# ENME 314 Fluids Tutorial solutions: 3 Fluid statics

## Hydraulic jacks

1. $F_2=F_1(A_2/A_1)$=100N x 0.5/0.1=500N (3 s.f.)

2. The change in volume of the smaller cylinder is the same as the change in volume of the larger cylinder, so 100m x 0.100m$^2$=Z x 0.500m$^2$ where Z is the stroke of the piston in the larger cylinder: Z=20mm. (2s.f.)

## Pressure measurement

3. P=-ρgz=-1000 x 9.81 x -1.00m=9810 Pa (3 s.f.)
4. 15m x 1000 x 9.81 = 150,000 Pa (2 s.f.)
5. 0.050m x 1000 x 9.81 = 490 Pa (2 s.f.)
6. $p_{piezo} = p_{static} + \rho g z$

   $p_{piezo}$ =80. kPa abs = $p_{static}$ +(1.2 kg/m$^3$)(9.81 m/s$^2$).(-1000. m)

   $p_{static}$ = 92,000 Pa abs

   Head = $p_{static}$ /($\rho_{water}$g) = 92,000 Pa/[(998.2 kg/m$^3$)(9.81 m/s$^2$)] =

   = 9.4 m of $H_2O$ abs (2 s.f.)
7. 1000Pa=ρgh. g=9.81ms$^{-2}$, ρ=900kgm$^{-3}$, so h=0.113m (3 s.f.)
8. $p_{static}$=ρgh h=200Pa / (900 kg/m$^3$ x 9.81 N/kg)=0.0226m or 22.6mm (3s.f.)
9.

$$p_A + \rho_A g \cdot 0.35m + \rho_m g \cdot 0.15m = p_B + \rho_B g \cdot 0.2m$$
$$p_B - p_A = \rho_A g \cdot 0.35m + \rho_m g \cdot 0.15m - \rho_B g \cdot 0.2m$$
$$= 998.2kgm^{-3} \cdot 9.81Nkg^{-1} \cdot 0.35m + 13{,}550kgm^{-3} \cdot 9.81Nkg^{-1} \cdot 0.15m - 998.2kgm^{-3} \cdot 9.81Nkg^{-1} \cdot 0.2m$$
$$= 3427Pa + 19{,}940Pa - 1958Pa$$
$$= 21{,}410Pa$$
$$p_B = p_A + 21{,}410Pa = 10{,}000Pa \text{ abs} + 21{,}410Pa = \begin{array}{l} 31{,}000Pa \text{ abs (2s.f.)} \\ \underline{\underline{\text{or } 101{,}325 - 31{,}0000 = -70{,}000Pa \text{ gauge (2 s.f.)}}} \end{array}$$

Omitting the pressures due to the columns of water in the manometer would introduce an error of 4%.

10.

$$p^* = p + \rho g z$$

So :

$$\Delta p^* = \Delta p + \rho g \Delta z$$

The change in static pressure is :

$$\Delta p = p_{10m} - p_{0m} = p^*_{10m} - p^*_{0m} - \rho g.10m + \rho g.0m$$
$$= 100{,}000Pa - 1000kgm^{-3}.9.81ms^{-2}.(10m - 0m)$$
$$\underline{\underline{= 1900Pa}}$$

11.

$$p_0 = p_{vap}(T) + \rho g H$$
$$H = \frac{p_0 - p_{vap}(T)}{\rho g} = \frac{100{,}000Pa - 10.0Pa}{900.kgm^{-3} \times 9.81Nkg^{-1}} = 11.3m$$

## Pitot-static tube and manometer

12.

$$U = K\sqrt{\frac{2(P_{stagnation} - P_{static})}{\rho}} = 0.99\sqrt{\frac{2 \cdot 100}{1.2}} = 13ms^{-1}$$

13.

