Demystifying Batch Normalization: Analysis of Normalizing Layer Inputs in Neural Networks

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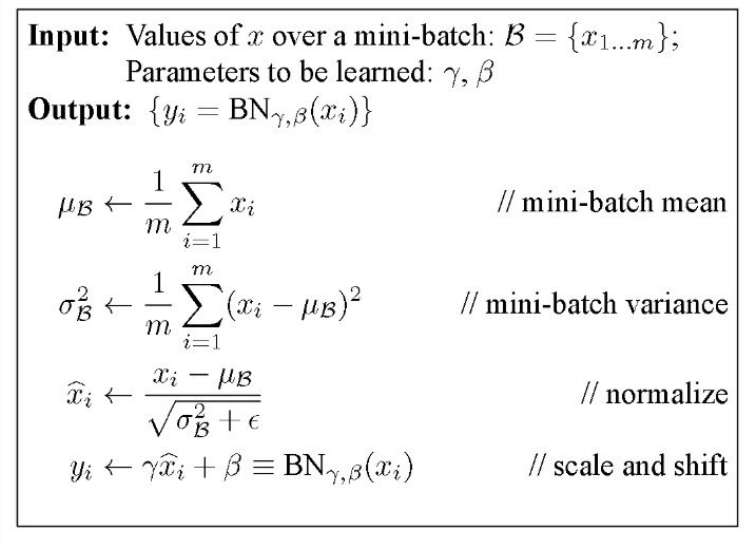
Introduction to Batch Normalization

- Ioffe, S., & Szegedy, C. (2015). Batch normalization: Accelerating deep network training by reducing internal covariate shift
  - Novel mechanism for dramatically accelerating the training of deep networks
  - Solution to “internal covariate shift”
  - Result: better performance than the best known system on ImageNet (4.8% test error)
- Batch Normalization (BN) -> ReLu

Introduction to Batch Normalization

- View 1: Batch Normalization -> ReLu (Ioffe, S., & Szegedy, C. (2015))
- View 2: ReLu -> Batch Normalization (Francois Chollet, Keras author)
- View 3: “There is no difference” (Ian Goodfellow)



Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

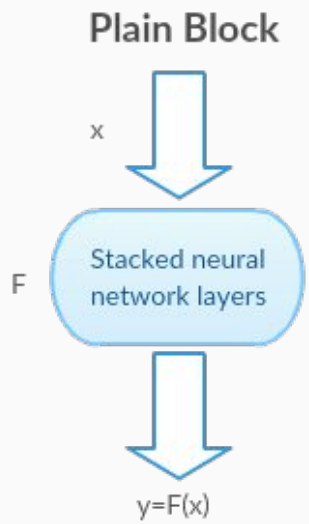
$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma^2_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

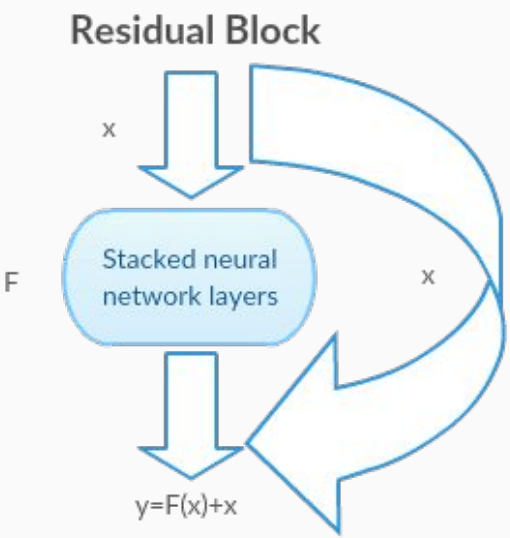
$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma^2_{\mathcal{B}} + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

