ENME 314 Fluids Tutorial problems: 4 Conservation laws in integral form

Terminology

1. It is everywhere parallel to the local velocity vector
2. None. It's walls are streamlines, so parallel to the flow
3. A control volume
4. No. A fluid particle may be a quantity of fluid much greater than one molecule. In fact new molecules may enter, and old ones leave, by diffusion, without changing the identity of the fluid particle
5. $1 m/s^2$. This is temporal acceleration.
6. $1 m/s^2$. This is spatial acceleration.

Conservation laws

7. Any three of: mass, momentum, angular momentum and energy

Flow rates

8. Cross sectional area is $2m \times 10m = 20m^2$ (1 s.f.). Q=AV so V=$120 m^3/s$ divided by $20m^2$=6m/s (1 s.f.)
9. Assume the density is $100 kg/m^3$, dynamic pressure=$\frac{1}{2}\rho V^2$=18,000Pa (20kPa to 1 s.f.)

Continuity equation

10. 10.0m/s*0.5=5.00m/s
11. 10.0m/s*0.25=2.50m/s
12. Inlet velocity is $1.00 m^3/s / 0.100 m^2$=10.0m/s. Outlet velocity=10.0m/s*0.5=5.00m/s.
13. Answer: 10.m/s*2=20.m/s (2 s.f.).
14. The two outlet flow rates must add up to the inlet flow rate so the other outlet pipe carries 1.0-0.3=$0.7 m^3/s$ (1 s.f.)

15.

Continuity equation:

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

Assume incompressible and divide by density to give the balance in terms of volumetric flow rates Q and the volume in the tank $\mathbb{V}$:

$$\frac{d\mathbb{V}}{dt} = Q_{in} - Q_{out}$$

$=0.01\ m^3s^{-1}-0.02m^3s^{-1}=-0.01\ m^3s^{-1}$

$=0.01\ m^3s^{-1}-0.02m^3s^{-1}=-0.01\ m^3s^{-1}$

So the volume contained in the cistern reduces by 0.01 m^3 every second. Starting with 1 m^3 it will take 100 s to empty.

16.

Continuity equation

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm}{dt}$$

Completely fills pipe and incompressible :

$$\frac{dm}{dt} = 0$$

$$\dot{m}_{in} = \dot{m}_{out}$$

Also

$$\dot{m} = \rho A \overline{V}$$

so :

$$\rho_{in} A_{in} \overline{V}_{in} = \rho_{out} A_{out} \overline{V}_{out}$$

Incompressible :

$$\rho_{in} = \rho_{out}$$

Thus :

$$A_{in} \overline{V}_{in} = A_{out} \overline{V}_{out}$$

If the pipe has constant cross section :

$$\overline{V}_{in} = \overline{V}_{out}$$

17.

If we assume the flow is incompressible, and does not separate from the walls, we can use the equation derived above:

$$A_{in} \overline{V}_{in} = A_{out} \overline{V}_{out}$$

Thus:

$$\overline{V}_{out} = \overline{V}_{in} \frac{A_{in}}{A_{out}}$$

$$\overline{V}_{out} = 10ms^{-1} \frac{\pi(0.020m)^2 / 4}{\pi(0.050m)^2 / 4}$$

$= 1.6ms^{-1}$ (2 s.f.)

Bernoulli's theorem

18.

Applying Bernoulli's principle:

$$p_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$$

We will assume steady inviscid incompressible flow along a streamline passing through the diffuser with no heat added or work done.

We will also assume the diffuser is horizontal i.e. $z_1 = z_2$
Therefore

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

Rearranging

$$p_2 - p_1 = \frac{1}{2}\rho(V_1^2 - V_2^2)$$

Assuming the pressure upstream is atmospheric i.e. 101,325 Pa, and using the ideal gas law:
$\rho = \frac{P}{R'T} = \frac{101{,}325\ Pa}{287\ Jkg^{-1}K^{-1}293K} = 1.20\ kgm^{-2}$ (temperature is only known to 2 s.f.)

$$p_2 - p_1 = \frac{1}{2}1.20kgm^{-3}((10\ ms^{-1})^2 - (1.6\ ms^{-1})^2) = 59\ Pa$$

19.

Applying Bernoulli's principle:

$$p_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$$

We will assume steady inviscid incompressible flow along a streamline passing through the diffuser with no heat added or work done.

$$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2) + \rho g(z_1 - z_2)$$

$$V_1 = \frac{2.0kgs^{-1}}{1000kgm^{-3}\,\pi(0.10m)^2/4} = 0.26ms^{-1}$$

$$V_2 = V_1\frac{A_1}{A_2} = 0.26ms^{-1}\frac{(0.10m)^2}{(0.050m)^2} = 1.0ms^{-1}$$

$$\begin{aligned} p_2 &= 25{,}000\ Pa\ gauge + \frac{1}{2}1000kgm^{-3}((0.26ms^{-1})^2 - (1.0ms^{-1})^2) \\ &\quad - 1000kgm^{-3}9.81Nkg^{-1}2.0m \\ &= 25{,}000\ Pa\ gauge - 488\ Pa - 19620\ Pa \\ &= 4{,}900\ Pa\ gauge\ (2\ s.f.) \end{aligned}$$

Add standard atmospheric pressure (101,325 Pa) to find absolute pressure

$$p_2 = 110{,}000\ Pa\ abs$$

Venturis and orifice plates

20.

