CE422 Project 1

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Contents

Problem Description **2**

Modeling Considerations **2**

Hand Calculations **3**

Determinate Model **4**

Indeterminate Model **6**

Conclusion **9**

Appendix **10**

Detailed Hand Calculations 10
APDL Model Setup 11
Determinate Model Inputs and Outputs 12
Indeterminate Model Inputs and Outputs 14

Problem Description

A 2D truss system was to be designed with boundary conditions that ensure the structural system is stable and determinate. The truss must contain 20 or more truss members. Using FEA software, the truss was to be modeled and subjected to static loading and evaluated for the nodal deflections, member stresses, and reaction forces. The simulated results were to be verified by hand calculations. The simulation was then to be repeated with a boundary condition change that results in an indeterminate structure and evaluated for the same results.

Modeling Considerations

The determinate truss modeled in this report is shown in Figure 1. The truss contains 12 nodes, labeled A to L, and 21 identical members, which will be referenced by its two nodes in alphabetical ordered (i.e. member FH rather than HF). Each member is $5m$ long and has a square cross-section of $2.54cm \times 2.54cm$; the members are all made of steel, which has a Young's modulus of $200 \times 10^9 Pa$ and yield stress of $350 \times 10^6 Pa$.[2] Two vertical point loads of $10kN$ and $25kN$ are applied at nodes B and L, respectively.

For the determinate model, a pinned support is applied to node A and a roller support to node H, resulting in three reaction forces (A_x, A_y, and H_y). A truss system is said to be statically determinate if the number of members plus reaction forces is equal to the number of nodes; the model used for this project meets those criteria.[1] For the indeterminate model, the same truss is used, except the roller support at node H is changed to a pinned support, resulting in four reaction forces (A_x, A_y, H_x, and H_y) such that the system does not meet the requirements for it to be considered statically determinate.

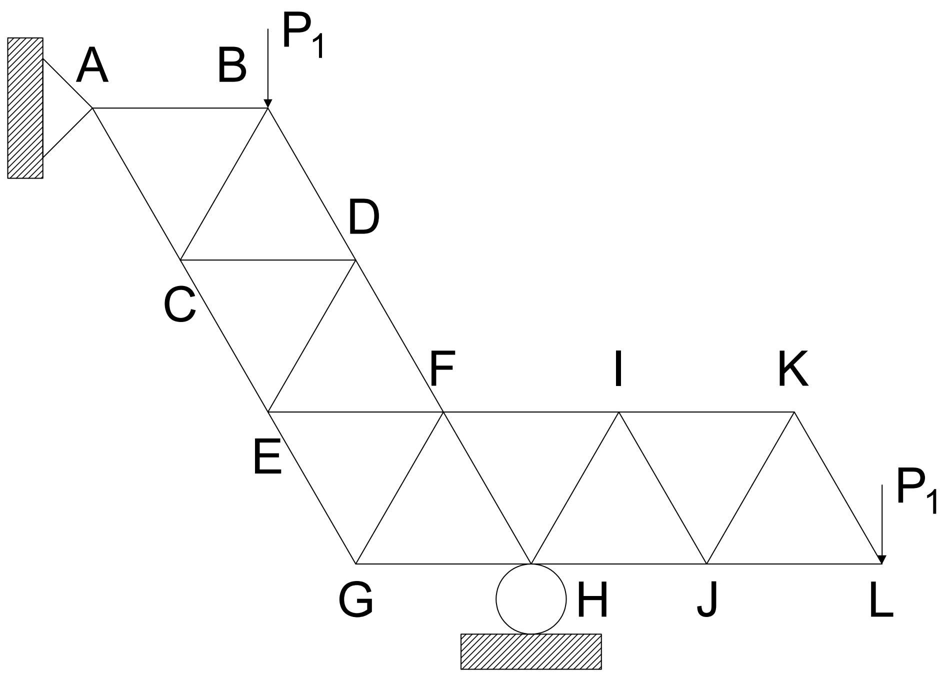

Figure 1: Determinate truss system used in this project. The indeterminate truss is identical except for a pinned support at node H rather than a roller support.

Hand Calculations

Hand calculations were performed for the determinate system to verify the simulation results. The system is static and determinate, so the sum of forces in the x and y directions should be zero for any node or collection of nodes. The x and y statics equations for each of the 24 nodes was determined, creating a system of equations describing the truss system (the 24 equations are reproduced in *Detailed Hand Calculations*).

