Chapter 8

Fundamental Sampling Distributions and Data Descriptions (2)

8.5 Sampling Distribution of S^2

Review: Sample Variance **S^2**

- **Sample variance**
  for n random variables X_1,, … , X_n

$$S^2 = \frac{1}{?}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

- Theorem 8.1

If S^2 is the variance of a random sample of size n, we may write

$$S^2 = \frac{1}{n(n-1)}\left[n\sum_{i=1}^{n}X_i^2 - \left(\sum_{i=1}^{n}X_i\right)^2\right].$$

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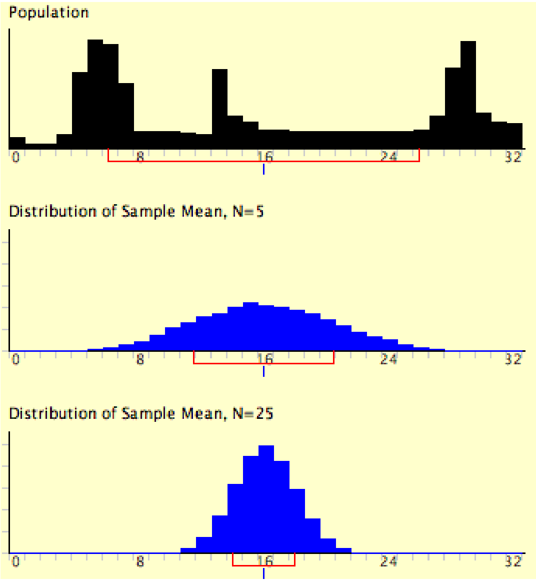
Review :
Central Limit Theorem

- **Sampling distribution of the Mean $\bar{X}$**

·If $X_1, X_2, \ldots, X_n$ is a random sample of size n from $N(\mu,\sigma)$, then

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \Leftrightarrow Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$



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Rather than mean μ, if ..

- if an engineer is interested in the population mean resistance of a certain type of resistor

$$\boxed{\text{the sampling distribution of } \bar{X}}$$

- if the variability in resistance is to be studied

$$\boxed{\text{the sampling distribution of } S^2 \text{ will be used}}$$

- If **the variability in samples** is to be studied, clearly the sampling distribution of $\boldsymbol{S}^2$ will be used in learning about the parametric counterpart, the population variance $\boldsymbol{\sigma}^2$.

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Question

- A manufacturer of car batteries guarantees that the batteries will last, on average, 3 years with a **standard deviation of 1 year**.

- If five of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5, and 4.2 years, should the manufacturer still be convinced that **the batteries have a standard deviation of 1 year**?

→ For this, we need to study "sample dist. of S^2"

Step to understand sample dist. of S^2

- 1. Understand **Chi-squared (χ^2) Distribution**
- **2.** Sample distribution of S^2
  - find the relationship between S^2 and Chi-squared dist.

Revisit: Chi-squared (χ^2) Distribution

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1. Chi-squared (χ^2) Distribution (ch.6)

//카이제곱분포

- Definition
  - If $Z_1, \ldots, Z_k$ are independent, standard normal random variables, then the sum of their squares,

$$X = Z_1^2 + Z_2^2 + \cdots + Z_n^2$$

  - is distributed according to the **chi-squared distribution** with k degrees of freedom.
  - Denoted as

$$X \sim \chi_n^2 \text{ or } X \sim \chi^2(n)$$

  - The chi-squared distribution has one parameter:
    - ***n*** (or ***v***) — a positive integer that specifies the number of degrees of freedom (i.e. the number of standard normal deviates being summed)

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Key Property

IF

- A random variable $\chi^2_{\nu_1}$ has a chi-squared distribution with ν_1 degrees of freedom, and
- A second independent random variable $\chi^2_{\nu_2}$ has a chi-squared distribution with ν_2 degrees of freedom,

THEN

$$\chi^2_{(\nu_1+\nu_2)} = \chi^2_{\nu_1} + \chi^2_{\nu_2}$$

their sum has a chi-squared distribution with $(\nu_1 + \nu_2)$ degrees of freedom.

- **Theorem 7.12**

If $X_1, X_2, \ldots, X_n$ are mutually independent random variables that have, respectively, chi-squared distributions with $v_1, v_2, \ldots, v_n$ degrees of freedom, then the random variable

$$Y = X_1 + X_2 + \cdots + X_n$$

has a chi-squared distribution with $v = v_1 + v_2 + \cdots + v_n$ degrees of freedom.

- **Corollary 7.1**

If $X_1, X_2, \ldots, X_n$ are independent random variables having identical normal distributions with mean μ and variance σ^2, then the random variable

$$Y = \sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma} \right)^2$$

has a chi-squared distribution with $v = n$ degrees of freedom.