$$\dot{m} = C_d A_2 \sqrt{\frac{2\rho\left(p_1 - p_2\right)}{\left(1 - \frac{A_2^2}{A_1^2}\right)}}$$

$$\left(p_1 - p_2\right) = \frac{1}{2\rho}\left(\frac{\dot{m}}{C_d A_2}\right)^2 \left(1 - \frac{A_2^2}{A_1^2}\right)$$

$$= \frac{1}{2.1000kgm^{-3}}\left(\frac{2.0kgs^{-1}}{0.60.\frac{\pi\left(0.025m\right)^2}{4}}\right)^2 \left(1 - \frac{\left(0.025\right)^4}{\left(0.050\right)^4}\right)$$

$$= \frac{1}{2000}\left(6791\right)^2.0.9400Pa$$

$$= 22,000Pa$$

21.

$$\dot{m} = C_d A_2 \sqrt{\frac{2\rho\left(p_1 - p_2\right)}{\left(1 - \frac{A_2^2}{A_1^2}\right)}}$$

$$= 0.99 \cdot \left(\frac{\pi\left(0.050m\right)^2}{4}\right)\sqrt{\frac{2 \cdot 1000kgm^{-3} \cdot 20,000Pa}{1 - \left(\frac{1}{3}\right)^2}}$$

$$= 0.99 \cdot 1.96 \times 10^{-3} m^2 \cdot 6708kgm^{-2}s^{-1} = \underline{\underline{13kgs^{-1}}} \text{ (2 s.f.)}$$

22.

a) *Bernoulli's equation*

Assume:

Steady, incompressible, inviscid, flow along a streamline with no heat added or work done. 3 out of 4 stated gives 0.5 mark

Restate the important assumption that the fuel tube reduces the throat area by a negligible amount, 0.5 mark.

$$p_1 + \tfrac{1}{2}\rho_{air}U_1^2 + \rho_{air}gz_1 = p_2 + \tfrac{1}{2}\rho_{air}U_2^2 + \rho_{air}gz_2$$

Station 1 is at the intake (atmospheric pressure) and station 2 at the throat.

Horizontal so $z_1=z_2$. $p_1=p_a$ atmospheric pressure.

$p_a - p_2 = \tfrac{1}{2}\rho_{air}\left(U_2^2 - U_1^2\right)$ *(1 mark)*

Applying the continuity equation:

$\rho_{air}A_1U_1 = \rho_{air}A_2U_2$ *(1 mark)*

$$U_2 = U_1\frac{A_1}{A_2}$$

So:

$p_a - p_2 = \tfrac{1}{2}\rho_{air}U_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$ *(1 mark for this or a correct equivalent. 0.5 mark if two different velocities or flow rates appear in the equation.)*

b) *The depression in the throat must equal the hydrostatic pressure at the base of the h=5cm column of fuel:*

$p_a - p_2 = \tfrac{1}{2}\rho_{air}U_1^2\left(\frac{A_1^2}{A_2^2} - 1\right) = \rho_{fuel}gh$ *(2 marks)*

Rearranging for U_1:

$U_1 = \sqrt{\frac{2gh}{\left(\frac{A_1^2}{A_2^2} - 1\right)}\frac{\rho_{fuel}}{\rho_{air}}}$ *(1 mark)*

And calculating the mass flow rate:

$\dot{m} = \rho_{air}A_1U_1 = A\sqrt{\frac{2\rho_{air}\rho_{fuel}gh}{\left(\frac{A_1^2}{A_2^2} - 1\right)}}$ *(1 mark. Must do more than $\dot{m} = \rho_{air}A_1U_1$)*

A common error was to apply Bernoulli's theorem to the fuel. This is invalid as the fuel is not moving- use fluid statics only.

c)

$$\dot{m} = \frac{\pi\left(0.050m\right)^2}{4}\sqrt{\frac{2\times 1.25kgm^{-3}0.70\times 1000kgm^{-3}9.81ms^{-2}0.050m}{\left(\left(\frac{\left(0.0500m\right)^2}{\left(0.02m\right)^2}\right)^2 - 1\right)}} = 9.3x10^{-3}kgs^{-1}$$

$Q = 7.5x10^{-3}m^3s^{-1}$ (2 s.f.)

(1 mark for either volumetric or mass flow rate.)

$$p_1 + \frac{1}{2}\rho u_1^2 + \rho g z_1 - 0.6L\frac{1}{2}\rho u_1^2 = p_2 + \frac{1}{2}\rho u_2^2 + \rho g z_2$$

(Assuming steady, incompressible, no heat added or work done)

$L =$ length of inlet pipe $= 10.m$

$z_1 = z_2$

23. $p_2 = p_1 + \frac{1}{2}\rho\left((-5)u_1^2 - u_2^2\right)$

$$u_1 = \frac{2.0x10^{-4}\,m^3 s^{-1}}{\pi(0.02m)^2/4} = 0.64ms^{-1}$$

$$u_2 = \frac{2.0x10^{-4}\,m^3 s^{-1}}{\pi(0.01m)^2/4} = 2.5ms^{-1}$$

$$p_2 = 10.Pag + 0.5.1.2.(-5.0.64^2 - 2.5^2) = 5.0Pag$$

24.

