8

Probability

TERMINOLOGY

Complement: The complement of an event *E* is when the event *E* does not occur

Equally likely outcomes: Each outcome has the same chance of occurring

Independent events: Events are independent if the result of one event does not influence the outcome of another event. There is no overlap between the events

Multi-stage events: A series of successive independent events

Mutually exclusive results: Two events with the same sample space that cannot both occur at the same time

Non-mutually exclusive results: Two events with the same sample space that can occur at the same time i.e. there is some overlap

Probability tree: A diagram that uses branches to show multi-stage events and sets out the probability on each branch with the sample space listed at the right of each branch

Random experiments: Experiments that are made with no pattern or order where each outcome is equally likely to occur

Sample space: The set of all possible outcomes in an event or series of events

Tree diagram: A diagram that uses branches to show multi-stage events where the probabilities on each branch are equal

Venn diagram: A special diagram to show the sample space for non-mutually exclusive events using circles for each event drawn inside a rectangle which represents the sample space

INTRODUCTION

PROBABILITY IS THE STUDY of how likely it is that something will happen. It is used to make predictions in different areas, ranging from gambling to determining the rate of insurance premiums. For example an actuary looking at death-rate statistics can estimate the probable age to which someone will live, and set life insurance premiums accordingly.

Another example of where probability is useful is in biology. The probability of certain diseases or genetic defects can be calculated in high-risk families.

Probability is also closely related to statistics and data analysis, as well as games of chance such as card games, tossing coins, backgammon and so on. It is also relevant to buying raffle and lottery tickets.

In this chapter, you will revise simple probability that you have learnt in previous years and extend this to more complex probability involving multi-stage events.

DID YOU KNOW?

Girolamo Cardano (1501–76) was a doctor and mathematician who developed the first theory of probability. He was a great gambler, and he wrote *De Ludo Aleae* ('On Games of Chance'). This work was largely ignored, and it is said that the first book on probability was written by **Christiaan Huygens** (1629–95).

The main study of probability was done by **Blaise Pascal** (1623–62), whom you have already heard about, and **Pierre de Fermat** (1601–65).

Simple Probability

Mutually exclusive events

Mutually exclusive events means that if one event occurs, the other cannot. For example, when rolling a die, a 6 cannot occur at the same time as a 2.

We can measure probability in theory as long as the events are random. However, even then, probability only gives us an approximate idea of the likelihood of certain events happening.

For example, in Lotto draws, there is a machine that draws out the balls at **random** and a panel of supervisors checks that this happens properly. Each ball is independent of the others and is **equally likely** to be drawn out.

In a horse race, it is difficult to measure probability as the horses are not all equally likely to win. Other factors such as ability, training, experience and weight of the jockey all affect it. The likelihood of any one horse winning is not random.

Class Discussion

Discuss these statements:

1. The probability of one particular football team winning a competition is $\frac{1}{16}$ as there are 16 teams.
2. The probability of Tiger Woods winning the US Open golf tournament is $\frac{1}{78}$ if there are 78 players in the tournament.
3. A coin came up tails 8 times in a row. So there is a greater chance that the next time it will come up heads.

The probability of an event E happening, $P(E)$, is given by the number of ways the event can occur, $n(E)$, compared with the total number of outcomes possible $n(S)$ (the size of the sample space)

$$P(E) = \frac{n(E)}{n(S)}$$

If $P(E) = 0$ the event is impossible

If $P(E) = 1$ the event is certain (it has to happen)

$$0 \le P(E) \le 1$$

The sum of all (mutually exclusive) probabilities is 1

EXAMPLES

1. A container holds 5 blue, 3 white and 7 yellow marbles. If one marble is selected at random, find the probability of getting

(a) a white marble
(b) a white or blue marble
(c) a yellow, white or blue marble
(d) a red marble.

Solution

The size of the sample space, or total number of marbles is 5 + 3 + 7 or 15.

