map



* 자료출처: 이영돈PD의 먹거리X파일
* 사용법: 표시클릭 - 상세정보 보기, 더블클릭 - 확대.
* 데이터 날짜: 2013.9.27.
* 댓글 남기기: 바로가기

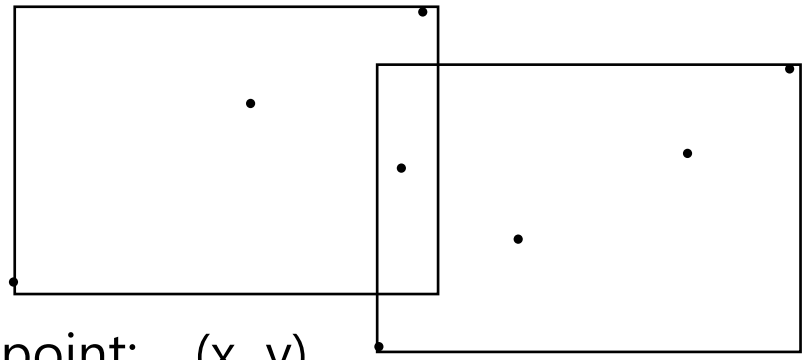
Spatial Data Structures

- QuadTree, OctTree
- k-d Tree, Grid File

Spatial vs. Non-Spatial Data Structures

- Non-Spatial Data Structures
  - Indexing 1-dimensional data
  - Key value is a single data item
    - e.g., 250, "Hong Gil Dong",…
- Spatial Data Structures
  - Indexing multi-dimensional data
  - Key value is a set of data items
    - E.g., (75, 127), (15, 46, 93), ("Hong Gil Dong", 25, thief, "running on the roof")

Point and Bounding Rectangle

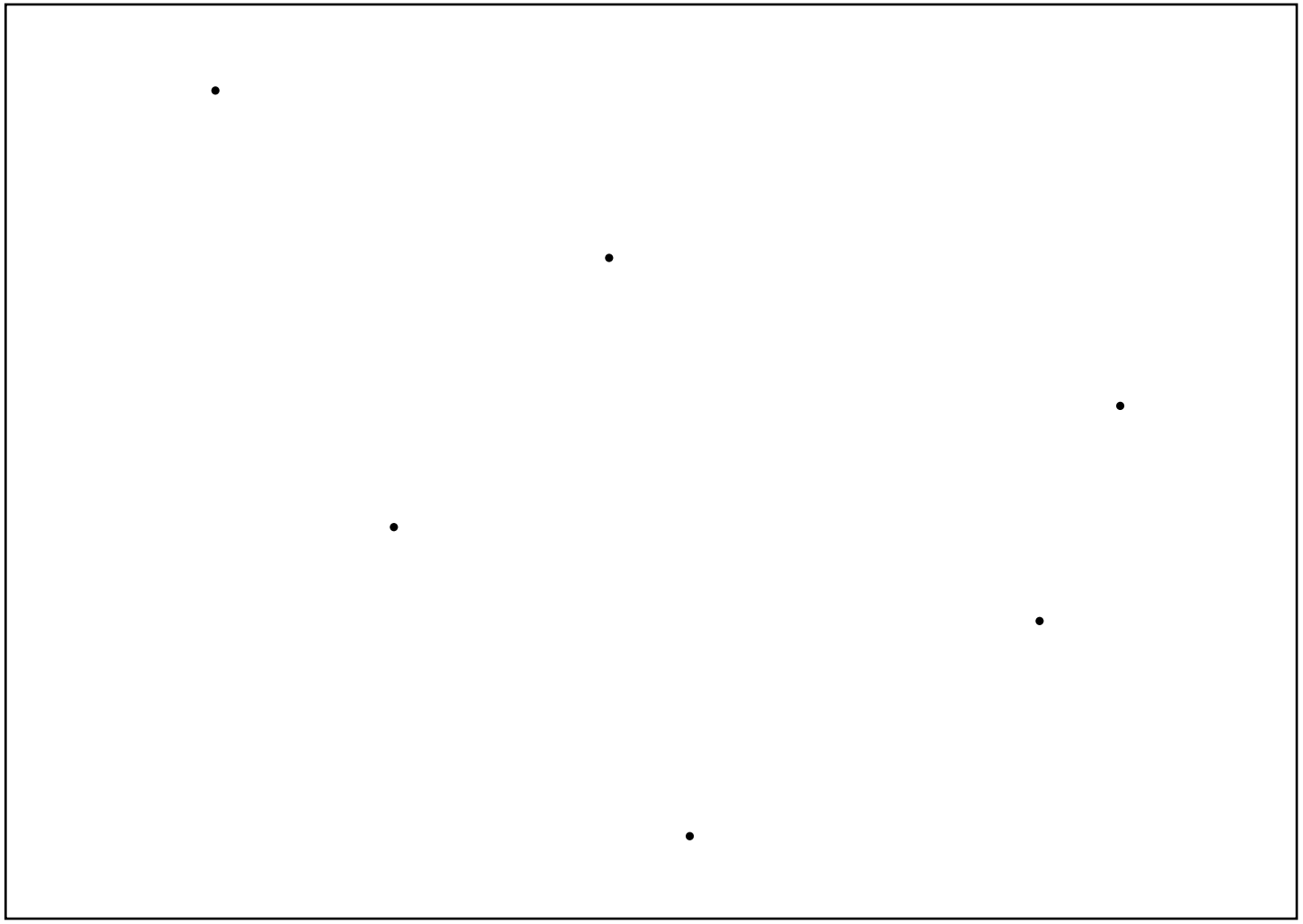


- point: (x, y)
- bounding rectangle (BR): ((x1,y1), (x2,y2))
- Relationship between a BR and a point
  - A point is contained in the BR
  - A point lies on an edge of the BR
  - Distance of the point from the center of the BR
- Relationship between two BRs
  - One BR contains the other BR
  - The two BRs touch each other
  - Distance between the BRs

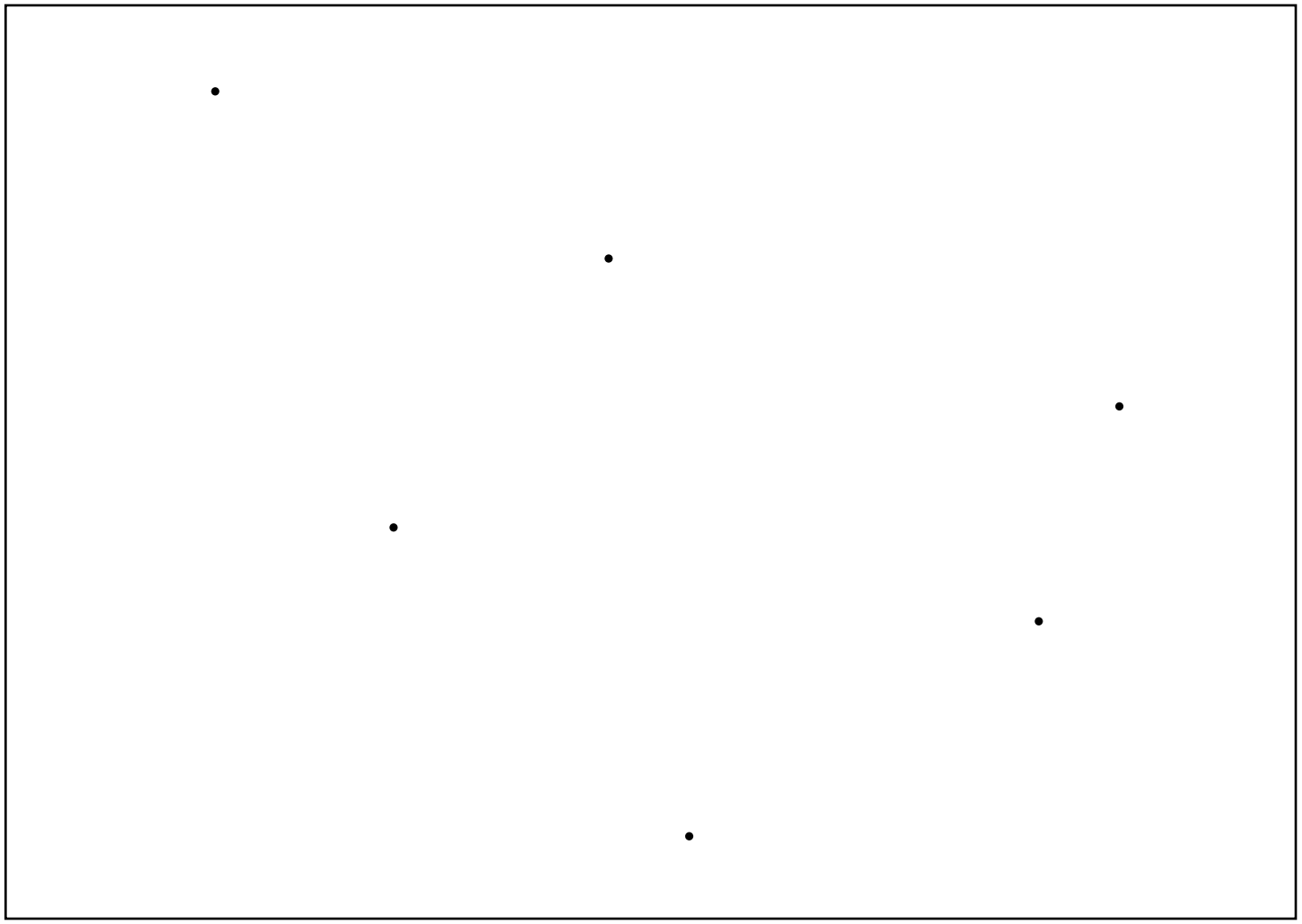
(Region) QuadTree

- Tree of Degree 4
- Performance: $O(\log_4 n)$
- Not Balanced
- Divides a 2-Dimensional Space into 4 Quadrants (Regions)
- Reading
  - http://www.cs.ubc.ca/~pcarbo/cs251/welcome.html

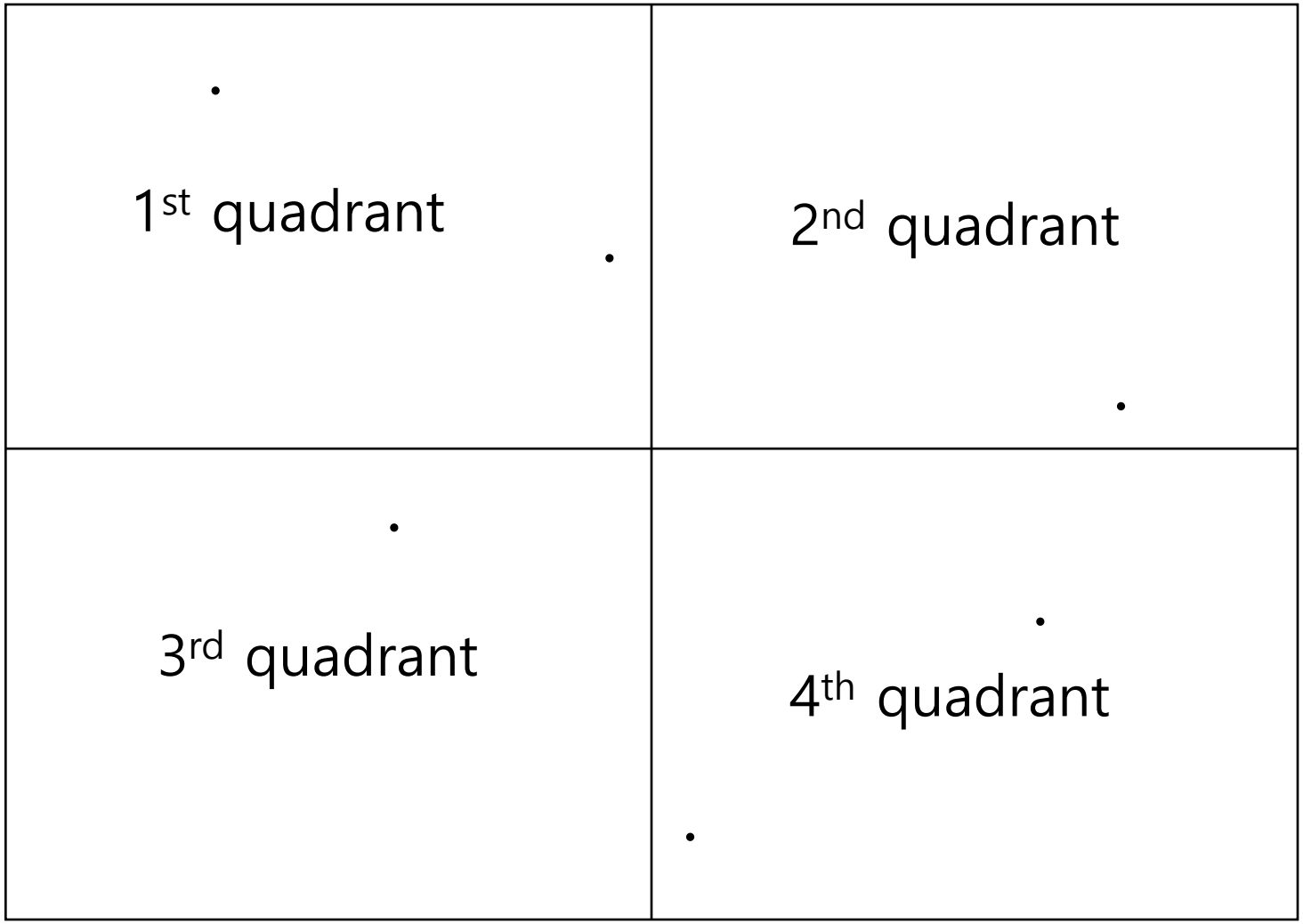
Example: Find Restaurants on the Map of a City (Each Point Represents All Data About a Restaurant.)



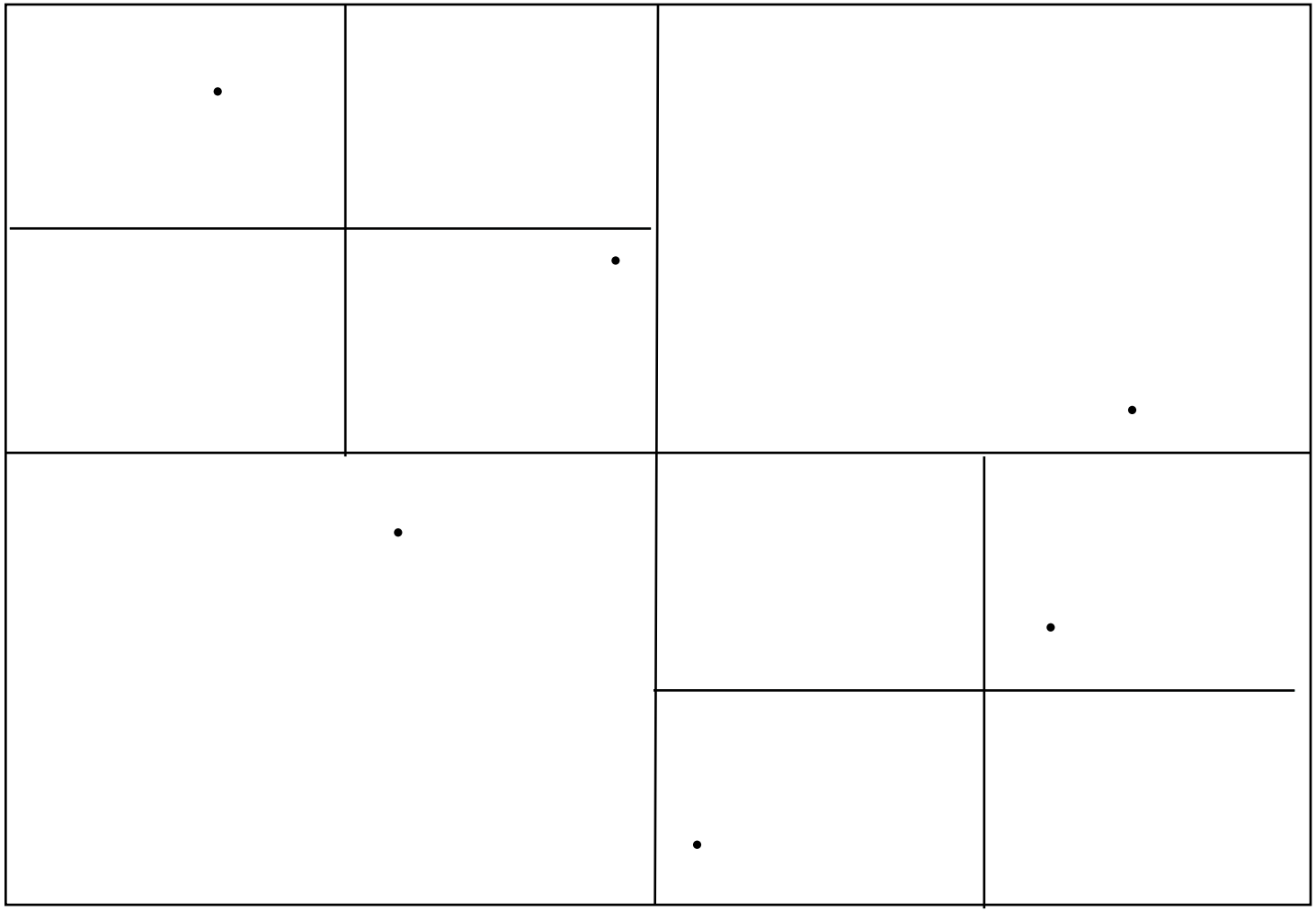
Divide a 2-D Space into Successive Quadrants



Divide a 2-D Space into Successive Quadrants

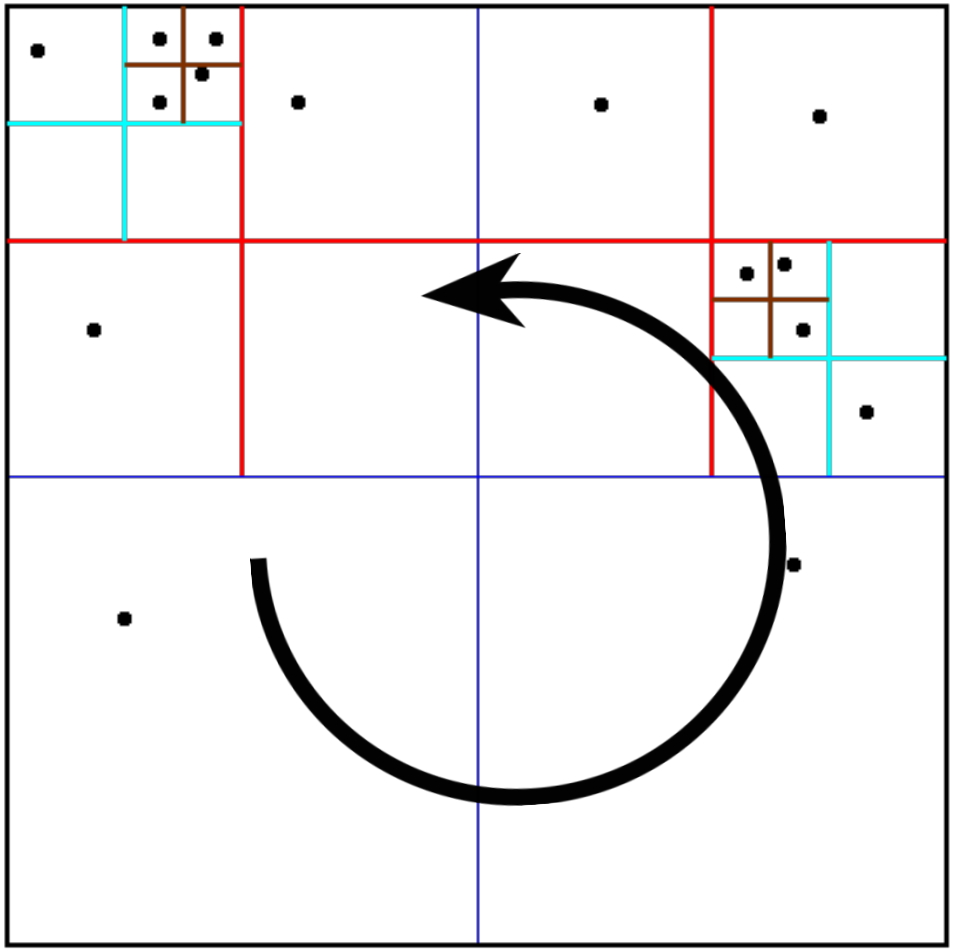


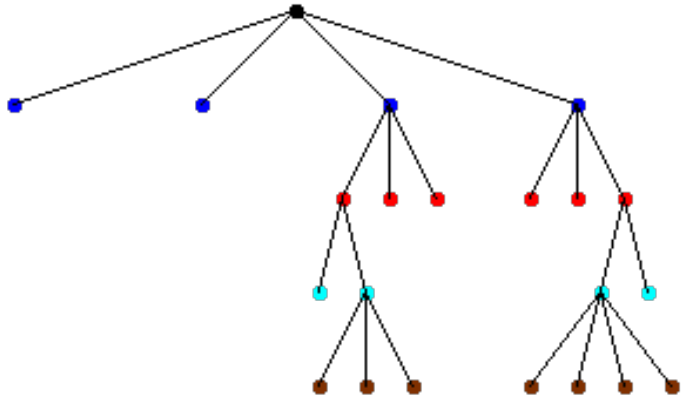
Divide a 2-D Space into Successive Quadrants (until each quadrant has only one point in it)



Map the Quadrants to Nodes of a QuadTree:
specify the starting quadrant and direction of mapping

Adaptive quadtree where no square contains more than 1 particle





<counter-clockwise
from the 3rd quadrant>

Region QuadTree Nodes

- Region quadtree is a type of trie (to learn later).
- Interior nodes contain pointers to child nodes, but no data.
- Data are stored on the leaf nodes.

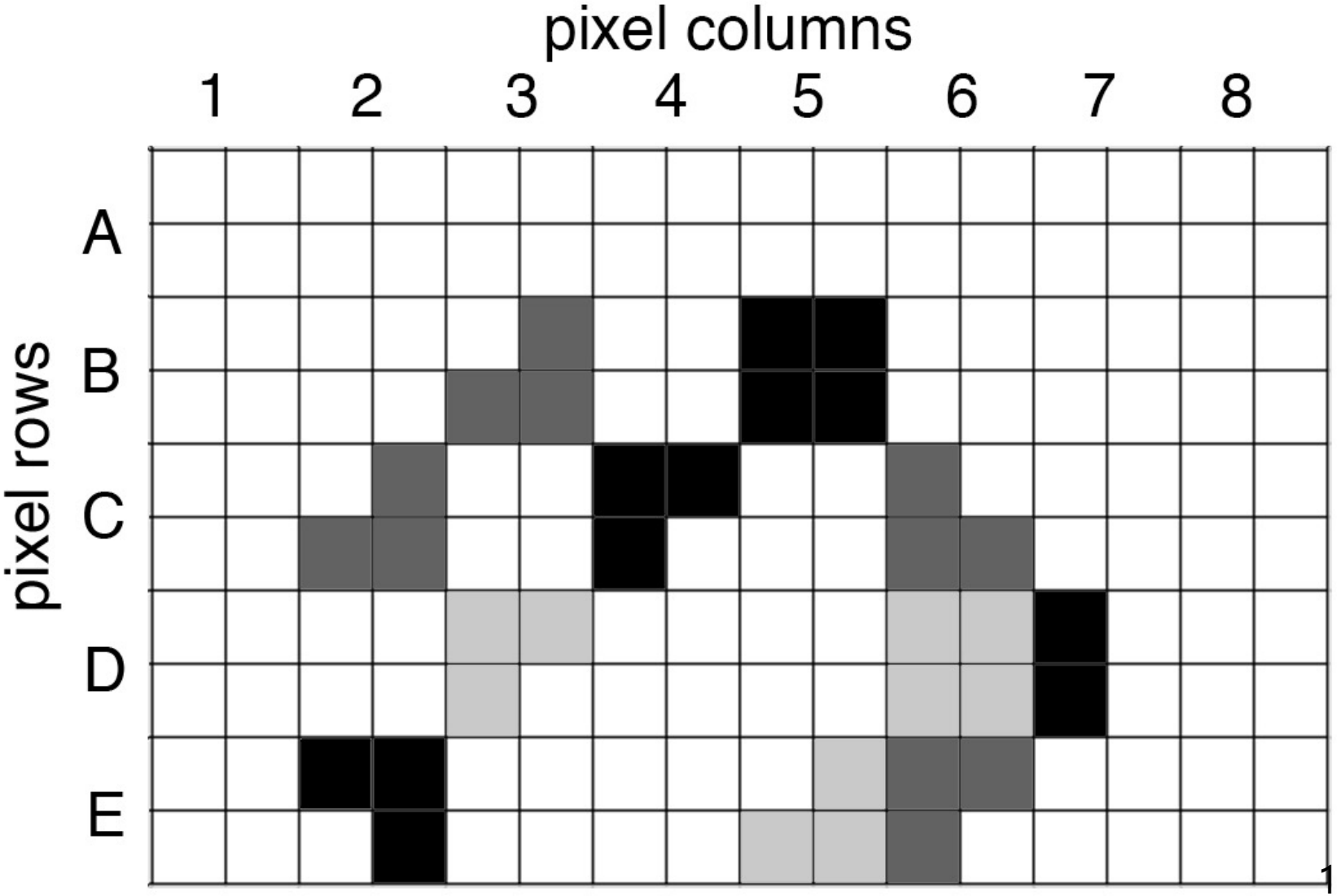
Leaf Node

- Each leaf node contains data corresponding to a specific subregion, for example,
  - point, number, string, pointer to a file,…
  - line, shape, pixels, image elements,…
  - the latitude and longitude of a set of cities
  - the average temperature over the subregion it represents