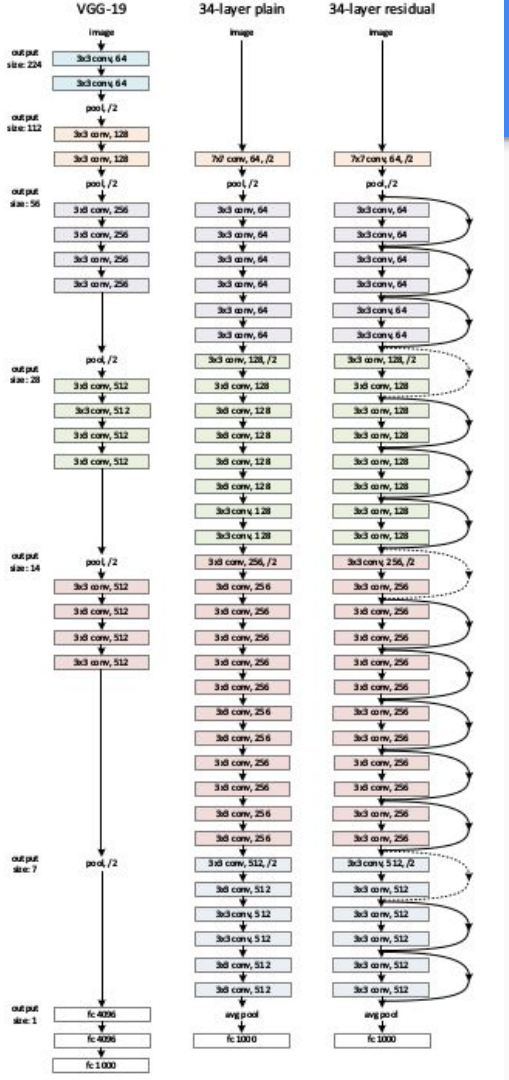
Resnet architecture



Hard to get F(x)=x and make y=x an identity mapping



Easy to get F(x)=0 and make y=x an identity mapping



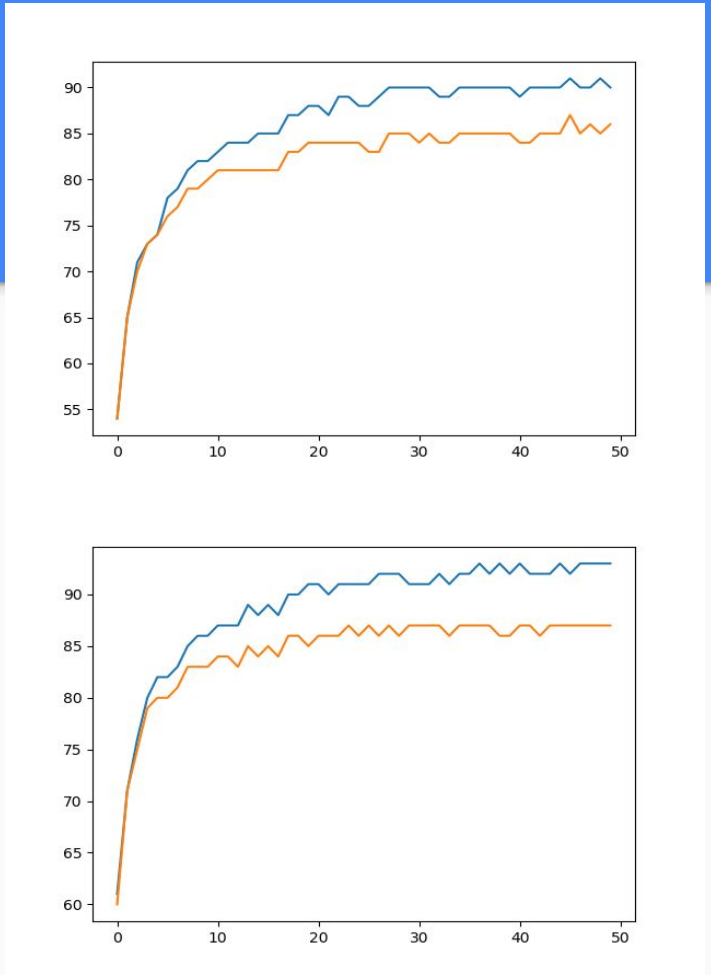
Experiments

- We performed experiments on CNN and Resnet architectures.
- Ran simulations for both the order Batch Normalization->ReLu and ReLU->Batch Normalization and analyzed the results
- Examined the gradients while training for each layer to gain further insight

Results: CNN

- BN->ReLu:
  - Max train accuracy- 91%
  - Max test accuracy- 86%
  - Loss at the end of training: 0.271911