(a) $P(W) = \frac{3}{15}$
$= \frac{1}{5}$

(b) $P(W \text{ or } B) = \frac{3+5}{15}$
$= \frac{8}{15}$

(c) $P(Y \text{ or } W \text{ or } B) = \frac{7+3+5}{15}$
$= \frac{15}{15}$
$= 1$

(d) $P(R) = \frac{0}{15}$
$= 0$

Getting a red marble is impossible!

2. The probability that a traffic light will turn green as a car approaches it is estimated to be $\frac{5}{12}$. A taxi goes through 192 intersections where there are traffic lights. How many of these would be expected to turn green as the taxi approached?

Solution

It is expected that $\frac{5}{12}$ of the traffic lights would turn green.

$\frac{5}{12} \times 192 = 80$

So it would be expected that 80 traffic lights would turn green as the taxi approached.

8.1 Exercises

1. Peter is in a class of 30 students. If one student is chosen at random to make a speech, find the probability that the student chosen will be Peter.

2. A pack of cards contains 52 different cards, one of which is the ace of diamonds. If one card is chosen at random, find the probability that it will be the ace of diamonds.

'Die' is singular for 'dice'.

3. There are 6 different newspapers sold at the local newsagent each day. Kim sends her little brother to buy her a newspaper one morning but forgets to tell him which one. What is the probability that he will buy the correct newspaper?

4. A raffle is held in which 200 tickets are sold. If I buy 5 tickets, what is the probability of my winning the prize in the raffle?

5. In a lottery, 200 000 tickets are sold. If Lucia buys 10 tickets, what is the probability of her winning first prize?

6. A bag contains 6 red balls and 8 white balls. If one ball is drawn out of the bag at random, find the probability that it will be
   (a) white
   (b) red.

7. A shoe shop orders in 20 pairs of black, 14 pairs of navy and 3 pairs of brown school shoes. If the boxes are all mixed up, find the probability that one box selected at random will contain brown shoes.

8. Abdul has 12 CDs in his car glovebox. The labels have become mixed up. If he chooses one of the CDs at random, find the probability that it is his favourite CD.

9. A bag contains 5 black marbles, 4 yellow marbles and 11 green marbles. Find the probability of drawing 1 marble out at random and getting
   (a) a green marble
   (b) a yellow or a green marble.

10. A die is thrown. Calculate the probability of throwing
   (a) a 6
   (b) an even number
   (c) a number less than 3.

11. A book has 124 pages. If the book is opened at any page at random, find the probability of the page number being
   (a) either 80 or 90
   (b) a multiple of 10
   (c) an odd number
   (d) less than 100.

12. In the game of pool, there are 15 balls, each with the number 1 to 15 on it. In Kelly Pool, each person chooses a number at random to determine which ball to sink. If Tracey chooses a number, find the probability that her ball will be
   (a) an odd number
   (b) a number less than 8
   (c) the 8 ball.

13. At a school dance, each girl is given a ticket with a boy's name on it. The girl then must dance with that boy for the next dance. If the tickets are given out at random and there are 50 boys at the dance, what is the probability that Jill will get to dance with her boyfriend?

14. Find the probability of a coin coming up heads when tossed. If the coin is double-headed, find the probability of tossing a head.

15. In a bag of caramels, there are 21 with red wrappers and 23 with blue wrappers. If Leila chooses a caramel at random from the bag, find the probability that she will choose one with a blue wrapper.

16. A student is chosen at random to write about his/her favourite sport. If 12 students like tennis best, 7 prefer soccer, 3 prefer squash, 5 prefer basketball and 4 prefer swimming, find the probability that the student chosen will write about
(a) soccer
(b) squash or swimming
(c) tennis.

17. A school has 875 students. If 5 students are chosen at random to help show some visitors around, find the probability that a particular student will be chosen.

18. A box containing a light globe has a $\frac{1}{20}$ probability of holding a defective globe. If 160 boxes are checked, how many would be expected to be defective?