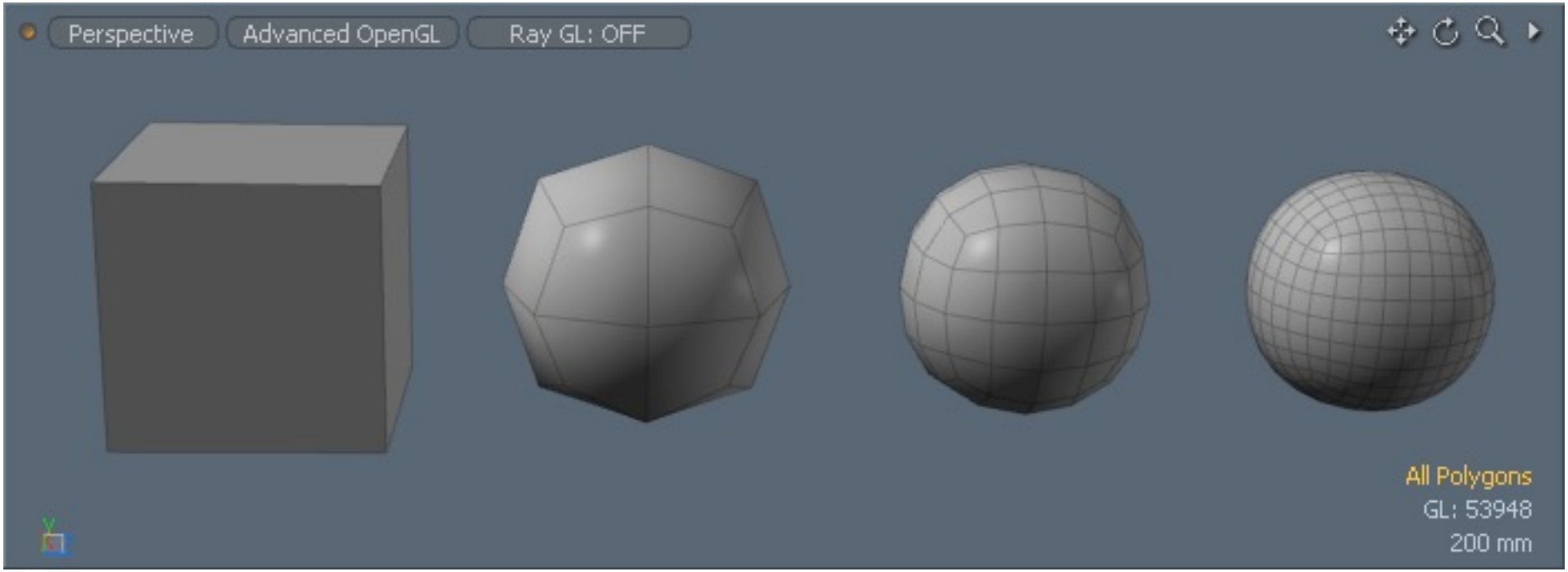
Applications

- software that searches for segments of stored color images
- software that searches for information about restaurants (schools, churches, car centers,…) within a region of a city (province, country)
- software that searches for parts of geometry objects
- supporting collision detection (intersection of two solid objects) in video games, physical simulations, computational geometry

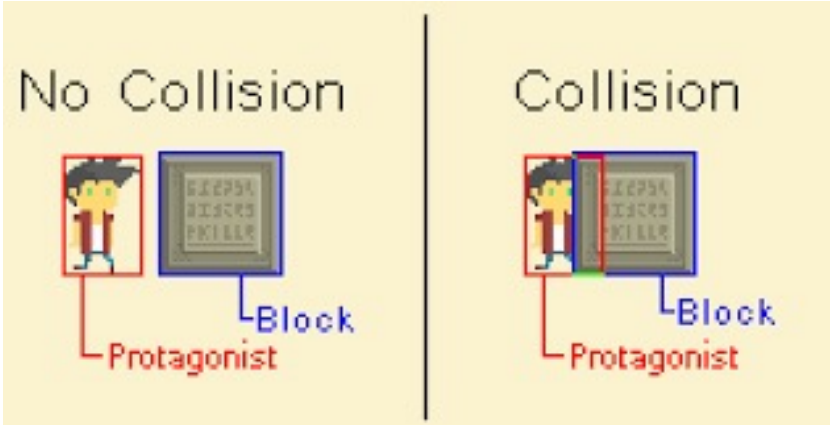
Image = Collection of Pixels (Picture Elements)

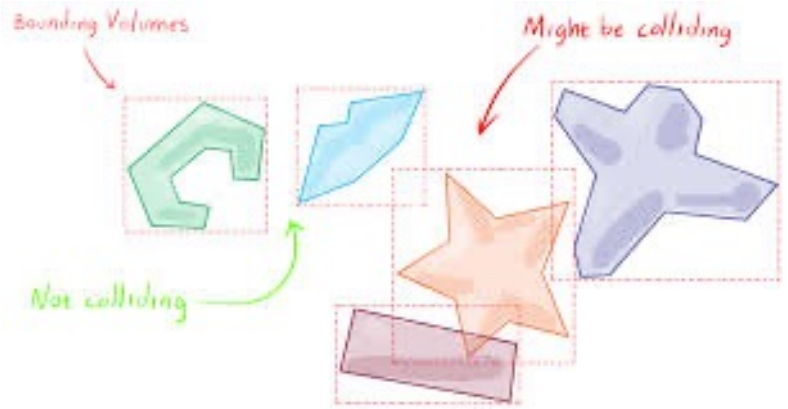


Subdividing Geometry Objects



Collision Detection (e.g., in Video Games)

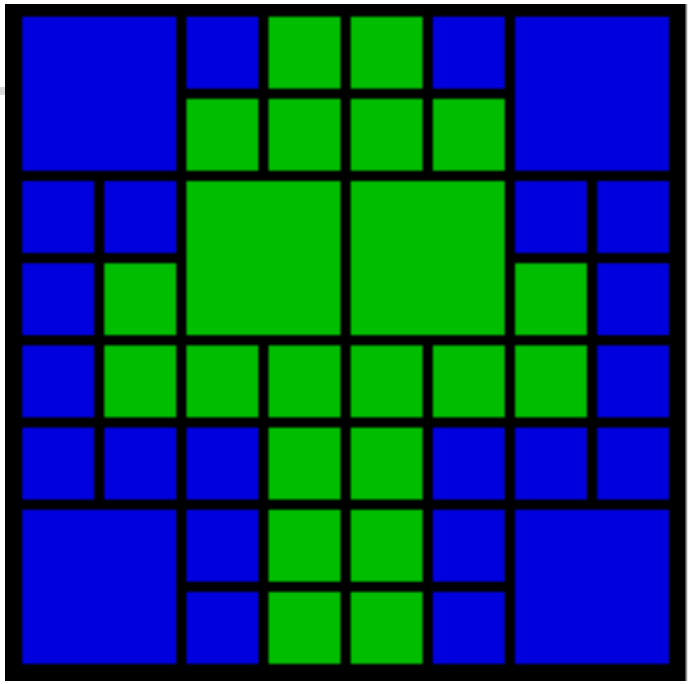




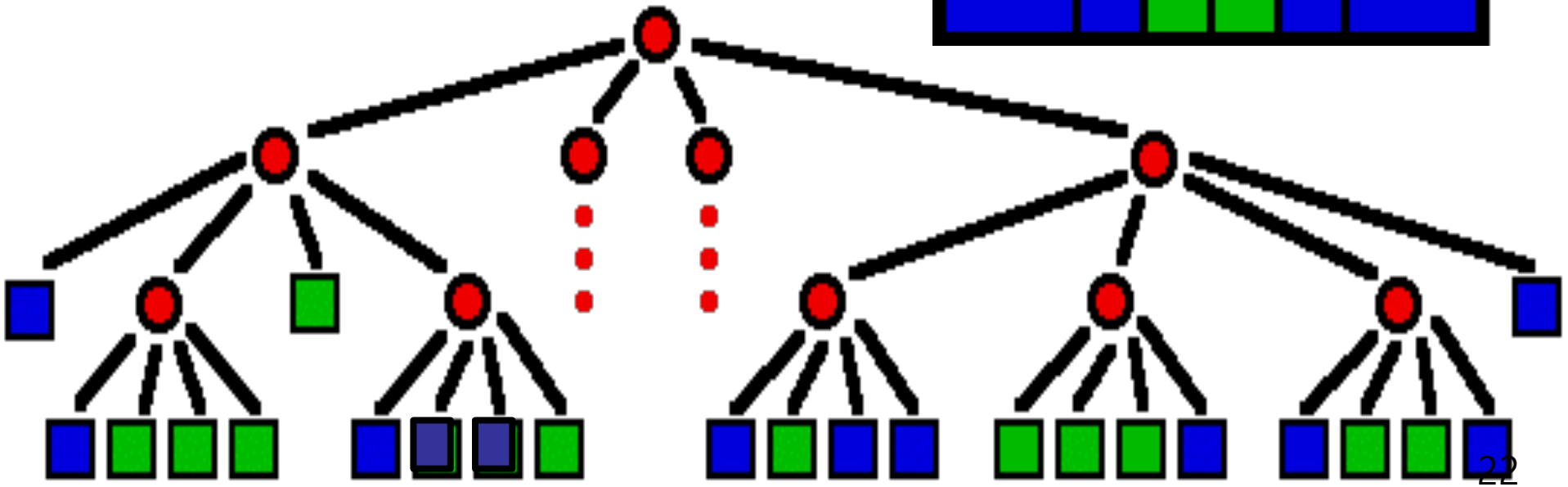
Applications: Representing an Image

- A region quadtree with a depth of n may represent an image consisting of $2^n \times 2^n$ pixels, where each pixel value is 0 or 1.
- The root node represents the entire image region. If the pixels in any region are not entirely 0s or 1s, it is subdivided.
- Each leaf node represents a block of pixels that are all 0s or all 1s.

A QuadTree for Storing 2-D Image Elements



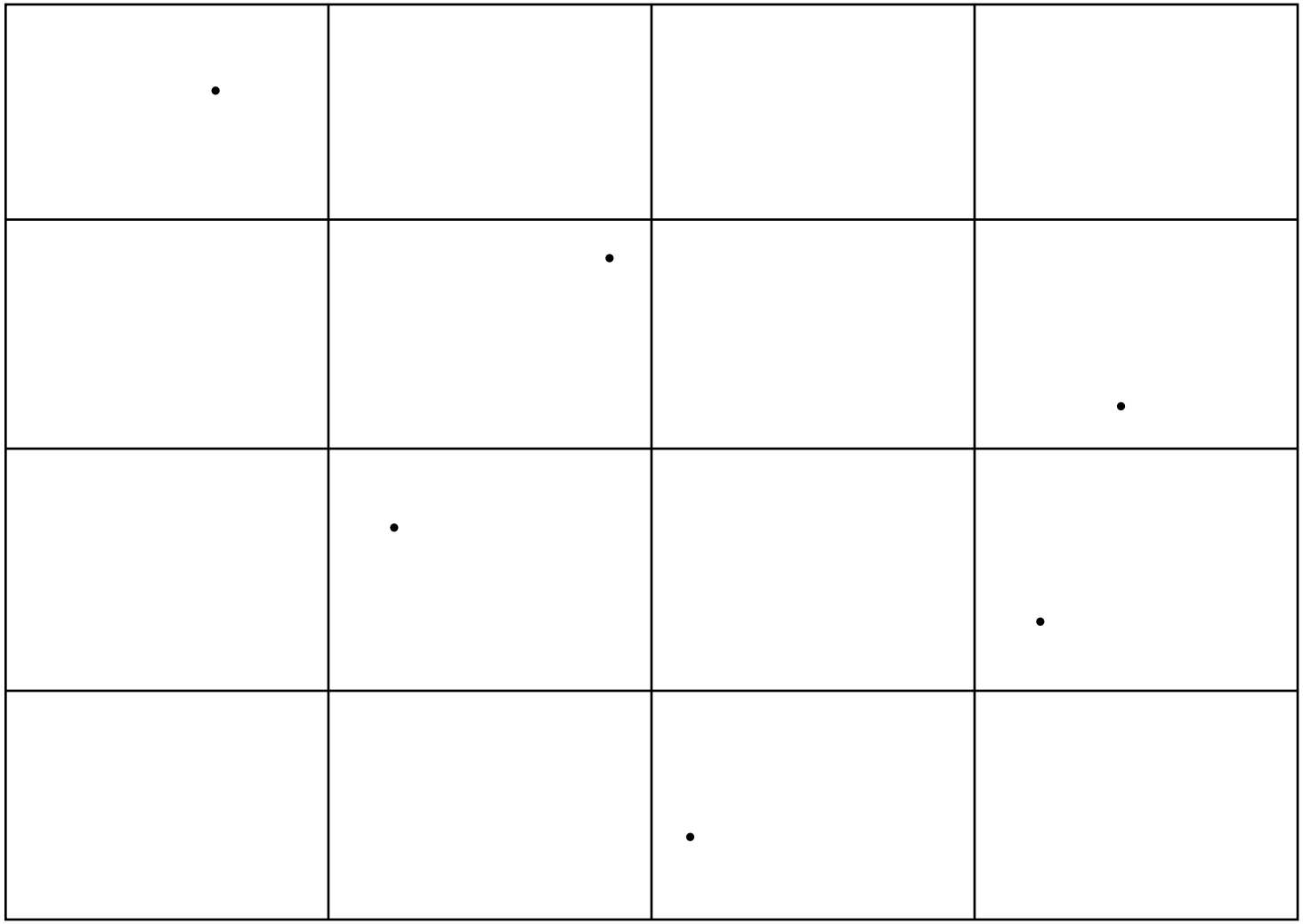
<clockwise from the 1st quadrant>



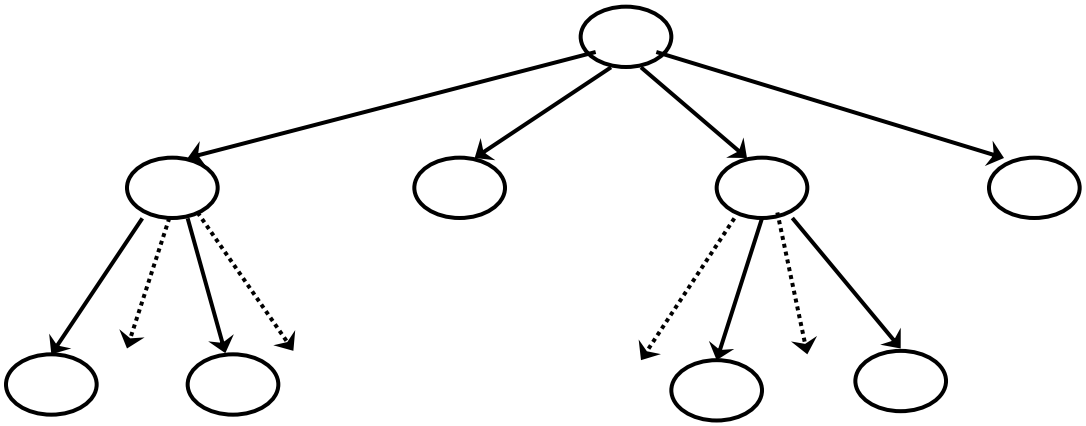
Storing and Accessing 2-D Image Elements

- Recursive subdividing of a quadrant
  - until each sub-quadrant has only one color
- Fast access to any region of an image at any level
  - any quadrant at any level of the tree
- Controlling the resolution of an image
  - raising or lowering the level of the tree

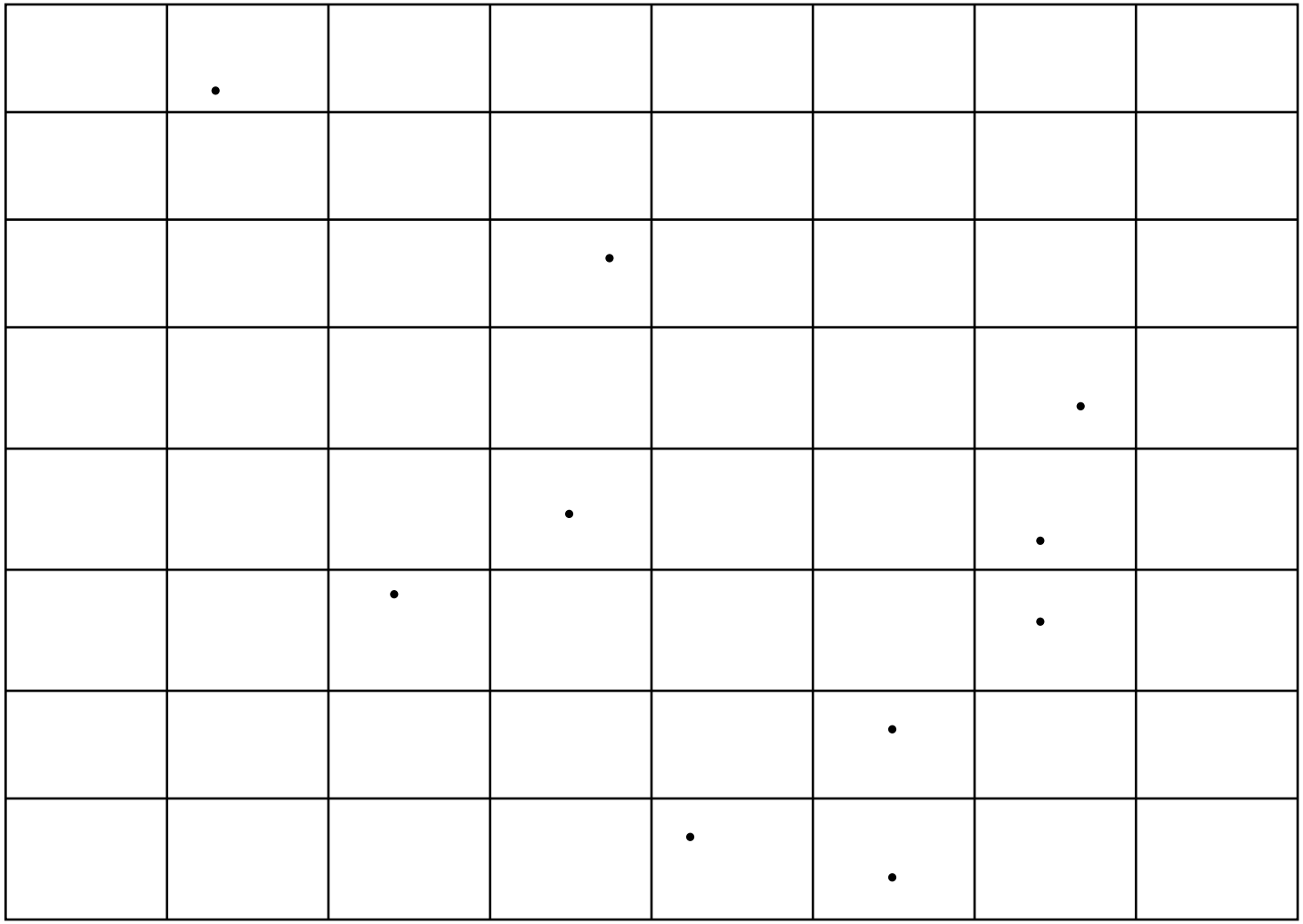
Exercise: Draw a Quad-Tree for the Following Point Data (clockwise from the 1st quadrant)



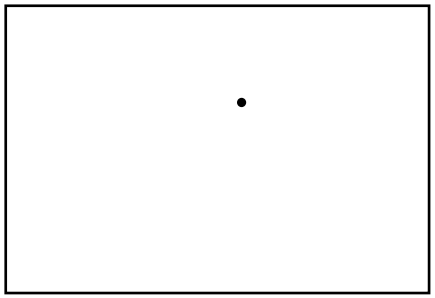
Exercise: Result



Exercise: Draw a Quad-Tree for the Following Point Data (clockwise from the 1st quadrant)



How to Determine Which Subtree to Go To?



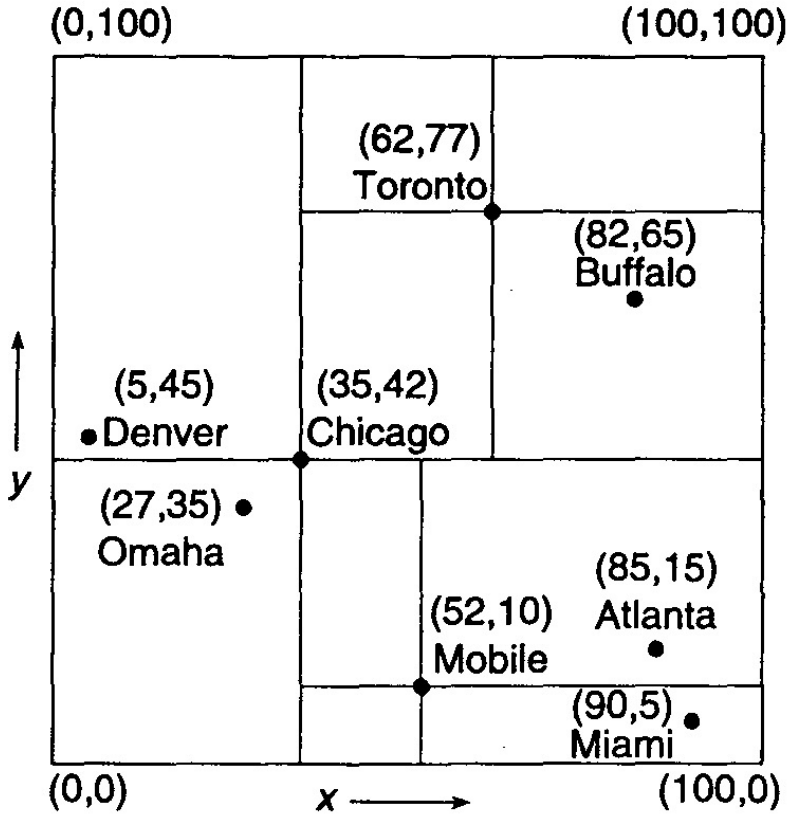
- point: (x, y)
- bounding rectangle (BR): ((x1,y1), (x2,y2))
- Each quadtree node has a corresponding BR.
- At each node, determine which of the child nodes (rectangles) can contain the search point or search rectangle.
- (The rectangle BR data may be stored in each non-leaf node or may be computed.)

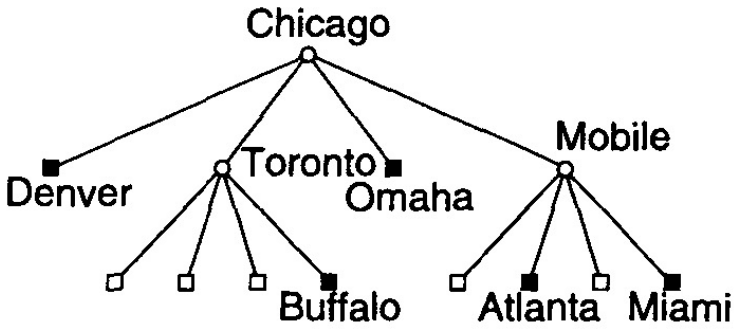
Point QuadTree

- Adaptation of a binary tree to represent 2-dimensional point data
- Each non-leaf node has 4 child nodes.
- Each node contains
  - 4 pointers (NW, NE, SW, SE)
  - key (x, y coordinates)
  - Value
- ** k-d tree is better (to be covered shortly)

Point Quadtree: Example

How do we determine the root node? (later..)