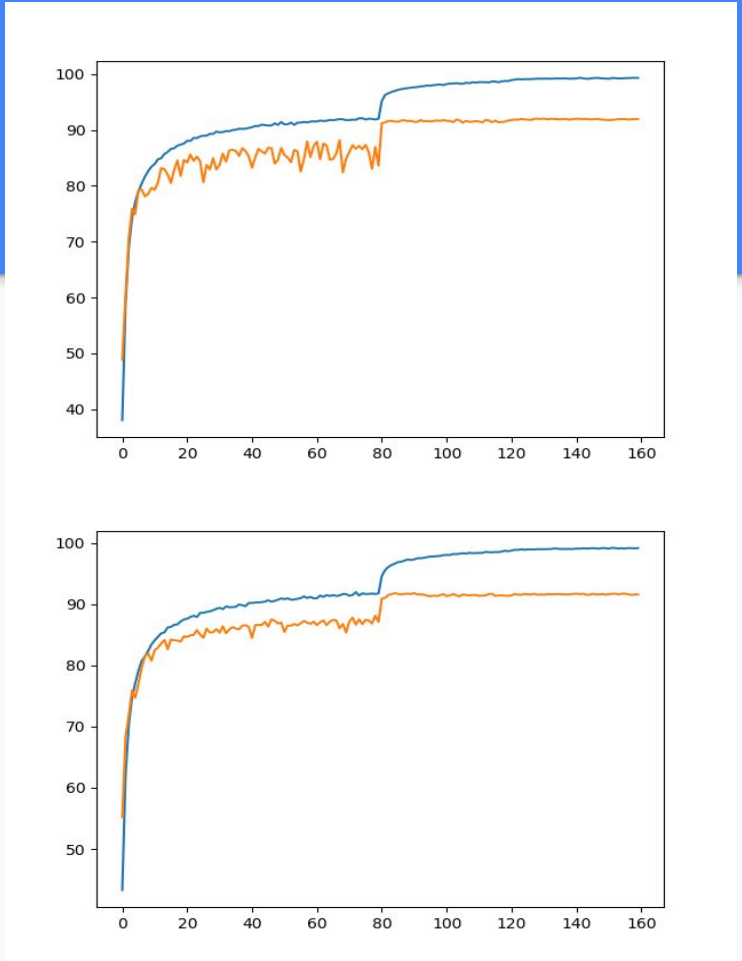
- ReLu->BN:
  - Max train accuracy- 93%
  - Max test accuracy- 87%
  - Loss at the end of training: 0.2120195



Results: Resnet

- BN->ReLu:
  - Max train accuracy- 98%
  - Max test accuracy- 90%
  - Loss at the end of training: 0.1632

- ReLu->BN:
  - Max train accuracy- 98%
  - Max test accuracy- 90%
  - Loss at the end of training: 0.1627