Distribution of the Statistic $(n-1)S^2/\sigma^2$

If a random sample of size n is drawn from a normal population with mean μ and variance σ^2, and the sample variance is computed, we obtain a value of the statistic S^2. We shall proceed to consider the distribution of the statistic $(n-1)S^2/\sigma^2$.

By the addition and subtraction of the sample mean $\bar{X}$, it is easy to see that

$$\begin{aligned}
\sum_{i=1}^{n}(X_i-\mu)^2 &= \sum_{i=1}^{n}[(X_i-\bar{X})+(\bar{X}-\mu)]^2 \\
&= \sum_{i=1}^{n}(X_i-\bar{X})^2+\sum_{i=1}^{n}(\bar{X}-\mu)^2+2(\bar{X}-\mu)\sum_{i=1}^{n}(X_i-\bar{X}) \\
&= \sum_{i=1}^{n}(X_i-\bar{X})^2+n(\bar{X}-\mu)^2.
\end{aligned}$$

So, ..

$$\sum_{i=1}^{n}(X_i-\mu)^2 = \sum_{i=1}^{n}(X_i-\bar{X})^2 + n(\bar{X}-\mu)^2.$$

Dividing each term of the equality by σ^2 and substituting $(n-1)S^2$ for $\sum_{i=1}^{n}(X_i-\bar{X})^2$, we obtain

$$\frac{1}{\sigma^2}\sum_{i=1}^{n}(X_i-\mu)^2 = \frac{(n-1)S^2}{\sigma^2} + \frac{(\bar{X}-\mu)^2}{\sigma^2/n}.$$

Corollary 7.1 :
v = n d.f.

Then v = n-1

Z² : v=1 d.f.

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Sampling Distribution of S^2

- **Theorem 8.4:** If S^2 is the variance of a random sample of size n taken from a *normal* population having the variance σ^2, then the statistic

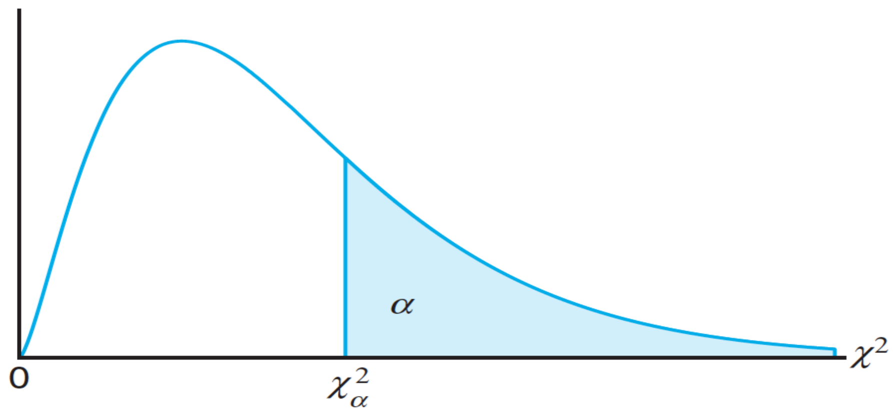
$$\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{\sigma^2}$$

- has a **chi-squared (χ^2)** distribution with $v = n - 1$ degrees of freedom *(자유도).

The Chi-Squared Distribution – Table A.5

- *Notation:* χ_α^2 represents the χ^2 value above which we find an area of α, that is, for which $P(\chi^2 > \chi_\alpha^2) = \alpha$.

This is illustrated by the shaded region below.



For tabulated values of the Chi-Squared distribution see the Chi-Squared table (Appendix Table A.5), which gives values of χ^2_α for various values of **α** and v. The areas, **α**, are the column headings; the degrees of freedom, v, are given in the left column, and the table entries are the χ^2 values.

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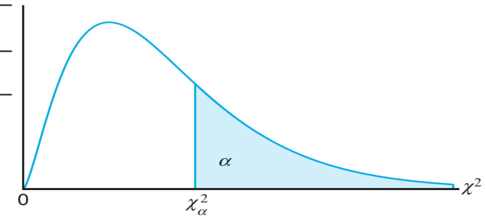
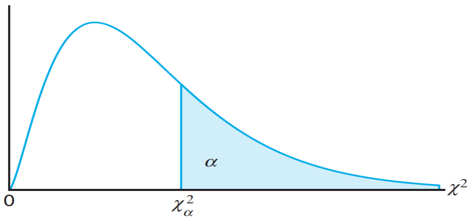
Table A.5 Critical Values of the Chi-Squared Distribution



v	α = 0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50
1	0.0^4393	0.0^3157	0.0^3628	0.0^3982	0.00393	0.0158	0.0642	0.102	0.148	0.455
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.647	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.041	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.878	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336
40	20.707	22.164	23.838	24.433	26.509	29.051	32.345	33.66	34.872	39.335
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335
60	35.534	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335