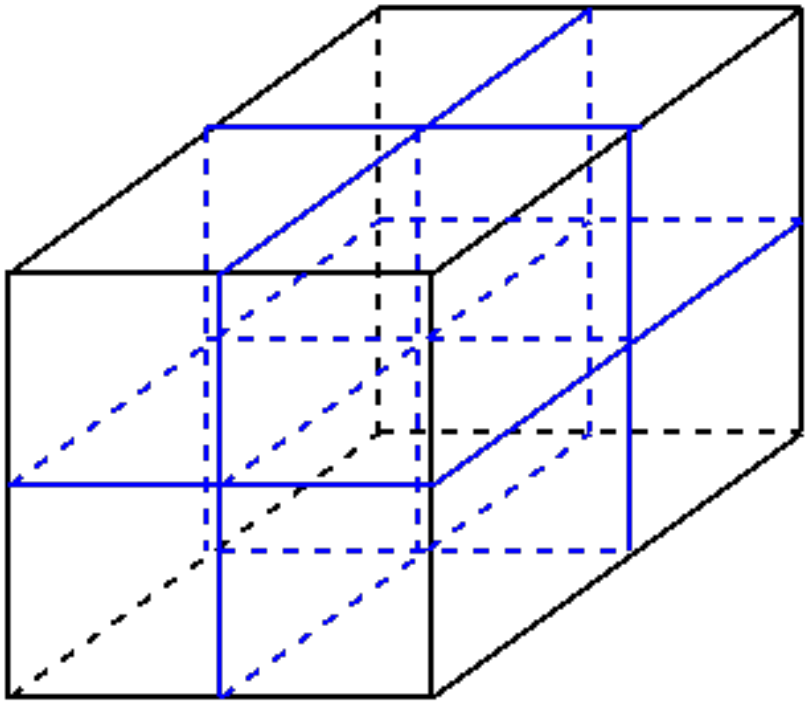


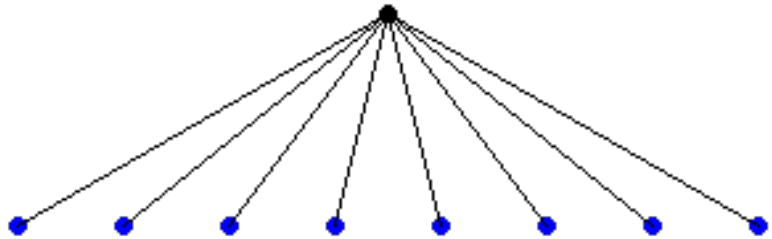
OctTree

- Tree of Degree 8
- Divides a 3-dimensional space into 8 sub-cubes
- Performance: $O(\log_8 n)$
- Leaf nodes can store pointers to 3-D geometry objects, 3-D image elements
- Reading
  - http://www.cs.berkeley.edu/~demmel/cs267/lecture26/lecture26.html

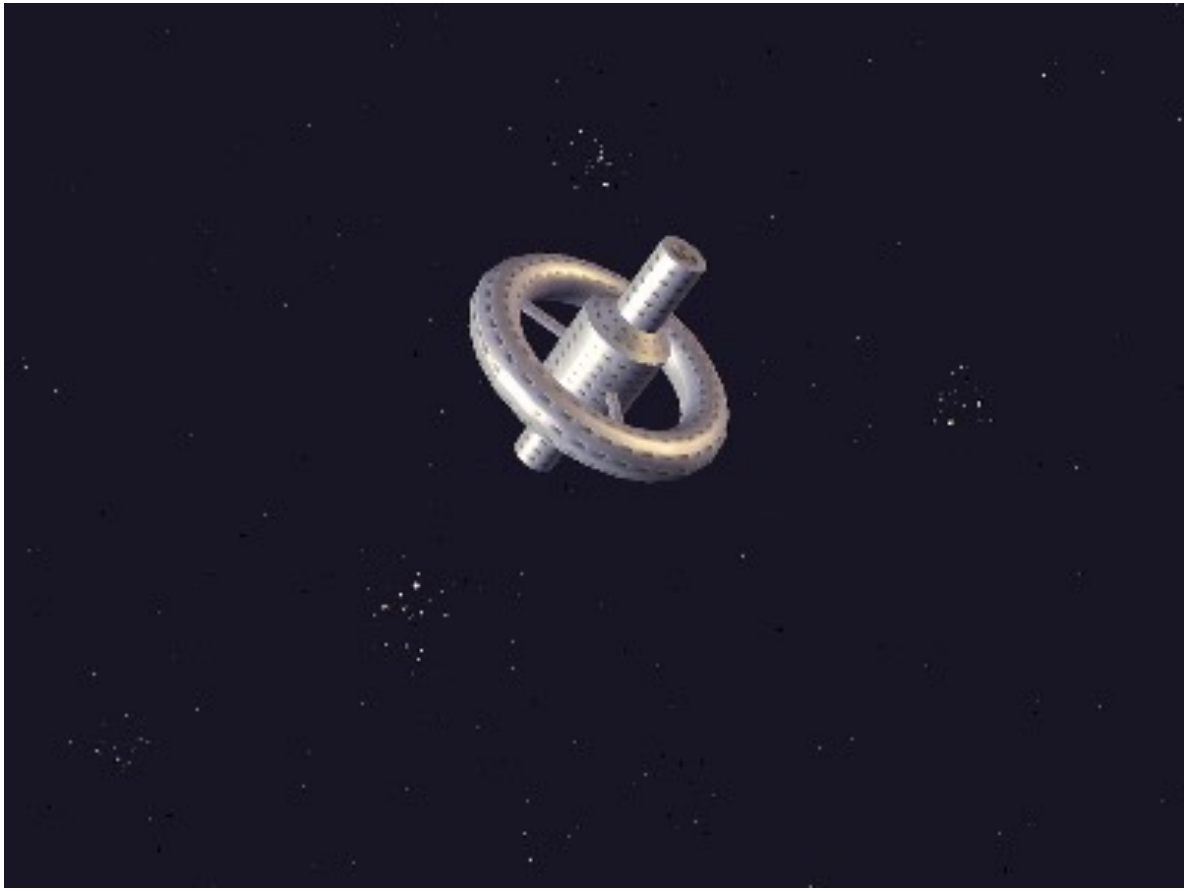
OctTree

2 Levels of an Octree





Storing and Accessing 3-D Image Elements



K-Dimensional Trees

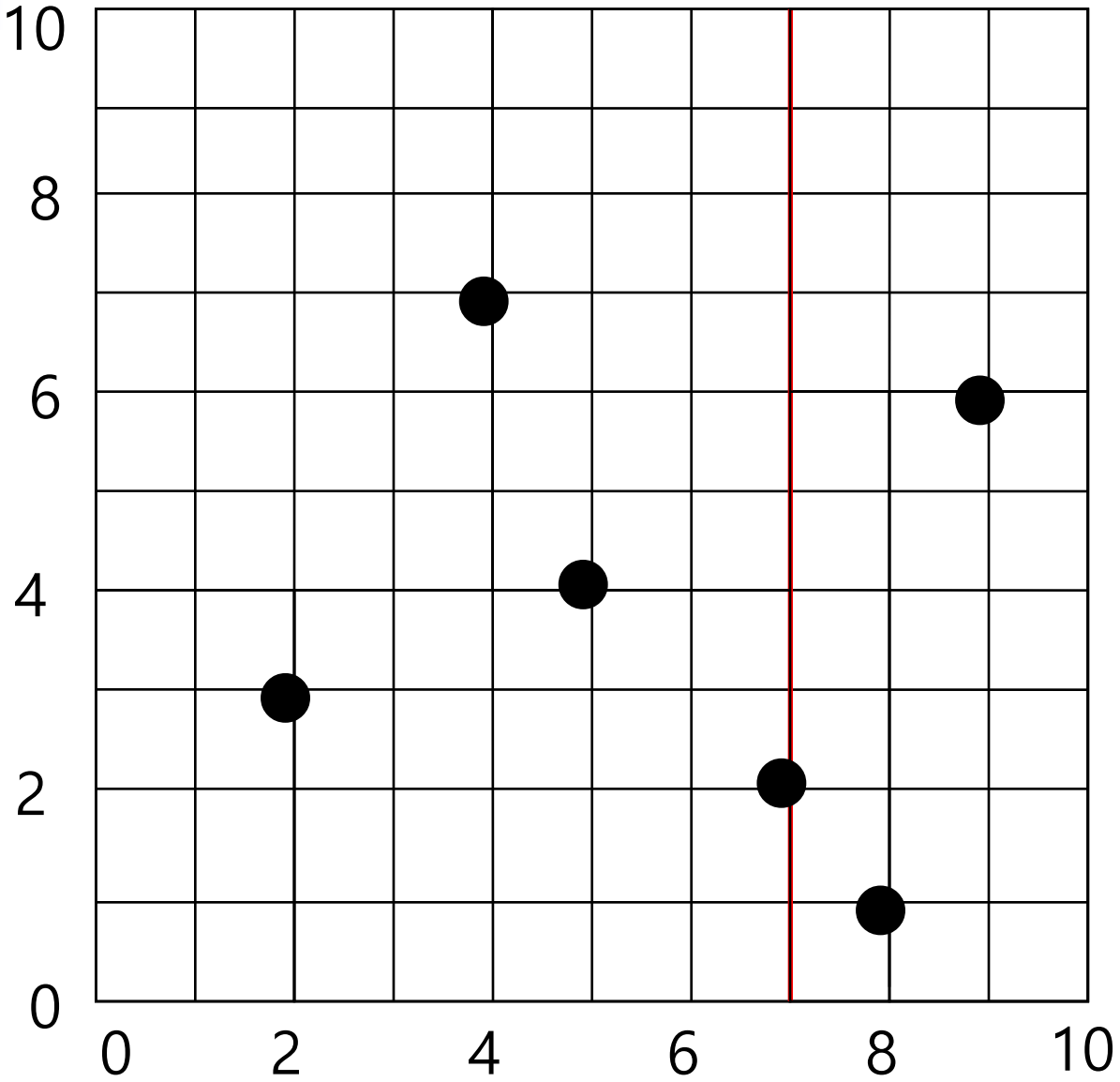
k-Dimensional Space

- 2-d
- 3-d
- 4-d
  - 3-d plus time
  - Person's (gender, age, education, ethnic origin)
- 5-d, 6-d, 7-d,…
  - Person's (gender, age, education, ethnic origin, religion, marital status, political party,…)

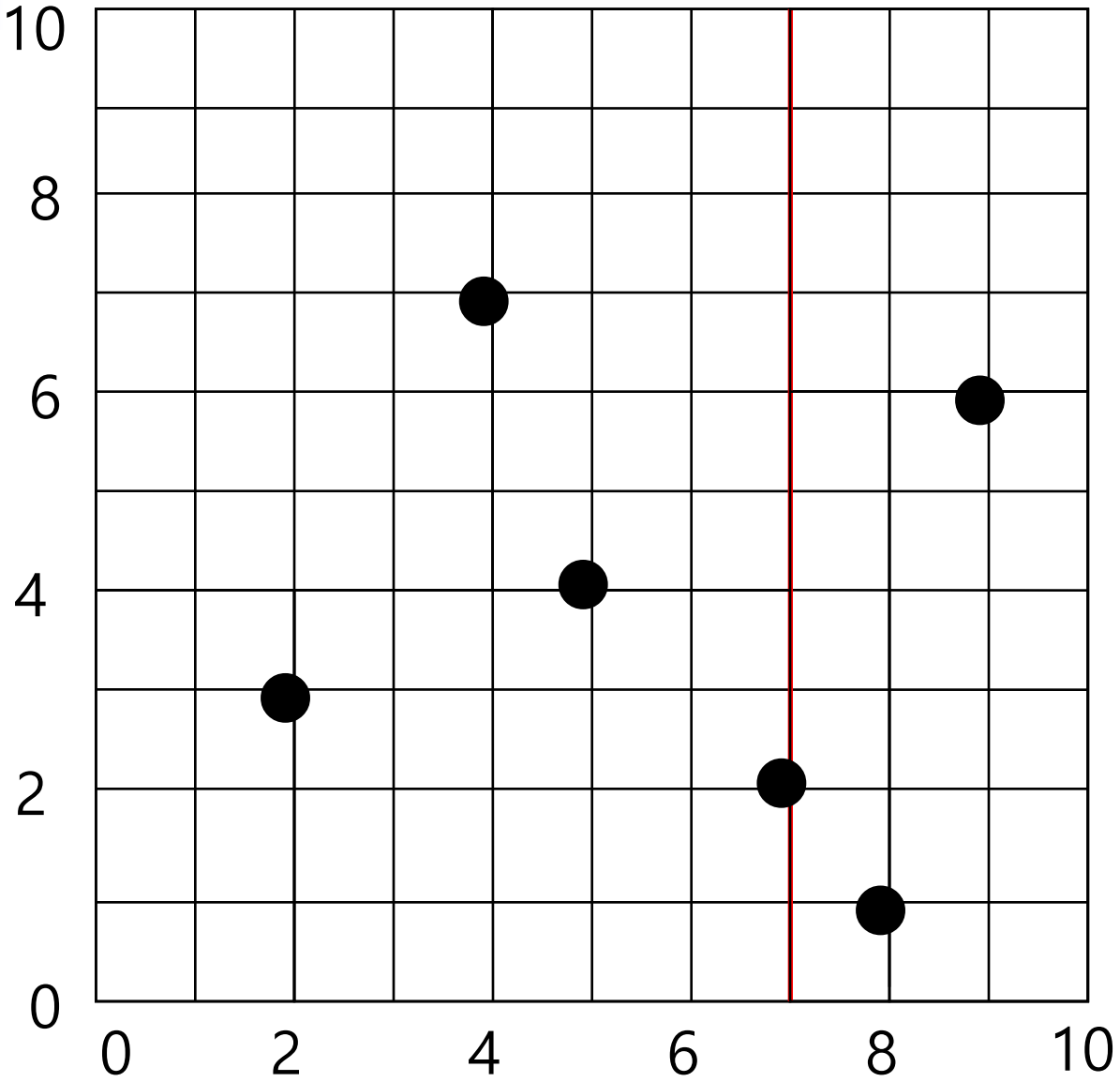
k-d Tree

- Store and search points in a k-dimensional space
  - k >= 2
- Point data (d1, d2, d3, d4,….)
  - ex. (5, 4), (70, 55, 37)
- Reading
  - http://www.answers.com/topic/kd-tree
  - http://www2.toki.or.id/book/AlgDesignManual/BOOK/BOOK3/NODE134.HTM
  - http://cis.poly.edu/~hakcan01/projects/kdtree/kdTree.html

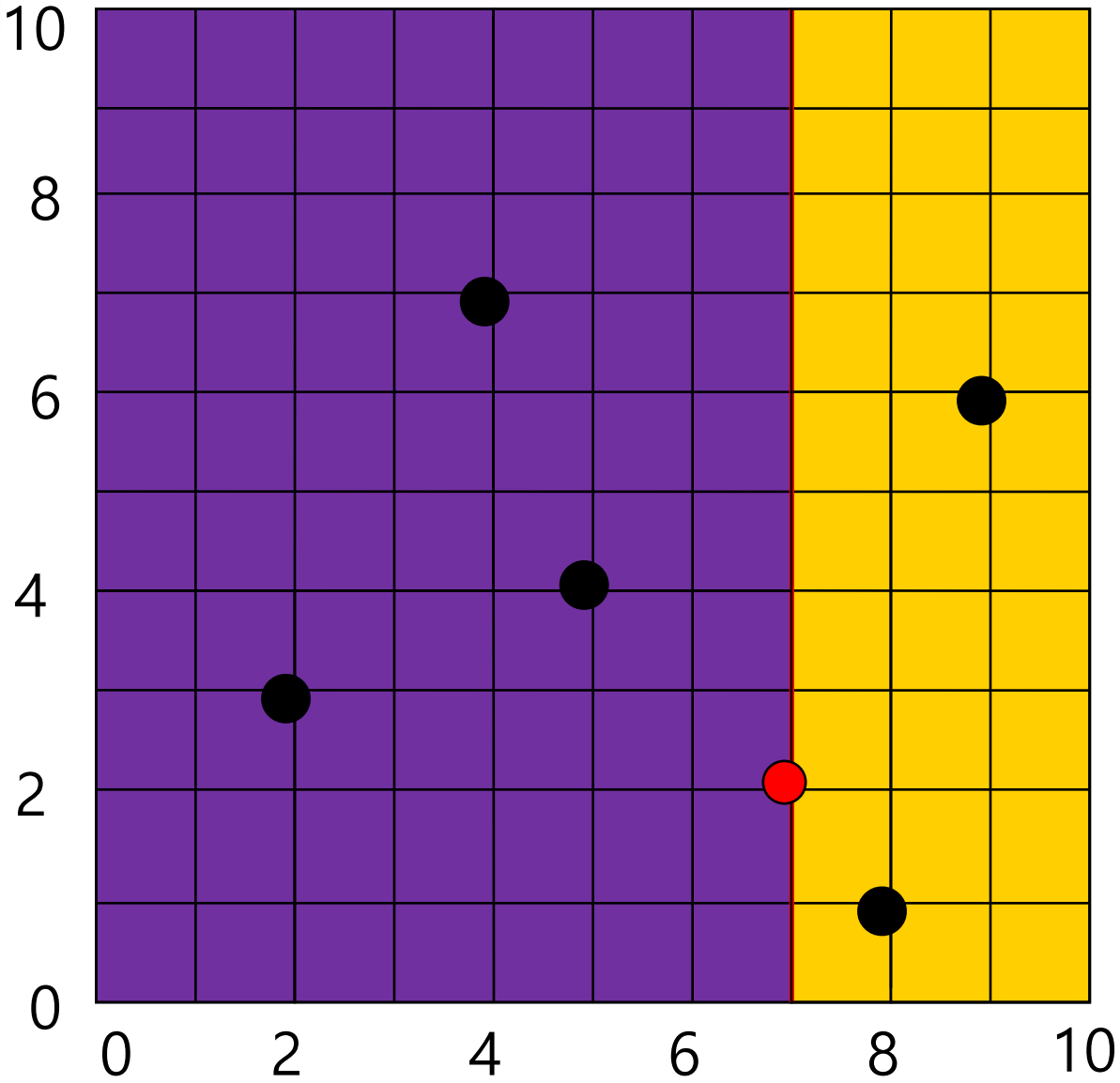
Successively Partition (Divide) a k-D Space Until No Partition Contains a Point



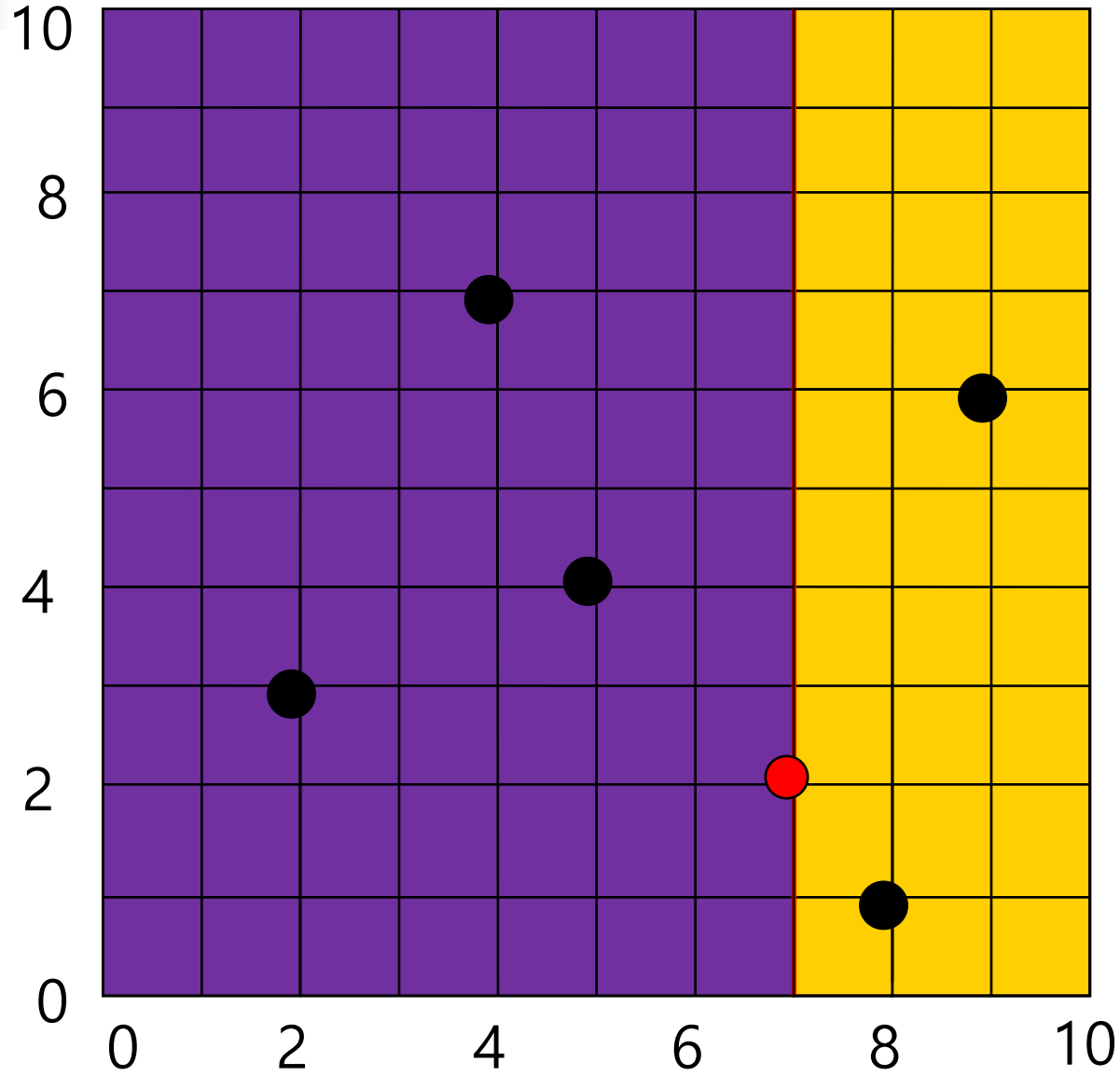
First, Select Point (7,2) and Partition the k-d Space along the Y Axis



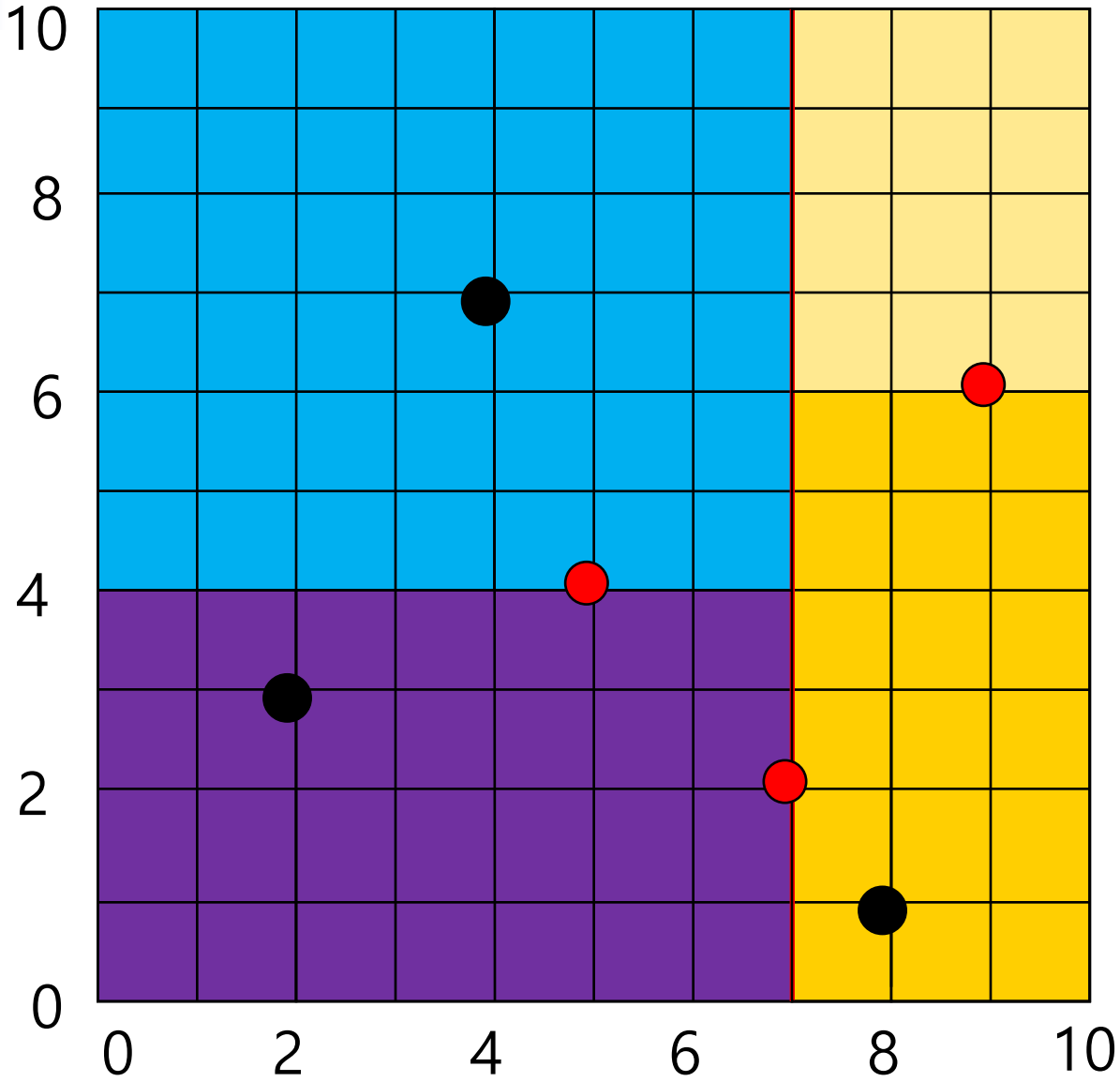
Result



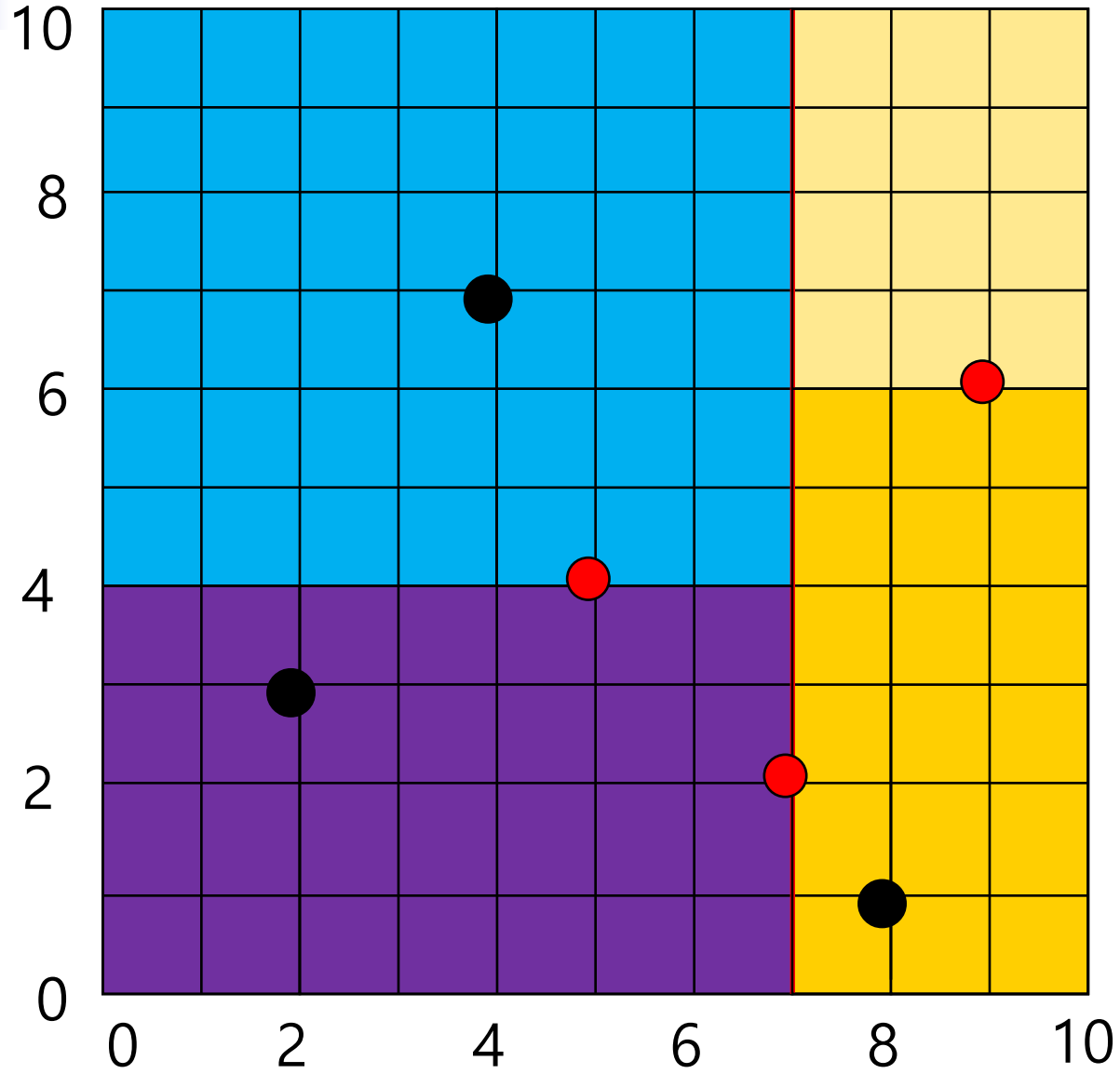
Second, Select Point (5,4) and (9,6) from Each k-d Space, and Partition Each k-d Space along the X Axis