The equations were transcribed into Python where the system of equations was transformed into the matrix equation $A\mathbf{x} = \mathbf{b}$, and solved for x, the force in each member. The stress, σ, of each member can be calculated using the following formula:

$$\sigma_i = \frac{F_i}{A} \tag{1}$$

where F_i is the force in the member i and A is the cross-sectional area of the member, which is identical for all members. The strain, ϵ, for each member can also be calculated using the Young's modulus as follows.

$$\epsilon_i = \frac{\sigma_i}{E} \tag{2}$$

where E is the Young's modulus of the material, which in this case was steel for all the members.

The computed reaction forces are summarized in Table 1. The member experiencing the least stress (in magnitude) was BD and the member experiencing the most stress (in magnitude) was FI. The load, stress, and strain for these two members are tabulated in Table 2. The remaining member loads, stresses, and strains can be found in *Detailed Hand Calculations* in Table 5.

Reaction	Force [N]
Ax	0.00
Ay	-14000.00
Hy	49000.00

Table 1: Reaction forces at nodes A and H.

Member	Load [N]	Stress [Pa]	Strain
BD	2.31×10^3	3.58×10^6	1.79×10^{-5}
FI	5.77×10^4	8.95×10^7	4.47×10^{-4}

Table 2: BD and FI member loads, stresses, and strains. A positive value denotes a tensile load, stress, or strain.

Determinate Model

The simulation was performed using Ansys APDL finite element software. The LINK180 element was chosen to model the members. LINK180 is a uniaxial compression and tension element with three degrees of freedom per node, which are translation in the x, y, and z directions. A structural, linear, elastic, isotropic material was chosen, and the Young's modulus for steel was input. The cross-sectional area was set to $6.4516 \times 10^{-4} m^2$.

Nodes were created at the corresponding nodal positions of the model, with node A being set as the origin (the nodal coordinates can be found in *APDL Model Setup*). Elements were then used to connect the nodes appropriately. Because this is a 2D analysis, all nodes were restricted from displacing in the z direction. Node A was also limited from displacing in the x and y directions and node H was limited from displacing in the y direction, modeling the pinned and roller supports, respectively. The appropriate loads were applied in the $-y$ direction at nodes B and L.

The simulation was run and quickly produced a solution. The undeformed (black) and deformed (navy) model can be seen in Figure 2. The member forces can be seen in Figure 3 and the member stresses in Figure 4 (unfortunately, I could not figure out how to change the coloring, so the yellow is difficult to see).

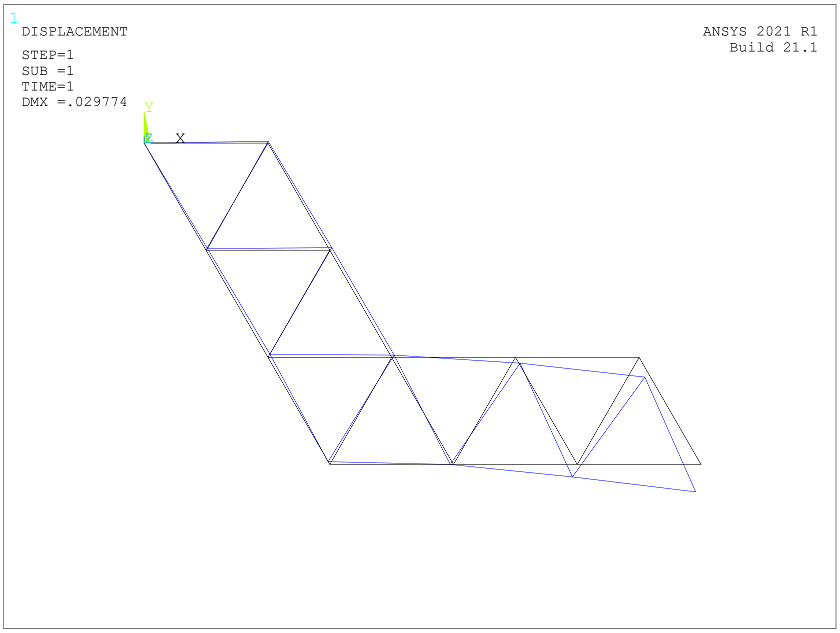

Figure 2: Undeformed and deformed determinate model.