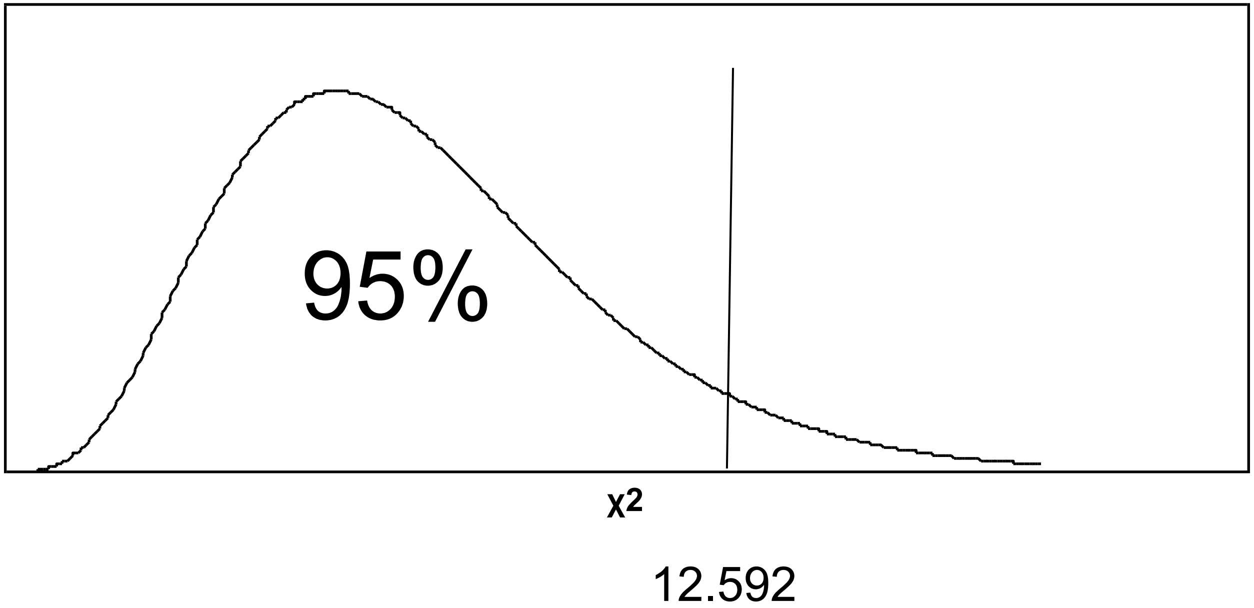
Table A.5 (continued) Critical Values of the Chi-Squared Distribution



	α									
v	**0.30**	**0.25**	**0.20**	**0.10**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.001**
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.466
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.515
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.321
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.471	27.688	29.819	34.527
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.124
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.698
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.791
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.819
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.314
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.796
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.619
26	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.051
27	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.475
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.994	56.892
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.335	58.301
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.702
40	44.165	45.616	47.269	51.805	55.758	59.342	60.436	63.691	66.766	73.403
50	54.723	56.334	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.660
60	65.226	66.981	68.972	74.397	79.082	83.298	84.58	88.379	91.952	99.608

Question

- If a sample of size 7 is taken from a normal population (i.e., n = 7), what value of χ^2 corresponds to $P(\chi^2 < \chi_\alpha^2)$ = 0.95? (Hint: first determine α.)



The Chi-Squared Distribution–Example

- Exercises 8.37, 8.39

8.37 For a chi-squared distribution, find

(a) $\chi^2_{0.025}$ when $v = 15$;

(b) $\chi^2_{0.01}$ when $v = 7$;

(c) $\chi^2_{0.05}$ when $v = 24$.

8.39 For a chi-squared distribution, find χ^2_α such that

(a) $P(X^2 > \chi^2_\alpha) = 0.99$ when $v = 4$;

(b) $P(X^2 > \chi^2_\alpha) = 0.025$ when $v = 19$;

(c) $P(37.652 < X^2 < \chi^2_\alpha) = 0.045$ when $v = 25$.

Answers

8.37 (a) 27.488.

(b) 18.475.

(c) 36.415.

8.39 (a) $\chi^2_\alpha = \chi^2_{0.99} = 0.297$.

(b) $\chi^2_\alpha = \chi^2_{0.025} = 32.852$.

(c) $\chi^2_{0.05} = 37.652$. Therefore, $\alpha = 0.05 - 0.045 = 0.005$. Hence, $\chi^2_\alpha = \chi^2_{0.005} = 46.928$.