19. There are 29 red, 17 blue, 21 yellow and 19 green chocolate beans in a packet. If Kate chooses one at random, find the probability that it will be red or yellow.

20. The probability of breeding a white budgerigar is $\frac{2}{9}$. If Mr Seed breeds 153 budgerigars over the year, how many would be expected to be white?

21. A biased coin is weighted so that heads comes up twice as often as tails. Find the probability of tossing a tail.

22. A die has the centre dot painted white on the 5 so that it appears as a 4. Find the probability of throwing
(a) a 2
(b) a 4
(c) a number less than 5.

23. Discuss these statements.
(a) The probability of one particular horse winning the Melbourne cup is $\frac{1}{20}$ if there are 20 horses in the race.
(b) The probability of Greg Norman winning a masters golf tournament is $\frac{1}{15}$ if there are 15 players in the tournament.
(c) A coin came up tails 8 times in a row. So the next toss must be a head.
(d) A family has three sons. There is more chance of getting a daughter next time.
(e) The probability of a Holden winning the car race at Bathurst this year is $\frac{6}{47}$ as there are 6 Holdens in the race and 47 cars altogether.

Complementary events

When we find the probabilities of events, the total of all the possible events will always add up to 1.

EXAMPLE

A ball is chosen at random from a bag containing 5 blue, 3 red and 7 yellow balls. The probabilities are as follows:

$$P(\text{blue}) = \frac{5}{15}$$

$$P(\text{red}) = \frac{3}{15}$$

$$P(\text{yellow}) = \frac{7}{15}$$

$$\begin{aligned}\text{Total probability} &= \frac{5}{15} + \frac{3}{15} + \frac{7}{15}\\ &= \frac{15}{15}\\ &= 1\end{aligned}$$

The complement of an event happening is the event *not* happening. That is, the complement of $P(E)$ is $P(\text{not } E)$. We can write this as $P(\widetilde{E})$

EXAMPLE

A die is thrown. Find the probability of

(a) throwing a 6
(b) not throwing a 6.

Solution

(a) $P(6) = \frac{1}{6}$

(b) $$\begin{aligned}P(\text{not } 6) &= P(1, 2, 3, 4, \text{ or } 5)\\ &= \frac{5}{6}\end{aligned}$$

In general,
$P(E) + P(\widetilde{E}) = 1$
or $P(E) = 1 - P(\widetilde{E})$

Proof

Let e be the number of ways E can happen out of a total of n events. Then the number of ways E will **not** happen is $n - e$.

$$\begin{aligned} \text{Then } P(E) &= \frac{e}{n} \\ P(\widetilde{E}) &= \frac{n-e}{n} \\ &= \frac{n}{n} - \frac{e}{n} \\ &= 1 - \frac{e}{n} \\ &= 1 - P(E) \end{aligned}$$

EXAMPLES

1. The probability of a win in a raffle is $\frac{1}{350}$. What is the probability of losing?

Solution

$$\begin{aligned} P(\text{lose}) &= 1 - P(\text{win}) \\ &= 1 - \frac{1}{350} \\ &= \frac{349}{350} \end{aligned}$$

2. The probability of a tree surviving a fire is 72%. Find the probability of the tree failing to survive a fire.

Solution

$$\begin{aligned} P(\text{failing to survive}) &= 1 - P(\text{surviving}) \\ &= 100\% - 72\% \\ &= 28\% \end{aligned}$$

8.2 Exercises

1. The probability of a bus arriving on time is estimated at $\frac{18}{33}$. What is the probability that the bus will not arrive on time?
2. The probability of a seed producing a pink flower is $\frac{7}{9}$. Find the probability of the flower producing a different colour.
3. If a baby has a 0.2% chance of being born with a disability, find the probability of the baby being born without any disabilities.
4. The probability of selecting a card with the number 5 on it is 0.27. What is the probability of not selecting this card?

