Let $U = \{U_0, U_1, U_2, U_3\}$ be an open covering indexed by I and $V = \{V_0, V_1\}$ be another open covering indexed by J. Say U refines V by

$$\begin{aligned} U_0 \subseteq V_1, &\quad U_1 \subseteq V_0 \\ U_2 \subseteq V_1, &\quad U_3 \subseteq V_0, \end{aligned}$$

call this refinement τ.

Consider the diagram

$$\begin{array}{ccc} C(U,0) & \xrightarrow{d} & C(U,1) \\ \tau\big\uparrow & & \tau\big\uparrow \\ C(V,0) & \xrightarrow{d} & C(V,1), \end{array}$$

where τ stands for the refinement map

$$\tau(f)(i_0, \ldots, i_p) = f(\tau(i_0), \ldots, \tau(i_p)) \mid_{U_{i_0} \cap \cdots \cap U_{i_p}} .$$

Let $f \in C(V,0)$, then $\tau df \in C(U,1)$, then

$$\begin{aligned} \tau df\{0,3\} &= df\{0,1\} \mid_{U_0 \cap U_3} \\ &= (f\{1\} \mid_{V_0 \cap V_1}) \mid_{U_0 \cap U_3} - (f\{0\} \mid_{V_0 \cap V_1}) \mid_{U_0 \cap U_3} \\ &= f\{1\} \mid_{U_0 \cap U_3} - f\{0\} \mid_{U_0 \cap U_3}, \\ d\tau f\{0,3\} &= \tau f\{3\} \mid_{V_0 \cap V_3} - \tau f\{0\} \mid_{V_0 \cap V_3} \\ &= (f\{0\} \mid_{U_0}) \mid_{V_0 \cap V_3} - (f\{1\} \mid_{U_0}) \mid_{V_0 \cap V_3} . \end{aligned}$$

They have different signs!