Example 8.7

- A manufacturer of car batteries guarantees that his batteries will last, on average, 3 years with a standard deviation of 1 year.
- A sample of five of the batteries yielded a sample variance of 0.815.
  Does the manufacturer have reason to suspect the standard deviation is no longer 1 year?
  (Hint: Use $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$.)

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Example 8.7

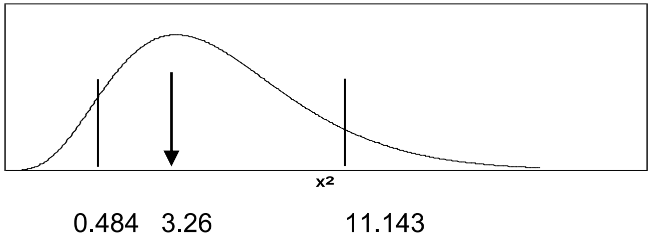
- Solution :

$\mu = \boxed{}\ \sigma = \boxed{}\ n = \boxed{}\quad \textit{Degrees of freedom } (v) = \boxed{}\ s^2 = \boxed{}$

$$\textit{calculated } \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(4)(0.815)}{1} = 3.26$$

  - If the χ^2 value fits within an interval that covers 95% of the χ^2 values with 4 degrees of freedom, then the estimate for σ is reasonable.

See Table A.5



For α = 0.025, χ_α^2 =11.143
The χ_α^2 value for α = 0.975 is 0.484.
Therefore, the computed value with $\sigma^2 = 1$ is reasonable!

8.6 t-Distribution

or Student's t-distribution

Basic Assumption for Central Limit Theorem

- Recall, by Central Limit Theorem:

$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \quad \text{is } N(z;\, 0,1)$$

- Assumption:

the population standard deviation <u>σ is known</u>

- in many experimental scenarios, knowledge of σ is certainly no more reasonable than knowledge of the population mean μ.

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What if we don't know σ?

- Then,
  - as we estimate mean μ through $\bar{X}$,
  - we should supply an estimate of σ
- As a result, a natural statistic to consider to deal with inferences on μ (without knowledge of σ) is required..

$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \quad \Longrightarrow \quad T = \frac{\overline{X} - \mu}{S / \sqrt{n}}$$

What if we **don't know** σ?

- New statistic:

$$T = \frac{\overline{X} - \mu}{S / \sqrt{n}}$$

Where,

$$\overline{X} = \sum_{i=1}^{n} \frac{X_i}{n} \quad \text{and} \quad S^2 = \sum_{i=1}^{n} \frac{(X_i - \overline{X})}{n-1}$$

follows a ***t*-distribution with *v* = *n* – 1** degrees of freedom.

t-distribution with $\nu = n - 1$ degrees of freedom?

$$T = \frac{\overline{X} - \mu}{S / \sqrt{n}}$$

$$T = \frac{(\bar{X} - \mu)/(\sigma/\sqrt{n})}{\sqrt{S^2/\sigma^2}} = \frac{Z}{\sqrt{V/(n-1)}},$$

where

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

has the standard normal distribution and

$$V = \frac{(n-1)S^2}{\sigma^2}$$

has a chi-squared distribution with $v = n - 1$ degrees of freedom.

Theorem 8.4: $$\boxed{\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{\sigma^2}}$$

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t-Distribution

- **Theorem 8.5**

Let Z be a standard normal random variable and V a chi-squared random variable with v degrees of freedom. If Z and V are independent, then the distribution of the random variable T, where

$$T = \frac{Z}{\sqrt{V/v}},$$

This is known as the ***t*-distribution** with v degrees of freedom.

t-Distribution

Corollary 8.1:

Let $X_1, X_2, \ldots, X_n$ be independent random variables that are all normal with mean μ and standard deviation σ. Let

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad \text{and} \qquad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2.$$

Then the random variable $T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$ has a t-distribution with $v = n-1$ degrees of freedom.

What Does the *t*-Distribution Look Like?