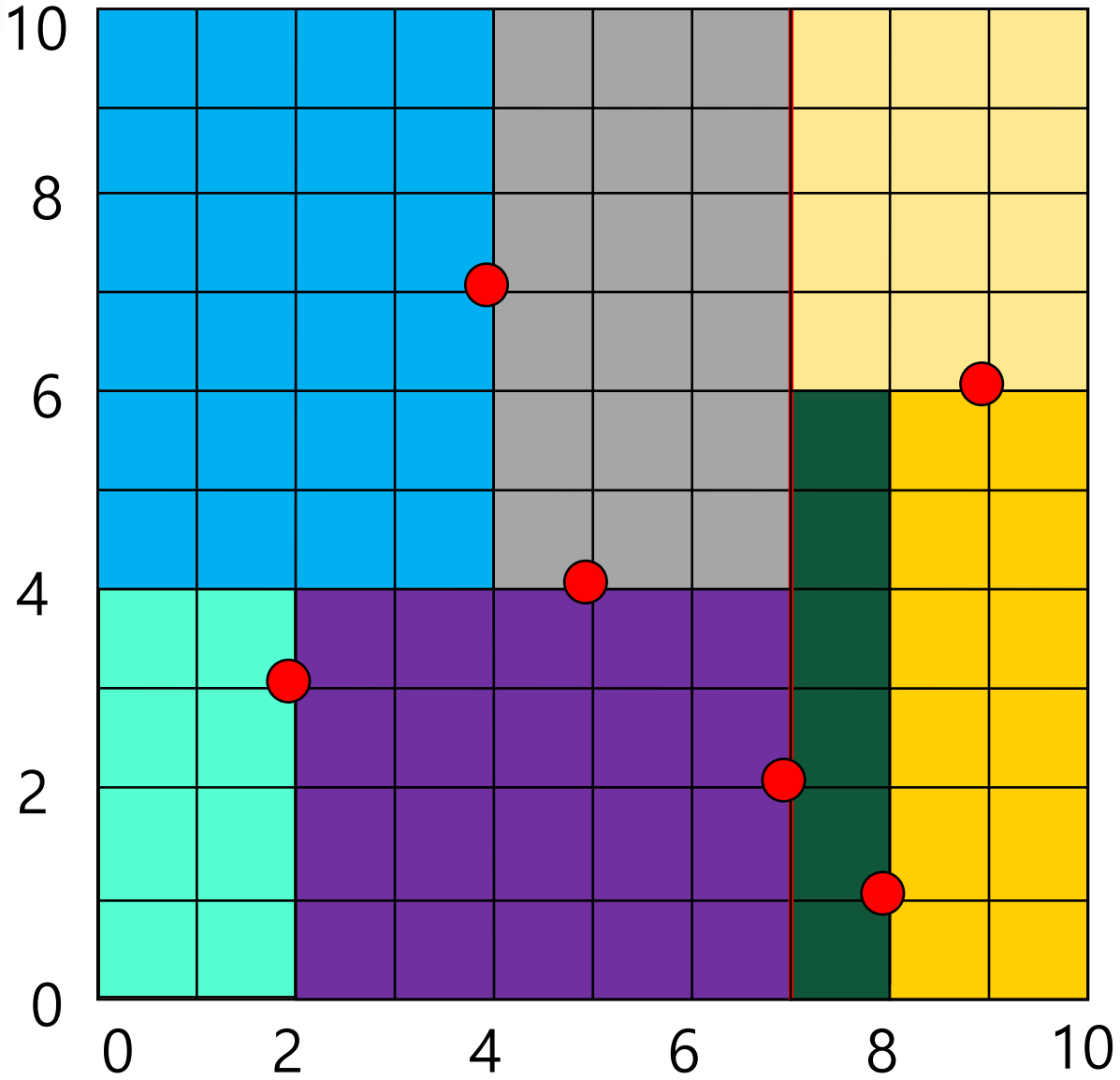
Result



Third, Select Point (2,3), (4,7), and (8,1) from Each k-d Space, and Partition Each k-d Space along the Y Axis

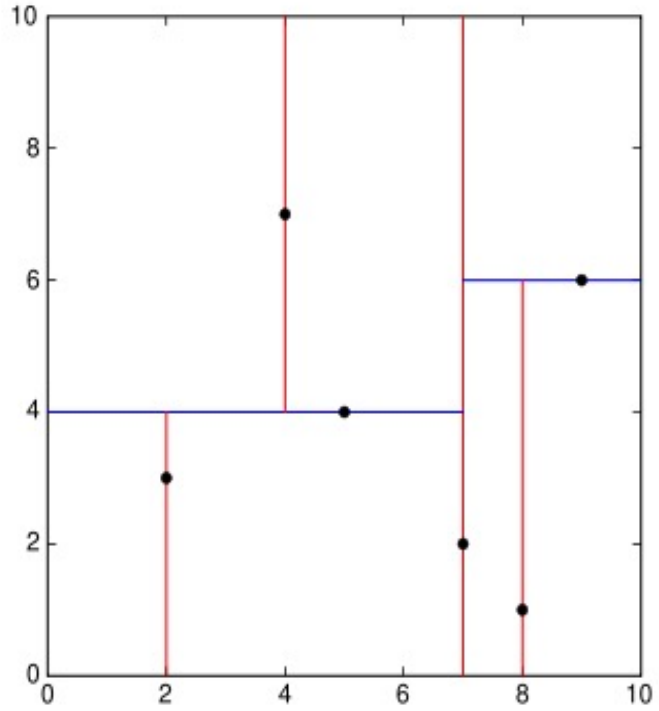


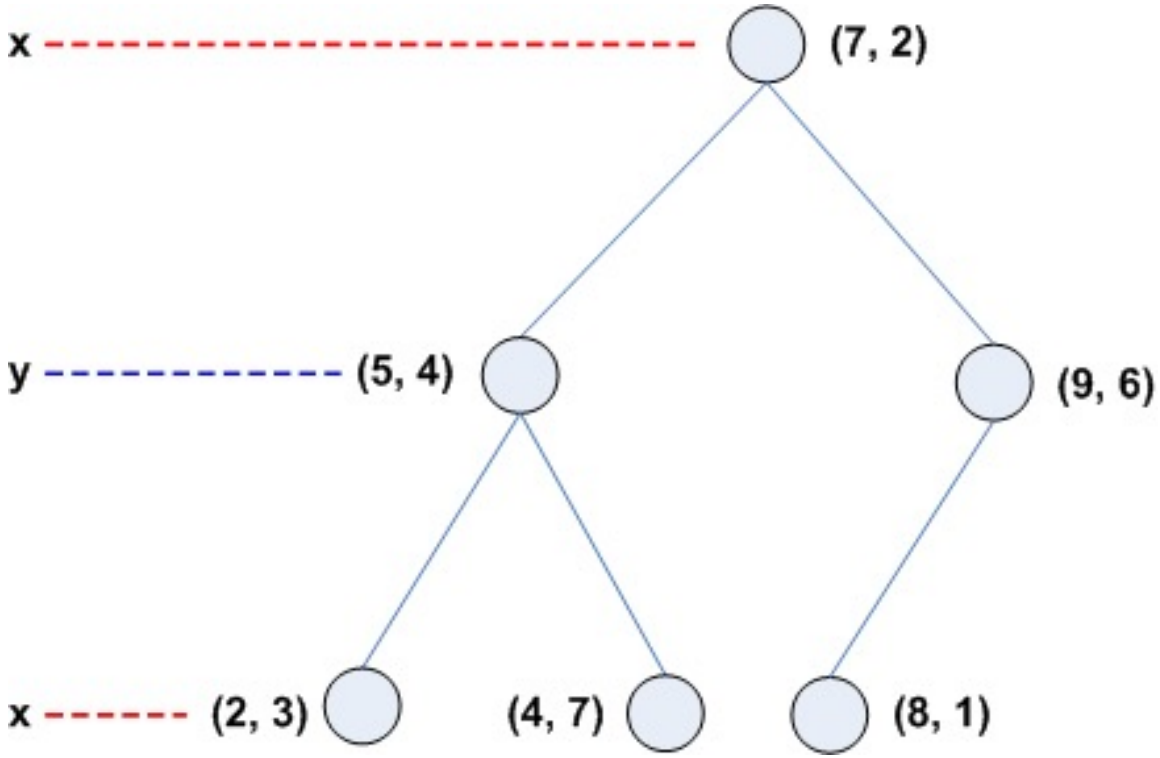
Result



Map Selected Points in a k-d Space to Nodes of a k-d Tree

How are these nodes selected? (We will see shortly…)





Constructing a k-d Tree

- In general, **static** construction with a set of given points in a k dimensional space.
- Each level of the tree represents a partitioning axis.
- The partitioning axis cycles through the k dimensions.
  - (e.g.) k=2 x,y,x,y,x,y,…
  - (e.g.) k=3 x,y,z,x,y,z,x,y,z,…
  - (e.g.) k=4 x,y,z,w,x,y,z,w,x,y,z,w,…
- Discriminator is the median key at each level of the k-d tree.
  - Median key is selected to distribute the data evenly on the tree

Computing the Median

- "Median"
  - equal number of values < and >
  - (e.g.) 4 7 9 10 11 15 30
- If the numbers are different,
  - Select the Largest of the Smaller Group or the Smallest of the Larger Group.
  - (e.g.) 4 7 10 11 15 30
    - 4 7 10 11 15 30
    - 4 7 10 11 15 30
- If there are multiple identical values in a median candidate?
  - Results in a skewed tree
  - (e.g.) 4 7 10 10 10 10 30
    - 4 7 10 10 10 10 30
- either "<= median" go to the left subtree
  - "> median" go to the right subtree
- or "< median" go to the left subtree
  - ">= median" go to the right subtree

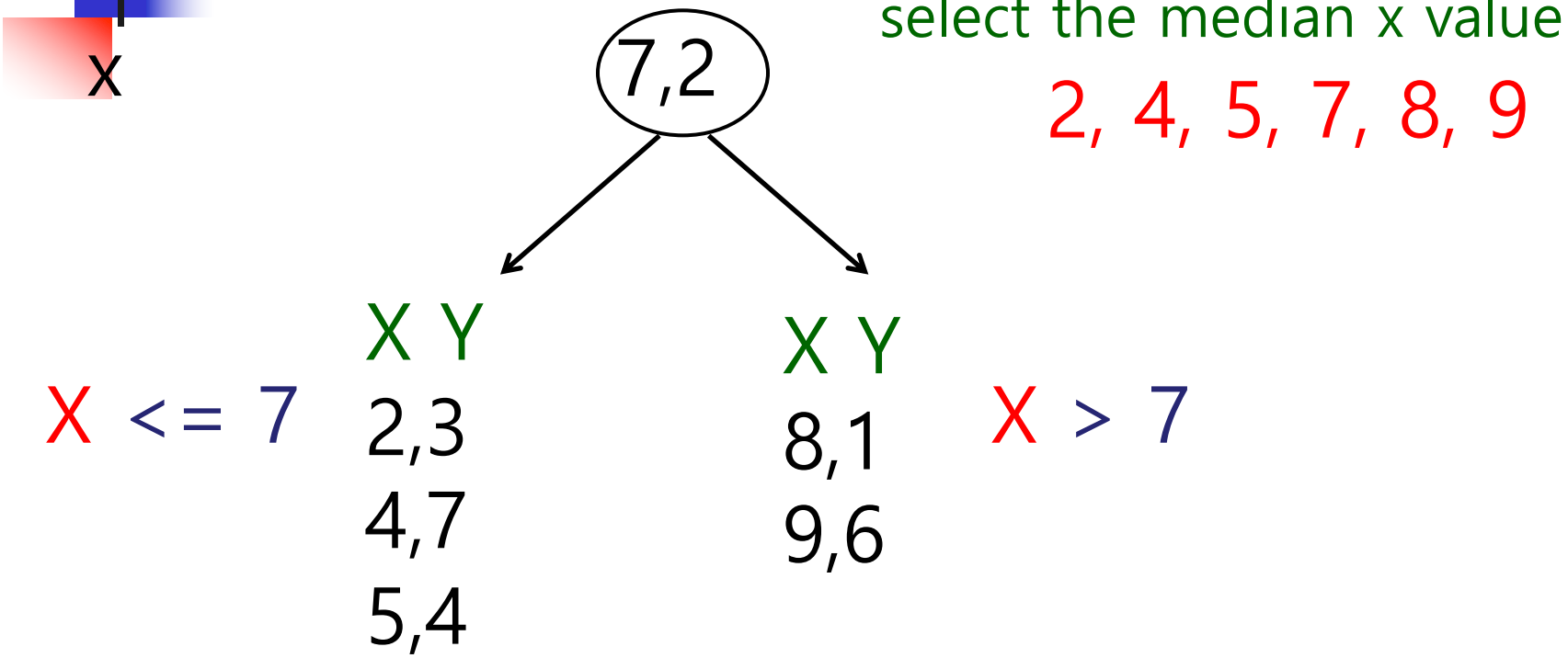
Example (1/5)

set of points in 2-dimension

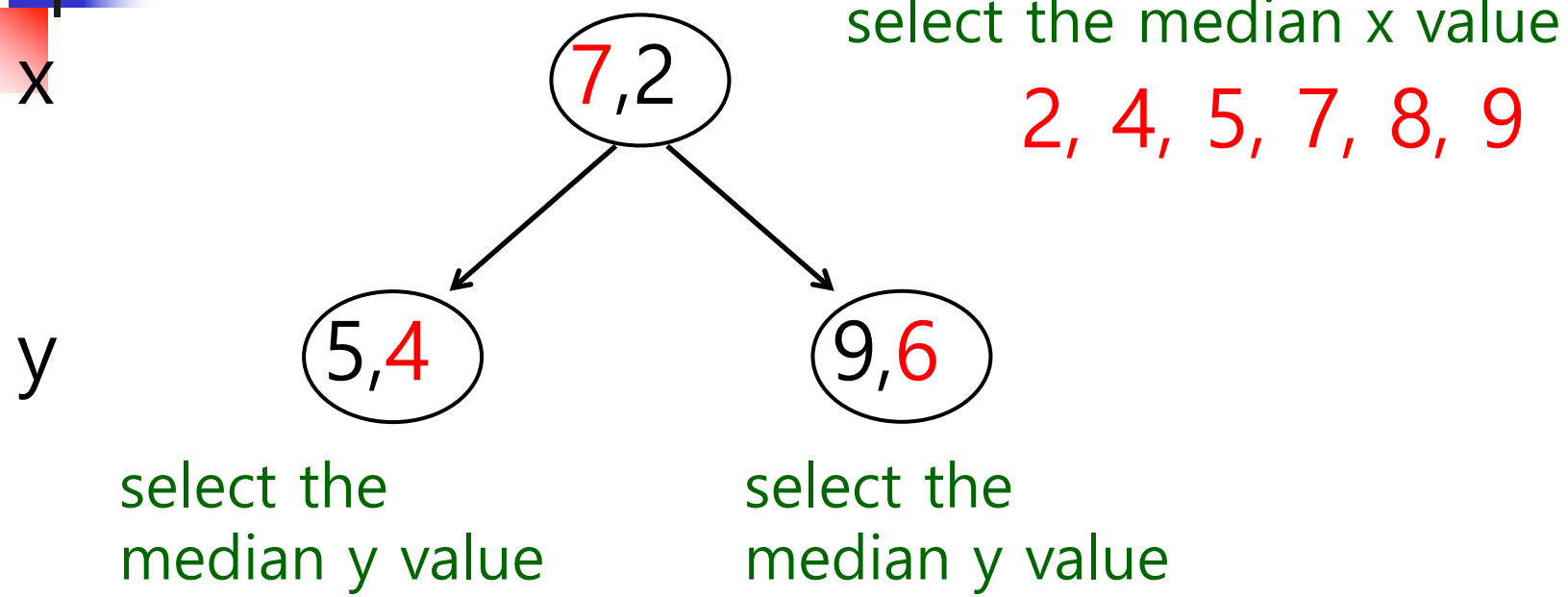
X	Y
2	3
4	7
5	4
7	2
8	1
9	6

select the median X value

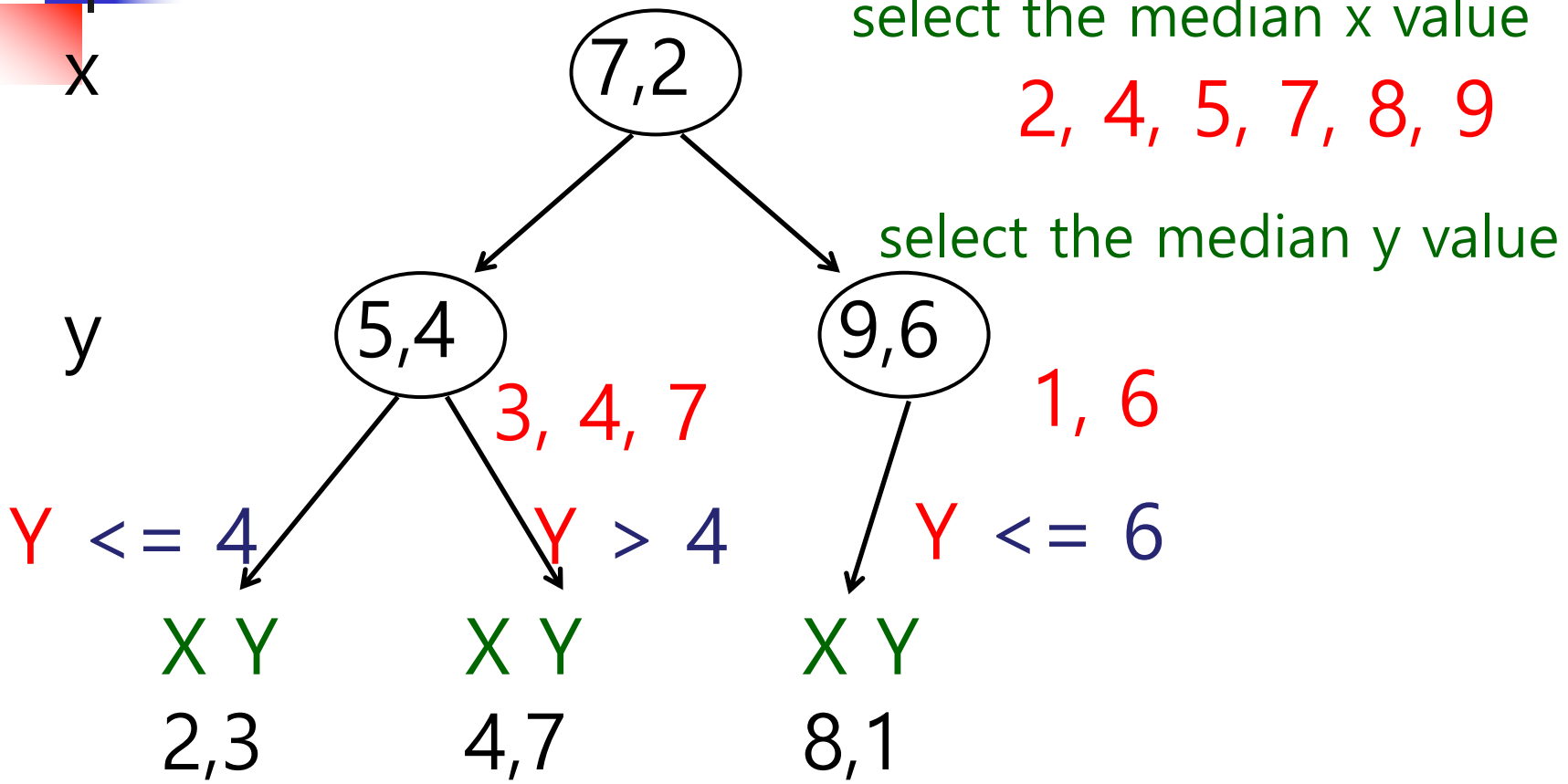
Example (2/5)


x
7,2
select the median x value
2, 4, 5, 7, 8, 9
X Y
2,3
4,7
5,4
X Y
8,1
9,6
X <= 7
X > 7
select the median Y value
select the median Y value

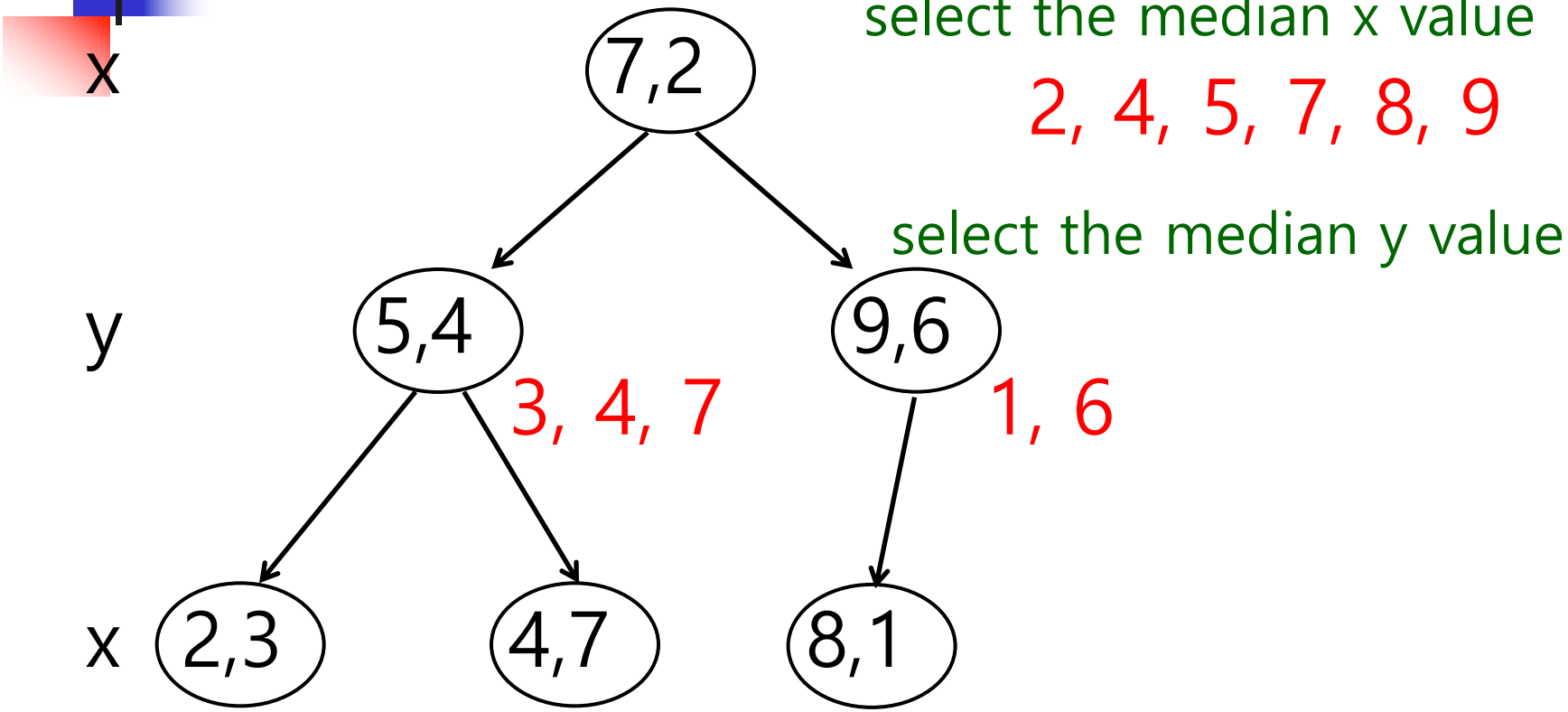
Example (3/5)


x
y
7,2
5,4
9,6
select the median x value
2, 4, 5, 7, 8, 9
select the median y value
select the median y value

Example (4/5)


x
y
7,2
5,4
9,6
select the median x value
2, 4, 5, 7, 8, 9
select the median y value
3, 4, 7
1, 6
Y <= 4
Y > 4
Y <= 6
X Y
2,3
X Y
4,7
X Y
8,1

Example (5/5)


x
7,2
select the median x value
2, 4, 5, 7, 8, 9
select the median y value
y
5,4
9,6
3, 4, 7
1, 6
x
2,3
4,7
8,1
select the median x value