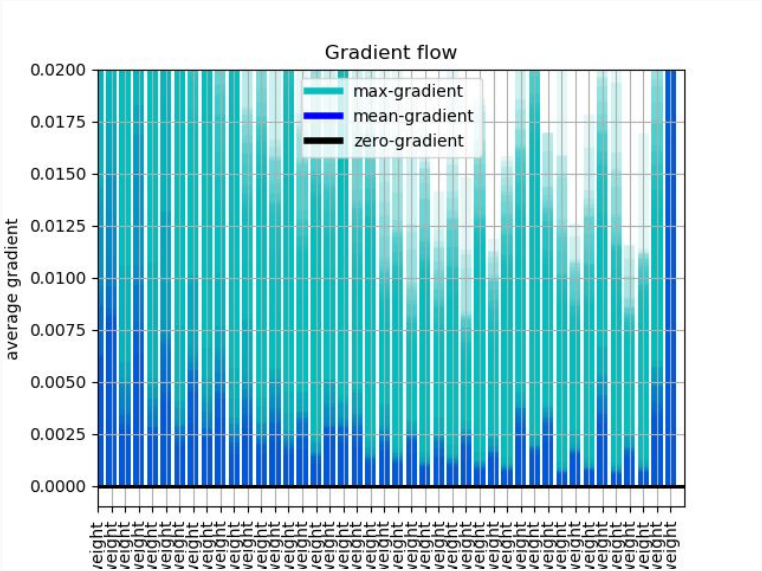


Results

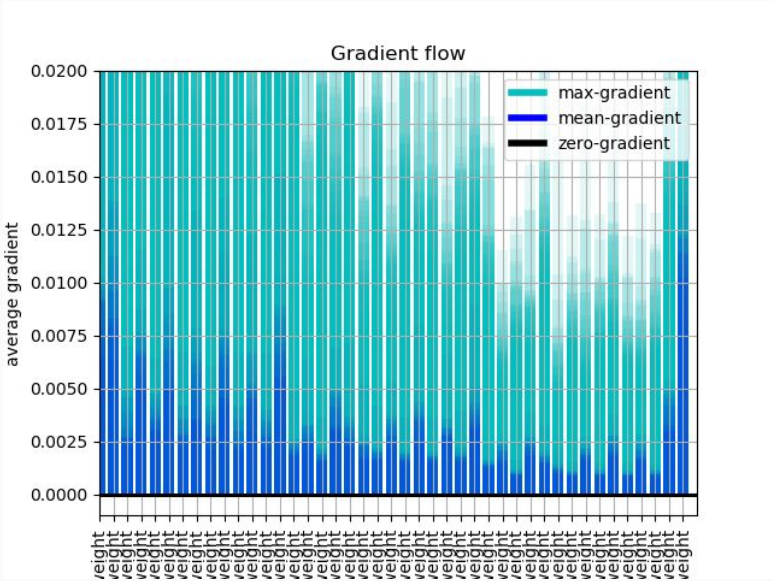
Why this behaviour?

- To explain our results we looked into the gradients while training for each layer. The results below are for Resnet

