CS 81, Logic and Computability
Problem Set 8: Context-Free Grammars

This problem set uses the Stanford CS 103 CFG developer:
https://web.stanford.edu/class/archive/cs/cs103/cs103.1156/tools/cfg/.

Challenge 1: Grammar Youngling [60 Points]

Using the online CFG tool, construct an **unambiguous** CFG that produces all the strings of **just zeroes** that have an even number of zeroes, and produces no other strings.

Then, test your CFG on the following strings:

```
1

00

0000
000000
00000000
000
0
00000
1
00100
```

Notice, the second item we test for is epsilon, the empty string. We test for that by including a blank line in our test box.

Include your grammar and a screenshot of the test results. The screenshot should look like Figure 3. **For full points** your grammar should only have unambiguous derivations for the strings in that language. Our sample solution uses two rules.

Challenge 2: Grammar Padawan [35 Points]

Let $L_1 = \{w\#x \mid w, x \in \{0,1\}^* \wedge (|w| = |x|) \wedge (w \neq x^R)\}$. Note that the input alphabet is $\Sigma = \{0, 1, \#\}$ and recall that x^R denotes the string x in reverse order.

For example, the following strings are in L_1: 000#001, 1100#0001, 00#01. However, the following strings are not in L_1: 00#000, 001#100.

Construct a context-free grammar for L_1 using the online tool and **include a screenshot of the results for tests** of the following strings:

```
00#00
00#01
00#000
1100#001
001#100
0
000#001
1#0
0010#0
001##101
```

Please include your grammar and your screenshot of the test results. Ambiguous grammars are fine for this problem.

Challenge 3: Grammar Jedi [5 Points]

This is a practice challenge, in case you want a more challenging problem to work on. It is only worth a small number of points, so don't sweat it if you can't figure it out.

Let $P(w)$ be a predicate on the domain of strings in $\{0,1\}^*$ where the interpretation of $P(w)$ is that the number of 0's in w is different from the number of 1's in w. Consider the language $L_2 = \{w \mid w \in \{0,1\}^* \wedge P(w)\}$. For example, the following strings are in L_2: 000, 001, 100, 10110.

Construct a context-free grammar for L_2 using the online tool, and **include a screenshot of the results for tests** of the following strings:

```
000
001
100
10110
001100
01
000001
10
11001001
1010101010
```

Please include your grammar and your screenshot of the test results. Ambiguous grammars are fine for this problem.

Answers

Answer 1:

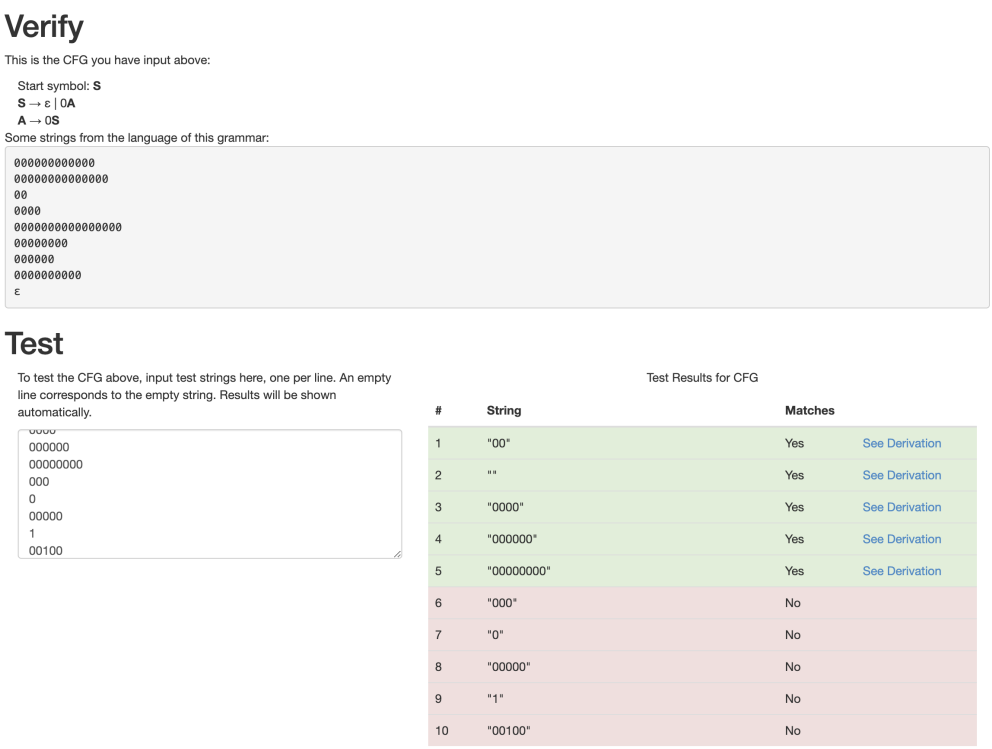

Figure 1: Screenshot of online tool tests.

