NAG Library Routine Document

F07AEF (DGETRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07AEF (DGETRS) solves a real system of linear equations with multiple right-hand sides,

$$AX = B \qquad \text{or} \qquad A^{\mathrm{T}}X = B,$$

where A has been factorized by F07ADF (DGETRF).

2 Specification

```
SUBROUTINE F07AEF(TRANS, N, NRHS, A, LDA, IPIV, B, LDB, INFO)
INTEGER           N, NRHS, LDA, IPIV(*), LDB, INFO
double precision  A(LDA,*), B(LDB,*)
CHARACTER*1       TRANS
```

The routine may be called by its LAPACK name ***dgetrs***.

3 Description

F07AEF (DGETRS) is used to solve a real system of linear equations $AX = B$ or $A^{\mathrm{T}}X = B$, the routine must be preceded by a call to F07ADF (DGETRF) which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If TRANS = 'N', the solution is computed by solving $PLY = B$ and then $UX = Y$.

If TRANS = 'T' or 'C', the solution is computed by solving $U^{\mathrm{T}}Y = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: TRANS – CHARACTER*1 *Input*

On entry: indicates the form of the equations.

TRANS = 'N'
$AX = B$ is solved for X.

TRANS = 'T' or 'C'
$A^{\mathrm{T}}X = B$ is solved for X.

Constraint: TRANS = 'N', 'T' or 'C'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

3: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides.

Constraint: NRHS ≥ 0.

4: A(LDA,$*$) – ***double precision*** array *Input*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the LU factorization of A, as returned by F07ADF (DGETRF).

5: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07AEF (DGETRS) is called.

Constraint: LDA $\geq \max(1, \mathrm{N})$.

6: IPIV($*$) – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least $\max(1, \mathrm{N})$.

On entry: the pivot indices, as returned by F07ADF (DGETRF).

7: B(LDB,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

8: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07AEF (DGETRS) is called.

Constraint: LDB $\geq \max(1, \mathrm{N})$.

9: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon$$

where $\mathrm{cond}(A, x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$, and $\mathrm{cond}\left(A^{\mathrm{T}}\right)$ can be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling F07AHF (DGERFS), and an estimate for $\kappa_\infty(A)$ can be obtained by calling F07AGF (DGECON) with NORM = 'I'.

8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This routine may be followed by a call to F07AHF (DGERFS) to refine the solution and return an error estimate.

The complex analogue of this routine is F07ASF (ZGETRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 1.80 & 2.88 & 2.05 & -0.89 \\ 5.25 & -2.95 & -0.95 & -3.80 \\ 1.58 & -2.69 & -2.90 & -1.04 \\ -1.11 & -0.66 & -0.59 & 0.80 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 9.52 & 18.47 \\ 24.35 & 2.25 \\ 0.77 & -13.28 \\ -6.22 & -6.21 \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by F07ADF (DGETRF).

9.1 Program Text

```
*     F07AEF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, LDA, NRHMAX, LDB
      PARAMETER        (NMAX=8,LDA=NMAX,NRHMAX=NMAX,LDB=NMAX)
      CHARACTER        TRANS
      PARAMETER        (TRANS='N')
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N, NRHS
*     .. Local Arrays ..
      DOUBLE PRECISION A(LDA,NMAX), B(LDB,NRHMAX)
      INTEGER          IPIV(NMAX)
*     .. External Subroutines ..
      EXTERNAL         DGETRF, DGETRS, X04CAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07AEF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, NRHS
      IF (N.LE.NMAX .AND. NRHS.LE.NRHMAX) THEN
*
*        Read A and B from data file
*
         READ (NIN,*) ((A(I,J),J=1,N),I=1,N)
         READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*
*        Factorize A
*
         CALL DGETRF(N,N,A,LDA,IPIV,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
*
*           Compute solution
*
            CALL DGETRS(TRANS,N,NRHS,A,LDA,IPIV,B,LDB,INFO)
*
```

```
*           Print solution
*
            IFAIL = 0
            CALL X04CAF('General',' ',N,NRHS,B,LDB,'Solution(s)',IFAIL)
         ELSE
            WRITE (NOUT,*) 'The factor U is singular'
         END IF
      END IF
*
      END
```

9.2 Program Data

```
F07AEF Example Program Data
  4  2                          :Values of N and NRHS
  1.80   2.88   2.05  -0.89
  5.25  -2.95  -0.95  -3.80
  1.58  -2.69  -2.90  -1.04
 -1.11  -0.66  -0.59   0.80    :End of matrix A
  9.52  18.47
 24.35   2.25
  0.77 -13.28
 -6.22  -6.21                  :End of matrix B
```

9.3 Program Results

```
 F07AEF Example Program Results

 Solution(s)
                1          2
 1       1.0000     3.0000
 2      -1.0000     2.0000
 3       3.0000     4.0000
 4      -5.0000     1.0000
```