5. There is a 62% chance of a student being chosen as a prefect. What is the probability of a student not being selected as a prefect?

6. A machine has a 1.5% chance of breaking down at any given time. What is the probability of the machine not breaking down?

7. The life of a certain type of computer is about 7 years. If the probability of its needing repairs in that time is $\frac{1}{23}$, find the probability that it will not need repairs.

8. A certain traffic light has a probability of $\frac{13}{18}$ of being green. Find the probability of the light not being green when a car comes to the traffic light.

9. A certain organism in a river has a probability of 0.79 of surviving a flood. What is the probability of its not surviving?

10. A city has an 8.1% chance of being hit by an earthquake. What is its chance of not having an earthquake?

11. The probabilities when 3 coins are tossed are as follows:

$$P(3 \text{ heads}) = \frac{1}{8}$$
$$P(2 \text{ heads and } 1 \text{ tail}) = \frac{3}{8}$$
$$P(1 \text{ head and } 2 \text{ tails}) = \frac{3}{8}$$
$$P(3 \text{ tails}) = \frac{1}{8}$$

Find the probability of tossing at least one head.

12. The probabilities of a certain number of seeds germinating when 4 seeds are planted are as follows:

$$P(4 \text{ seeds}) = \frac{4}{49}$$
$$P(3 \text{ seeds}) = \frac{8}{49}$$
$$P(2 \text{ seeds}) = \frac{16}{49}$$
$$P(1 \text{ seed}) = \frac{18}{49}$$
$$P(0 \text{ seeds}) = \frac{3}{49}$$

Find the probability of at least 1 seed germinating.

13. The probabilities of 4 friends being chosen for a soccer team are as follows:

$$P(4 \text{ chosen}) = \frac{1}{15}$$
$$P(3 \text{ chosen}) = \frac{4}{15}$$
$$P(2 \text{ chosen}) = \frac{6}{15}$$
$$P(1 \text{ chosen}) = \frac{2}{15}$$

Find the probability of
(a) none of the friends being chosen
(b) at least 1 of the friends being chosen.

14. A dog breeder tries to produce a dog with a curly tail. If 2 puppies are born, the probabilities are as follows:

$$P(\text{no curly tails}) = \frac{4}{11}$$
$$P(1 \text{ curly tail}) = \frac{5}{11}$$
$$P(2 \text{ curly tails}) = \frac{2}{11}$$

Find the probability that at least 1 puppy will have a curly tail.

15. The probabilities of 3 new cars passing a quality control check are as follows:

$$P(3 \text{ passing}) = \frac{1}{16}$$
$$P(2 \text{ passing}) = \frac{7}{16}$$
$$P(1 \text{ passing}) = \frac{3}{16}$$
$$P(0 \text{ passing}) = \frac{5}{16}$$

Find the probability that at least 1 car will fail the check.

Non-mutually exclusive events

All the examples of probability given so far in this chapter are mutually exclusive. This means that if one event occurs, then another one cannot. For example, if a die is thrown, a 6 cannot occur at the same time as a 2.

Sometimes, there is an overlap where more than one event can occur at the same time. We call these non-mutually exclusive events.

It is important to count the possible outcomes carefully when this happens. If there are not too many outcomes, we can simply list them, but if this is difficult, we can use a Venn diagram to help.

EXAMPLES

1. One card is drawn from a set of cards numbered 1 to 10. Find the probability of drawing out an odd number or a multiple of 3.

Solution

The odd cards are 1, 3, 5, 7 and 9.
The multiples of 3 are 3, 6 and 9.
The numbers 3 and 9 are both odd and multiples of 3.
So there are 6 numbers that are odd or multiples of 3: 1, 3, 5, 6, 7 and 9

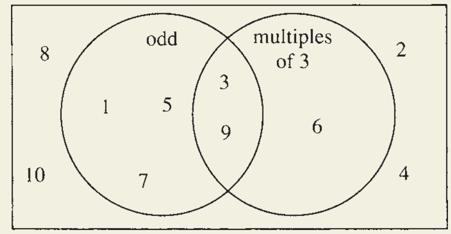
The trick with this question is not to count the 3 and the 9 twice.

$$P(\text{odd or multiple of 3}) = \frac{6}{10} = \frac{3}{5}$$

CONTINUED

Using a Venn diagram:

Count all the numbers inside the two circles.