$$\rho = 1.247kg/m^3$$

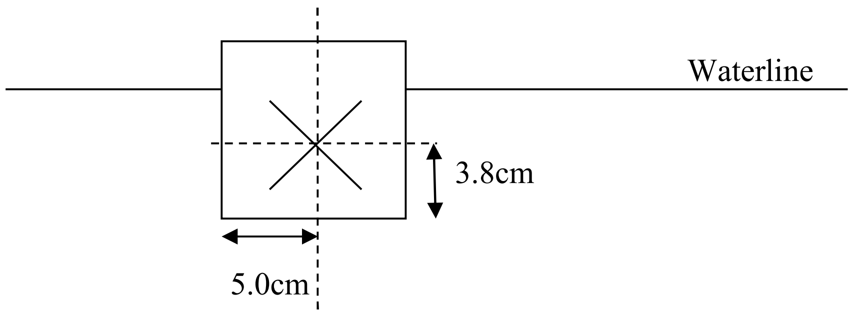
$$U = K\sqrt{\frac{2(P_{stagnation} - P_{static})}{\rho}} = 0.99\sqrt{\frac{2.150}{1.247}} = 15.3ms^{-1}$$

## Archimedes and buoyancy

14. Submerged volume of beam= 0.10m x 0.075m x 2.0m =0.015m$^3$
Mass of water displaced by submerged volume of beam =1000.kg/m$^3$ x 0.015m$^3$=15.kg

15. For the beam to float in this way: mass of entire beam=mass of displaced water=15.kg
Mass of entire beam= density of beam x 0.10m x 0.10m x 2.0m= density of beam x 0.020m$^3$
Therefore density of beam=15.kg/0.020m$^2$=750kg/m$^3$ (2 s.f.)

16. The line of action is through the centroid of the submerged portion of the beam. The centroid will lie 5cm from either vertical face, and 7.5cm/2=3.8cm from the bottom face of the beam, and 1.0m along the beam, at the point marked with a cross below. The line of action will pass vertically upwards through this point.

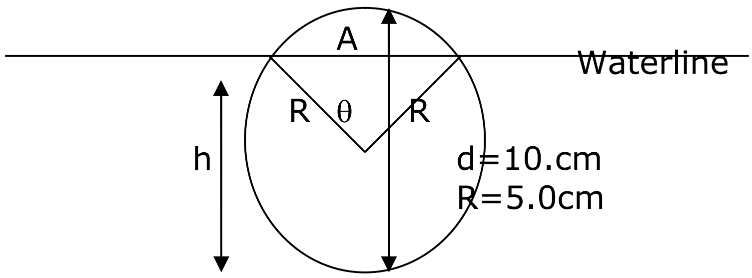



17.

Length of log (into plane of paper) =L

Area of the segment above the waterline is $A = \frac{R^2}{2}(\theta - \sin\theta)$ (Look up "circular segment" in Wikipedia

R=radius=d/2=0.050m




Upthrust=weight

Upthrust=density of water x g x immersed volume=density of water x g x ($\pi$R$^2$-A)L

Weight=density of wood x g x total volume of beam=density of wood x g x $\pi R^2 L$

$$\frac{\rho_{wood}}{\rho_{water}}=\frac{\left(\pi R^2-A\right)}{\pi R^2}=\frac{\left(\pi R^2-\frac{R^2}{2}(\theta-\sin\theta)\right)}{\pi R^2}=1-\frac{R^2}{2\pi R^2}(\theta-\sin\theta)=1-\frac{(\theta-\sin\theta)}{2\pi}$$

$$0.8=1-\frac{(\theta-\sin\theta)}{2\pi}$$

$$\theta-\sin\theta=2\pi(1-0.8)=\frac{2\pi}{5}$$

By trial and error (remember $\theta$ must be in radians) $\theta\approx 2.11$ rad=121°

$$h=R+R\cos\left(\frac{\theta}{2}\right)=0.05m+0.05m\cdot\cos\left(\frac{2.11rad}{2}\right)=\underline{\underline{0.075m}}\text{ or }7.5\text{cm}$$

## Forces on plane submerged surfaces

18. First calculate the force on the gate due to the water.

Force=pressure at centroid x area

Area= $4m\cdot 10m=40m^2$

Pressure at centroid= $\rho g h_{centroid}$ where $h_{centroid}$=depth of centroid=5m i.e. depth of a point halfway between the top and bottom edges of the gate

Thus force=1000kgm$^{-3}$.9.81Nkg$^{-1}$.5m.40m$^2$=1.962x10$^6$N

Direction of force is normal to the gate

Line of action of the force passes through the centre of pressure.

