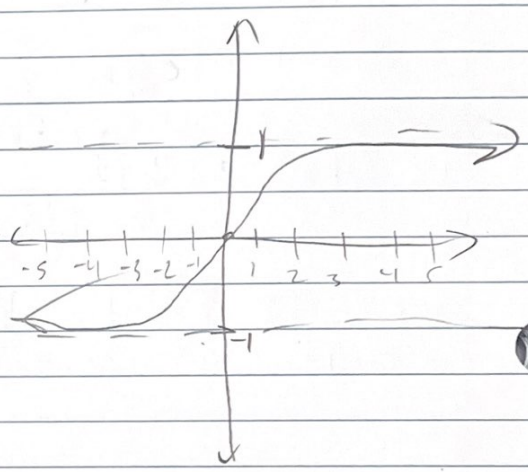
Machine Learning

① $\tanh(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$ inputs $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

$\tanh(-5) = \frac{e^{2(-5)} - 1}{e^{2(-5)} + 1} = \frac{e^{-10} - 1}{e^{-10} + 1} = -0.999$

$\tanh(-4) = -0.999$
$\tanh(-3) = -0.995$
$\tanh(-2) = -0.964$
$\tanh(-1) = -0.762$
$\tanh(0) = 0$
$\tanh(1) = 0.762$
$\tanh(2) = 0.964$
$\tanh(3) = 0.995$
$\tanh(4) = 0.999$
$\tanh(5) = 0.999$



② $s_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $s_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $s_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

1) $2s_1 + 3s_2 + 4s_3 = v$

$2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = v$

$\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} = v$

$v = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$

$$\underset{S}{\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}}\underset{X}{\begin{bmatrix}2\\5\\9\end{bmatrix}}=\begin{bmatrix}2\\7\\16\end{bmatrix}$$

2) $M=\begin{bmatrix}2&0&0\\2&3&0\\2&3&4\end{bmatrix}$ $S=\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}$

$$\left[\begin{array}{ccc|ccc}1&0&0&2&0&0\\1&1&0&2&3&0\\1&1&1&2&3&4\end{array}\right]$$

r_2-r_1 & r_3-r_1

$$\left[\begin{array}{ccc|ccc}1&0&0&2&0&0\\0&1&0&0&3&0\\0&1&1&0&3&4\end{array}\right]$$

r_3-r_2

$$\left[\begin{array}{ccc|ccc}1&0&0&2&0&0\\0&1&0&0&3&0\\0&0&1&0&0&4\end{array}\right]$$

$A=\begin{bmatrix}2&0&0\\0&3&0\\0&0&4\end{bmatrix}$

$$\underset{S}{\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}}\underset{A}{\begin{bmatrix}2&0&0\\0&3&0\\0&0&4\end{bmatrix}}=\begin{bmatrix}2&0&0\\2&3&0\\2&3&4\end{bmatrix}=M \quad \checkmark$$

(3) $\bar{x}=\dfrac{25+26+38+45+31+30+29+43}{8}=33.375$

$$s^2=\frac{(25-\bar{x})^2+(26-\bar{x})^2+(38-\bar{x})^2+(45-\bar{x})^2+(31-\bar{x})^2+(30-\bar{x})^2+(29-\bar{x})^2+(43-\bar{x})^2}{8-1}$$

$s^2\approx 59.55$

(4) $P(\text{die} \mid \text{forgot}) = 0.8$, $P(\text{die} \mid \text{rem}) = 0.15$, $P(\text{Rem}) = 0.9$,
$P(\text{forgot}) = 0.1$

a) $P(\text{alive}) = 1 - P(\text{dead})$
$= 1 - (P(d \mid f) \cdot P(f) + P(d \mid r) \cdot P(r))$
$= 1 - (0.8 \cdot 0.1 + 0.15 \cdot 0.9)$
$= 1 - 0.215$
$= 0.785$

b) $P(\text{forgot} \mid \text{dead}) = \frac{P(d \mid f) \cdot P(f)}{P(d)}$
$= \frac{0.8 \cdot 0.1}{0.215}$
$= 0.372$