$\dot{m}_{in1} + \dot{m}_{in2} = \dot{m}_{out} = 1.0kgs^{-1} + 1.5kgs^{-1} = 2.5kgs^{-1}$

By conservation of mass of salt,

$\dot{m}_{salt,in1} = 35gkg^{-1} \cdot 1.0kgs^{-1} = 35gs^{-1}$

$\dot{m}_{salt,in2} = 70gkg^{-1} \cdot 1.5kgs^{-1} = 110gs^{-1}$

$\dot{m}_{salt,out} = \dot{m}_{salt,in1} + \dot{m}_{salt,in2} = 145gs^{-1}$

Pressure variation in curved streamlines

25.

First calculate the speed of the flow:

$$u = \frac{\dot{m}}{\rho A} = \frac{1.0kgs^{-1}}{1000kgm^{-3}\left(\pi(0.050m)^2/_4\right)} = 0.51ms^{-1}$$

Now the pressure gradient:

$$\frac{\partial p^*}{\partial n} = \rho\frac{u^2}{r_c} = 1000kgm^{-3}\frac{(0.51ms^{-1})^2}{0.20m} = 1300\ Pa\,m^{-1}$$

Note that the piezometric pressure gradient is equal to the static pressure gradient as there is no change in height.

$$\frac{\partial p^*}{\partial n} = \frac{\partial p}{\partial n}$$

Now multiply this by the diameter of the pipe to give the pressure difference between the inside and outside walls.

$$d \times \frac{\partial p}{\partial n} = 0.050m \times 1300\ Pa\,m^{-1} = 65\ Pa\ (2\ s.f.)$$

Vortices

26.

$$\frac{\partial p^*}{\partial n} = \rho \frac{V_\theta^2}{r}$$

for $r \le R$

$$V_\theta(r) = {V_o r}/{R}$$

$$\frac{\partial p^*}{\partial n} = \rho \frac{\left({V_0 r}/{R} \right)^2}{r} = \underline{\underline{\rho \frac{V_0^2 r}{R^2}}}$$

and for $r > R$

$$V_\theta(r) = {V_0 R}/{r}$$

$$\frac{\partial p^*}{\partial n} = \rho \frac{\left({V_0 R}/{r} \right)^2}{r} = \underline{\underline{\rho \frac{V_0^2 R^2}{r^3}}}$$

27.

Free vortex portion: $V_\theta = \Gamma/(2\pi r) = V_o R/r \quad r>R$

Forced vortex portion: $V_\theta = \omega r = V_o r/R \quad r \le R$

Therefore: $\Gamma/(2\pi) = V_o R$

and: $\omega = V_o/R$

Solving these last two equations for R and equating them gives: $R = \Gamma/(2\pi V_o) = V_o/\omega$

or: $V_o^2 = \Gamma\omega/(2\pi)$ which is: $V_o = [\Gamma\omega/(2\pi)]^{0.5}$

so: $V_o = [(10\ m^2 s^{-1})(5\ Hz)(2\pi)[2\pi)]^{0.5} = 7.07107$ m/s

$R = V_o/\omega = (7.07107\ m/s)/[(5\ Hz)(2\pi)] = 0.225079$ m

(a)

At $r = R/2 = (0.225079\ m)/2 = 0.1125395$ m

And $V_\theta = \omega r = V_o r/R = (7.07107\ m/s)\ (0.5) = 3.53554$ m/s

$\rho = 789\ kg/m^3$

$\partial p/\partial r = \rho V_\theta^2/r = (789\ kg/m^3)\ (3.53554\ m/s)^2 /(0.1125395\ m) = 87.3258$ kPa/m

= **<u>87 kPa/m (2 SF)</u>**

(b)

At $r = R = 0.225079$ m

And $V_\theta = \omega r = V_o r/R = 7.07107$ m/s

$\partial p/\partial r = \rho V_\theta^2/r = (789\ kg/m^3)\ (7.07107\ m/s)^2 /(0.225079\ m) = 175.2719$ kPa/m

= **<u>180 kPa/m (2 SF)</u>**

(c)

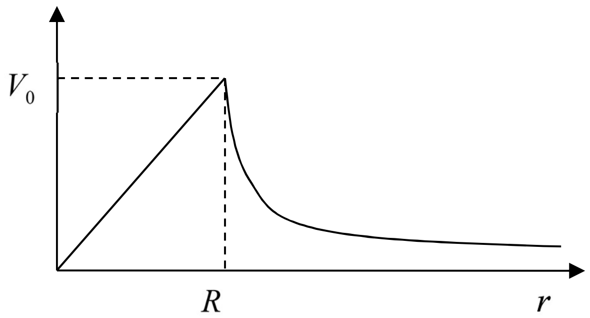
At r = 1.5*R = 1.5(0.225079) m = 0.3376185 m

And $V_\theta = \Gamma/(2\pi r) = V_o R/r$ = (7.07107 m/s)(0.6666667) = 4.71405 m/s

$\partial p/\partial r = \rho V_\theta^2/r$ = (789 kg/m^3) (4.71405 m/s)^2 /(0.3376185 m) = 51.9324 kPa/m

= **<u>52 kPa/m (2 SF)</u>**

28.



a)

b) The central part is a forced vortex, the surrounding is a free vortex. This might be found anywhere where there is a stirring mechanism driving a forced vortex, far from solid boundaries. A stirrer in a large mixing tank for example. (Other answers are possible)

c) Assuming circular motion with negligible change in height:

The pressure gradient normal to a curved streamline is given by

$$\frac{\partial p}{\partial r} = \rho \frac{u^2}{r}$$ (2 marks)

$u(r) = \frac{V_o r}{R}$ for the central forced vortex

$$\frac{\partial p}{\partial r} = \rho \frac{V_o^2 r}{R^2}$$

separate variables and integrate

$$\int_{P(r=0)}^{0\,Pagauge} dp = 0 - P(r=0) = \int_{r=0}^{r=R} \rho \frac{V_o^2 r}{R^2} dr = \left[\rho \frac{V_o^2 r^2}{2R^2} \right]_{r=0}^{r=R} = \rho \frac{V_o^2}{2} = 1000 kgm^{-3} \frac{\left(1.0 ms^{-1}\right)^2}{2} = 500 Pa$$

(2 marks for correct procedure)

$P(r=0) = -500 Pa$ gauge to 2 s.f.