Centre of pressure lies at a point at depth $h_p$ vertically below the free surface.

$$h_p=\frac{\left(Ak^2\right)_{Cx}}{A\bar{h}}+\bar{h}$$

Where A=area of gate=40m$^2$

$\left(Ak^2\right)_{Cx}$ =second moment of gate area= $\frac{bd^3}{12}=\frac{4m\cdot(10m)^3}{12}=333.3m^4$

$\bar{h}$ =depth of centroid=5m

Thus $h_p=\frac{333.3m^4}{40m^2\cdot 5m}+5m=\underline{\underline{6.667m}}$

Now equate the moments about the hinge due to the pressure force and due to $F_r$

$$1.962\times 10^6 N\times 6.667\ m=F_r\times 10\ m$$
$$F_r=1.3\times 10^6\ N\ (2\ s.f.)$$

19.

$$F=\rho g h A \qquad \ell_p=k^2_{cent-x}/\bar{\ell}+\bar{\ell} \qquad k^2_{cent-x}=[\int \ell^2 dA]/A$$

$$F=\rho g h A=(998.2\ kg/m^3)(9.81\ m/s^2)(8m+2.5m)(7m\ x\ 3m)=2.159x10^6\ N$$

$$\bar{\ell}=(8\ m)(7/5)+3.5\ m=14.7\ m$$

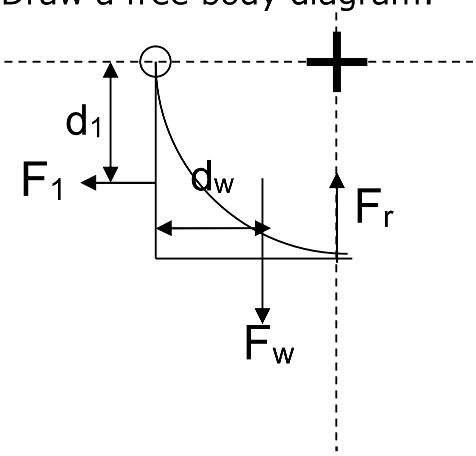
$$\ell_p=k^2_{cent-x}/\bar{\ell}+\bar{\ell}=(d^2/12)/\bar{\ell}+\bar{\ell}=[(7\ m)^2/12]/14.7\ m+14.7\ m=14.9778\ m$$

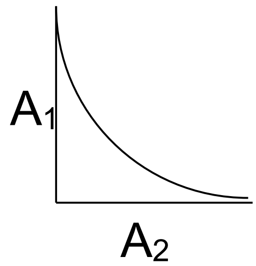
$F_r$ = F (14.9778 m – 11.2 m)/(7 m) = 1.1652 x 10$^6$ N = **1.2 x 10$^6$ N (2 SF)**

## Forces on curved submerged surfaces

20.

Draw a free body diagram:




$F_1$ is due to the hydrostatic pressure distribution on vertical area $A_1$
$F_1 = \rho g h A_1$ where $h$ is the depth of the centroid of $A_1$, and $h$=3m.
$F_1 = 1000 kgm^{-3}.9.81 ms^{-2}.3m.2m.4m = 235,440N$

$F_1$ is acting at a centre of pressure which is at distance $d_1$ below the hinge