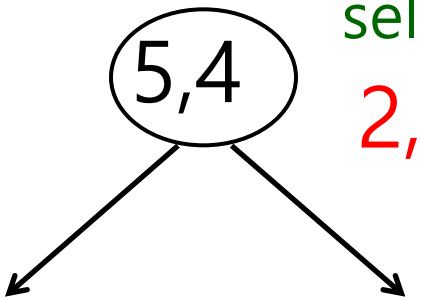
Example (1/6)

set of points in 2-dimension

X	Y
2	3
5	1
5	2
5	4
5	6
5	7
9	8
10	6

Example 2 (2/6)

x



select the median x value

2,5,5,5,5,5,9,10

X Y

2,3

5,1

5,2

5,4

5,6

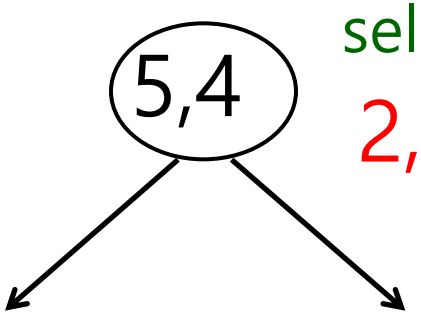
5,7

9,8

10,6

Example 2 (3/6)

x



select the median x value

2,5,5,5,5,5,9,10

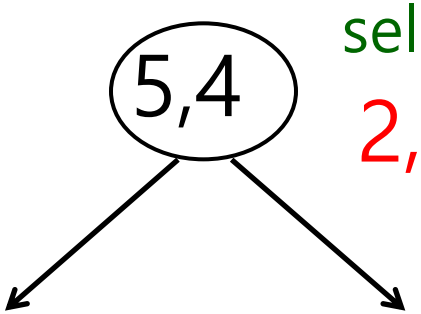
X Y
2,3
5,1
5,2
5,6
5,7

X Y
9,8
10,6

X Y
2,3
5,1
5,2
5,4
5,6
5,7
9,8
10,6

Example 2 (4/6)

X



select the median x value

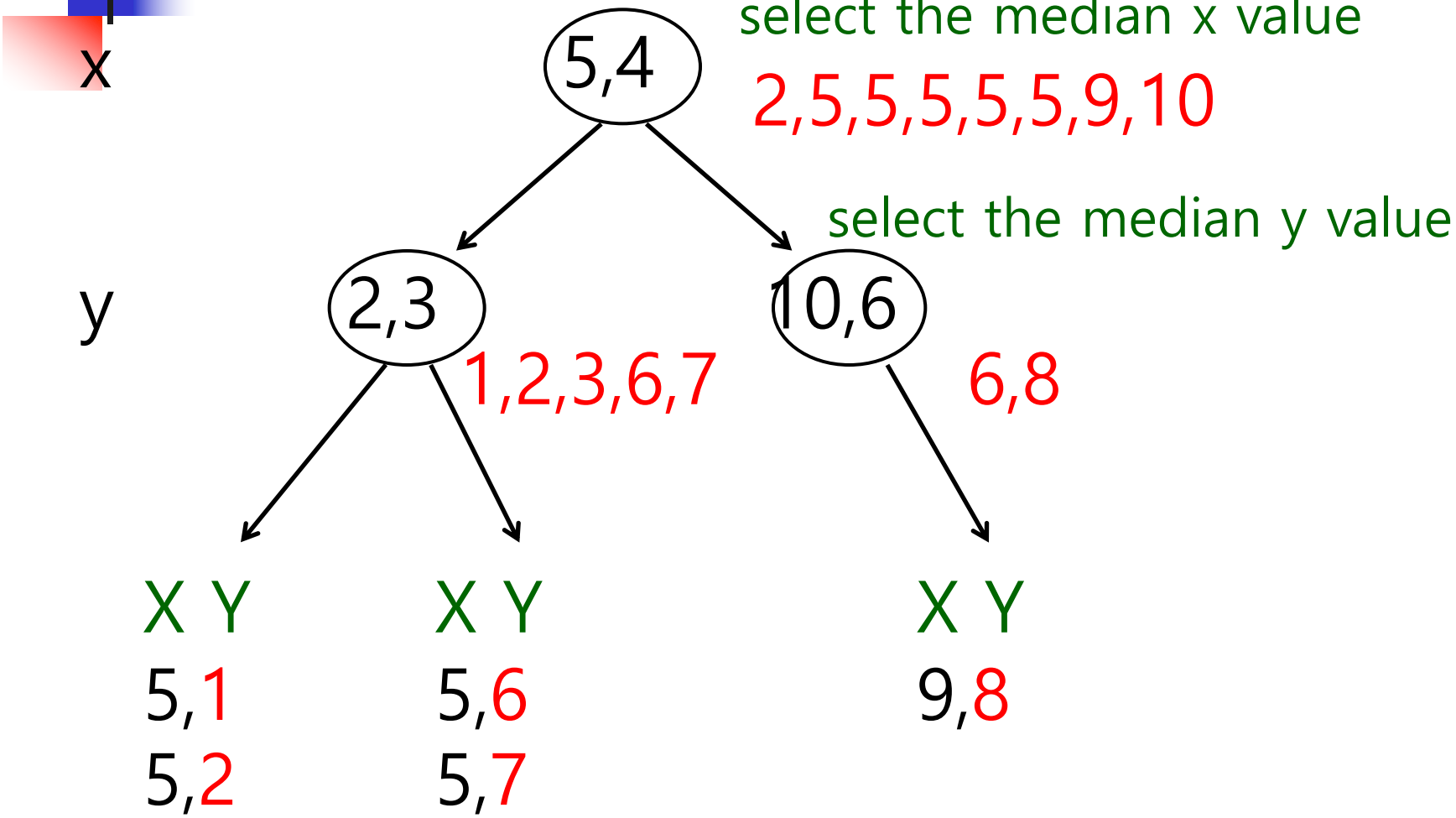
2,5,5,5,5,5,9,10

X Y
2,3
5,1
5,2
5,4
5,6
5,7
9,8
10,6

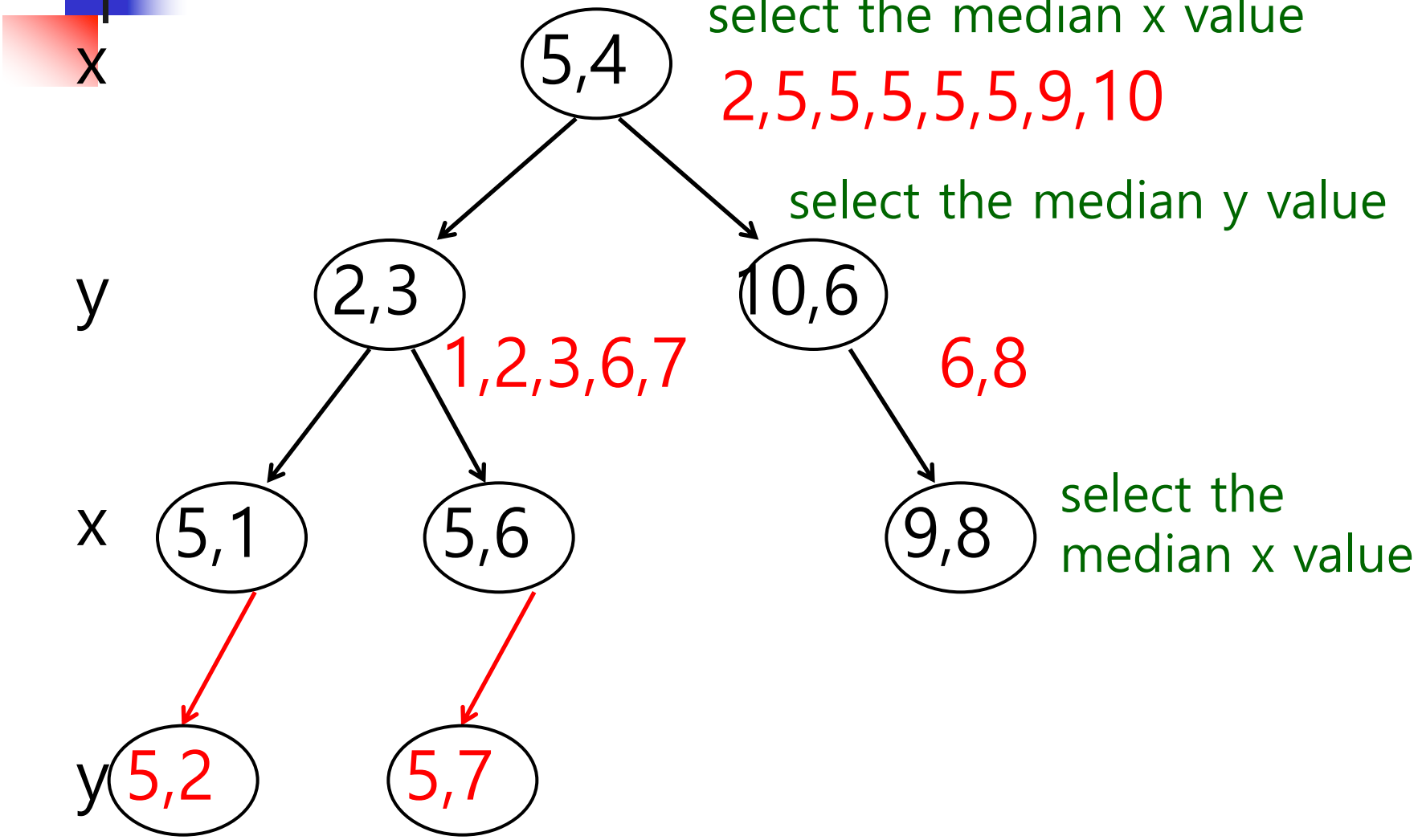
X Y
5,1
5,2
2,3
5,6
5,7

X Y
10,6
9,8

Example 2 (5/6)


x
y
5,4
select the median x value
2,5,5,5,5,5,9,10
select the median y value
2,3
10,6
1,2,3,6,7
6,8
X Y
5,1
5,2
X Y
5,6
5,7
X Y
9,8

Example 2 (6/6)


x
5,4
select the median x value
2,5,5,5,5,5,9,10
select the median y value
y
2,3
10,6
1,2,3,6,7
6,8
x
5,1
5,6
9,8
select the median x value
y
5,2
5,7

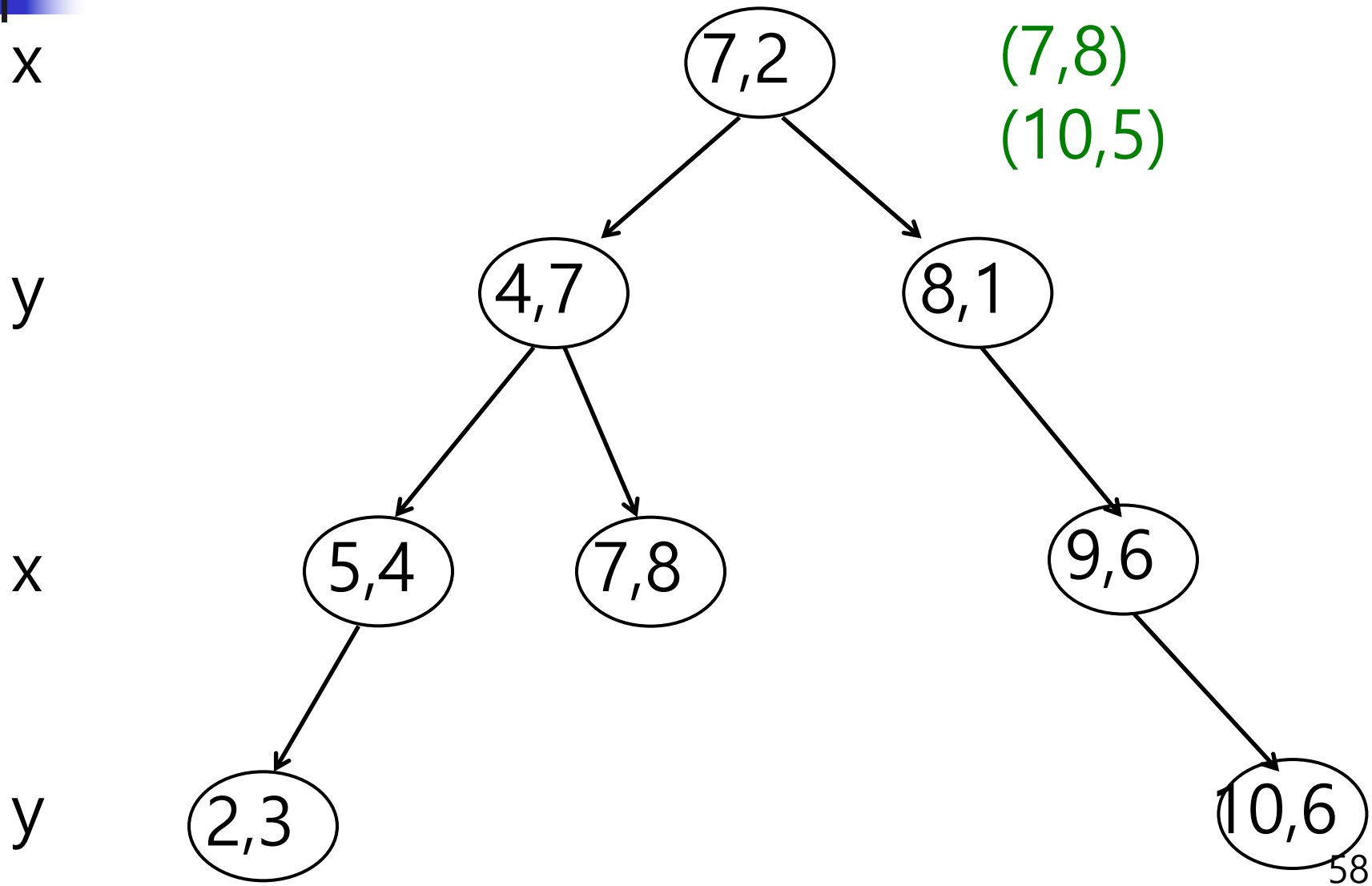
Exercise

set of points in 3-dimension

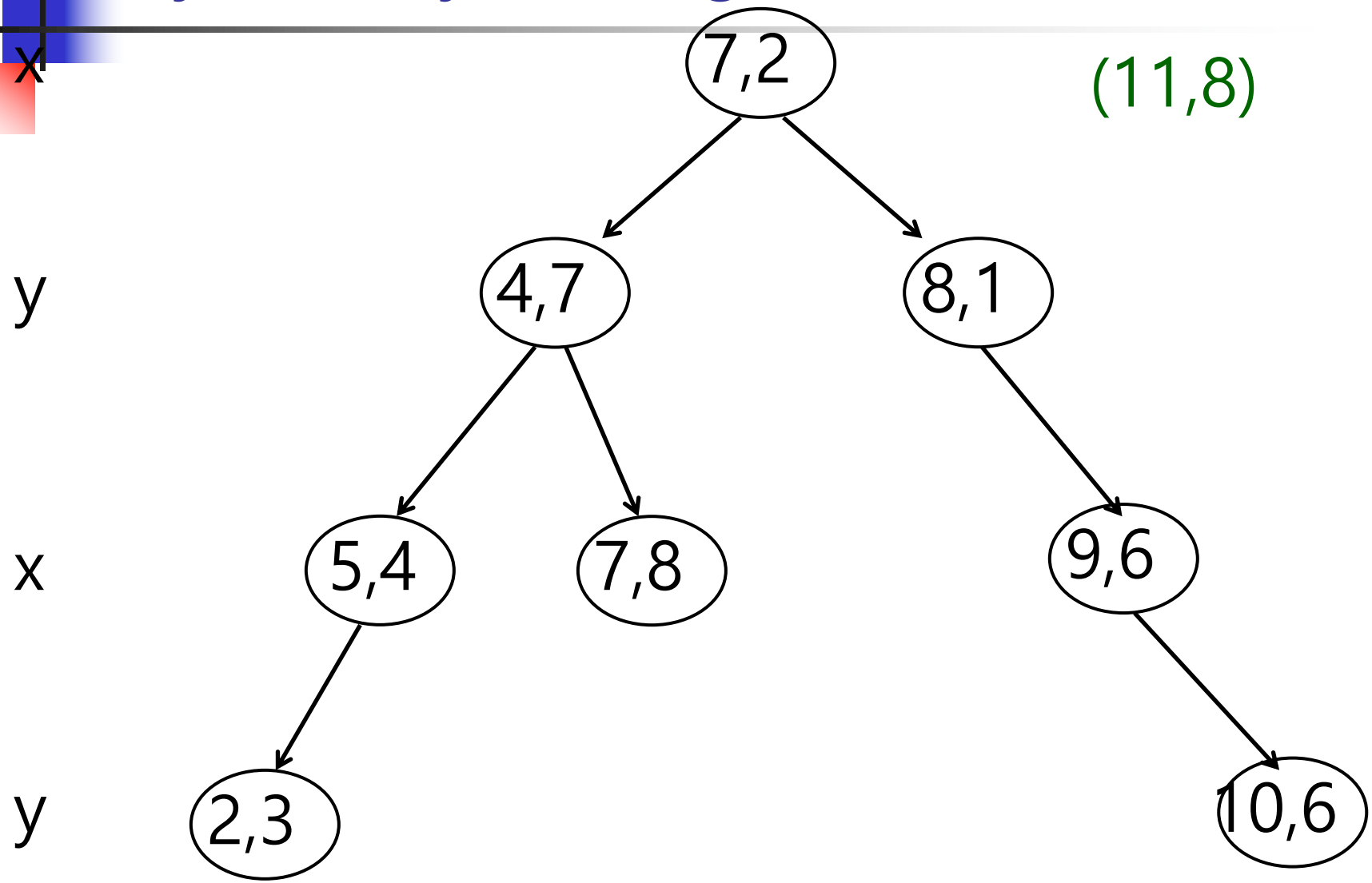
X Y Z

1,4,5
2,3,11
3,7,4
4,10,6
5,12,9
6,2,8
7,8,4
9,5,3
10,6,5
11,11,11
12,1,1

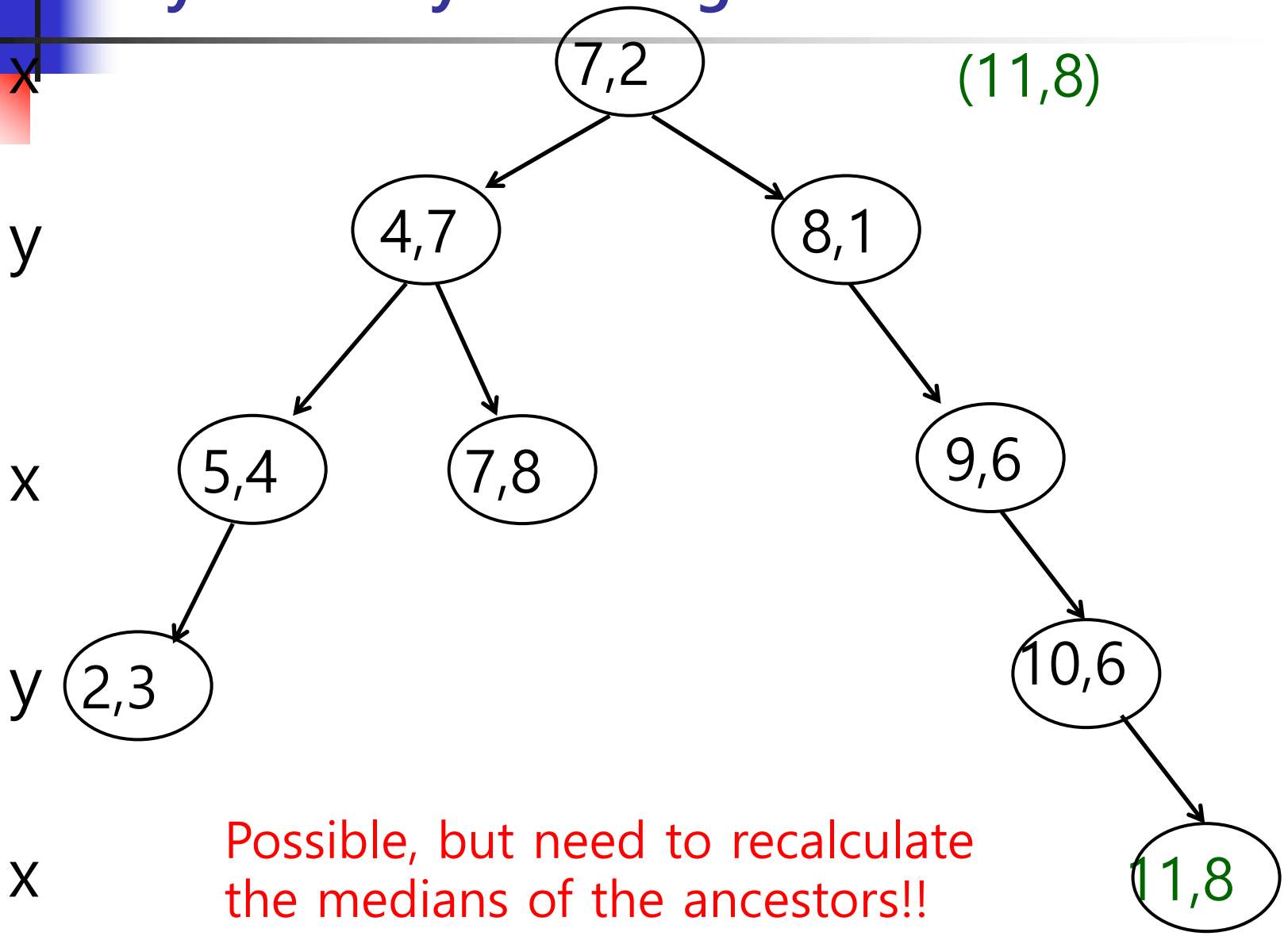
Searching a k-d Tree: (similar to constructing a k-d tree)



Dynamically Inserting a Node to a k-d Tree

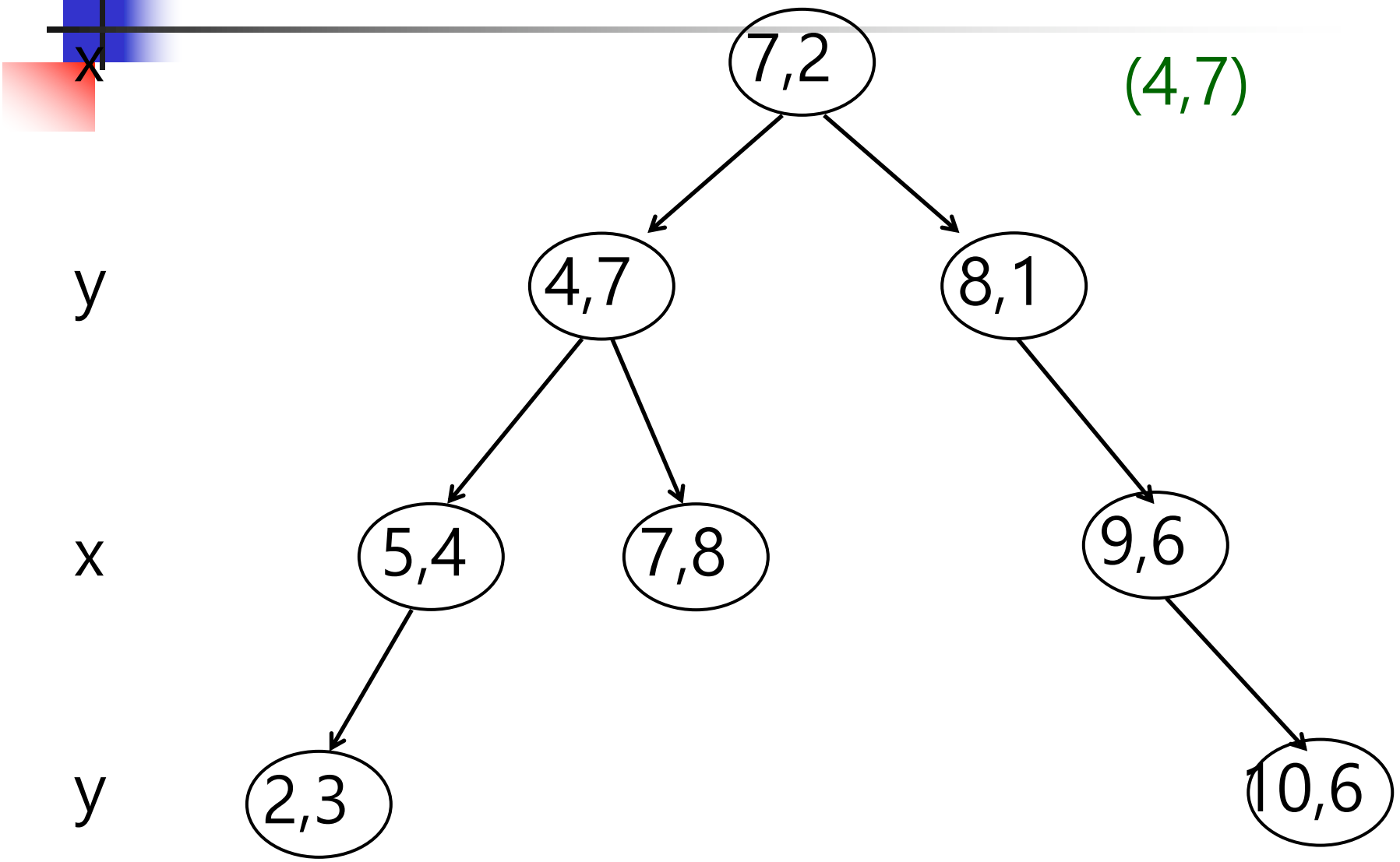


Dynamically Inserting a Node to a k-d Tree

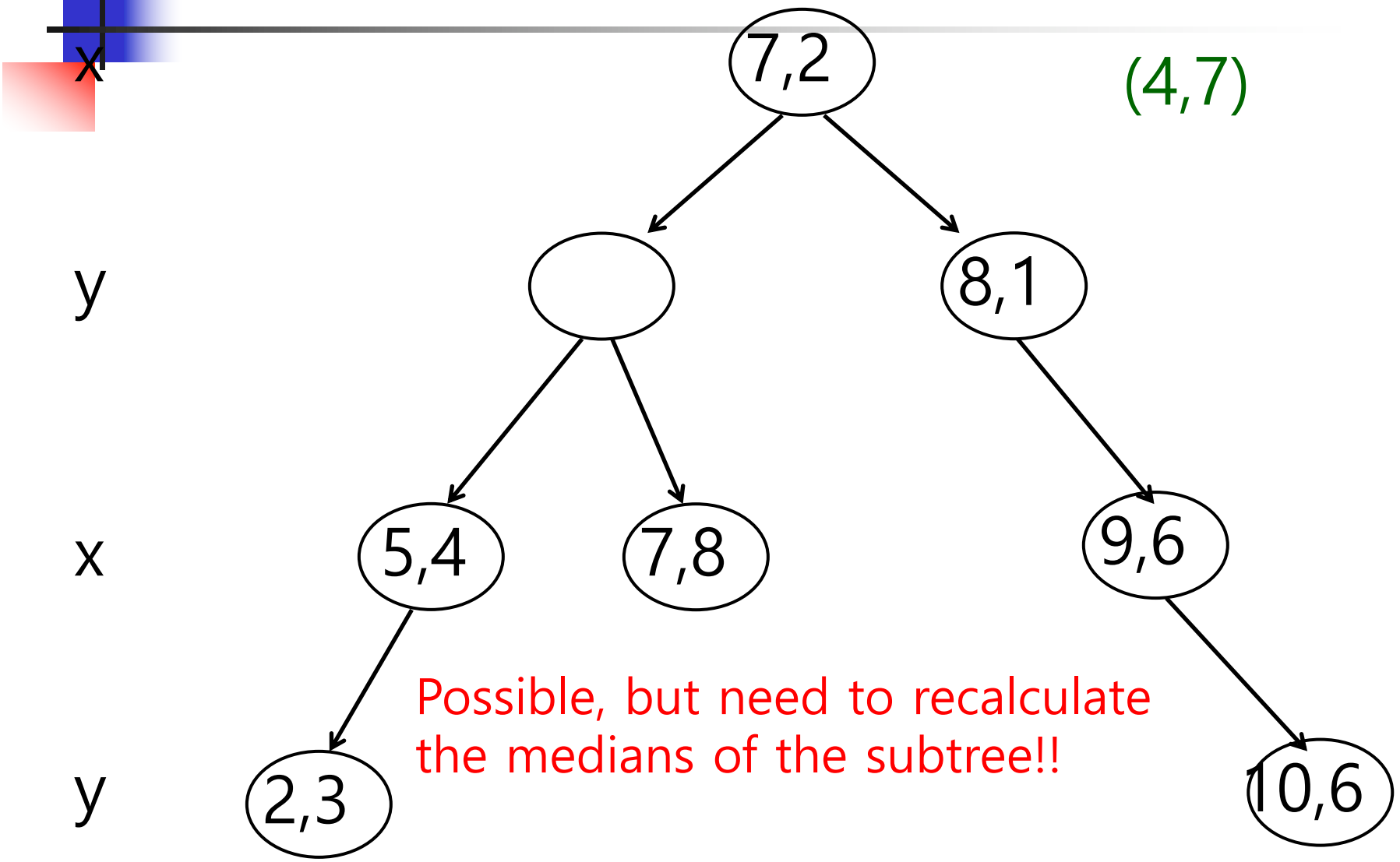


Possible, but need to recalculate the medians of the ancestors!!