Answer 2:

Answer 3:

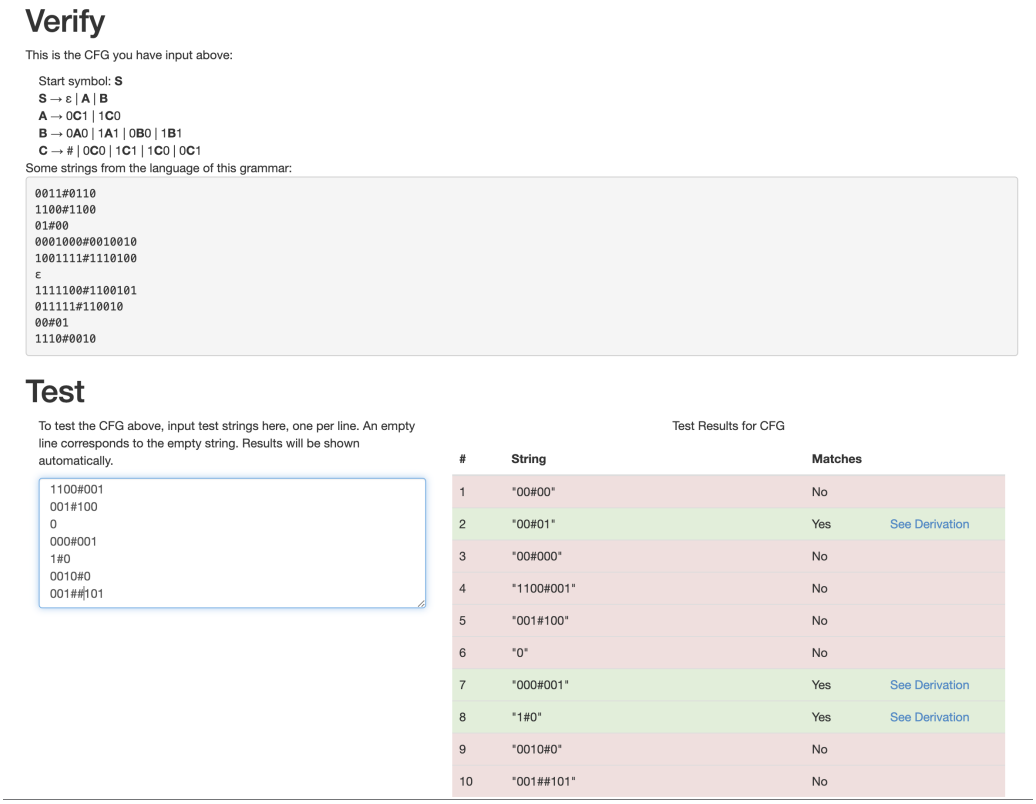

Figure 2: Screenshot of online tool tests.

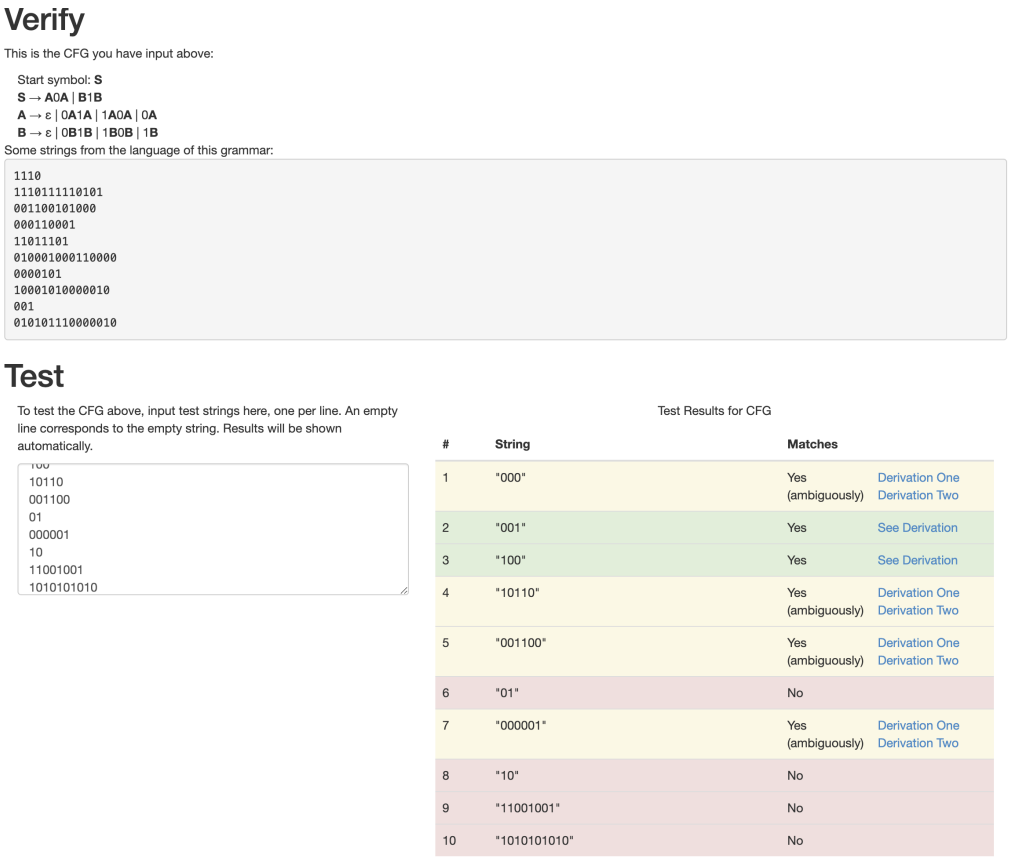

Figure 3: Screenshot of online tool tests.