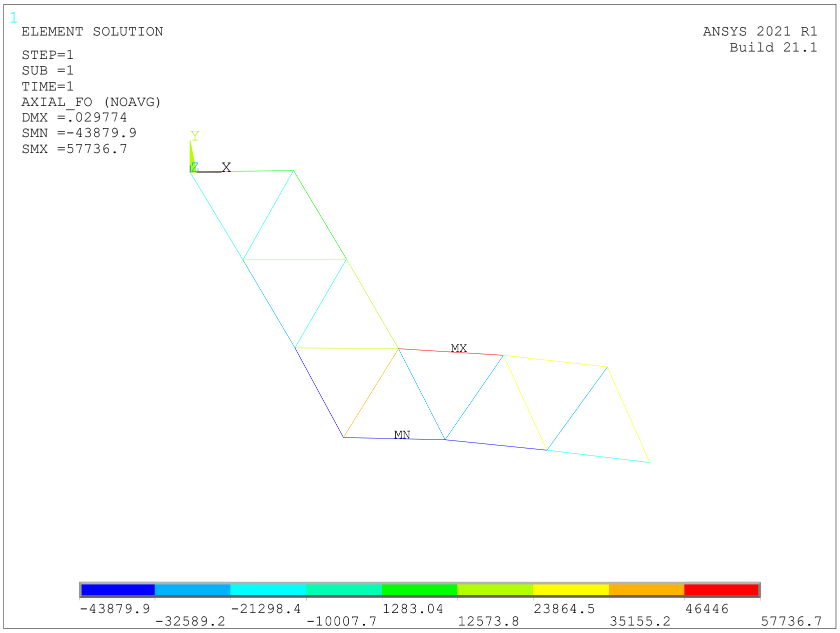

Figure 3: Deformed determinate model with coloring based on axial force.

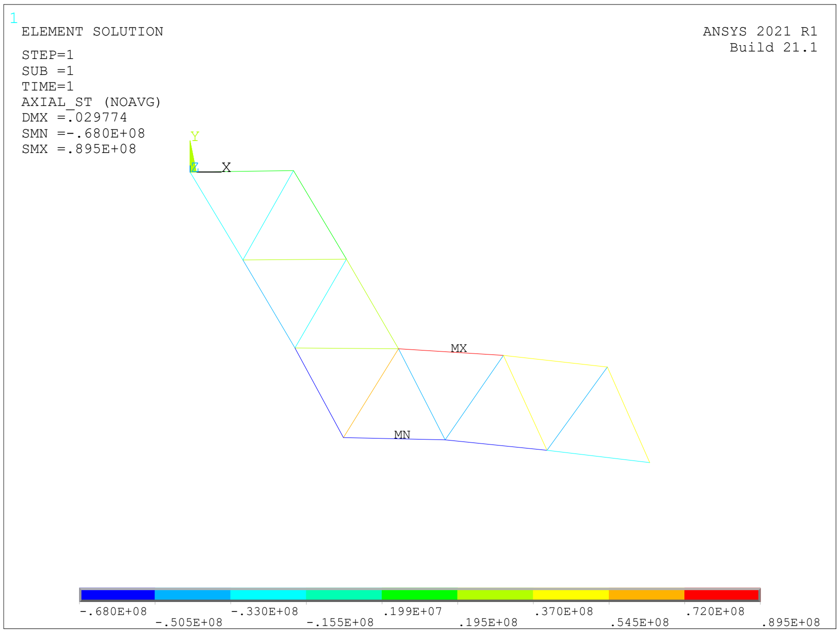

Figure 4: Deformed determinate model with coloring based on axial stress.

Because the static hand calculations exactly model the physics, the reaction forces and member load, stress, and strain results from Ansys were identical to those produced by hand calculation, verifying the simulation results. Because the simulation results were verified by the hand calculations, the displacement results can be trusted. The displacement results showed that node L displaced the most, displacing about 29.8mm from its original position; this is not surprising since node L is extended away from the supports and has a load directly acting on it. The tabulated simulation results can be found in Tables 7 - 9 in *Determinate Model Inputs and Outputs*.