Note: Bernoulli should not be used as the pressure gradient of interest lies perpendicular to the streamlines.

Reynolds' Transport Theorem

29.

$$\underline{F}_{cv} = \int_{cv} \frac{\partial}{\partial t}(\rho \underline{V}) dV + \oint_{cs} \rho \underline{V}\, \underline{\hat{n}} \bullet \underline{V} dA$$

Steady flow $\frac{\partial}{\partial t} = 0$:

$$\underline{F}_{cv} = \oint_{cs} \rho \underline{V}\, \underline{\hat{n}} \bullet \underline{V} dA$$

Incompressible $\rho = \text{const.}$:

$$\underline{F}_{cv} = \rho \oint_{cs} \underline{V}\, \underline{\hat{n}} \bullet \underline{V} dA$$

Uniform flow in the z - direction, only the z - component of velocity is nonzero:

$\underline{V} = V_z$

Inflow and outflow are through one area perpendicular to the flow only:

$\underline{\hat{n}} \bullet \underline{V} = V_z$

Note the inflow and outflow areas must be parallel to each other and will have unit vectors pointing in opposite directions

$\left(\underline{\hat{n}} \bullet \underline{V}\right)_{\inf low} = V_{z \inf low}$

$\left(\underline{\hat{n}} \bullet \underline{V}\right)_{outflow} = -V_{zoutlow}$

$$\underline{F}_{cv} = \rho \int_{\inf low} V_z^2 dA - \rho \int_{outflow} V_z^2 dA$$

$$= \rho V_{z \inf low}^2 A_{\inf low} - \rho V_{zoutlow}^2 A_{outflow}$$

$$\underline{\underline{\underline{F}_{cv} = \dot{m}_{\inf low} V_{z \inf low} - \dot{m}_{outflow} V_{zoutflow}}}$$

Finally note that for a uniform flow you would draw a control volume with $A_{outflow} = A_{\inf low}$ and also that $V_{zoutflow} = V_{\inf low}$

Thus:

$$\underline{F}_{cv} = \rho V_{z \inf low}^2 A_{\inf low} - \rho V_{zoutlow}^2 A_{outflow}$$

$$= \rho V_{z \inf low}^2 A_{\inf low} - \rho V_{z \inf low}^2 A_{\inf low}$$

$$\underline{\underline{\underline{F}_{cv} = 0}}$$

Pipe bends

30.

a)

Continuity (or conservation of mass) and

Conservation of momentum 1 mark (It is not enough to say only "Newton's Laws")

b)

Steady

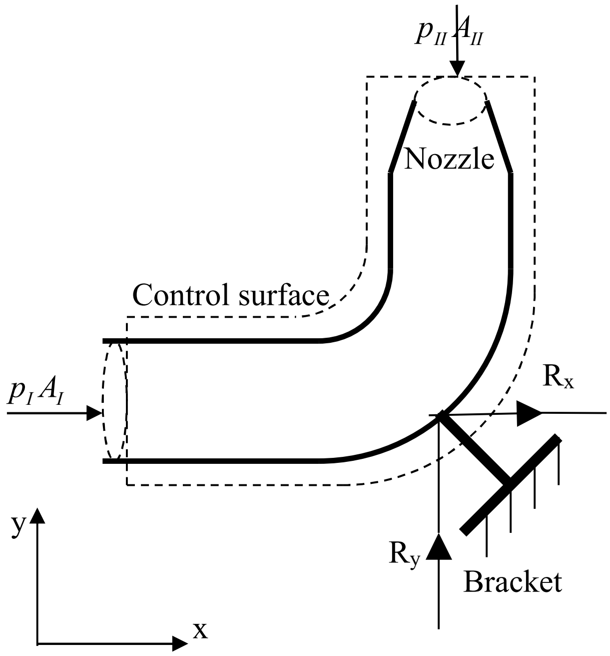
Inviscid

Incompressible

c)

$$V_{II} = V_I \frac{A_I}{A_{II}}$$

d)



Control surface must cut the pipe at the entrance and touch the pipe at the exit. p_I and p_{II} OR $p_I A_I$ and $p_{II} A_{II}$ should be indicated correctly with correct directions

e)

x-direction: $p_I A_I + R_x = -\dot{m} V_I$

y-direction: $-p_{II} A_{II} + R_y = \dot{m} V_{II}$

or equivalent expressions: it is acceptable to take pII=0 gauge but need to see in working that it is considered.