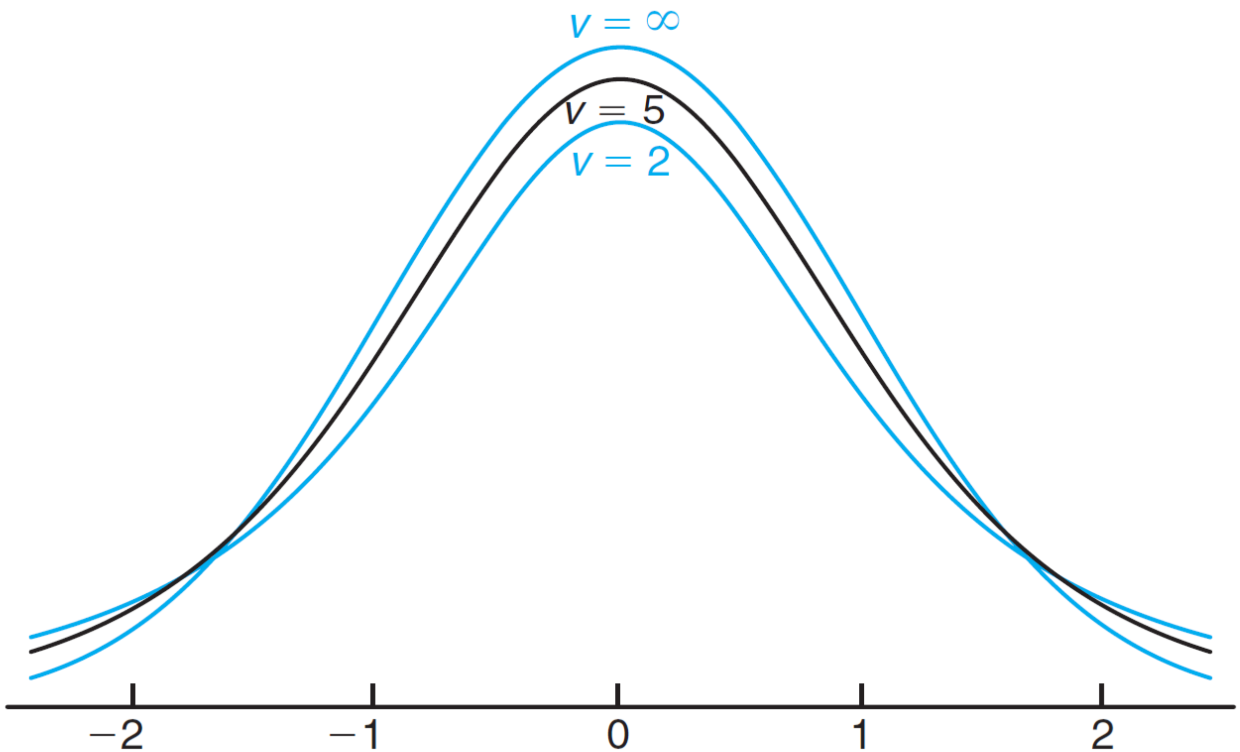
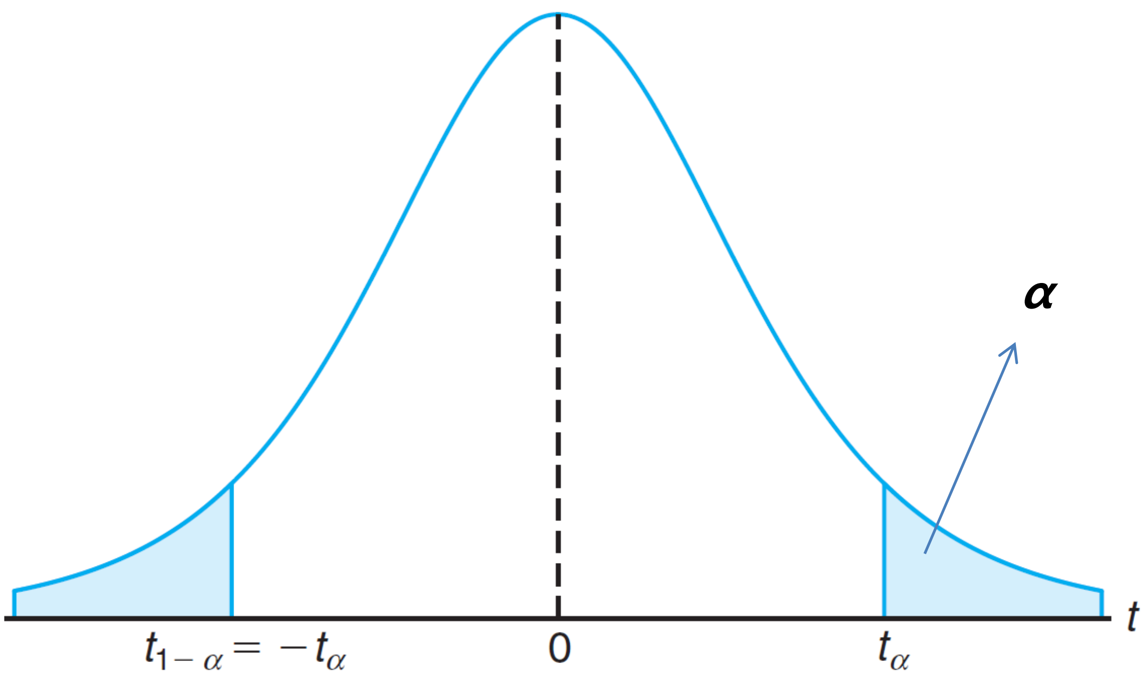
- 

