NAME:______________________________ DUE DEC 17, 2021, 10:00 AM

FINAL EXAM

ADVANCED FLUID MECHANICS, CHE/EID/ME 440, COOPER UNION

Ground Rules:

Complete problem 1, and either problem 2 or problem 3 (your choice). Or do all three if you have the time (for extra credit).

Direct all questions about the exam to the instructor.

Please submit an electronic copy of the exam (pdf preferred) on Teams, or bring a hard copy to class on Friday Dec 17.

By submitting this exam solution with your name, you affirm that you have abided by the code of conduct of Cooper Union. You affirm that this exam solution is your own work, and that you did not receive unauthorized assistance from any person.

This final exam is <u>open book and notes</u>.

Show All Work.

USE UNITS when numerical calculations are required, and check your units and dimensions.

3. (80 Pts) Solve text problem 6-22, to derive the turbulent Couette flow velocity profile and wall shear stress for 2D flow between parallel plates, where the lower plate is stationary, and flow is driven by the velocity U of the upper plate. The gap between the plates is $2h$. (See text figure 3-5). For calculations, you may assume the fluid is air with a density of 1.2 kg/m^3, and viscosity 1.8 x 10^{-5} kg/m-s, and a half gap width of 1 cm. Assume:

   i. Incompressible flow
   ii. Steady mean flow
   iii. Negligible gravity effects (or use total hydrostatic pressure)
   iv. Negligible pressure gradients
   v. The law of the wall adequately describes the velocity profile, except in the viscous sublayer
   vi. The viscous sublayer thickness is much smaller than h.

   a. Using a control volume approach, derive an expression for the mean shear stress profile, $\underline{\tau}_{yx}(y)$, given the mean wall shear stress on the upper wall, τ_w.
   b. Set a velocity boundary condition for the midpoint of the channel, y=0, based on symmetry arguments.
   c. Identify an appropriate (and tractable) wall boundary condition that lets you define all terms in the law of the wall velocity profile
   d. Solve for the velocity profile, $\underline{u}(y)$ or $\frac{\underline{u}}{U}\left(\frac{y}{h}\right)$ (ie, not just a function of y^+).
   e. Derive a friction law, i.e. a simple (but implicit) expression relating the coefficient of friction C_f to the Reynolds number, $Re_h = Uh/\nu$.
   f. Solve for and plot the mean velocity profile for $Re_h = 10^4$. (This will require an iterative solution to the friction law expression.) Include the laminar velocity profile in your plot for comparison.
   g. Derive an expression for the eddy viscosity, i.e. the ratio of the mean shear stress to the mean velocity gradient. Plot the eddy viscosity profile for $Re_h = 10^4$, and explain its shape. Also, show the molecular viscosity on your plot for comparison.
   h. Estimate the viscous sublayer thickness under these conditions. Briefly discuss this thickness in the context of your velocity profile, and recommend a mesh density if you wished to model this flow in CFD with an enhanced wall function.