talk

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```
[2]: %load_ext notexbook
     %texify
```

```
The notexbook extension is already loaded. To reload it, use:
  %reload_ext notexbook
```

```
[2]: <IPython.core.display.HTML object>
```

1 Proj as a Scheme

1.1 Definitions

1.1.1 Scheme

Scheme is a locally ringed space X that is locally affine, i.e. for any $x \in X$, there is an open neighourhood $x \in U$, such that $X|_U \cong \mathrm{Spec} R$ for some commutative ring R

```
structure Scheme extends to_LocallyRingedSpace : LocallyRingedSpace :=
(local_affine :  x : to_LocallyRingedSpace,  (U : open_nhds x) (R : CommRing),
  nonempty (to_LocallyRingedSpace.restrict U.open_embedding
    Spec.to_LocallyRingedSpace.obj (op R)))
```

1.1.2 Locally ringed space

A locally ringed space is a sheafed space X such that for any $x \in X$, the stalk at x is a local ring.

```
structure LocallyRingedSpace extends SheafedSpace CommRing :=
(local_ring :  x, local_ring (presheaf.stalk x))
```

1.1.3 Sheafed space

A sheafted space is a presheafed space whose structure presheaf is a sheaf.

```
structure SheafedSpace extends PresheafedSpace C :=
(is_sheaf : presheaf.is_sheaf)
```

1.1.4 Presheafed space

A presheafed space is a topological space X with a structure presheaf on it.

```
structure PresheafedSpace :=
(carrier : Top)
(presheaf : carrier.presheaf C)
```

Given a $\mathbb{N}$-graded ring, we need a - a topology - a sheaf - a proof that stalks are local ring - an affine cover

1.2 Zariski Topology