Indeterminate Model

The same simulation file generated for the determinate model was used as the starting point for the indeterminate model. Only a single change was made, which was that the for the indeterminate model, node H was additionally limited from displacing in the x direction (in total, H was limited from displacing in the x, y, and z directions), modeling a pinned support at node H.

Again, the simulation was run and quickly produced a solution. The undeformed (black) and deformed (navy) model can be seen in Figure 5. The member forces can be seen in Figure 6 and the member stresses in Figure 7 (again, I could not figure out how to change the coloring, so the yellow is difficult to see).

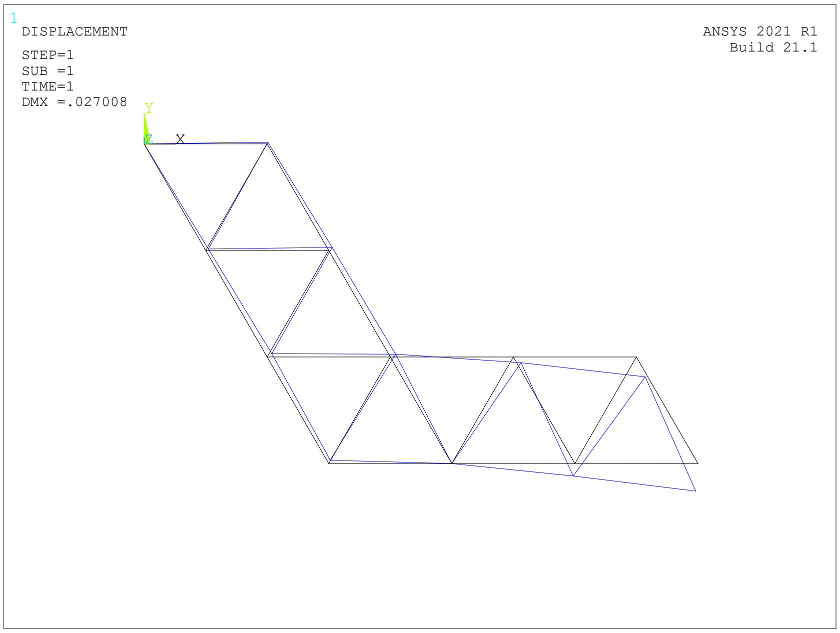

Figure 5: Undeformed and deformed indeterminate model.

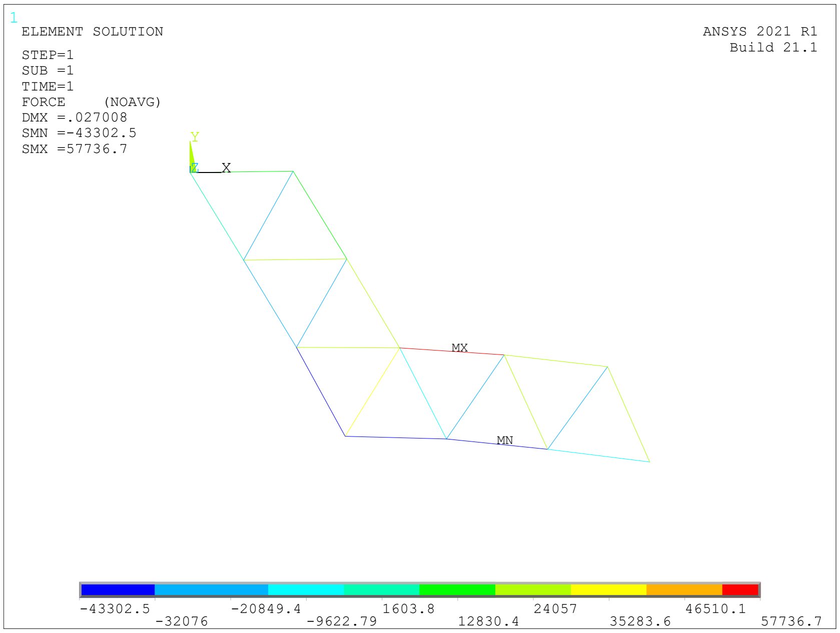

Figure 6: Deformed indeterminate model with coloring based on axial force.