$\underline{F} = \dot{m}(\underline{U}_2 - \underline{U}_1)$ is not sufficient f)

$p_I = 100Pa$ gauge

$p_{II} = 0Pa$ gauge

$\rho = 1.20kgm^{-3}$

$\dot{m} = 0.50kgs^{-1}$

$$V_I = \frac{\dot{m}}{\rho A_I} = \frac{0.50kgs^{-1}}{1.20kgm^{-3} \cdot 0.050m^2} = 8.3ms^{-1}$$

$$V_{II} = \frac{\dot{m}}{\rho A_{II}} = \frac{0.50kgs^{-1}}{1.20kgm^{-3} \cdot 0.010m^2} = 42ms^{-1}$$

$$R_x = -p_I A_I - \dot{m}V_I = -100Pa \cdot 0.050m^2 - 0.50kgs^{-1} \cdot 8.3ms^{-1} = -\underline{\underline{9.2N}}$$

$$R_y = p_{II} A_{II} + \dot{m}V_{II} = 0 + 0.50kgs^{-1} \cdot 42ms^{-1} = \underline{\underline{21N}}$$

31.

$p_I = 100kPa$ gauge

$p_{II} = 0.0$ gauge

$\rho = 1000kgm^{-3}$

$\dot{m} = 1.0kgs^{-1}$

$$Q = \frac{\dot{m}}{\rho} = \frac{1.0kgs^{-1}}{1000kgm^{-3}} = 0.0010m^3s^{-1}$$

$$V_I = \frac{Q}{A_I} = \frac{0.0010m^3s^{-1}}{0.0100m^2} = 0.10ms^{-1}$$

$$V_{II} = \frac{Q}{A_{II}} = \frac{0.0010m^3s^{-1}}{0.0050m^2} = -0.20ms^{-1} \text{ (negative : pointing downwards)}$$

$$R_x = -p_I A_I - \dot{m}V_I = -100{,}000Pa \cdot 0.0100m^2 - 1.0kgs^{-1} \cdot 0.10ms^{-1} = -\underline{\underline{1.0kN}}$$

$$R_y = p_{II} A_{II} + \dot{m}V_{II} = 0 - 1.0kgs^{-1} \cdot 0.20ms^{-1} = -\underline{\underline{0.20N}} \text{ (negative due to the sign of } V_{II}\text{)}$$

Vanes

32.

$$\dot{W}_x = R_x V_B = \dot{m}_r \left(V - V_B\right) V_B (\cos\theta - 1)$$

$$V = 5.0ms^{-1}$$

$$V_B = 4.0ms^{-1}$$

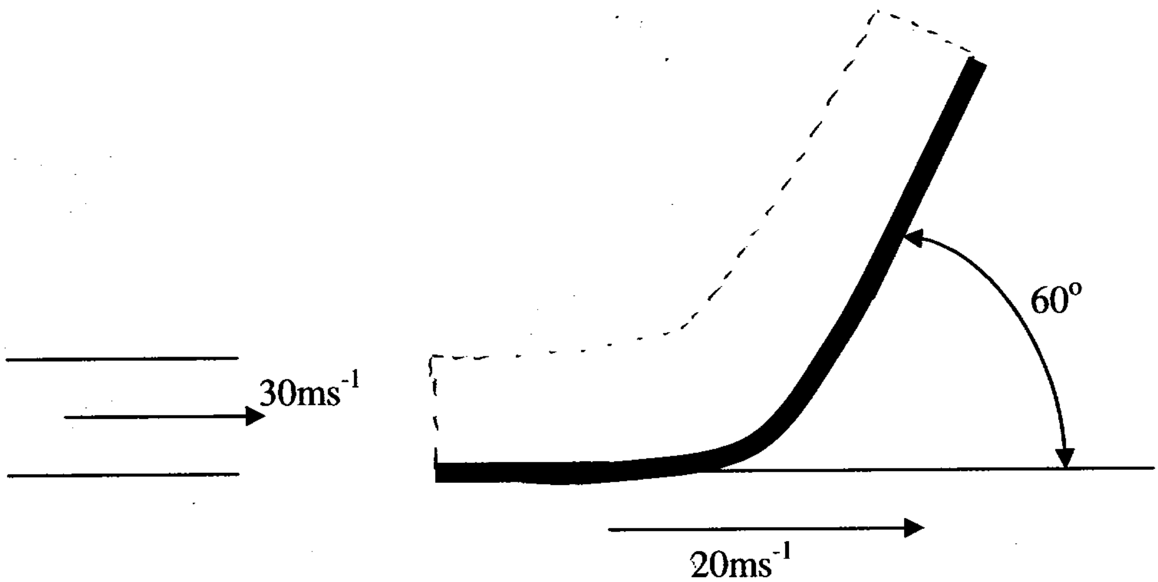
$$\theta = 35.^o$$

$$\dot{m}_r = \rho A\left(V - V_B\right) = 1000kgm^{-3}\frac{\pi\left(0.050m\right)^2}{4}\left(5.0 - 4.0\right)ms^{-1} = 2.0kgs^{-1}$$

$$\dot{W}_x = 2.0kgs^{-1}(5.0 - 4.0)ms^{-1}.4.0ms^{-1}\left(\cos 35.^o - 1\right) = \underline{\underline{-1.4W}}$$

33.

a) Draw an appropriate control volume for analysing the force on the vane.



b)

Assume the fluid is incompressible, the flow is steady, there is no spreading of the jet, the static pressure is the same everywhere, the flow is inviscid, and the weight of the fluid is negligible.

X direction is horizontal, from left to right.
Y direction is vertically upwards.