$$d_1 = \frac{(Ak^2)_{Cx}}{A\overline{y}} + \overline{y}$$

$$(Ak^2)_{Cx} = \frac{bd^3}{12} = \frac{4m.(2m)^3}{12} = 2.67m^4$$

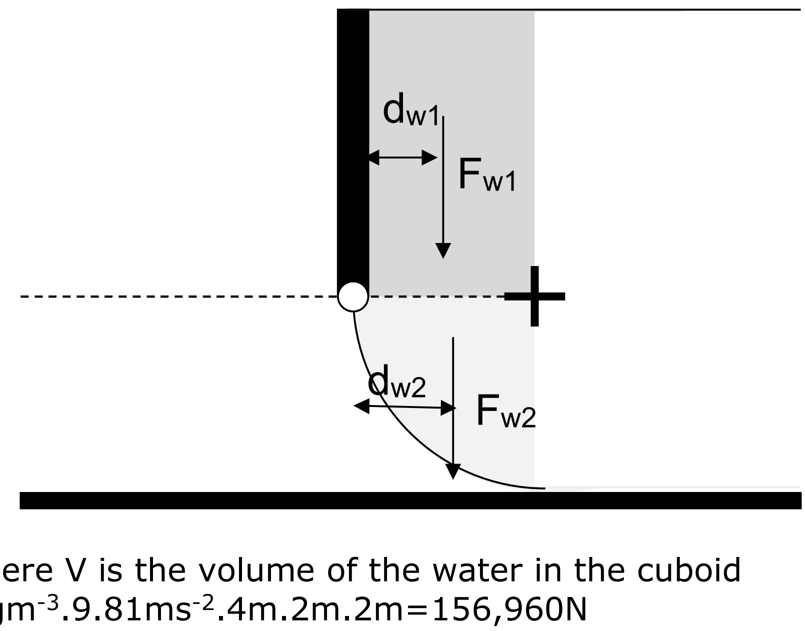
$$A = 4m.2m = 8m^2$$

$$\overline{y} = 1m$$

$$\therefore d_1 = 1.33m$$

$F_w$ is the weight of the water above the gate
$F_w$ consists of two parts, $F_{w1}$ and $F_{w2}$ where:

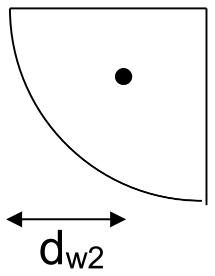
$F_{w1}$ is the weight of a cuboid volume 4m.2m.2m adjacent to the wall.
$F_{w2}$ is the weight of the quarter-cylinder of water adjacent to the gate




$F_{w1} = \rho g V$ where V is the volume of the water in the cuboid
$F_{w1} = 1000 kgm^{-3}.9.81 ms^{-2}.4m.2m.2m = 156,960N$

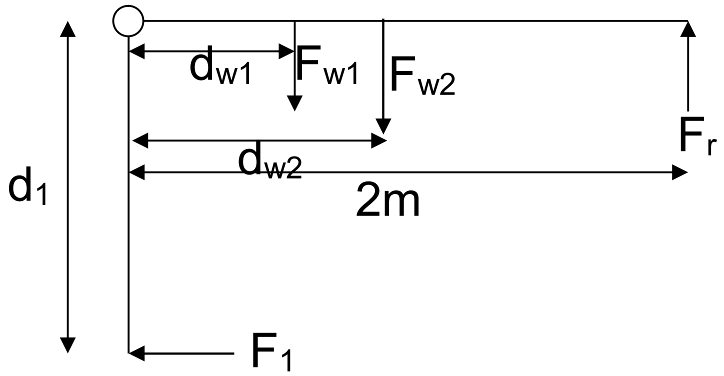
This acts at the centre of the cuboid volume, which is 1m horizontally from the hinge so $d_{w1}$=1m.

$F_{w2}$ is the weight of the quarter-cylinder of water adjacent to the gate
This volume is 4m.$\pi$.(2m)$^2$/4=12.6m$^3$
$F_{w2}$=1000kgm$^{-3}$.9.81ms$^{-2}$.12.6m$^3$=123,606N
$F_{w2}$ acts at the centre of this quarter-cylinder volume, which is $d_{w2}$ horizontally from the hinge.




$d_{w2}$=2m-4r/3$\pi$=2m-4.2m/3.$\pi$=1.15m
(where r is the radius of the quarter-cylinder)

Drawing the moments acting on the hinge:




Clockwise: $F_1.d_1+F_{w1}d_{w1}+ F_{w2}d_{w2}=F_r.2m$ Anticlockwise

$F_r=(F_1d_1+F_{w1}d_{w1}+F_{w2}d_{w2})/2m$
=(235,440N.1.33m +156,960N.1m+123,606N.1.15m)/2m
=306,000N to 3 s.f.

**Pressure distribution in incompressible fluids**

22.

$$tan\theta = \frac{a}{g} = \frac{1}{9.81}$$
$$\theta = 6^o$$

The density wasn't needed, as it affects both the weight force and the inertial force due to acceleration, and the effect of density cancels. The same angle would be found if the tank held water, or mercury.