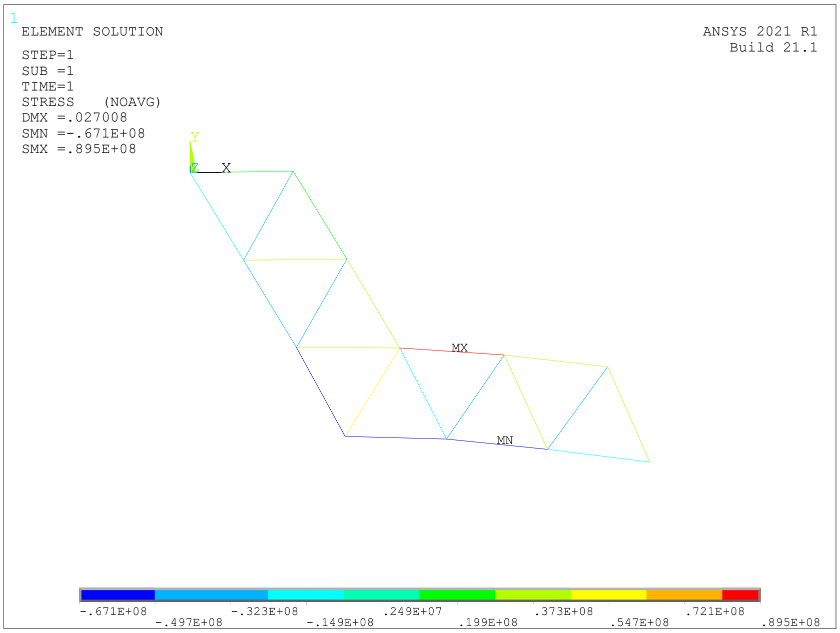

Figure 7: Deformed indeterminate model with coloring based on axial stress.

Because the boundary conditions changed, the reaction forces and nodal displacements as well as the member loads, stresses, and strains changed, now that the loads are redistributed. The indeterminate model reaction forces can be found below in Table 3. Because the node H boundary condition now restricts motion in the x direction, there is now a non-zero x reaction at A and H. Like in the determinate model, the highest load (in magnitude) was found in member FI, but the minimum load (in magnitude) was now found in member AC; the load, stress, and strain for members FI and AC can be found in Table 4. The largest displacement was still node L, but it is reduced to $27.0mm$. The tabulated simulation results can be found in Tables 10 - 12 in *Indeterminate Model Inputs and Outputs*.

Reaction	Force [N]
Ax	-16084
Ay	2714.2
Hx	16084
Hy	32286

Table 3: Indeterminate model reaction forces.

Member	Load [N]	Stress [Pa]	Strain
AC	3.13×10^3	4.86×10^6	2.43×10^{-5}
FI	5.77×10^4	8.95×10^7	4.47×10^{-4}

Table 4: AC and FI member loads, stresses, and strains.A positive value denotes a tensile load, stress, or strain

Because the hand calculations verified the determinate model, and the indeterminate model used the same simulation except for a single change, the indeterminate simulation results can be trusted to be accurate.

Conclusion

Overall, the structural truss was designed and simulated in Ansys APDL. The determinate truss simulation results were verified using hand calculations that were based on the static structure and solved in Python; these results matched exactly. The indeterminate simulation used the same simulation file but changed the node H boundary condition from roller to pinned. In the end, the indeterminate system was more rigid, causing less nodal displacement than the determinate model.

Appendix

Detailed Hand Calculations

Below are the 24 x and y statics equations performed for the determinate truss model.

$$\sum F_{A,x} = 0 = F_{AB} + F_{AC}\cos(\theta) + A_x \tag{3}$$

$$\sum F_{A,y} = 0 = -F_{AC}\sin(\theta) + A_y \tag{4}$$

$$\sum F_{B,x} = 0 = -F_{AB} - F_{BC}\cos(\theta) + F_{BD}\cos(\theta) \tag{5}$$

$$\sum F_{B,y} = 0 = -F_{BC}\sin(\theta) - F_{BD}\sin(\theta) - P_1 \tag{6}$$

$$\sum F_{C,x} = 0 = -F_{AC}\cos(\theta) + F_{BC}\cos(\theta) + F_{CD} + F_{CE}\cos(\theta) \tag{7}$$

$$\sum F_{C,y} = 0 = F_{AC}\sin(\theta) + F_{BC}\sin(\theta) - F_{CE}\sin(\theta) \tag{8}$$