Figure 8.8: The t-distribution curves for $v = 2, 5,$ and ∞.

t-value t_α

- *Notation :* Let t_α represent the t-value above which we find an area equal to α.
- i.e., $P(T \geq t_\alpha) = \alpha$



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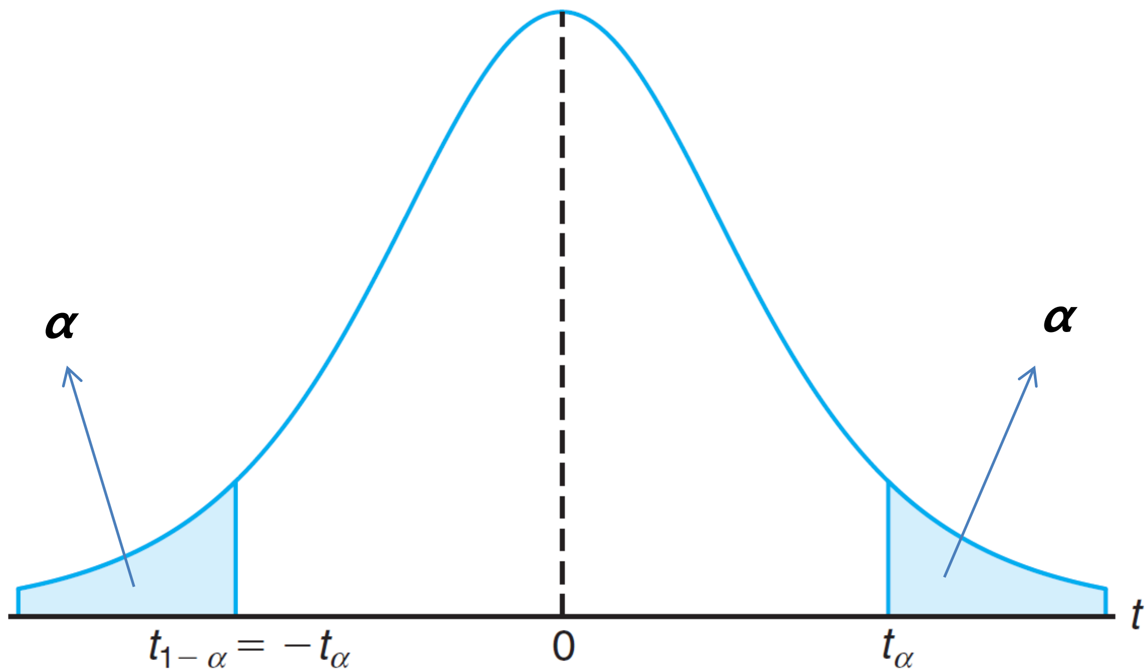
Symmetry property

- Symmetry property (about 0) of the *t*-distribution



- Example 8.8
  - The *t*-value with *v* = 14 degrees of freedom that leaves an area of 0.025 to the left, and therefore an area of 0.975 to the right, is

$$t_{0.975} = -t_{0.025} = -2.145.$$

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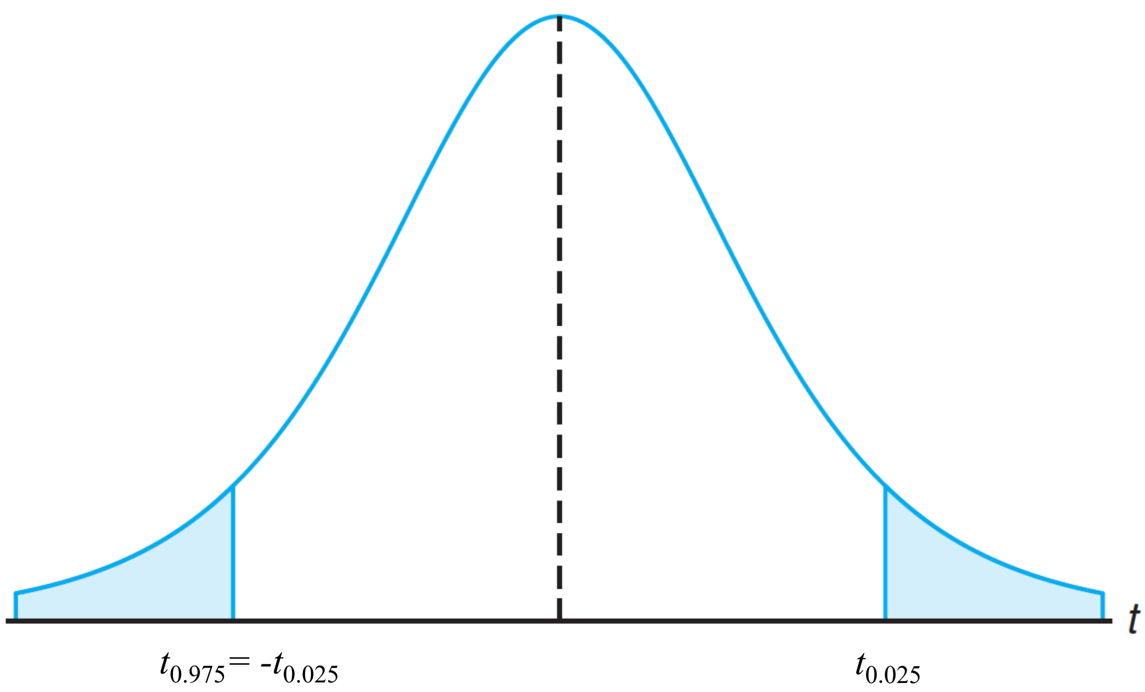
Example 8.8: The t-value with $v = 14$ degrees of freedom that leaves an area of 0.025 to the left, and therefore an area of 0.975 to the right, is