From the Venn diagram:

Probability is $\frac{6}{10} = \frac{3}{5}$.

There are 6 numbers inside the circles and 10 numbers altogether.

2. In year 7 at Mt Random High School, every student must do art or music. In a group of 100 students surveyed, 47 do music and 59 do art. If one student is chosen at random from year 7, find the probability that this student does

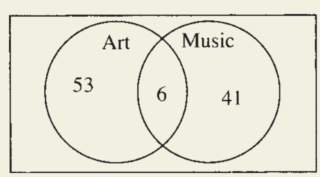
(a) both art and music
(b) only art
(c) only music.

Solution

$47 + 59 = 106$

But there are only 100 students!
This means 6 students have been counted twice.
i.e. 6 students do both art and music.

Students doing music only: $47 - 6 = 41$
Students doing art only: $59 - 6 = 53$
A Venn diagram shows this information.



(a) $P(\text{both}) = \frac{6}{100} = \frac{3}{50}$

(b) $P(\text{art only}) = \frac{53}{100}$

(c) $P(\text{music only}) = \frac{41}{100}$

DID YOU KNOW?

Venn diagrams are named after **John Venn** (1834–1923), an English probabilist and logician.

There is a formula that can be used for non-mutually exclusive events.

$$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}$$

Notice that $P(A) + P(B)$ counts $P(A \text{ and } B)$ twice, since it occurs in both $P(A)$ and $P(B)$. This can be adjusted by subtracting $P(A \text{ and } B)$.

EXAMPLE

From 100 cards, numbered from 1 to 100, one is selected at random. Find the probability that the card selected is even or less than 20.

Solution

Some cards are both even and less than 20 (i.e. 2, 4, 6, 8, 10, 12, 14, 16, 18).

$$P(\text{even and} < 20) = \frac{9}{100}$$

$$P(\text{even}) = \frac{50}{100}$$

$$P(< 20) = \frac{19}{100}$$

$$\begin{aligned} P(\text{even or} < 20) &= P(\text{even}) + P(< 20) - P(\text{even and} < 20) \\ &= \frac{50}{100} + \frac{19}{100} - \frac{9}{100} \\ &= \frac{60}{100} \\ &= \frac{3}{5} \end{aligned}$$

8.3 Exercises

1. A number is chosen at random from the numbers 1 to 20. Find the probability that the number chosen will be
   (a) divisible by 3
   (b) less than 10 or divisible by 3
   (c) a composite number
   (d) a composite number or a number greater than 12.

2. A set of 50 cards is labelled from 1 to 50. One card is drawn out at random. Find the probability that the card will be
   (a) a multiple of 5
   (b) an odd number
   (c) a multiple of 5 or an odd number
   (d) a number greater than 40 or an even number
   (e) less than 20.

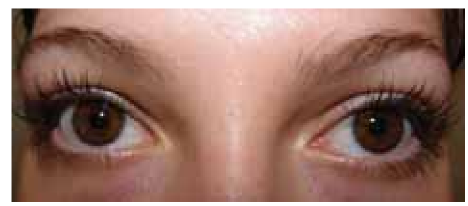
3. A set of 26 cards, each with a different letter of the alphabet on it, is placed in a box and one is drawn out at random. Find the probability that the letter on the card drawn will be
   (a) a vowel
   (b) a vowel or one of the letters in the word 'random'
   (c) a consonant or one of the letters in the word 'movies'.

4. A set of discs is numbered 1 to 100 and one is chosen at random. Find the probability that the number on the disc will be
   (a) less than 30
   (b) an odd number or a number greater than 70
   (c) divisible by 5 or less than 20.