$$\sum F_{D,x} = 0 = -F_{CD} - F_{BD}\cos(\theta) - F_{DE}\cos(\theta) + F_{DF}\cos(\theta) \tag{9}$$

$$\sum F_{D,y} = 0 = F_{BD}\sin(\theta) - F_{DE}\sin(\theta) - F_{DF}\sin(\theta) \tag{10}$$

$$\sum F_{E,x} = 0 = -F_{CE}\cos(\theta) + F_{DE}\cos(\theta) + F_{EF} + F_{EG}\cos(\theta) \tag{11}$$

$$\sum F_{E,y} = 0 = F_{CE}\sin(\theta) + F_{DE}\sin(\theta) - F_{EG}\sin(\theta) \tag{12}$$

$$\sum F_{F,x} = 0 = -F_{DF}\cos(\theta) - F_{EF} - F_{FG}\cos(\theta) + F_{FH}\cos(\theta) + F_{FI} \tag{13}$$

$$\sum F_{F,y} = 0 = F_{DF}\sin(\theta) - F_{FG}\sin(\theta) - F_{FH}\sin(\theta) \tag{14}$$

$$\sum F_{G,x} = 0 = -F_{EG}\cos(\theta) + F_{FG}\cos(\theta) + F_{GH} \tag{15}$$

$$\sum F_{G,y} = 0 = F_{EG}\sin(\theta) + F_{FG}\sin(\theta) \tag{16}$$

$$\sum F_{H,x} = 0 = -F_{FH}\cos(\theta) - F_{GH} + F_{HI}\cos(\theta) + F_{HJ} \tag{17}$$

$$\sum F_{H,y} = 0 = F_{FH}\sin(\theta) + F_{HI}\sin(\theta) + H_y \tag{18}$$

$$\sum F_{I,x} = 0 = -F_{FI} - F_{HI}\cos(\theta) + F_{IJ}\cos(\theta) + F_{IK} \tag{19}$$

$$\sum F_{I,y} = 0 = -F_{HI}\sin(\theta) - F_{IJ}\sin(\theta) \tag{20}$$

$$\sum F_{J,x} = 0 = -F_{HJ} - F_{IJ}\cos(\theta) + F_{JK}\cos(\theta) + F_{JL} \tag{21}$$

$$\sum F_{J,y} = 0 = F_{IJ}\sin(\theta) + F_{JK}\sin(\theta) \tag{22}$$

$$\sum F_{K,x} = 0 = -F_{IK} - F_{JK}\cos(\theta) + F_{KL}\cos(\theta) \tag{23}$$

$$\sum F_{K,y} = 0 = -F_{JK}\sin(\theta) - F_{KL}\sin(\theta) \tag{24}$$

$$\sum F_{L,x} = 0 = -F_{JL} - F_{KL}\cos(\theta) \tag{25}$$

$$\sum F_{L,y} = 0 = F_{KL}\sin(\theta) - P_2 \tag{26}$$

In the above equations, θ is the angle off of the x-axis.

The elemental load, stress, and strain results are tabulated below.

Member	Load [N]	Stress [Pa]	Strain
AB	8.08×10^3	1.25×10^7	6.26×10^{-5}
AC	-1.62×10^4	-2.51×10^7	-1.25×10^{-4}
BC	-1.39×10^4	-2.15×10^7	-1.07×10^{-4}
BD	2.31×10^3	3.58×10^6	1.79×10^{-5}
CD	1.39×10^4	2.15×10^7	1.07×10^{-4}
CE	-3.00×10^4	-4.65×10^7	-2.33×10^{-4}
DE	-1.39×10^4	-2.15×10^7	-1.07×10^{-4}
DF	1.62×10^4	2.51×10^7	1.25×10^{-4}
EF	1.39×10^4	2.15×10^7	1.07×10^{-4}
EG	-4.39×10^4	-6.80×10^7	-3.40×10^{-4}
FG	4.39×10^4	6.80×10^7	3.40×10^{-4}
FH	-2.77×10^4	-4.30×10^7	-2.15×10^{-4}
FI	5.77×10^4	8.95×10^7	4.47×10^{-4}
GH	-4.39×10^4	-6.80×10^7	-3.40×10^{-4}
HI	-2.89×10^4	-4.47×10^7	-2.24×10^{-4}
HJ	-4.33×10^4	-6.71×10^7	-3.36×10^{-4}
IJ	2.89×10^4	4.47×10^7	2.24×10^{-4}
IK	2.89×10^4	4.47×10^7	2.24×10^{-4}
JK	-2.89×10^4	-4.47×10^7	-2.24×10^{-4}
JL	-1.44×10^4	-2.24×10^7	-1.12×10^{-4}
KL	2.89×10^4	4.47×10^7	2.24×10^{-4}