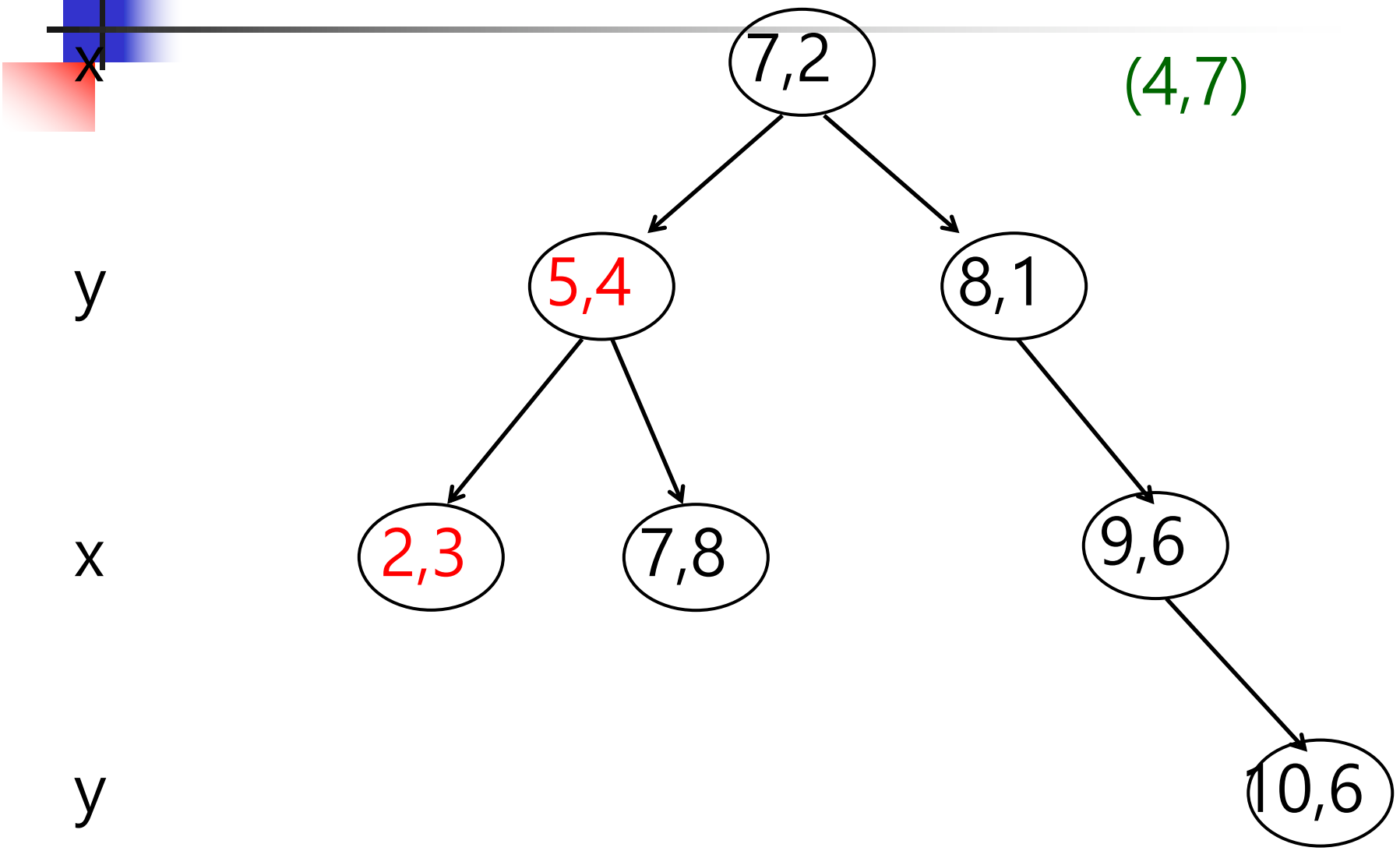
Dynamically Deleting a Node from a k-d Tree



Dynamically Deleting a Node from a k-d Tree



Dynamically Deleting a Node from a k-d Tree



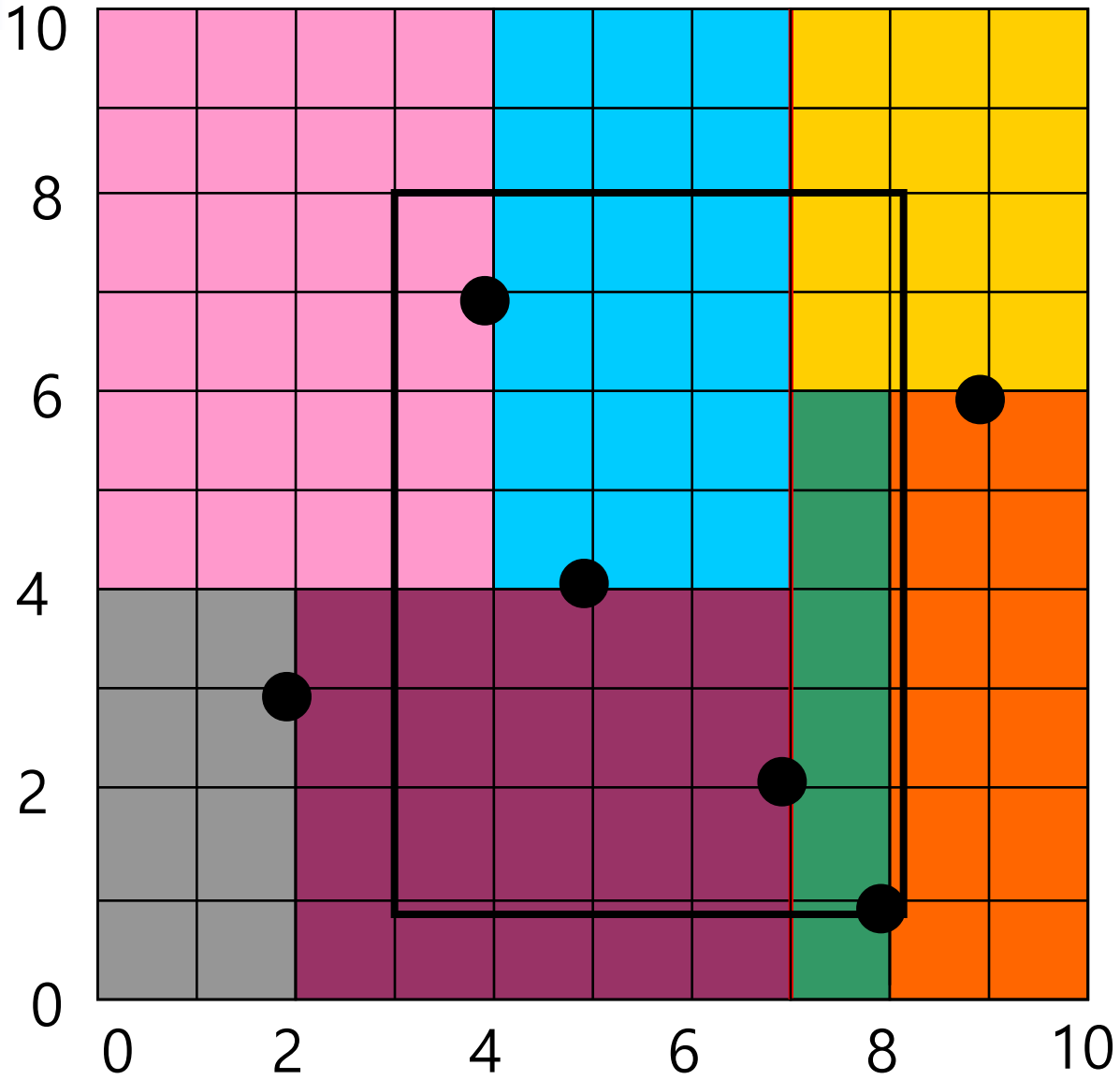
k-d Tree: Properties

- It is a binary search tree for k-d keys.
- It is not height-balanced
- It can be badly skewed if there are many identical median values at any level.
- It is a static tree for searches; (i.e.,) it is constructed once for repeated searches.
- Dynamic insertion and deletion of keys can require partial reorganizations of the tree.

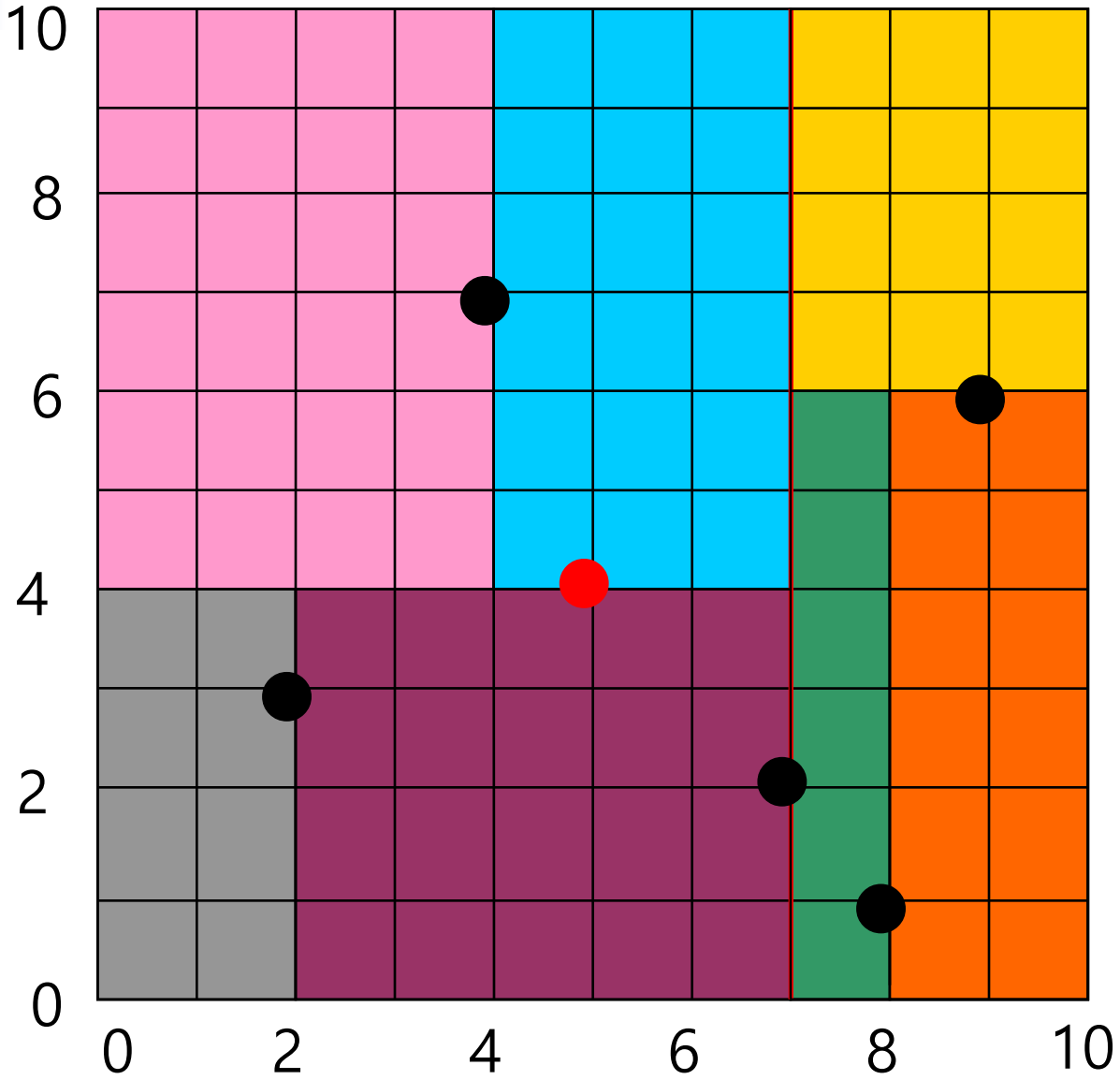
k-d Tree: Uses

- Point search
- Range (region) search
- Nearest neighbor search
- Partial key search

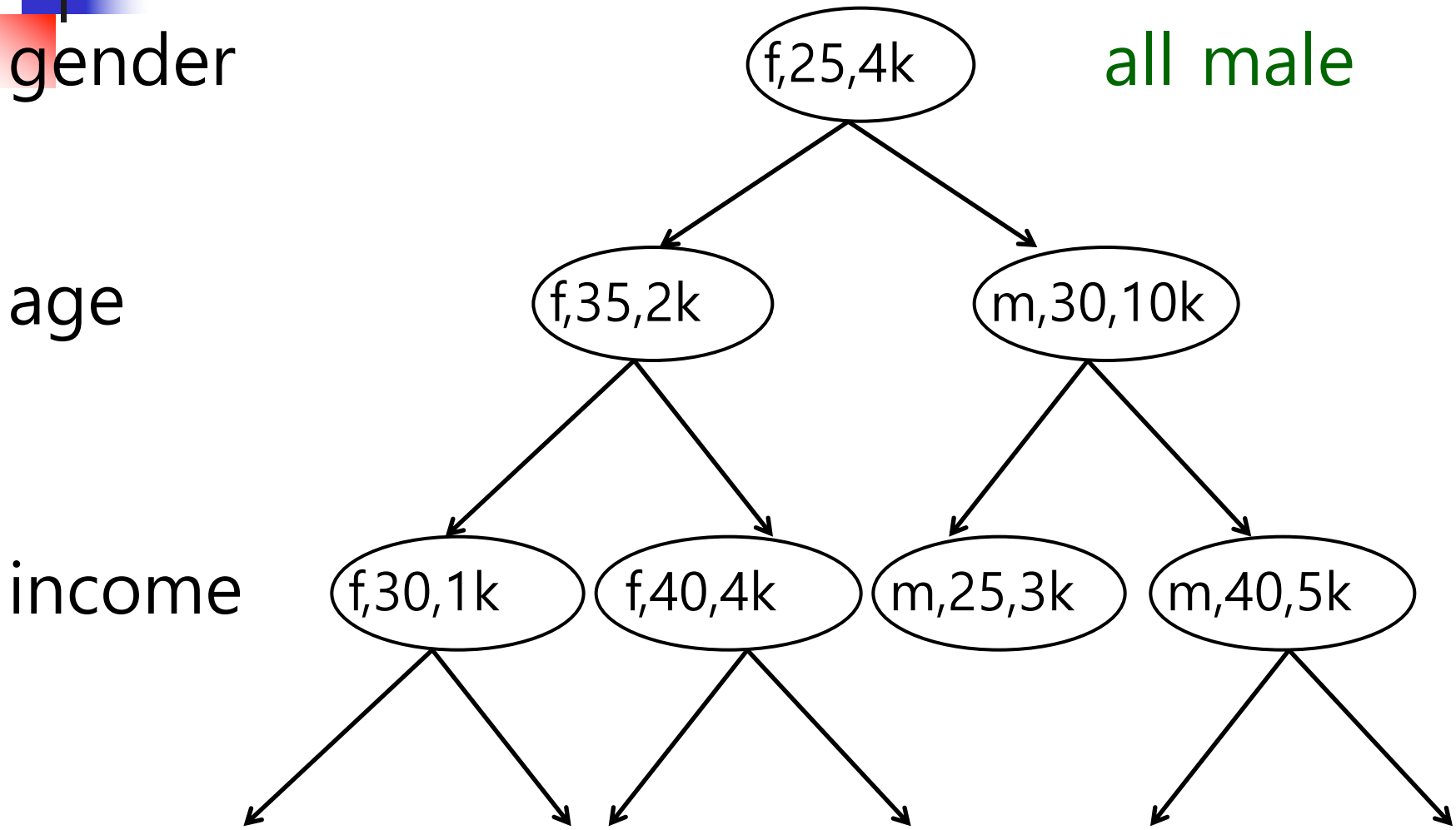
Example: Range Search



Example: Nearest Neighbor Search



Example: Partial Key Search



Tries

Binary Trie

- Retrieval
- A binary search tree
  - Not height-balanced
- Can be used as a replacement for hashing
- Reading:
  - pp. 561-564

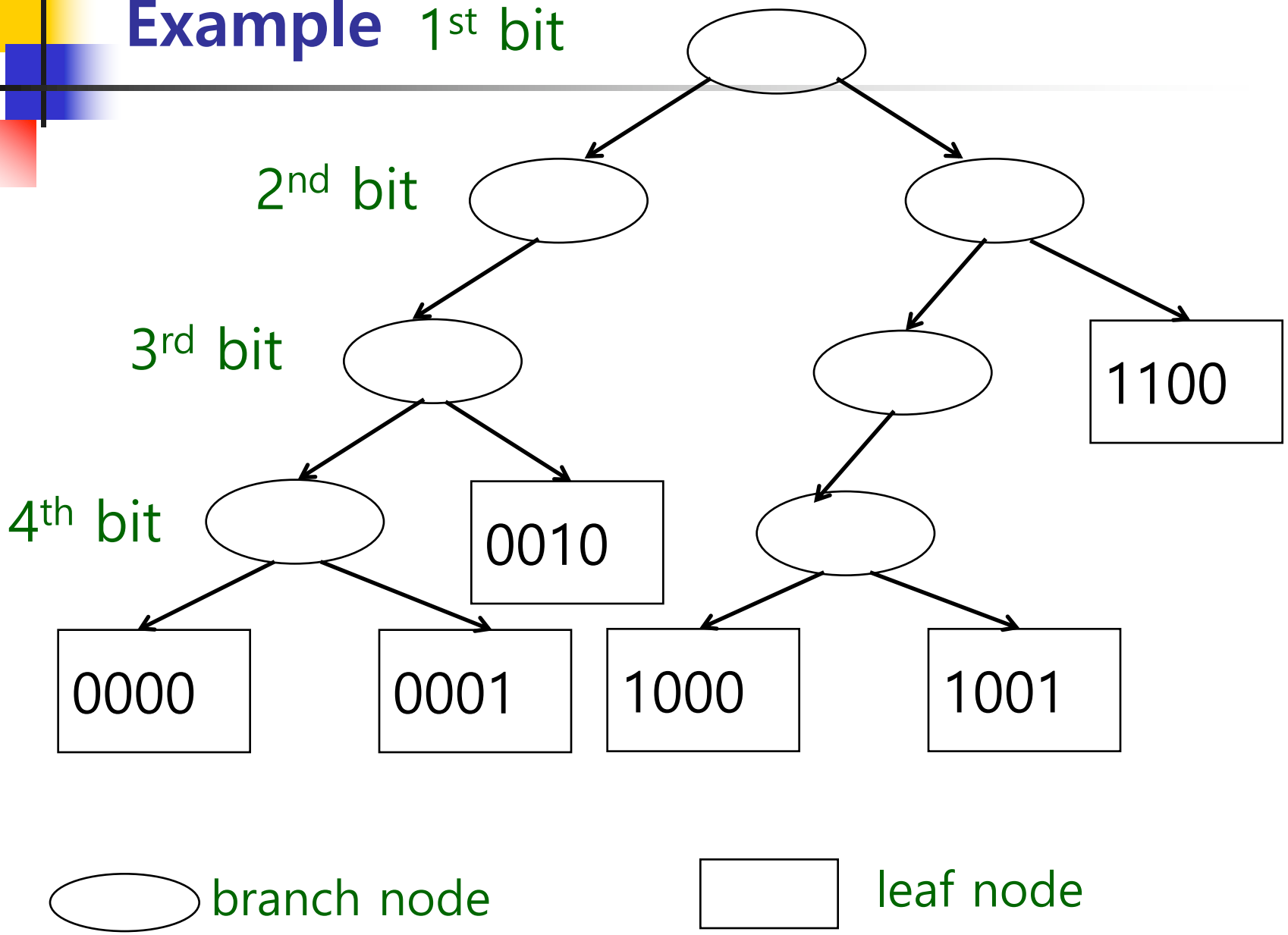
Binary Trie

- Two Types of Node
  - Branch (link, interior, non-leaf) node
    - left-child ptr, right-child ptr
    - no data
  - Element (data, leaf) node
    - contains the full key

Binary Trie: Search

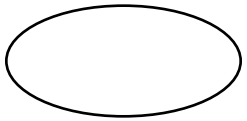
- For a Search Key k
  - At level i
    - if k's i^{th} bit is 0, move to the left subtree;
    - if it is 1, move to the right subtree.
  - At a leaf node, the search key is compared with the data stored in the node.
- End of Search
  - success: at a leaf node
  - failure: at a leaf node or a NULL pointer

Example


1st bit
2nd bit
3rd bit
4th bit
1100
0010
0000
0001
1000
1001
branch node
leaf node

