1. Of three cards, one is painted red on both sides; one is painted black on both sides; and one is painted red on one side and black on the other. A card is randomly chosen and placed on a table. If the side facing up is red, what is the probability that the other side is also red?

A1 is the event that the front side of the card is red, and A2 is the event that the back side of the card is also red.

P(A2|A1) = P(A1∩A2)/P(A1)

P(A1∩A2) = 1/3 since there’s only one card which has red on both side.

P(A1) = (1/2)\*(1/3) + (1/2)\*(2/3) = ½

(1/2)\*(1/3) : when there are 1 red card and 2 black cards on the table.

(1/2)\*(2/3) : when there are 2 red cards and 1 black card on the table.

P(A1∩A2)/P(A1) = (1/3)/(1/2) = 2/3

2. The following is the famous Monty Hall problem. Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?” What would be your choice? Explain why. (Hint: link1, link2)

Let’s say you choose door1. Then, the probability of winning if we switch my choice is P(car is behind door2 | host reveals door 3). And The probability if you stay is P(car is behind door 1| host reveals door3).

P(host reveals door 3) = (1/3) \* (1/2) + (1/3) \* 1 = **1/2** . Because for the car behind the door1, the host can open either door 1 or door 2 which is probability (1/3) \* (1/2). And for the car behind the door 2, the host should open door 3 which is probability (1/3) \* 1.

Let’s compute P(car is behind door2 | host reveals door 3) first.

P(car is behind door2 | host reveals door 3) = P(car is behind the door 2 and host reveals door 3)/P(host reveals door 3) = (1/3)/(1/2) = **2/3** since the probability of the car behind the door 2 is 1/3 and the host then has to reveal door 3. So it’s (1/3)\*1 = 1/3.

Next, P(car is behind door 1| host reveals door3) is P(car is behind door 1 and host reveals door 3)/(host reveals door 3). {(1/3)\*(1/2)}/(1/2) = 1/3

This shows you gets 2/3 chance of winning if you switch and 1/3 chance of winning if you stay. So Switching your choice is better.

3. Prostate cancer is the most common type of cancer found in males. As an indicator of whether a male has prostate cancer, doctors often perform a test that measures the level of the PSA protein (prostate specific antigen) that is produced only by the prostate gland. Although higher PSA levels are indicative of cancer, the test is notoriously unreliable. Indeed, the probability that a noncancerous man will have an elevated PSA level is approximately .135, with this probability increasing to approximately .268 if the man does have cancer. If, based on other factors, a physician is 70 percent certain that a male has prostate cancer, what is the conditional probability that he has the cancer given that

(a) the test indicates an elevated PSA level;

=(0.7\*0.268)/(0.7\*0.268 + 0.3 \* 0.135) = 1876/2281

(b) the test does not indicate an elevated PSA level?

(0.7 \* 0.732) / (0.7\*0.732 + 0.3 \* 0.865) ≈ 0.6638

(c) Repeat the preceding (a), this time assuming that the physician initially believes there is a 30 percent chance the man has prostate cancer.

(0.3\*0.268)/(0.3\*0.268 + 0.7\*0.135) ≈ 0.4596

4. Suppose that an insurance company classifies people into one of three classes — good risks, average risks, and bad risks. Their records indicate that the probabilities that good, average, and bad risk persons will be involved in an accident over a 1-year span are .05, .15, and .30, respectively. 20 percent of the population are “good risks,” 50 percent are “average risks,” and 30 percent are “bad risks”. A policy holder Diana had no accidents in 1987.

(a) What is the probability that Diana is a good risk class?

(0.2 \* 0.95)/(0.2\*0.95+0.5\*0.85+0.3\*0.7) ≈ 0.230303

(b) What is the probability that Diana is an average risk class?

(0.5\*0.85)/(0.2\*0.95+0.5\*0.85+0.3\*0.7)

= 0.51515

5. An investment firm offers its customers municipal bonds that mature after varying numbers of years. Given that the cumulative distribution function of T, the number of years to maturity for a randomly selected bond, is find

(a) P(T = 5); 1/4

(b) P(T > 3); 1/2

(c) P(1.4 < T < 6); 1/2

(d) P(T ≤ 5 | T ≥ 2). = (f(5)-f(2))/(1-f(2)) =2/3

6. If the density function of X equals

(1) Find c.

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C = 2

(2) What is P(X > 2)?

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e-4

7. Suppose it is known from large amounts of historical data that X, the number of cars that arrive at a specific intersection during a 20-second time period, is characterized by the following discrete probability function:

(a) Find the probability that in a specific 20-second time period, more than 8 cars arrive at the intersection.

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0.1527625

(b) Find the probability that only 2 cars arrive.

18 \* e-6

8. The joint density of X and Y is given by

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자동 생성된 설명 (a) Compute the marginal probability density of X, i.e., fX(x).

6x/7 + 12x2/7

(b) Compute the marginal probability density of Y, i.e., fY (y).

2/7 + 3y/14