Table 5: Member loads, stresses, and strains. A positive value denotes a tensile load, stress, or strain while a negative value denotes a compressive load, stress, or strain.

APDL Model Setup

Below are the nodes and their x and y coordinates in the Cartesian plane.

Node	X	Y
A	0	0
B	5	0
C	2.5	-4.33013
D	7.5	-4.33013
E	5	-8.66025
F	10	-8.66025
G	7.5	-12.9904
H	12.5	-12.9904
I	15	-8.66025
J	17.5	-12.9904
K	20	-8.66025
L	22.5	-12.9904

Table 6: Nodal coordinates.

Determinate Model Inputs and Outputs

Determinate model simulation settings.

Model	Structural
Material	Linear>Elastic>Isotropic
Young's Modulus	200GPa
Element Type	LINK180
Cross-Sectional Area	$6.4516 \times 10^4 m^2$
Node A Node H	0 displacement in x, y, and z 0 displacement in y and z
Loads	10kN downward at B and 25kN downward at H

Nodal reaction forces.

Reaction	Force [N]
Ax	0.0
Ay	-14000.0
Hy	49000.0

Table 7: Determinate reaction forces at nodes A and H.

Nodal displacements where UX, UY, and UZ indicate translation in the x, y, and z directions. USUM is the Pythagorean sum of the three displacements.

NODE	UX [m]	UY [m]	UZ [m]	USUM [m]
A	0	0	0	0
B	3.13×10^{-4}	1.78×10^{-3}	0	1.81×10^{-3}
C	1.61×10^{-3}	1.65×10^{-3}	0	2.31×10^{-3}
D	2.15×10^{-3}	2.74×10^{-3}	0	3.48×10^{-3}
E	2.19×10^{-3}	3.33×10^{-3}	0	3.99×10^{-3}
F	2.73×10^{-3}	2.35×10^{-3}	0	3.60×10^{-3}
G	-1.79×10^{-3}	3.00×10^{-3}	0	3.49×10^{-3}
H	-3.49×10^{-3}	0	0	3.49×10^{-3}
I	4.97×10^{-3}	-6.17×10^{-3}	0	7.92×10^{-3}
J	-5.17×10^{-3}	-1.33×10^{-2}	0	1.43×10^{-2}
K	6.09×10^{-3}	-2.11×10^{-2}	0	2.20×10^{-2}
L	-5.73×10^{-3}	-2.92×10^{-2}	0	2.98×10^{-2}

Table 8: Determinate model nodal displacements.

Simulation results for the determinate model showing the member loads, stresses, and strains.

Member	Axial Force [N]	Stress [Pa]	Strain
AB	8083.1	1.25×10^{7}	6.26×10^{-5}
AC	-16166	-2.51×10^{7}	-1.25×10^{-4}
BC	-13857	-2.15×10^{7}	-1.07×10^{-4}
BD	2309.4	3.58×10^{6}	1.79×10^{-5}
CD	13857	2.15×10^{7}	1.07×10^{-4}
CE	-30022	-4.65×10^{7}	-2.33×10^{-4}
DE	-13857	-2.15×10^{7}	-1.07×10^{-4}
DF	16166	2.51×10^{7}	1.25×10^{-4}
EF	13857	2.15×10^{7}	1.07×10^{-4}
EG	-43879	-6.80×10^{7}	-3.40×10^{-4}
FG	43879	6.80×10^{7}	3.40×10^{-4}
FH	-27713	-4.30×10^{7}	-2.15×10^{-4}
FI	57737	8.95×10^{7}	4.47×10^{-4}
GH	-43880	-6.80×10^{7}	-3.40×10^{-4}
HI	-28868	-4.47×10^{7}	-2.24×10^{-4}
HJ	-43303	-6.71×10^{7}	-2.24×10^{-4}
IJ	28868	4.47×10^{7}	2.24×10^{-4}
IK	28868	4.47×10^{7}	2.24×10^{-4}
JK	-28868	-4.47×10^{7}	-2.24×10^{-4}
JL	-14434	-2.24×10^{7}	-1.12×10^{-4}
KL	28868	4.47×10^{7}	2.24×10^{-4}

Table 9: Determinate model member force, stress, and strain.