BN->ReLu

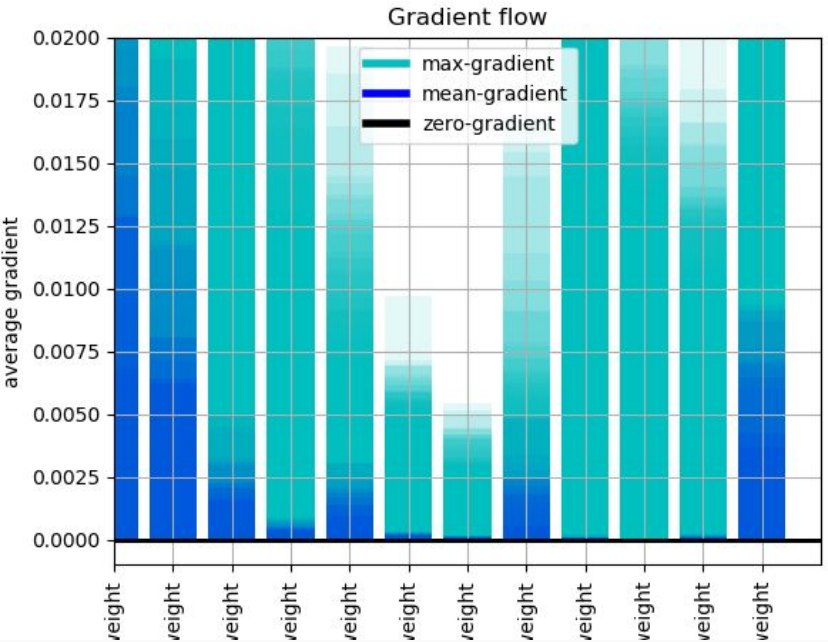


ReLu->BN

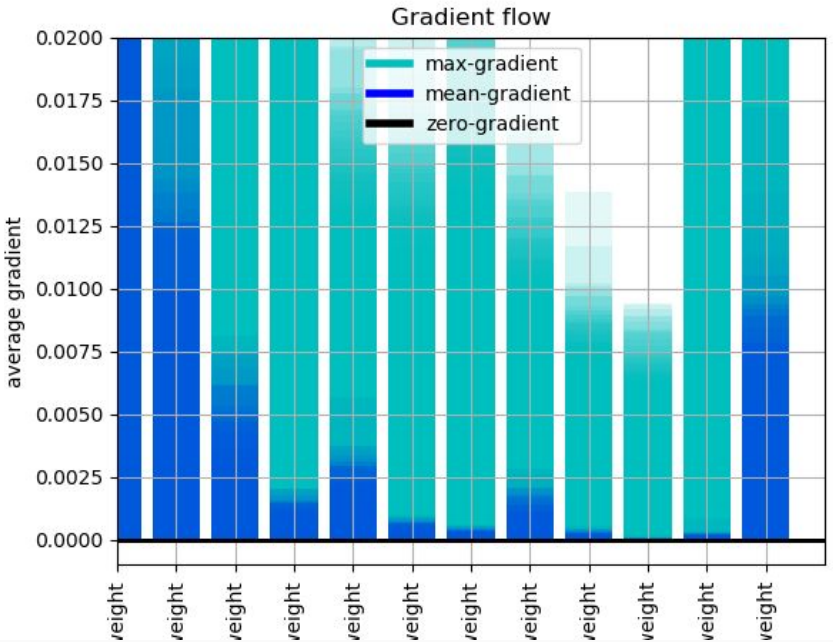


Gradients for CNN

BN->ReLu



ReLu->BN



Inference from the experiments

- Our experiments clearly show that in Convolutional Neural Networks there is an improvement in performance when we switch the order of BN -> ReLu to ReLu->BN.
- This effect was not observed in Residual Nets which have skip connections.
- Examination of gradients shows that in CNN there is an improvement in gradient flow when the ordering is ReLu->BN.
- There is no improvement in gradients for Residual Nets.

Backpropagation of Batch Normalization Overview

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;
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$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma^2_{\mathcal{B}} + \epsilon}} \qquad \text{// normalize}$$

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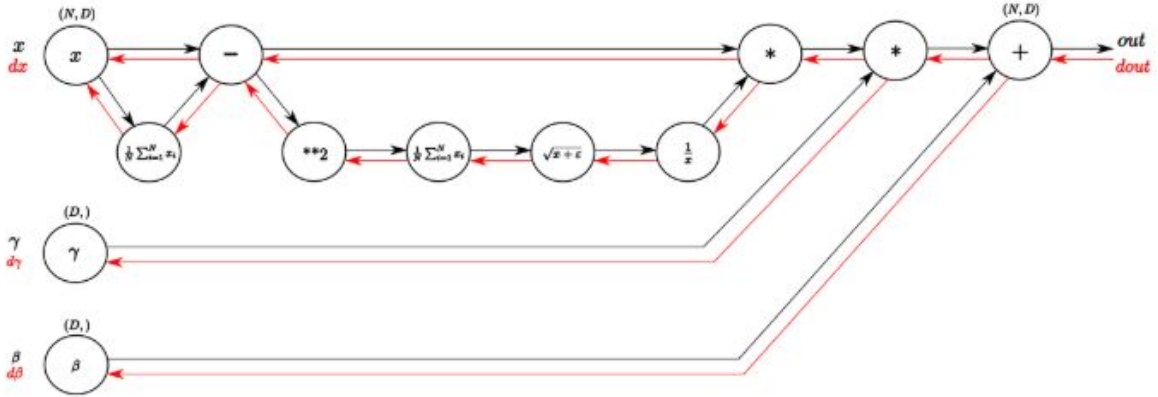
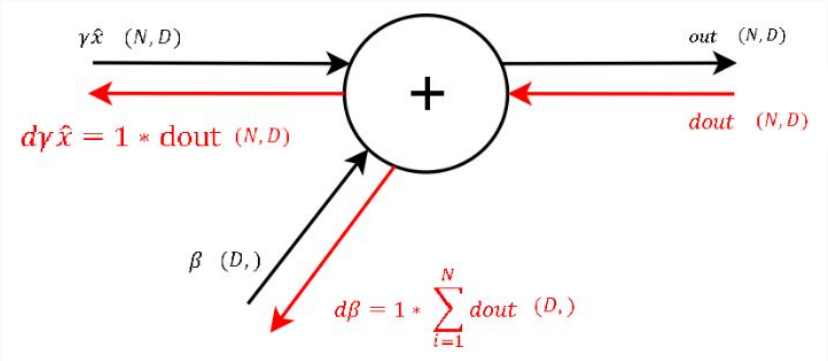




$$BN = \gamma\hat{x} + \beta$$

$$\frac{\partial Out}{\partial \beta} = \frac{\partial Out}{\partial BN}\frac{\partial BN}{\partial \beta} = \frac{\partial Out}{\partial BN}$$

$$\frac{\partial Out}{\partial \gamma} = \frac{\partial Out}{\partial BN}\frac{\partial BN}{\partial \gamma} = \frac{\partial Out}{\partial BN}\hat{x}$$

