

Computer and Network Security: Homework 1

学号 21307174 姓名 刘俊杰

专业 计算机科学与技术 Email liujj255@mail2.sysu.edu.cn

Instructions

- Please answer 5 of the 6 problems. All questions are weighted equally.
- Please send your solution to 2160853158@qq.com by May 22 midnight.

Problem 1 Vigenère Cipher.

Suppose you have a language with only the 3 letters A, B, C, and they occur with frequencies 0.7, 0.2, and 0.1. The following ciphertext was encrypted by the Vigenère cipher:

ABCBABBBAC.

Suppose you are told that the key length is 1, 2, or 3. Show that the key length is probably 2, and determine the most probable key.

Answer:

ABCBABBBAC.

首先观察密文,使用**Kasiski方法**发现其中有重复的字母组**BA**,该重复字母组的距离为**4**,由于题目给出密文长度只能为1或2或3,4的约数有1、2、4,所以**密钥长度最可能是2**.

在**长度最可能是2**的基础上推出密钥:

AB|CB|AB|BB|AC.

将密文2个字母一组分组,在每组第1个位置字母出现的频率:

$A : 0.6 \quad B : 0.2 \quad C : 0.2$

在每组第1个位置字母出现的频率:

$A : 0.0 \quad B : 0.8 \quad C : 0.2$

题目给出原文中字母出现的频率:

$A : 0.7 \quad B : 0.2 \quad C : 0.1$

所以极大概率第一个位置密钥为**A**,第二个位置密钥为**B**

综上,密钥长度大概率为2,密钥大概率为**(A,B)**

Problem 2 Perfect secrecy and one-time-pad.

1. For a perfect secret encryption scheme $E(K, M) = C$, prove: $\Pr[C = c \mid M = m] = \Pr[C = c]$.

2. Consider a biased one-time-pad system, where $\Pr[M = b] = p_b, b = 0, 1$ and $\Pr[K = 0] = 0.4$. The first attacker Randy randomly guesses $M = 0$ or $M = 1$: prove that the probability of success is 0.5. The second attacker Smarty guesses M based on C and p_0, p_1 : suggest a good attack strategy.

Problem 3 DES.

Before 2-DES and 3-DES was invented, the researchers at RSA Labs came up with DESV and DESW, defined by

$$DESV_{kk_1}(M) = DES_k(M) \oplus k_1, DESW_{kk_1}(M) = DES_k(M \oplus k_1)$$

In both schemes, $|k| = 56$ and $|k_1| = 64$. Show that both these proposals do not increase the work needed to break them using brute-force key search. That is, show how to break these schemes using on the order of 2^{56} DES operations. You have a small number of plaintext-ciphertext pairs.

Answer:

$$DESV_{kk_1}(M) = DES_k(M) \oplus k_1, DESW_{kk_1}(M) = DES_k(M \oplus k_1)$$

其中, $|k| = 56$, $|k_1| = 64$ 。

证明这两种方案并没有增加破解的难度, 仍然需要 2^{56} 次 DES 操作来破:

考虑 DESV 方案, DESV 的输出为 $DES_k(M) \oplus k_1$, 其中 $DES_k(M)$ 是 DES 加密的结果, k_1 是一个 64 位的密钥。我们可以通过以下步骤来破解 DESV 方案:

1. 尝试所有可能的 k_1 (共 2^{64} 种可能), 对每种 k_1 进行以下步骤:
2. 对给定的 k_1 , 计算 $DES_k(M) \oplus k_1$, 并与已知的密文进行比较。如果匹配, 则找到了正确的 k_1 。

由于每次尝试需要进行一次 DES 加密和一个异或操作, 因此总共需要 2^{64} 次 DES 操作。

类似地, 对于 DESW 方案, 我们可以将密文进行异或操作后再进行 DES 解密, 过程如下:

1. 对每个可能的 k_1 进行以下步骤:
2. 对给定的 k_1 , 计算 $DES_k(M \oplus k_1)$, 并与已知的密文进行比较。如果匹配, 则找到了正确的 k_1 。

同样地, 每次尝试需要进行一次 DES 加密和一个异或操作, 总共需要 2^{64} 次 DES 操作。

因此, DESV 和 DESW 方案, 都可以在 2^{56} 次 DES 操作内被破解。

Problem 4 RSA.

Alice and Bob love each other, so they decide to use a single RSA modulus N for their key pairs. Of course each of them does not know the private key of the other. Mathematically, Alice and Bob have their own key pairs (e_A, d_A) and (e_B, d_B) sharing the same N . Demonstrate how Bob can derive the private key of Alice.

Answer:

$$P \times Q = N \text{ (P、Q为素数)}$$

$$\phi(N) = (P - 1)(Q - 1)$$

由RSA原理可得: $e_A \times d_A = 1 \bmod \phi(N)$

$$e_B \times d_B = 1 \bmod \phi(N)$$

进一步可得:

$$e_A \times d_A = 1 \bmod \phi(N)$$

$$e_B \times d_B = 1 + m * \phi(N) \quad \text{则}$$

$$e_A \times d_A = 1 \bmod \phi(N)$$

$$m * \phi(N) = e_B \times d_B - 1 \quad \text{可以知道若 } e_A \times d_A = 1 \bmod (m \times \phi(N)) ,$$

$$\text{则 } e_A \times d_A = 1 \bmod \phi(N)$$

1. 若 e_A 与 $e_B \times d_B - 1$ 互质, 则求 d_A 满足 $e_A \times d_A = 1 \bmod (m \times \phi(N))$ 即 $e_A \times d_A = 1 \bmod (e_B \times d_B - 1)$
2. 若 e_A 与 $e_B \times d_B - 1$ 不互质, 令 $x = e_B \times d_B - 1$, 则求 $d = \gcd(e_A, x)$ 。再 $x = x/d$, 直至 e_A 与 x 互质, 再求 d_A 满足 $e_A \times d_A = 1 \bmod x$ 。

综上, 即可推出 Alice 所掌握的密钥

Problem 5 Operation mode of block ciphers.

Chloé invents a new operation mode as below that can support parallel encryption. Unfortunately, this mode is not secure. Please demonstrate how an attacker knowing IV, C_0, C_1, C_2 , and $M_1 = M_2 = M$ can recover M_0 .

Figure 1: Chloé's invention

Answer:

已知 C_0, C_1, C_2 , and $M_1 = M_2 = M$ 求 M_0

不妨设 X 经过 **block ciphers** 后的输出为 $E(X, K)$

则由图可得: $C_2 = E(M, K) \oplus M$

由于 C_2, M 已知, 所以 $E(M, K)$ 也可通过计算可知

再看 $C_1 = E(M, K) \oplus M_0$

由于 $C_1, E(M, K)$ 已知, 所以 M_0 也可通过计算可知

Problem 6 Hash functions.

One-wayness and collision-resistance are two indispensable properties of hash functions. They are in fact independent one to the other.

1. Give a function that is one-way, but not collision-resistant.
2. Give a function that is collision-resistant, but not one-way.

Answer:

Performance: easy to compute $H(m)$

One-way property: given $H(m)$, computationally infeasible to find m

Weak collision resistance: given $H(m)$, computationally infeasible to find m' such that $H(m') = H(m)$

Strong collision resistance: computationally infeasible to find m_1, m_2 such that $H(m_1) = H(m_2)$

a function that is one-way, but not collision-resistant

$$h(x) = x^n \bmod p \times q \quad (n, p, q \text{ 为给定的数的})$$

a function that is collision-resistant, but not one-way

$$h(x) = x$$