5. In Lotto, a machine holds 45 balls, each with a number between 1 and 45 on it. The machine draws out one ball at a time at random. Find the probability that the first ball drawn out will be
   (a) less than 10 or an even number.
   (b) between 1 and 15 inclusive, or divisible by 6
   (c) greater than 30 or an odd number.

6. A class of 28 students puts on a concert with all class members performing. If 15 dance and 19 sing in the performance, find the probability that any one student chosen at random from the class will
   (a) both sing and dance
   (b) only sing
   (c) only dance.

7. A survey of 80 people with dark hair or brown eyes showed that 63 had dark hair and 59 had brown eyes. If one of the people surveyed is chosen at random, find the probability that the person will
   (a) have dark hair but not brown eyes
   (b) have brown eyes but not dark hair
   (c) have both brown eyes and dark hair.



8. A list is made up of people with experience of either computers or digital cameras. On the list of 20 people, 13 have computer experience while 9 have experience with a digital camera. If one name is chosen at random from the list, find the probability that the person will have experience with
   (a) both computers and digital cameras
   (b) computers only
   (c) digital cameras only.

9. In a group of 75 students, all do either History or Geography. Altogether 54 do History and 31 do Geography. If I select one student at random, find the probability that he/she will do
(a) only Geography
(b) both History and Geography
(c) History but not Geography.

10. In a group of 20 dogs at obedience school, 14 dogs will walk to heel and 12 will stay when told. If one dog is chosen at random, find the probability that the dog will
(a) both walk to heel and stay
(b) walk to heel but not stay
(c) stay but not walk to heel.

Multi-Stage Events

Product rule of probability

Class Discussion

Break up into pairs and try these experiments with one doing the activity and one recording the results.

1. Toss two coins as many times as you can in a 5 minute period and record the results in the table:

Result	Two heads	One head and one tail	Two tails
Tally			

Compare your results with others in the class. What do you notice? Is this surprising?

2. Roll two dice as many times as you can in a 5 minute period, find the total of the two uppermost numbers on the dice and record the results in the table:

Total	2	3	4	5	6	7	8	9	10	11	12
Tally											

Compare your results with others in the class. What do you notice? Is this surprising?
Why don't these results appear to be equally likely?

The counting of all possible outcomes (the size of the sample space) is important. This is why we use tables and tree diagrams.

You learned about tree diagrams in your earlier studies.

EXAMPLES

Compare these probabilities with your results in the experiments.

Find the size of the sample space and the probability of each outcome for each question by using a table or tree diagram.

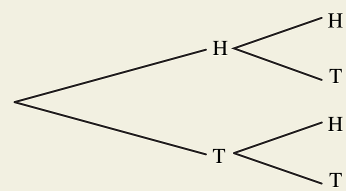
1. Tossing two coins

Solution

Using a table gives

	H	T
H	HH	HT
T	TH	TT

Using a tree diagram gives



Since there are four separate outcomes (HH, HT, TH, TT) each outcome has a probability of $\frac{1}{4}$.

Remember that each outcome when tossing 1 coin is $\frac{1}{2}$.

Notice that $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

2. Rolling 2 dice and recording the sum of the uppermost numbers.

Solution

Can you see why a tree diagram is too difficult here?

A tree diagram would be too difficult to draw for this question.

Using a table:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Since there are 36 outcomes, each has a probability of $\frac{1}{36}$

Remember that each outcome when rolling 1 die is $\frac{1}{6}$.

Notice that $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

If A and B are independent events, then the probability of both occurring is the product of their probabilities.

$$\boxed{P(AB) = P(A) \cdot P(B)}$$

EXAMPLES

1. The probability of getting a 6 when rolling a die is $\frac{1}{6}$. Find the probability of getting a double 6 when rolling two dice.