Backpropagation of BN/ReLU and ReLU/BN

- BN->ReLu:
  - Sparse Gradient
  - Semi-Definite Positive Matrix

$$\frac{\partial Out}{\partial \beta} = \frac{\partial Out}{\partial BN}$$

$$\sim \frac{\partial ReLU_i}{\partial BN} = \begin{bmatrix} 0 & 0.2 & \dots \\ 1 & 0.6 & \dots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$\frac{\partial Out}{\partial \gamma} = \frac{\partial Out}{\partial BN}\hat{x}$$

$$\sim \frac{\partial ReLU_i}{\partial BN} \cdot \hat{x} = \begin{bmatrix} 0 & 0.2 & \dots \\ 0 & 0.6 & \dots \\ \vdots & \vdots & \vdots \end{bmatrix} \cdot \hat{x} = \begin{bmatrix} 0 & 0.2 & \dots \\ 0 & -0.1 & \dots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

- ReLu->BN:
  - More Non-zero gradient

$$\frac{\partial Out}{\partial \beta} = \frac{\partial Out}{\partial BN}$$

$$\sim \frac{\partial w_i}{\partial BN} = \begin{bmatrix} 0 & -0.2 & \dots \\ -1 & 0.6 & \dots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$\frac{\partial Out}{\partial \gamma} = \frac{\partial Out}{\partial BN}\hat{x}$$

$$\sim \frac{\partial w_i}{\partial BN} \cdot \hat{x} = \frac{\partial w_i}{\partial BN} \cdot BN(ReLU(Input)) = \begin{bmatrix} -0.8 & 0.2 & \dots \\ -1 & -0.1 & \dots \\ \vdots & \vdots & \vdots \end{bmatrix} \cdot \frac{\partial w_i}{\partial BN}$$

$$= \begin{bmatrix} -0.1 & 0.7 & \dots \\ -1.3 & 0.8 & \dots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$BN = \gamma\hat{x} + \beta$$

$$\frac{\partial Out}{\partial \beta} = \frac{\partial Out}{\partial BN}\frac{\partial BN}{\partial \beta} = \frac{\partial Out}{\partial BN}$$

$$\frac{\partial Out}{\partial \gamma} = \frac{\partial Out}{\partial BN}\frac{\partial BN}{\partial \gamma} = \frac{\partial Out}{\partial BN}\hat{x}$$

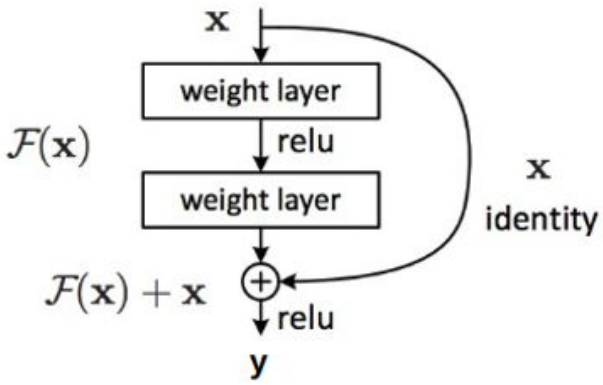
Backpropagation of Residual Networks

- When Gradient of Residual is Dominating
  - Gradient of Error has no effect on overall gradient

$$\frac{\partial E}{\partial y} \ll F^*(x)$$
$$\frac{\partial E}{\partial y} \sim 0$$

- When Gradient of Error is Dominating
  - Gradient of Residual has no effect on overall gradient

$$\frac{\partial E}{\partial y} \gg F^*(x)$$
$$\frac{\partial E}{\partial y} \cdot F^*(x) \sim 0$$



$$y = x + F(x)$$

$$\frac{\delta E}{\delta x} = \frac{\delta E}{\delta y} * \frac{\delta y}{\delta x} = \frac{\delta E}{\delta y} * (1 + F'(x))$$
$$= \frac{\delta E}{\delta y} + \frac{\delta E}{\delta y} * F'(x)$$

Residual

References

- Ioffe, S., & Szegedy, C. (2015). Batch normalization: Accelerating deep network training by reducing internal covariate shift. *arXiv preprint arXiv:1502.03167*.
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