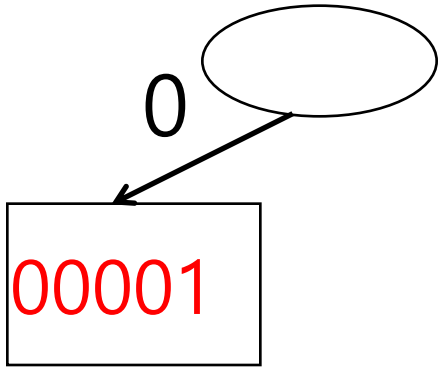
Building a Binary Trie

Insert 00001



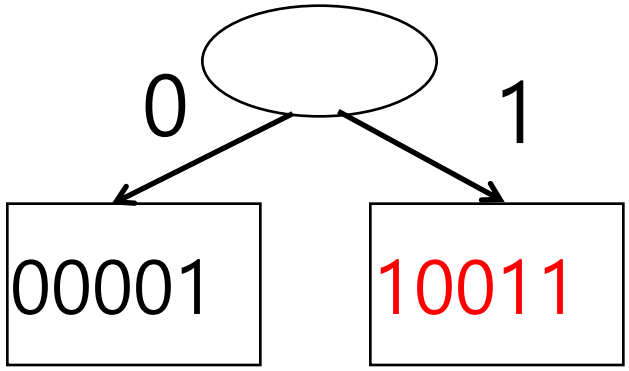
Building a Binary Trie

Insert 10011

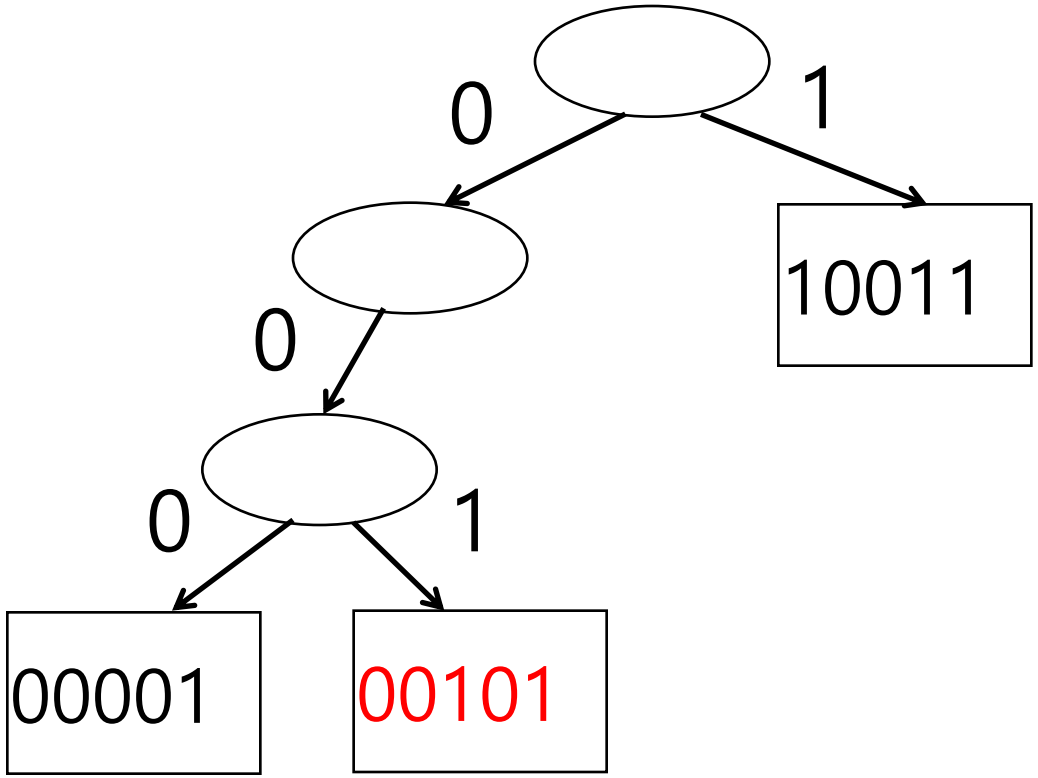


Building a Binary Trie

00101

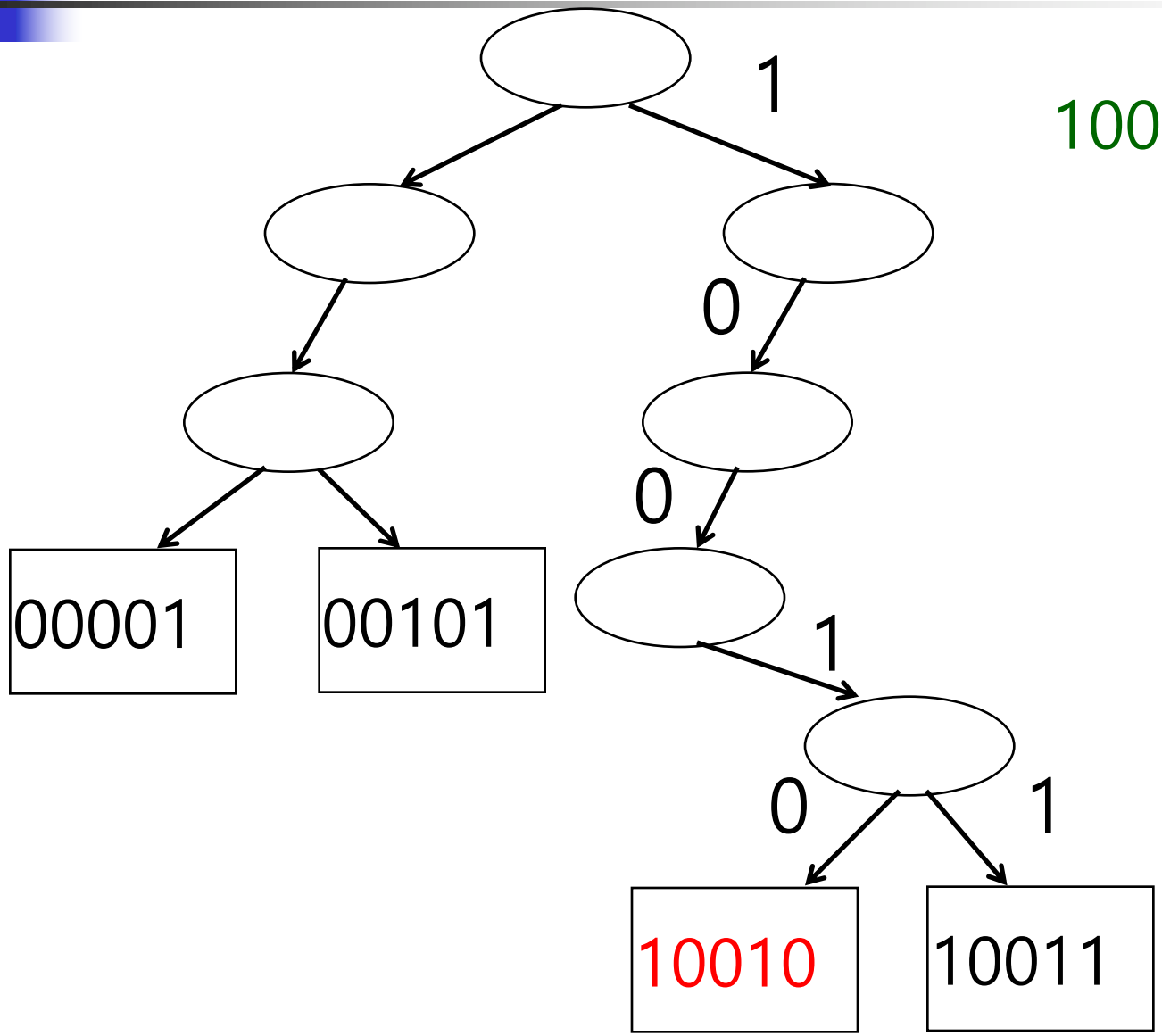


Building a Binary Trie – cont'd



10010

Building a Binary Trie – cont'd



10010

Exercise

Insert 10010

10011

00001

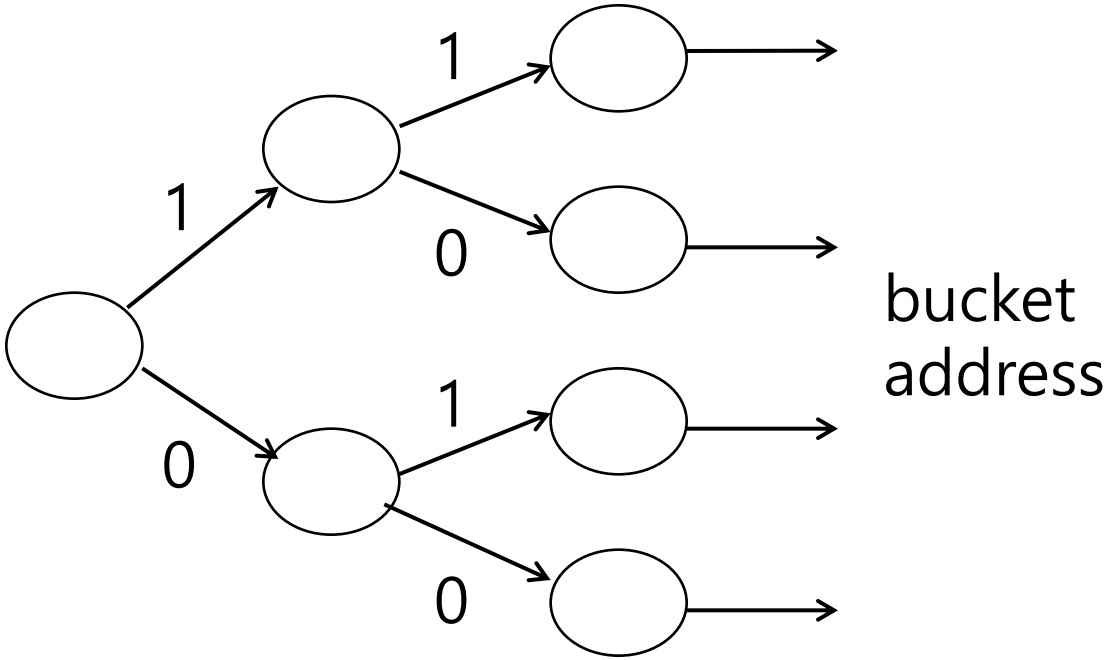
00101

Binary Trie: Performance and Properties

- Avg. performance: o(k)
  - K is the maximum number of bits in the keys
- One comparison in a search
  - Only at the leaf node
- Efficient for exact key match and partial key match.
- Unbalanced tree
- The shape of the tree is independent of the order of insertions.

Binary Trie In Use

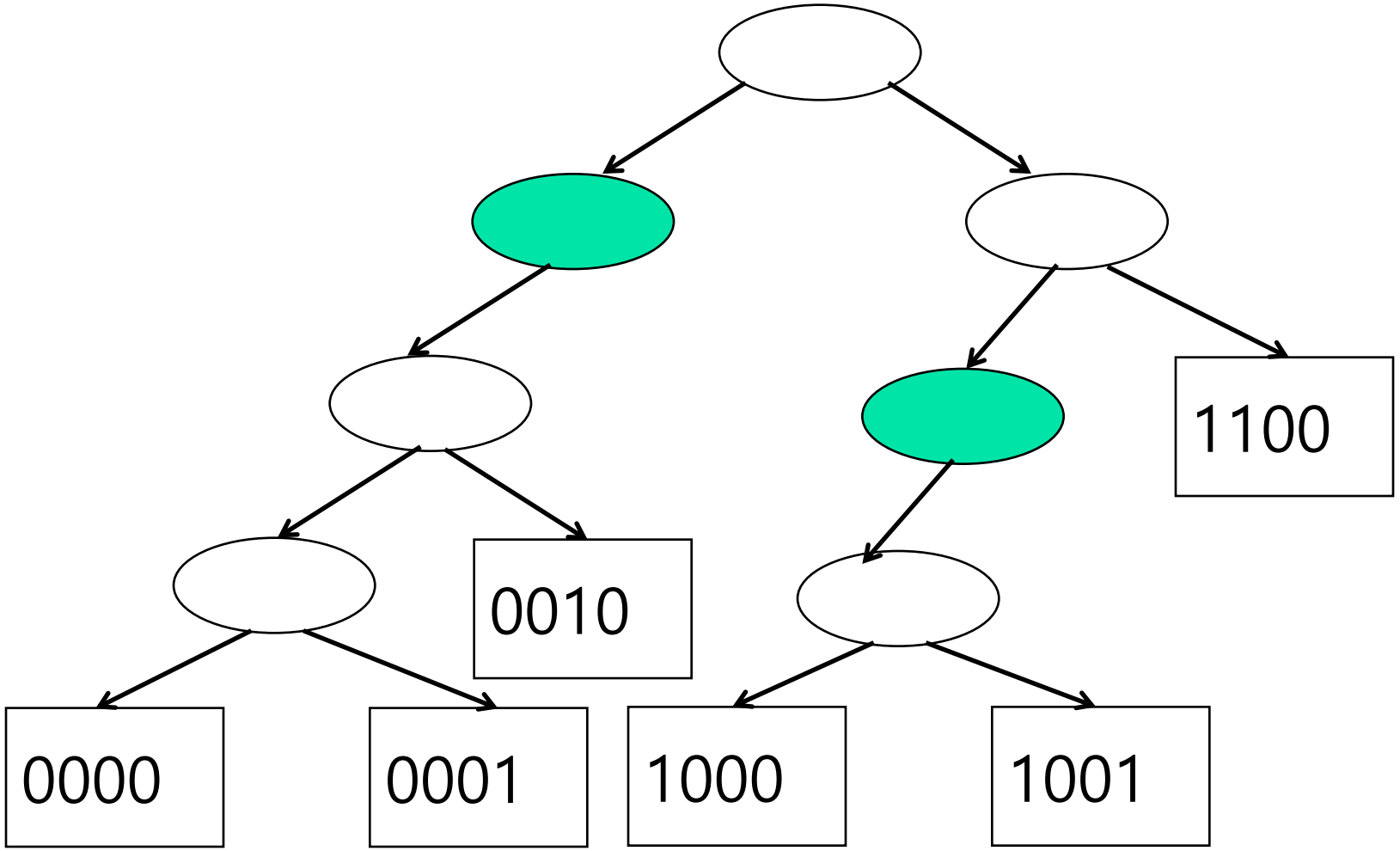
- Bucket directory in extendible hashing (to learn later)



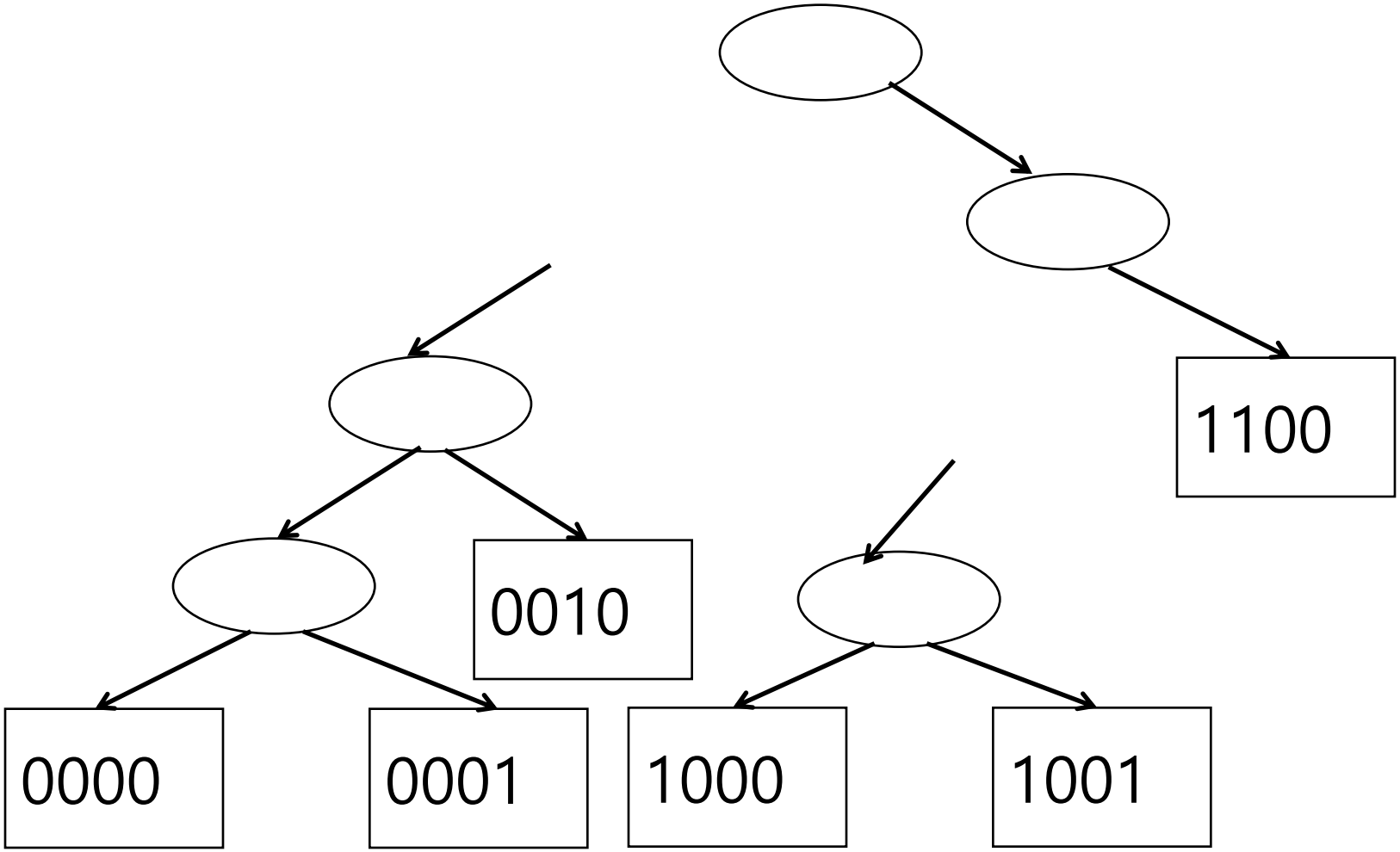
Compressed Binary Trie

- Branch nodes in a binary trie may be degree 1.
- Compressing a binary trie
  - Binary trie with degree-1 branch nodes eliminated.
  - Tree height is reduced.
- (On a compressed binary trie) each branch node is degree 2
- Branch node
  - left-child ptr, right-child ptr, bit-number

Eliminating Degree 1 Branch Nodes

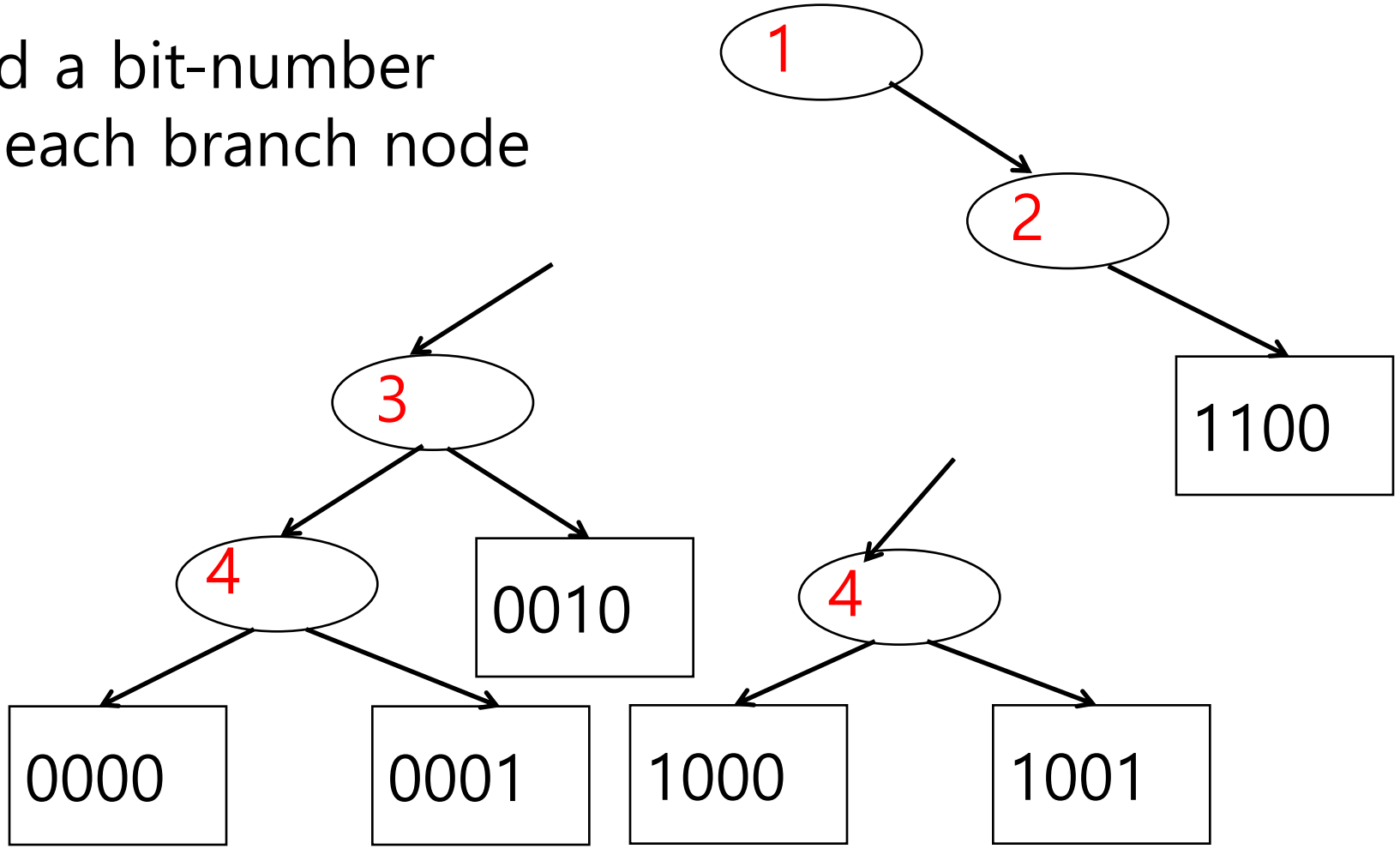


Eliminating Degree 1 Branch Nodes (cont'd)

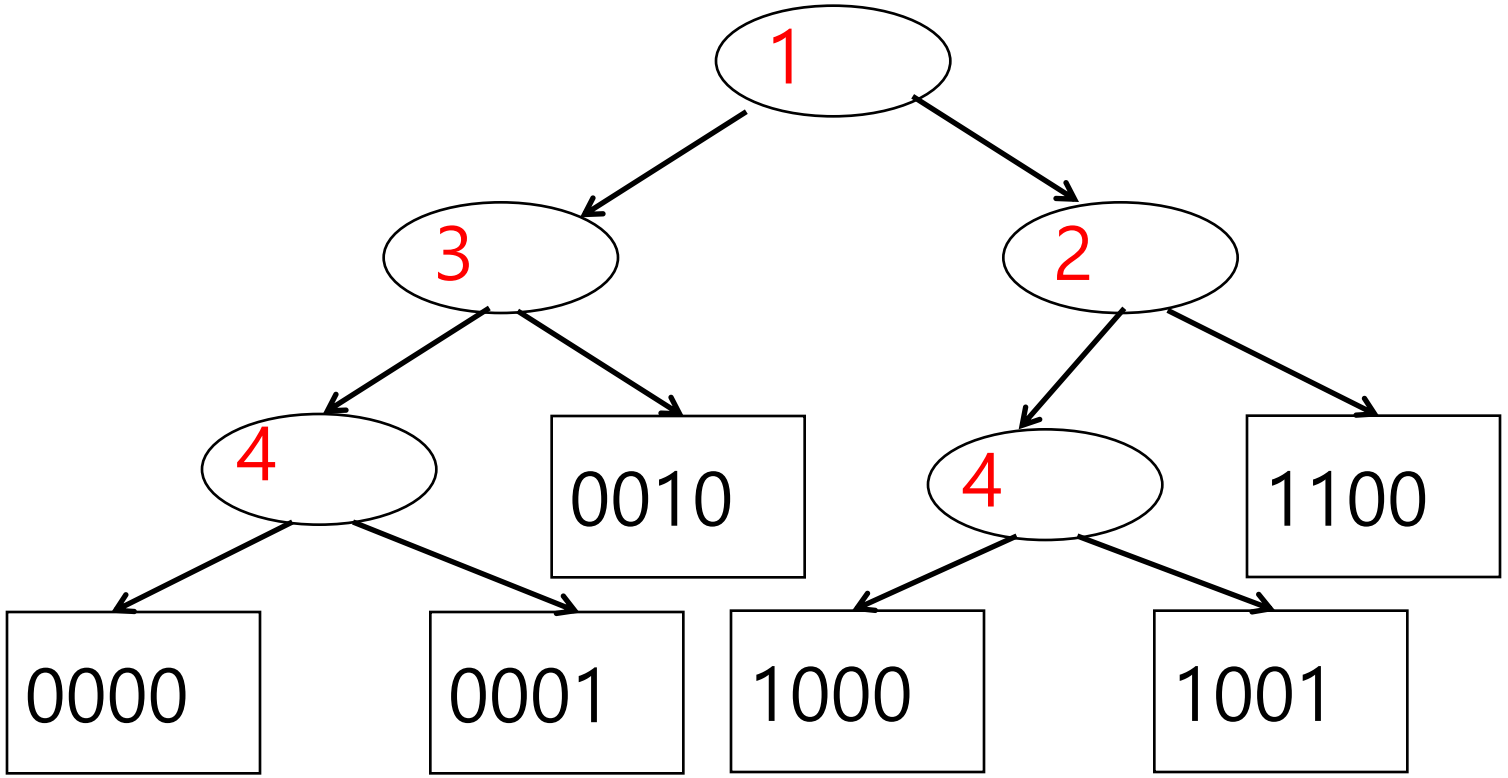


Eliminating Degree 1 Branch Nodes (cont'd)

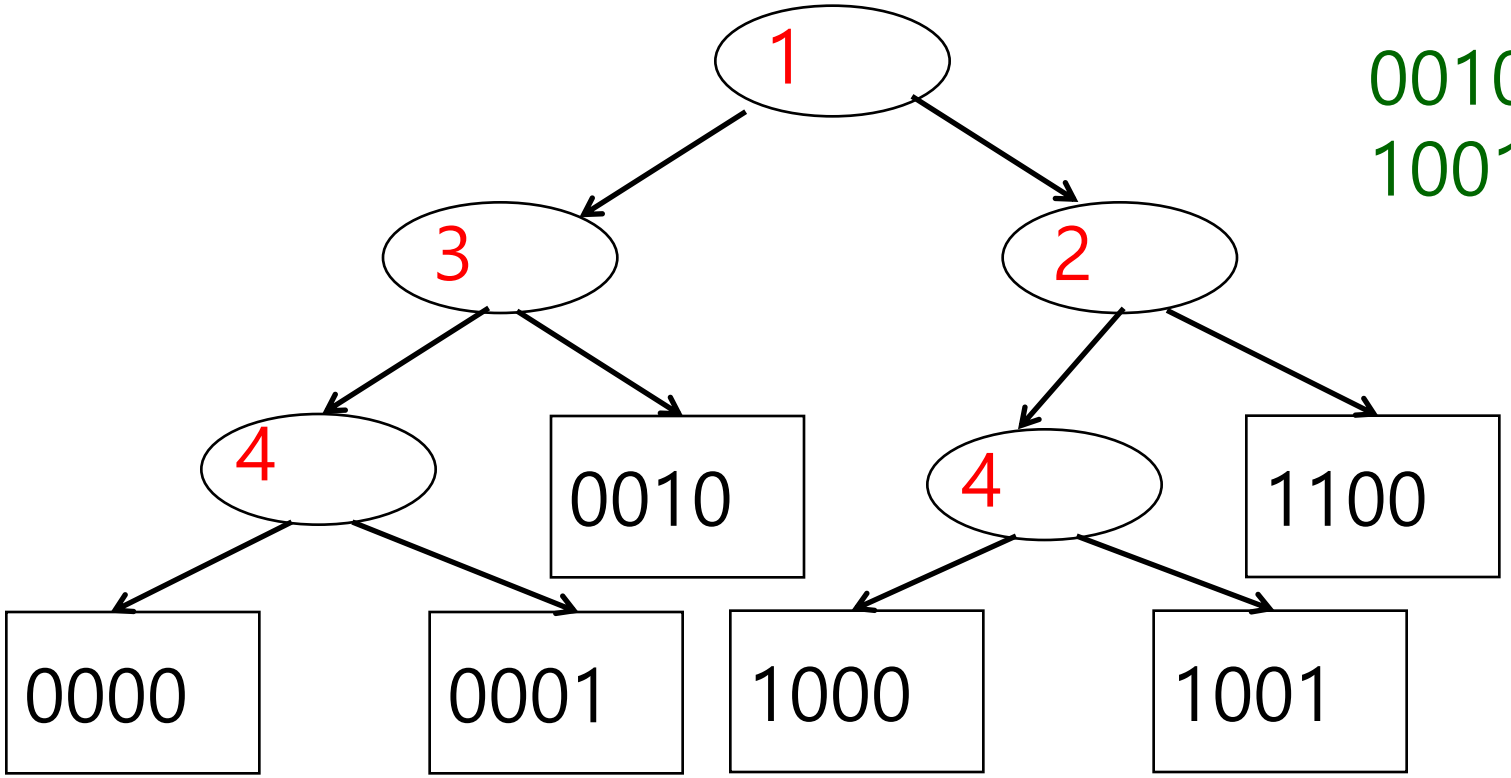
add a bit-number
to each branch node



Eliminating Degree 1 Branch Nodes (cont'd)