Solution

$$\begin{aligned} P(\text{double } 6) &= \frac{1}{6} \times \frac{1}{6} \\ &= \frac{1}{36} \end{aligned}$$

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2. The probability that a certain missile will hit a target is $\frac{7}{8}$. Find the probability that the missile will

(a) hit two targets

(b) miss two targets.

Solution

(a) $$\begin{aligned} P(2 \text{ hits}) &= \frac{7}{8} \times \frac{7}{8} \\ &= \frac{49}{64} \end{aligned}$$

(b) $$\begin{aligned} P(\text{miss}) &= 1 - \frac{7}{8} \\ &= \frac{1}{8} \end{aligned}$$

$$\begin{aligned} P(2 \text{ misses}) &= \frac{1}{8} \times \frac{1}{8} \\ &= \frac{1}{64} \end{aligned}$$

Using these answers, could you calculate the probability that the missile hits one target and not the other?

Sometimes the outcomes change when looking at more than one event.

EXAMPLES

1. Maryam buys 5 tickets in a raffle in which 95 tickets are sold altogether. There are two prizes in the raffle. What is the probability of her

(a) winning both first and second prizes?
(b) winning neither prize?
(c) winning at least one of the prizes?

Solution

(a) Probability of winning first prize $= \frac{5}{95}$

After winning first prize, Maryam's winning ticket is taken out of the draw. She then has 4 tickets left in the raffle out of a total of 94 tickets left.
Probability of winning second prize $= \frac{4}{94}$

$$P(\text{WW}) = \frac{5}{95} \times \frac{4}{94}$$
$$= \frac{2}{893}$$

(b) Probability of not winning first prize $= 1 - \frac{5}{95}$

$$= \frac{90}{95}$$

After not winning first prize, Maryam's 5 tickets are all left in the draw, but the winning ticket is taken out, leaving 94 tickets in the raffle.
Probability of winning second prize $= \frac{5}{94}$

Probability of not winning second prize $= 1 - \frac{5}{94}$

$$= \frac{89}{94}$$

$$P(\text{LL}) = \frac{90}{95} \times \frac{89}{94}$$
$$= \frac{801}{893}$$

(c) $P(\text{at least one W}) = 1 - P(\text{LL})$

$$= 1 - \frac{801}{893}$$
$$= \frac{92}{893}$$

2. I choose 3 balls at random from a bag containing 7 blue and 5 red balls.

(a) Find the probability of getting 3 blue balls if
(i) I replace each ball before choosing the next one
(ii) I don't replace each ball before choosing the next one.

(b) Find the probability of getting at least one red ball (without replacement).

Solution

(a) (i) $P(B) = \frac{7}{12}$

So $P(BBB) = \frac{7}{12} \times \frac{7}{12} \times \frac{7}{12}$

$= \frac{343}{1728}$

(ii) $P(B) = \frac{7}{12}$

After the first blue ball has been chosen, the bag now contains 6 blue and 5 red balls.

$P(2\text{nd}\, B) = \frac{6}{11}$

After the second blue ball has been chosen, the bag contains 5 blue and 5 red balls.

$P(3\text{rd}\, B) = \frac{5}{10}$

So $P(BBB) = \frac{7}{12} \times \frac{6}{11} \times \frac{5}{10}$

$= \frac{7}{44}$

(b) $P(\text{at least one}\, R) = 1 - (\text{no}\, R)$

$= 1 - P(BBB)$

$= 1 - \frac{7}{44}$

$= \frac{37}{44}$

8.4 Exercises

1. If 2 dice are thrown, find the probability of throwing two 6s.
2. Find the probability of getting 2 heads if a coin is tossed twice.
3. A coin is tossed 3 times. Find the probability of tossing 3 tails.
4. A card has a picture on one side and is blank on the other. If the card is thrown into the air twice, find the probability that it will land with the picture side up both times.
5. A box contains 2 black balls, 5 red balls and 4 green balls. If I draw out 2 balls at random, replacing the first before drawing out the second, find the probability that they will both be red.
6. The probability of a conveyor belt in a factory breaking down at any one time is 0.21. If the factory has 2 conveyor belts, find the probability that at any one time
   (a) both machines will break down
   (b) neither machine will break down.