Indeterminate Model Inputs and Outputs

Indeterminate model simulation settings.

Model	Structural
Material	Linear>Elastic>Isotropic
Young's Modulus	200GPa
Element Type	LINK180
Cross-Sectional Area	$6.4516 \times 10^4 m^2$
Node A Node H	0 displacement in x, y, and z 0 displacement in x, y, and z
Loads	10kN downward at B and 25kN downward at H

Nodal reaction forces.

Reaction	Force [N]
Ax	-16084.0
Ay	2714.2
Hx	16084.0
Hy	32286.0

Table 10: Indeterminate model reaction forces at nodes A and H.

Nodal displacements where UX, UY, and UZ indicate translation in the x, y, and z directions. USUM is the Pythagorean sum of the three displacements.

NODE	UX [m]	UY [m]	UZ [m]	USUM [m]
A	0	0	0	0
B	5.63×10^{-4}	1.64×10^{-3}	0	1.73×10^{-3}
C	2.61×10^{-3}	1.37×10^{-3}	0	2.94×10^{-3}
D	3.39×10^{-3}	2.88×10^{-3}	0	4.45×10^{-3}
E	4.44×10^{-3}	3.19×10^{-3}	0	5.46×10^{-3}
F	5.22×10^{-3}	2.64×10^{-3}	0	5.85×10^{-3}
G	1.45×10^{-3}	3.14×10^{-3}	0	3.46×10^{-3}
H	0	0	0	0
I	7.46×10^{-3}	-5.60×10^{-3}	0	9.33×10^{-3}
J	-1.68×10^{-3}	-1.22×10^{-2}	0	1.23×10^{-2}
K	8.58×10^{-3}	-1.94×10^{-2}	0	2.12×10^{-2}
L	-2.24×10^{-3}	-2.69×10^{-2}	0	2.70×10^{-2}

Table 11: Indeterminate model nodal displacements

Simulation results for the indeterminate model showing the member loads, stresses, and strains.

Member	Axial Force [N]	Stress [Pa]	Strain
AB	14517	2.25×10^7	1.13×10^{-4}
AC	3134.1	4.86×10^6	2.43×10^{-5}
BC	-20290	-3.14×10^7	-1.57×10^{-4}
BD	8742.8	1.36×10^7	6.78×10^{-5}
CD	20290	3.15×10^7	1.57×10^{-4}
CE	-17156	-2.66×10^7	-1.33×10^{-4}
DE	-20290	-3.14×10^7	-1.57×10^{-4}
DF	29033	4.50×10^7	2.25×10^{-4}
EF	20290	3.15×10^7	1.57×10^{-4}
EG	-37446	-5.80×10^7	-2.90×10^{-4}
FG	37446	5.80×10^7	2.90×10^{-4}
FH	-8413	-1.30×10^7	-6.52×10^{-5}
FI	57737	8.95×10^7	4.47×10^{-4}
GH	-37446	-5.80×10^7	-2.90×10^{-4}
HI	-28868	-4.47×10^7	-2.24×10^{-4}
HJ	-43303	-6.71×10^7	-3.36×10^{-4}
IJ	28868	4.47×10^7	2.24×10^{-4}
IK	28868	4.47×10^7	2.24×10^{-4}
JK	-28868	-4.47×10^7	-2.24×10^{-4}
JL	-14434	-2.24×10^7	-1.12×10^{-4}
KL	28868	4.47×10^7	2.24×10^{-4}

Table 12: Indeterminate model member force, stress, and strain.

References

[1] *Statically Determinate and Indeterminate Trusses*. URL: https://www.ae.msstate.edu/vlsm/truss/statically_det_indet_trusses/statically_det_indet_trusses.htm.

[2] *Steels Material Properties*. URL: http://www.matweb.com/search/datasheet.aspx?bassnum=MS0001&ckck=1.