Searching a Compressed Binary Trie

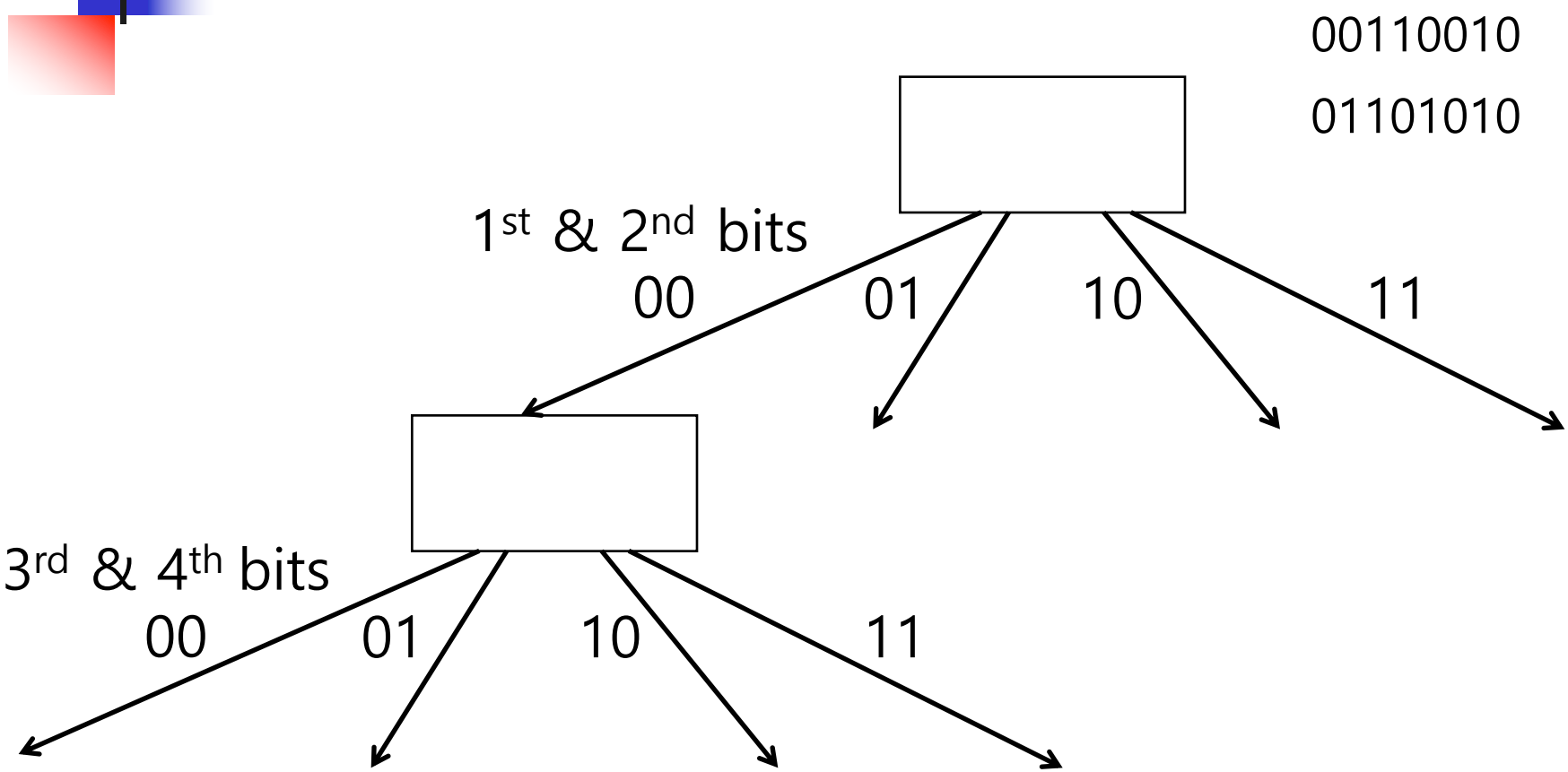


Multi-Way Tries

Multi-Way Trie

- Trie of degree m >= 2
- Tree height is reduced (vs. Binary trie)
- Branching is based on a portion of the key.
  - May be a bit combination or digit or an alphabet
- Reading
  - pp. 571-578

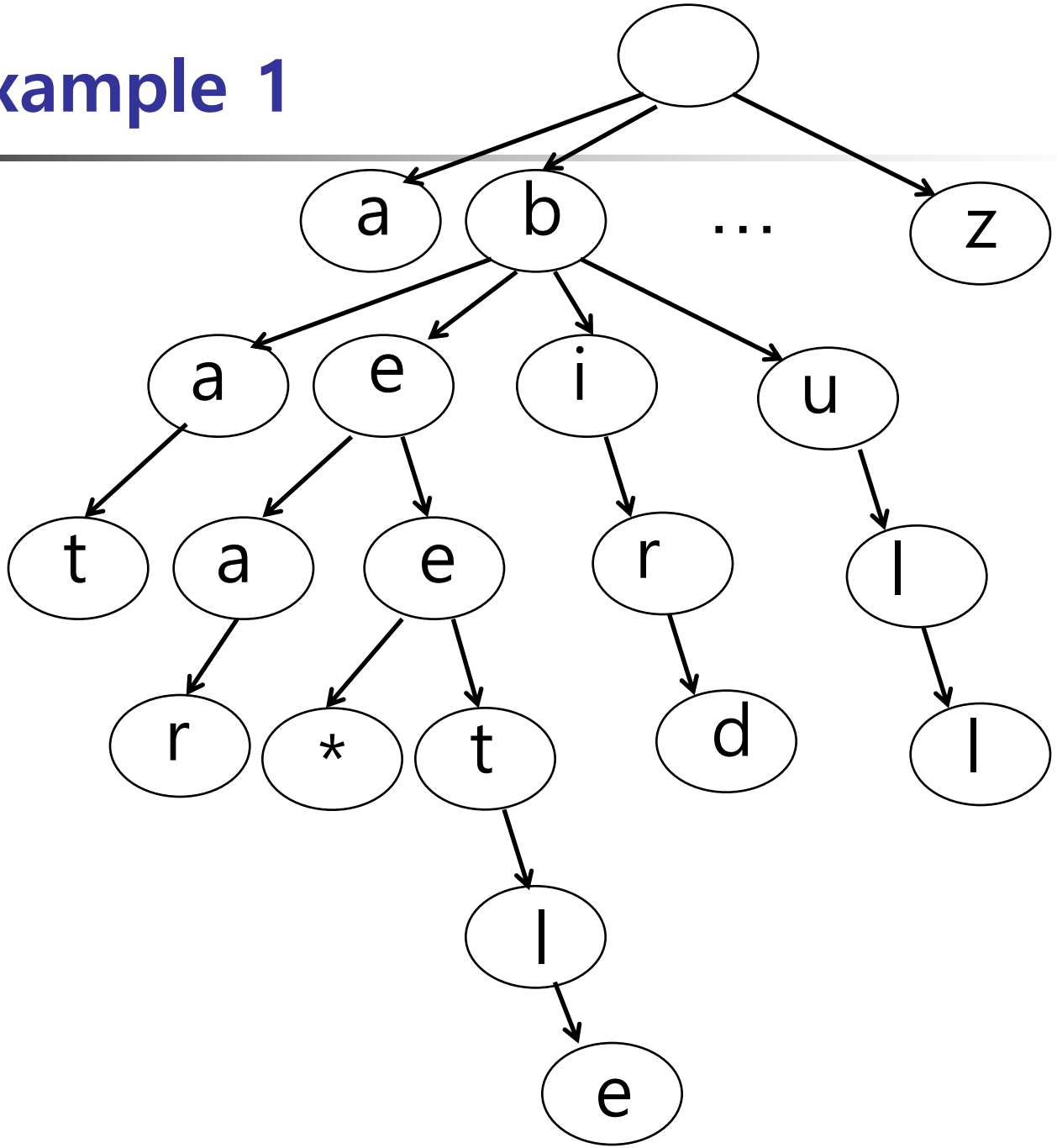
A 4-Way Trie


00110010
01101010
1st & 2nd bits
00
01
10
11
3rd & 4th bits
00
01
10
11

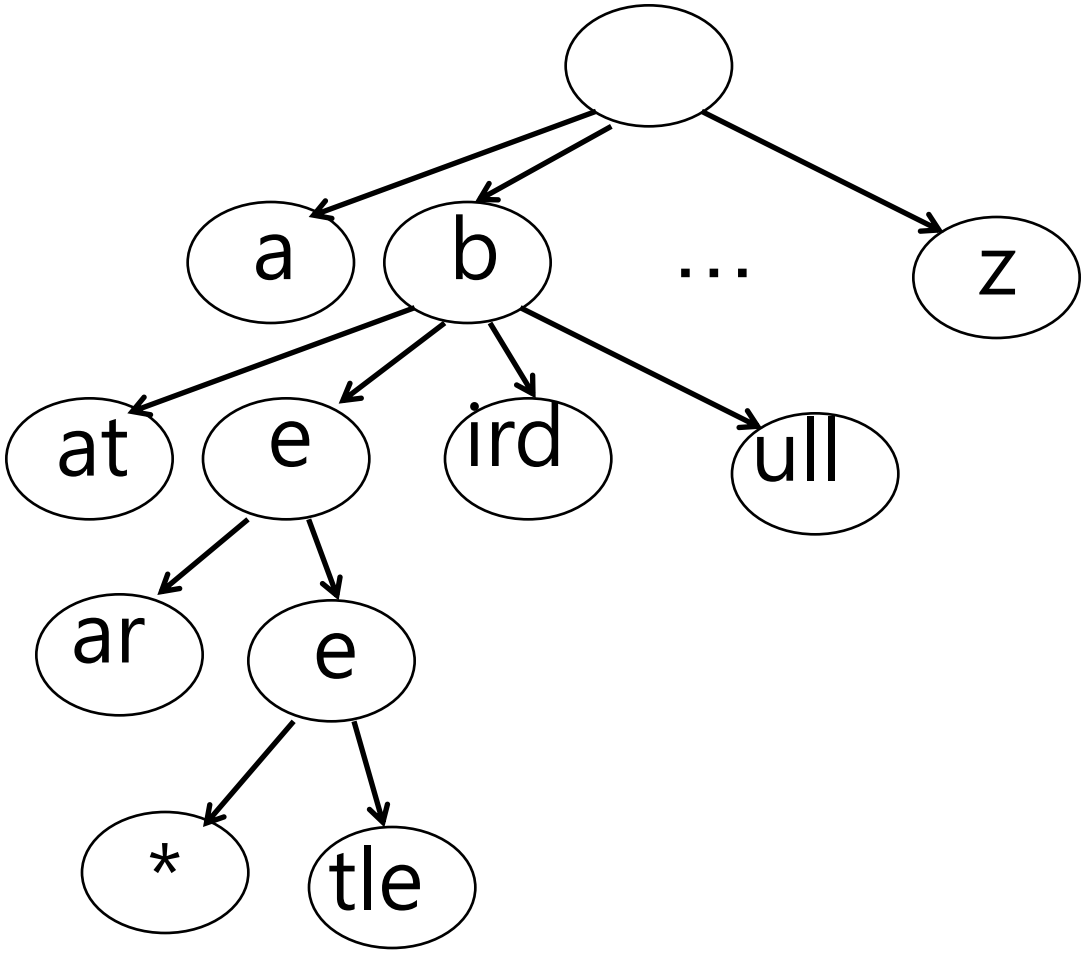
Multi-Way Trie: Use Example

- Stores a string over the alphabet across a chain of nodes.
- Usage
  - Storing a dictionary of words
  - Auto-complete dictionary, spell-checking and hyphenation software
- Keys at any level are the alphabet.
  - M = 26 + 1
- Keys are not stored in the branch nodes.
- Final portion of the keys are stored in the leaf nodes.

Example 1

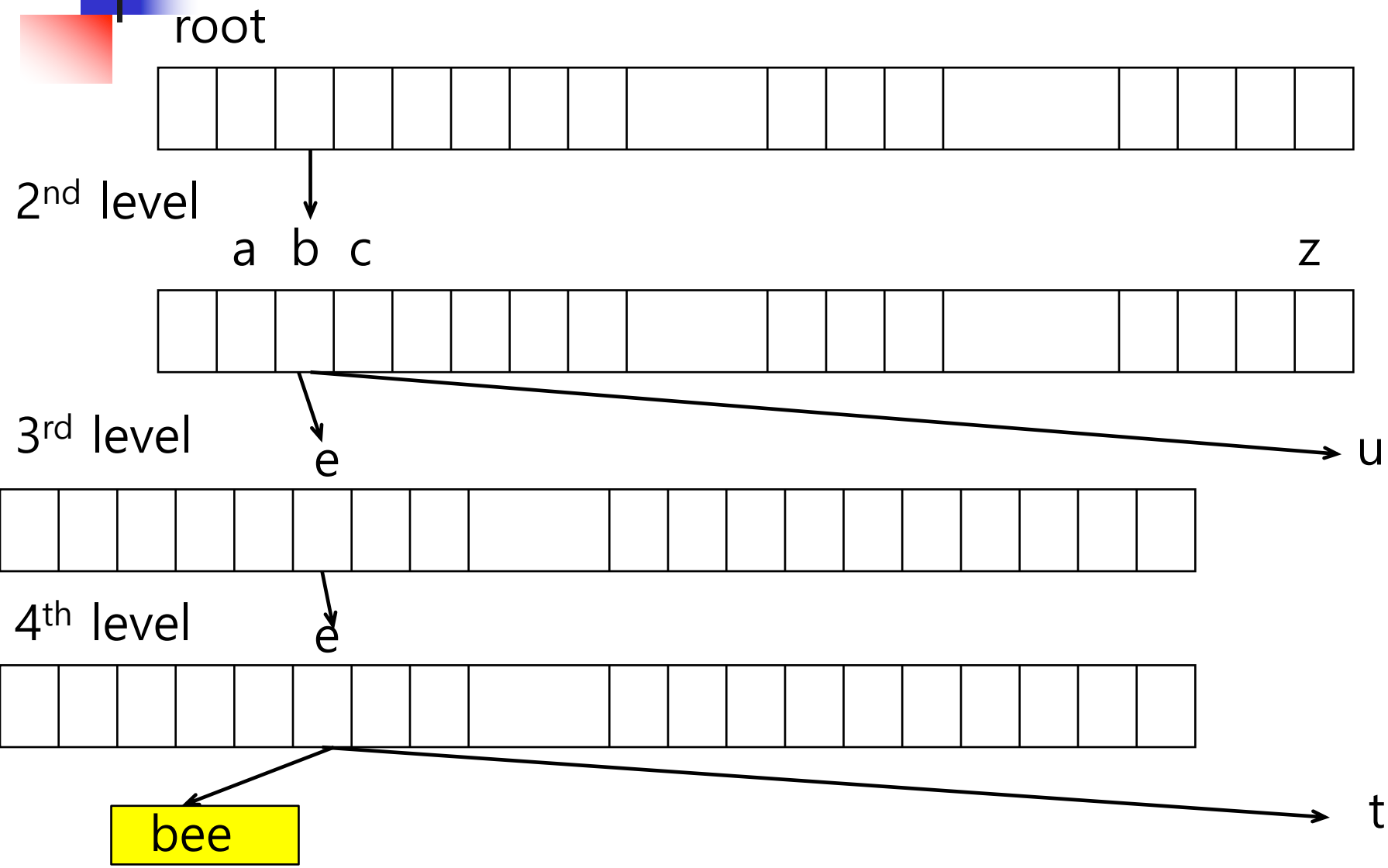


Example 2



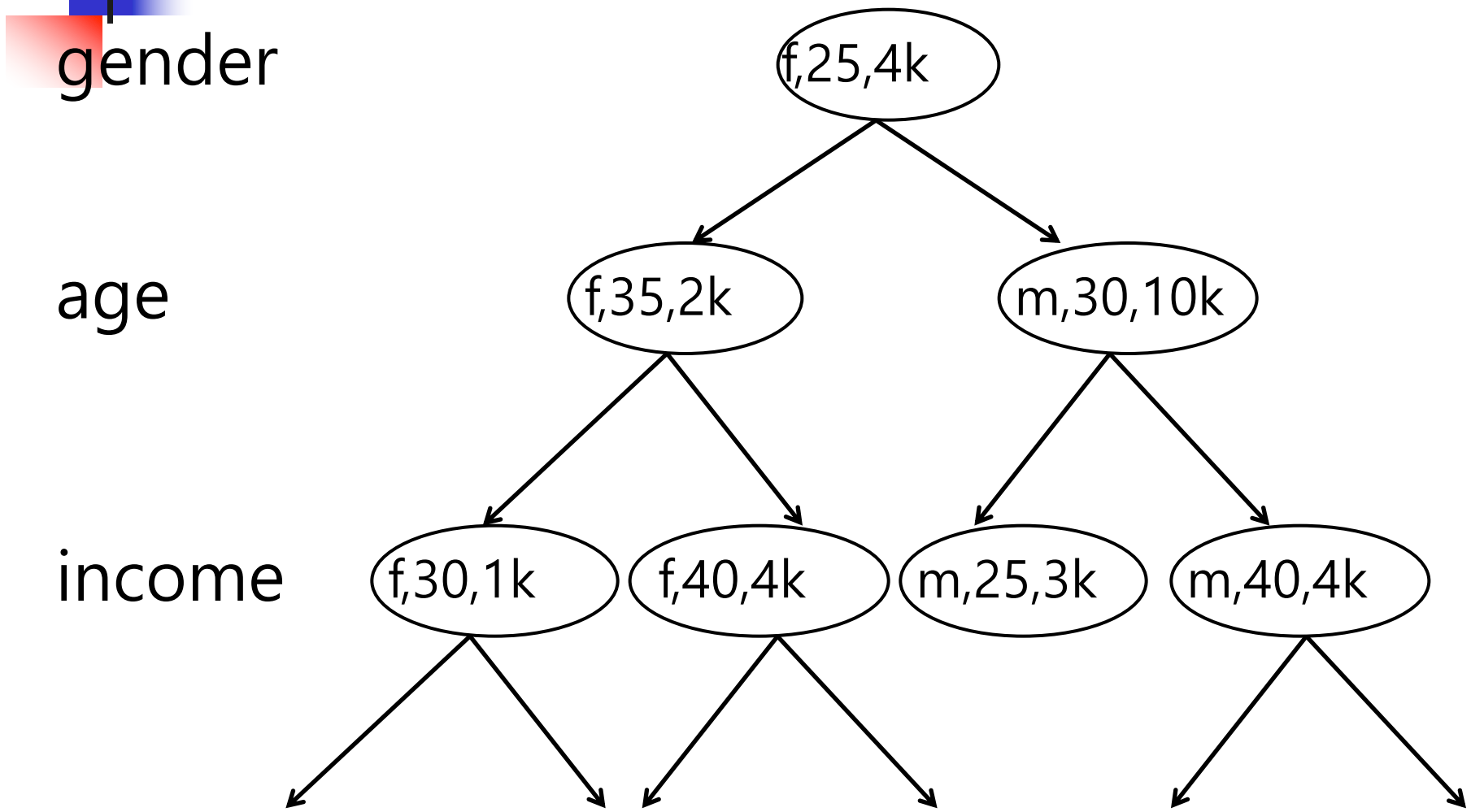
Implementation

bee beetle

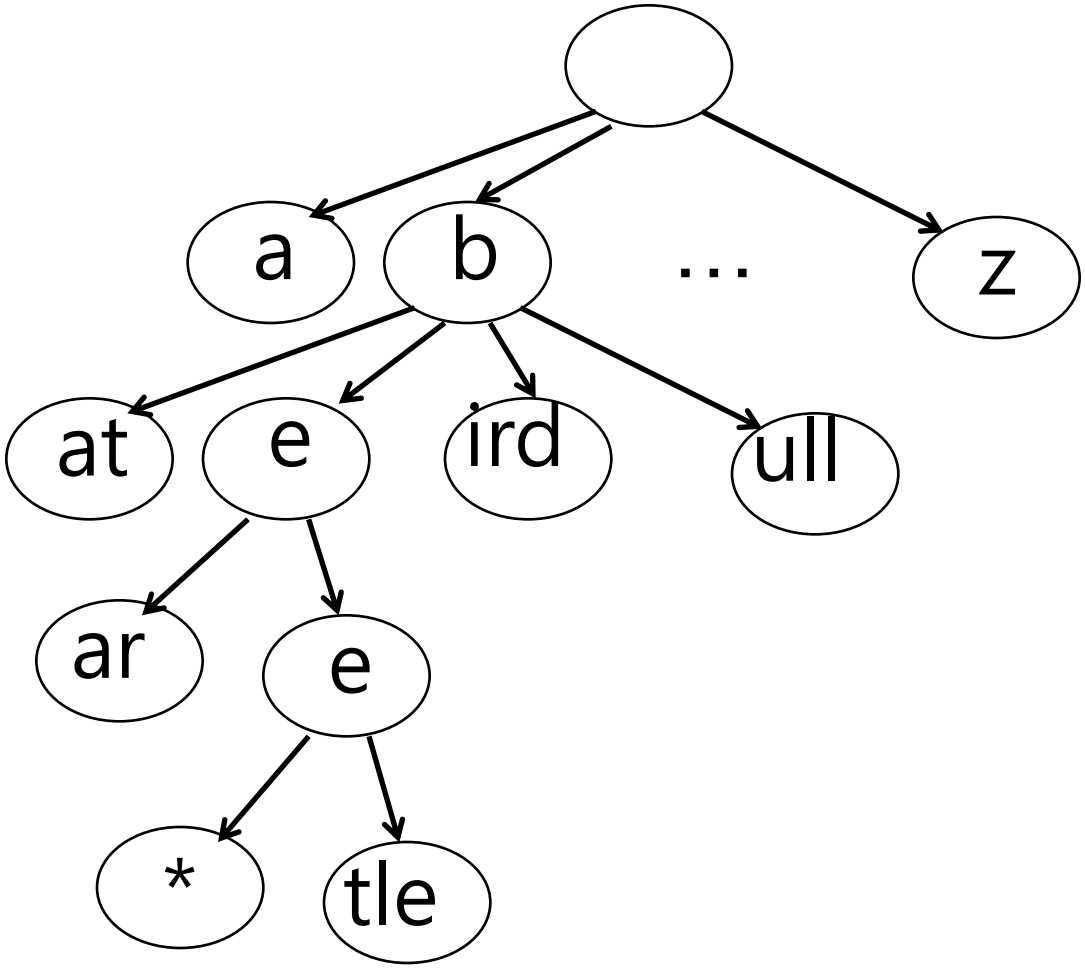


Data Structures for Partial Key Search

k-d Tree: Partial Key Search



M-Way Trie

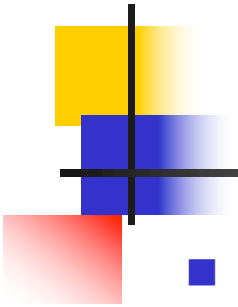


Reprised

- Binary Search Trees
  - AVL tree, binary trie
  - k-d tree, T-tree
- Multi-Way Search Trees
  - m-way trie, quad tree, oct tree
- Height-Balanced Search Trees
  - AVL tree, T-tree, red-black tree
- Perfectly Height-Balanced Search Trees
  - 2-3 tree, (B tree, R-tree)
- Spatial Search Trees
  - quad tree, oct tree, k-d tree
- Trees that Support Partial Searches
  - all search trees
- Trees that Support Partial Key Searches
  - k-d trees, tries (binary, m-way)

Search Performance: Best, Avg, Worst (1)

- Array
  - O(1), O(n), O(n)
- Binary Search
  - O(1), $O(\log_2 n)$, $O(\log_2 n)$
- Binary Search Tree
  - O(1), $O(\log_2 n)$, O(n)
- AVL, Red-Black (Binary Search) Tree
  - O(1), $O(\log_2 n)$, $O(\log_2 n)$
- Quad Tree
  - O(1), $O(\log_4 n)$, O(n)
- k-d Tree
  - O(1), $O(\log_2 n)$, O(n)
- T-Tree (with m-element array)
  - O(1), $O(\log_2 n/m)$, $O(\log_2 n/m)$



Search Performance: Best, Avg, Worst (2)

- Binary Trie (with k-bit keys)
  - O(1), O(k), O(k)
- M-Way Trie (with k-digit keys)
  - O(1), O(k), O(k)

End of Lecture