Relative to laboratory frame of reference:
Mass flow rate $\dot{m} = \rho AU$ =1000kgm^{-3}*π*(0.050m)^2/4*30.ms^{-1}=59kgs^{-1}
V_{1x}=30.ms^{-1}
V_{2x}=30.ms^{-1}*cos(60.°)
V_{1y}=0

V_{2y}=30.ms^{-1}*sin(60.°)

Relative to a frame of reference moving with the vane:
Mass flow rate $\dot{m} = \rho A U$ =1000kgm^{-3}*π*(0.050m)^2/4*(30.ms^{-1}-20.ms^{-1})=20.kgs^{-1}
V_{1x}=10.ms^{-1}
V_{2x}=10.ms^{-1}*cos(60.°)
V_{1y}=0
V_{2y}=10.ms^{-1}*sin(60.°)

Force in x direction= $F_x = \dot{m}(\underline{V}_{2x} - \underline{V}_{1x})$ =20.kgs^{-1}*10.ms^{-1}*(cos(60°)-1)=-98N (2 s.f.)
Force in y direction= $F_y = \dot{m}(\underline{V}_{2y} - \underline{V}_{1y})$ =20.kgs^{-1}*10.ms^{-1}*sin(60°) =170N (2 s.f.)

These are the forces on the fluid in the control volume. The forces on the vane are equal and opposite to these i.e. F_x=98N (directed from left to right) and F_y=-170N (directed vertically downwards) (2 s.f.).

Common errors: calculating the force on the fluid, not on the vane
Not showing the direction of the force on the vane by the correct sign, a statement or arrows.
Not stating assumptions.

Other applications of the momentum equation

34.

We need only consider one direction, the direction of flow, which we will call the x-direction.

We will assume the flow is inviscid.

$$F_x = \dot{m}\left(u_{2x} - u_{1x}\right)$$

We will also assume the flow is incompressible.

$$u_{1x} = \frac{\dot{m}}{\rho A_1} = \frac{5.0kgs^{-1}}{1000kgm^{-3}\,\pi\left(0.10m\right)^2/4} = 0.64s^{-1}$$

$$u_{2x} = u_{1x}\frac{A_1}{A_2} = 0.64ms^{-1}\frac{\left(0.10m\right)^2}{\left(0.025m\right)^2} = 10.ms^{-1}$$

So:

$$F_x = R_x = 5.0kgs^{-1}\left(10.ms^{-1} - 0.64ms^{-1}\right) = 48N$$

positive i.e. in the direction of flow

35.

We need only consider one direction, the z-direction of flow.

We will assume the flow is inviscid.

$F_z = \dot{m}\left(u_{2z} - u_{1z}\right)$

We will also assume the flow is incompressible.

The inflow has no z-component velocity.

$u_{1z} = 0$

There are in fact two outflows:

$$\dot{m}_A = 1.0kgs^{-1} \qquad u_{2zA} = \frac{\dot{m}_A}{\rho A_A} = \frac{1.0kgs^{-1}}{1000kgm^{-3}\,\pi\left(0.050m\right)^2/4} = 0.51ms^{-1}$$

$$\dot{m}_B = 0.50kgs^{-1} \qquad u_{2zB} = -\frac{\dot{m}_A}{\rho A_A} = -\frac{0.50kgs^{-1}}{1000kgm^{-3}\,\pi\left(0.030m\right)^2/4} = -0.71ms^{-1}$$

So the force on the fluid in the control volume:

$F_z = \dot{m}_A u_{2zA} + \dot{m}_B u_{2zB} = 1kgs^{-1}.0.51ms^{-1} - 0.50kgs^{-1}.0.71ms^{-1} = 0.16N$

i.e. in the $+z$ direction.

The force on the pipe will be equal in magnitude and opposite in direction.

Remarks on solutions submitted in previous years:

Assumptions

The necessary assumptions are: incompressible, inviscid, steady. Pressure variations due to curved streamlines are neglected. No heat is added or work done. Atmospheric pressure is constant with height.

It is not necessary to assume the flow is laminar. In fact a high Reynolds number flow more closely satisfies the inviscid assumption. Neither is it necessary to assume uniform flow, the equation will give the correct answer (subject to the other assumptions being valid) even if the flow is non-uniform, provided the velocities used are averages.

Sign

It is easy to get the sign of the force incorrect, if you are not careful and consistent with the signs of the velocity components at each inlet or outlet.