$$t_{0.975} = -t_{0.025} = -2.145.$$



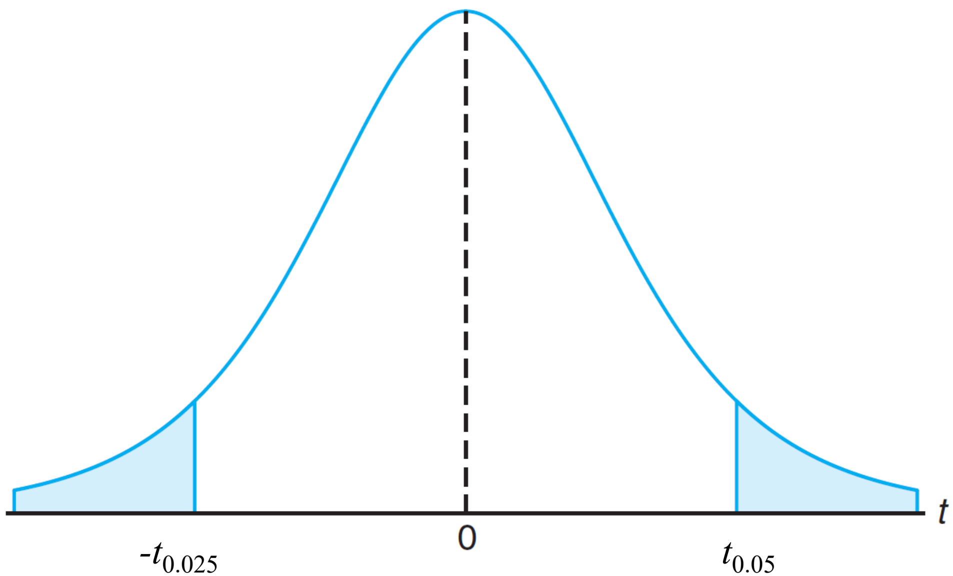
Example 8.9: Find $P(-t_{0.025} < T < t_{0.05})$.

***Solution*:** Since $t_{0.05}$ leaves an area of 0.05 to the right, and $-t_{0.025}$ leaves an area of 0.025 to the left, we find a total area of

$$1 - 0.05 - 0.025 = 0.925$$

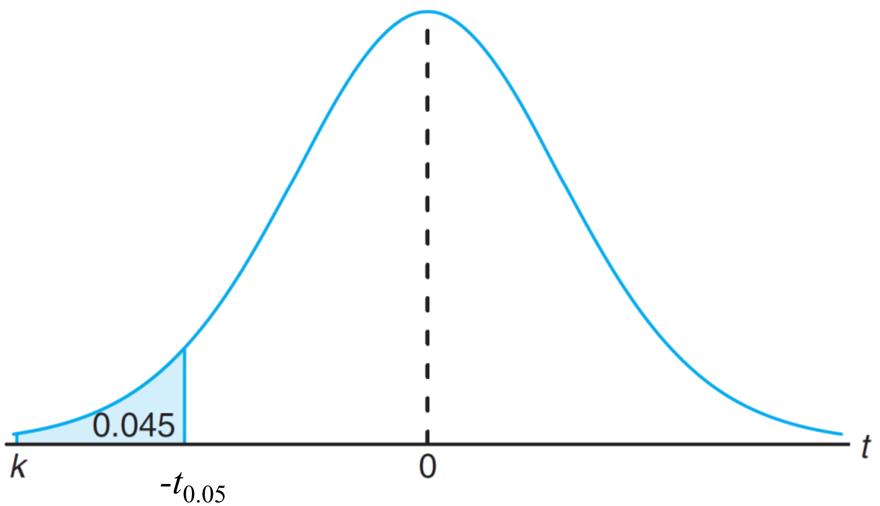
between $-t_{0.025}$ and $t_{0.05}$. Hence

$$P(-t_{0.025} < T < t_{0.05}) = 0.925.$$



- # Example 8.10

Find k such that $P(k < T < -1.761) = 0.045$ for a random sample of size 15 selected from a normal distribution and $\frac{\overline{X}-\mu}{s/\sqrt{n}}$.



From Table A.4 we note that 1.761 corresponds to $t_{0.05}$ when $v = 14$. Therefore, $-t_{0.05} = -1.761$. Since k in the original probability statement is to the left of $-t_{0.05} = -1.761$, let $k = -t_\alpha$. Then, from Figure 8.10, we have

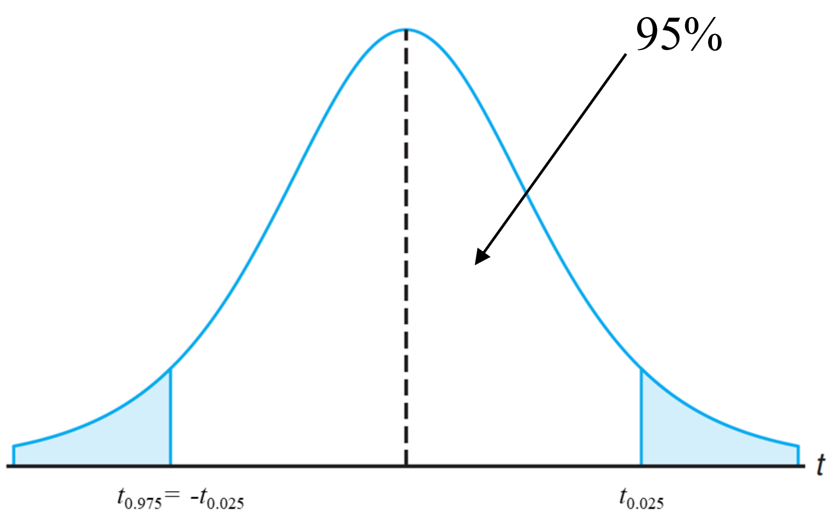
$$0.045 = 0.05 - \alpha, \text{ or } \alpha = 0.005.$$

Hence, from Table A.4 with $v = 14$,

$$k = -t_{0.005} = -2.977 \text{ and } P(-2.977 < T < -1.761) = 0.045.$$

95% of the values of a t-distribution

- Exactly 95% of the values of a t-distribution with $v = n-1$ degrees of freedom lie between $-t_{0.025}$ and $t_{0.025}$.
- A t-value that falls below $-t_{0.025}$ or above $t_{0.025}$ would tend to make us believe either that a very rare event has taken place or that our assumption about μ is in error.
- Should this happen, we shall make the decision that our assumed value of μ is in error.



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Question

- A chemical engineer claims that the population mean yield of a certain batch process is 500 grams per milliliter of raw material. To check this claim he samples 25 batches each month. If the computed t-value falls between $-t_{0.05}$ and $t_{0.05}$, he is satisfied with this claim. What conclusion should he draw from a sample that has a mean $\bar{x} = 518$ grams per milliliter and a sample standard deviation $s = 40$ grams? Assume the distribution of yields to be approximately normal.

From Table A.4 we find that $t_{0.05} = 1.711$ for 24 degrees of freedom. Therefore, the engineer can be satisfied with his claim if a sample of 25 batches yields a t-value between -1.711 and 1.711. If $\mu = 500$, then

$$\boxed{T = \frac{\overline{X} - \mu}{S/\sqrt{n}}}$$

$$t = \frac{518 - 500}{40/\sqrt{25}} = 2.25,$$

a value well above 1.711. The probability of obtaining a t-value, with $v = 24$, equal to or greater than 2.25 is approximately 0.02. If $\mu > 500$, the value of t computed from the sample is more reasonable. Hence, the engineer is likely to conclude that the process produces a better product than he thought.

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What is the t-Distribution Used for?

- The t-distribution is used extensively in problems that deal with inference about the population mean or in problems that involve comparative samples (i.e., in cases where one is trying to determine if means from two samples are significantly different).
- We should note that the use of the t-distribution for the statistic

$$T = \frac{\overline{X} - \mu}{S / \sqrt{n}}$$

requires that $X_1;X_2, \ldots, X_n$ be normal.

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Table A.4 Critical Values of the t-Distribution

	α						
v	**0.40**	**0.30**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

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Table A.4 (continued) Critical Values of the t-Distribution

	α						
v	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.689
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.660
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.290

https://qigongwest.blog/2020/11/09/qa-session/