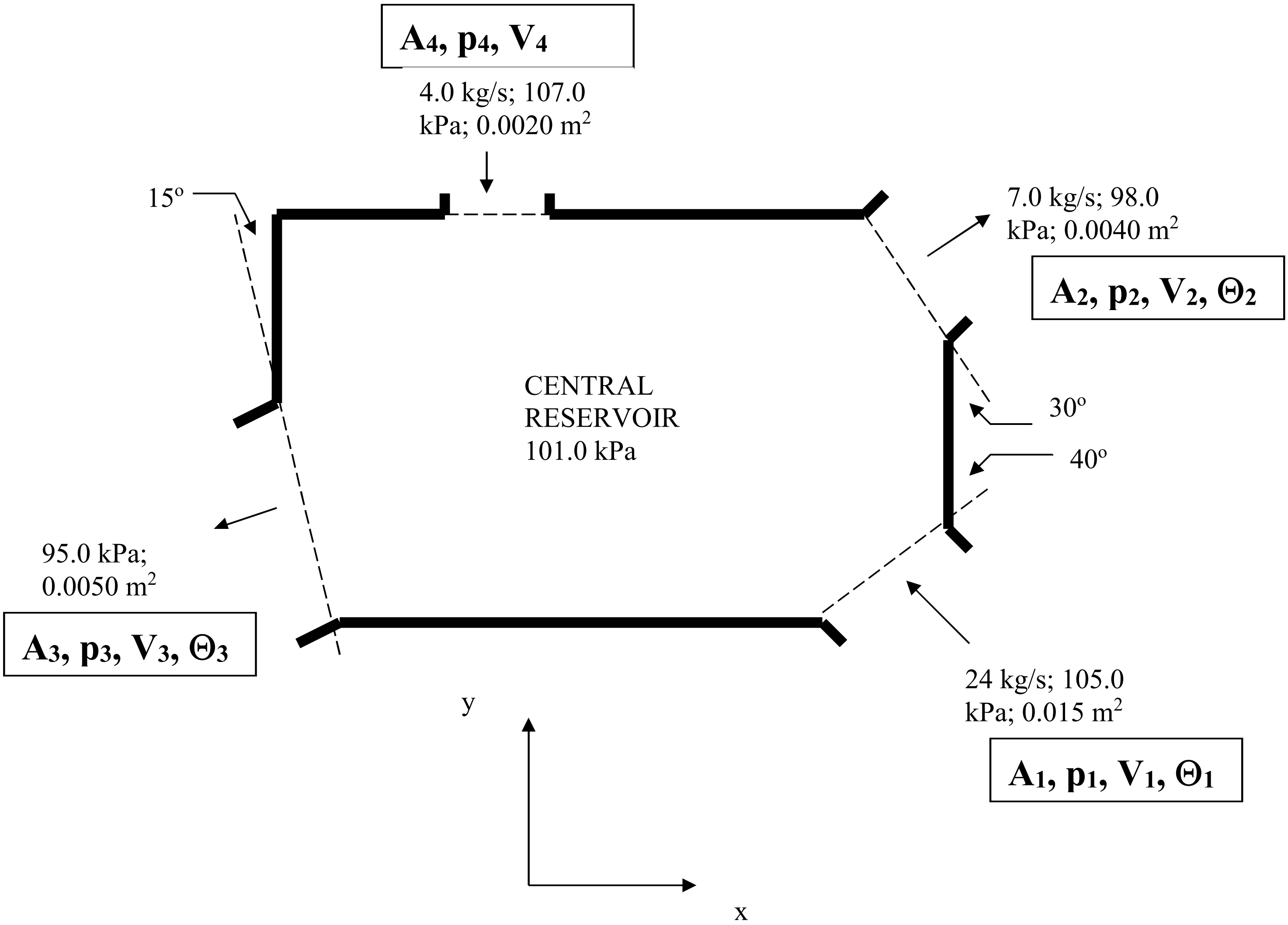
Resolving into components

To apply the momentum equation correctly you must write a separate equation for each orthogonal direction: here, horizontal and vertical. You only need the equation for vertical forces to solve this problem. A common error is to combine horizontal and vertical speeds or forces in the same equation.

Pressure

Because the pipe exits are open, atmospheric pressure acts everywhere, in all directions, in addition to any non-zero gauge pressure at the inlet which drives the flow. Therefore atmospheric pressure acts both inwards onto the control volume and outwards onto the surrounding fluid. Atmospheric pressure must be deducted from any pressures appearing on the left hand side (F_{cv}) of the momentum equation. If you don’t do this it is possible to calculate that there is an unbalanced pressure force which in this case, with both exits open to the atmosphere, is not physical.

36. The central water reservoir shown below [in the horizontal plane] takes in and distributes water through the inlet and exit tubes as shown. Assume steady flow, a full reservoir at all times, and water and surroundings temperatures of 5.0 ºC. Determine the horizontal and vertical forces which are needed to hold the reservoir in place.



Solution:

Sum of flow rates = 0; so flow rate a lower left (station 3) = 21 kg/s (out)

(Flow rate)$_i$ = ρ A$_i$ V$_i$ at 5 °C, ρ = 1000 kg/m^3 (p. 167)

Lower Right (station 1): 24 kg/s = (1000 kg/m^3)(0.015 m^2) V$_1$ ==> V$_1$ = 1.6 m/s
Upper Right (station 2): 7 kg/s = (1000 kg/m^3)(0.004 m^2) V$_2$ ==> V$_2$ = 1.75 m/s
Lower Left (station 3): 21 kg/s = (1000 kg/m^3)(0.005 m^2) V$_3$ ==> V$_3$ = 4.2 m/s
Top (station 4): 4 kg/s = (1000 kg/m^3)(0.002 m^2) V$_4$ ==> V$_4$ = 2 m/s

$$\sum \underline{F}_{cv} = \sum \dot{m} V_{in\ or\ out}$$

$F_x = B_x - (p_1 - p_{ext})(A_1)(\cos \Theta_1) - (p_2 - p_{ext})(A_2)(\cos \Theta_2) + (p_3 - p_{ext})(A_3)(\cos \Theta_3)$
$= (\text{flow rate})_1 (V_1)(\cos \Theta_1) + (\text{flow rate})_2 (V_2)(\cos \Theta_2) - (\text{flow rate})_3 (V_3)(\cos \Theta_3)$

(inflow mass flow rates, where mass flows into the control volume, are negative, and outflows positive. The velocity also has a sign, where motion to the right is positive).

$F_x = B_x$ - (105.0 kPa - 101.0 kPa)(0.015 m^2)(cos 40) - (98.0 kPa – 101.0 kPa)(0.0040 m^2)(cos 30) + (95.0 kPa – 101.0 kPa)(0.0050 m^2)(cos 15)

= (24 kg/s)(1.6 m/s)(cos 40) + (7 kg/s)(1.75 m/s)(cos 30) - (21 kg/s)(4.2 m/s)(cos 15)

Calculating and solving for B_x gives:

B_x – 64.548 N = -45.16974 N ==> B_x = 19.3784 N = **19 N (2 SF)**

$F_y = B_y + (p_1 - p_{ext})(A_1)(\sin \Theta_1) - (p_2 - p_{ext})(A_2)(\sin \Theta_2) + (p_3 - p_{ext})(A_3)(\sin \Theta_3) - (p_4 - p_{ext})(A_4)$
$= - (\text{flow rate})_1 (V_1)(\sin \Theta_1) + (\text{flow rate})_2 (V_2)(\sin \Theta_2) - (\text{flow rate})_3 (V_3)(\sin \Theta_3) + (\text{flow rate})_4 (V_4)$

$F_y = B_y$ + (105.0 kPa - 101.0 kPa)(0.015 m^2)(sin 40) - (98.0 kPa – 101.0 kPa)(0.0040 m^2)(sin 30) + (95.0 kPa – 101.0 kPa)(0.0050 m^2)(sin 15) - (107.0 kPa – 101.0 kPa)(0.0020 m^2)

= - (24 kg/s)(1.6 m/s)(sin 40) + (7 kg/s)(1.75 m/s)(sin 30) - (21 kg/s)(4.2 m/s)(sin 15) + (4 kg/s)(2 m/s)

Calculating and solving for B_y gives:

B_y + 24.80268 N = -33.385884 N ==> B_y = -58.18857 N = **-58 N (2 SF)**

Propeller efficiency

37.

(a) $\eta = 2*V_I/(V_I + V_{II})$,

or $V_{II} = 2*V_I/\eta - V_I = 2*(300\ \text{km/hr})/0.75 - 300\ \text{km/hr} = 500\ \text{km/hr}$

Since: $F_c = (\text{mass flow rate})(V_{II} - V_I) = \rho A_{III} V_{III} (V_{II} - V_I)$

$A_{III} = \pi r^2 = \pi (1.5\ \text{m})^2 = 7.06858\ \text{m}^2$

Need $\rho = p/RT$; $R = (8.314\ \text{kJ/kmol-K})/(28.97\ \text{kg/kmol}) = 0.28698\ \text{kJ/kg-K}$;
at 3050 m, $T = -5\ ^oC = -5\ ^oC + 273.15 = 268.15\ \text{K}$; and $p = 70\ \text{kPa}$;

Therefore:

$$\rho = (70\ \text{kPa})/[(0.28698\ \text{kJ/kg-k})(268.15\ \text{K}) = 0.9095746\ \text{kg/m}^3$$

And: $F_c = \rho A_{III} V_{III} (V_{II} - V_I)$, where $V_{III} = 0.5*(V_{II} + V_I)$

$F_c = (0.9095746\ \text{kg/m}^3)(7.06858\ \text{m}^2)(\mathbf{400000}\ \text{m/hr})(500000\ \text{m/hr} – 300000\ \text{m/hr})/(3600\ \text{s/hr})^2$
$F_c = 39.6872\ \text{kN} =$ **<u>40 kN (2 SF)</u>**

(b) power = Force x velocity = F_c x V_I
= 39.6872 kN x 300000 m/hr (1 hr/3600 s) = 3.307305 MW = **<u>3.3 MW (2 SF)</u>**

(c) air flow rate = $A_{III} V_{III} = A_{III} (V_{II} + V_I)/2 = A_{prop} (V_{II} + V_I)/2$
$= (7.06858\ \text{m}^2)(500000\ \text{m/hr} + 300000\ \text{m/hr})/[2*3600\ \text{s/hr}] = 785.398\ \text{m}^3/\text{s}$
= **<u>790m³/s (2SF)</u>**

(d) $(\text{mass flow rate})_I = \rho A_I V_I = (\text{mass flow rate})_{III} = \rho A_{III} V_{III}$ ==> $A_I = A_{III} V_{III}/V_I =$
$A_I = A_{III} V_{III}/V_I = (7.06858\ \text{m}^2)(400000\ \text{m/hr})/[(3600\ \text{s/hr})(300000\ \text{m/hr})/(3600\ \text{s/hr})]$
$A_I = 9.424778\ \text{m}^2$ ==> $r_I = 1.732051$ m; $d_I = 3.464102$ m = **<u>3.5 m (2 SF)</u>**

(e) $(\text{mass flow rate})_{II} = \rho A_{II} V_{II} = (\text{mass flow rate})_{III} = \rho A_{III} V_{III}$ ==> $A_{II} = A_{III} V_{III}/V_{II} =$
$A_{II} = A_{III} V_{III}/V_{II} = (7.06858\ \text{m}^2)(400000\ \text{m/hr})/[(3600\ \text{s/hr})(500000\ \text{m/hr})/(3600\ \text{s/hr})]$
$A_{II} = 5.654867\ \text{m}^2$ ==> $r_{II} = 1.341640$ m; $d_I = 2.683281$ m = **<u>2.7 m (2 SF)</u>**