7. The probability of a certain plant flowering is 93%. If a nursery has 3 of these plants, find the probability that they will all flower.

8. An archery student has a 69% chance of hitting a target. If she fires 3 arrows at a target, find the probability that she will hit the target each time.

9. A bag contains 8 yellow and 6 green lollies. If I choose 2 lollies at random, find the probability that they will both be green
(a) if I replace the first lolly before selecting the second
(b) if I don't replace the first lolly.

10. I buy 10 tickets in a raffle in which 250 tickets are sold. Find the probability of winning both first and second prizes.

11. Two cards are drawn from a deck of 20 red and 25 blue cards (without replacement). Find the probability that they will both be red.

12. Find the probability of winning the first 3 prizes in a raffle if Peter buys 5 tickets and 100 tickets are sold altogether.

13. The probability of a pair of small parrots breeding an albino bird is $\frac{2}{33}$. If they lay three eggs, find the probability of the pair
(a) not breeding any albinos
(b) having all three albinos
(c) breeding at least one albino.

14. A photocopier has a paper jam on average around once every 2400 sheets of paper.
(a) What is the probability that a particular sheet of paper will jam?
(b) What is the probability that two particular sheets of paper will jam?
(c) What is the probability that two particular sheets of paper will not jam?

15. In Yahtzee, 5 dice are rolled. Find the probability of rolling
(a) five 6's
(b) no 6's
(c) at least one 6.

16. The probability of a faulty computer part being manufactured at Omega Computer Factory is $\frac{3}{5000}$.
If 2 computer parts are examined, find the probability that
(a) both are faulty
(b) neither are faulty
(c) at least one is faulty.

17. A set of 10 cards is numbered 1 to 10 and two are drawn out at random with replacement. Find the probability of drawing
(a) two odd numbers
(b) two numbers that are divisible by 3
(c) two numbers less than 4.

18. A bag contains 5 white, 4 black and 3 red marbles. If 2 marbles are selected from the bag at random without replacement, find the probability of selecting
    (a) two red marbles
    (b) two black marbles
    (c) no white marbles
    (d) at least one white marble.

19. The probability of an arrow hitting a target is 85%. If 3 arrows are shot, find the probability as a percentage, correct to 2 decimal places, of
    (a) all arrows hitting the target
    (b) no arrows hitting the target
    (c) at least one arrow hitting the target.

20. A coin is tossed n times. Find the probability in terms of n of tossing
    (a) all heads
    (b) no tails
    (c) at least one tail.

Tree diagrams and probability trees

When using the product rule to find the probability of successive events occurring, sometimes there is more than one possible result. For example, when tossing two coins, there are two ways of getting a head and a tail (HT and TH). We add these results together.

$$\boxed{P(A \text{ or } B) = P(A) + P(B)}$$

This is called the addition rule of probability.

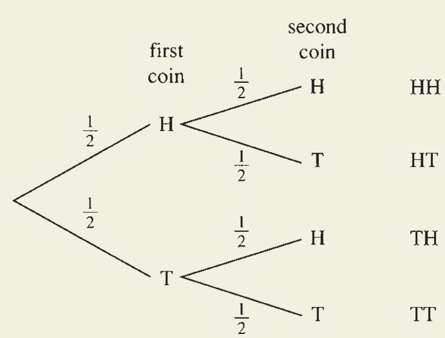
We use tree diagrams or probability trees to combine the product and addition rules.

We use the product rule by multiplying along the branches and the addition rule by adding up the probabilities from different branches.

EXAMPLES

1. If 2 coins are tossed, find the probability of tossing a head and a tail.

Solution



This is a probability tree as it has the probabilities listed.

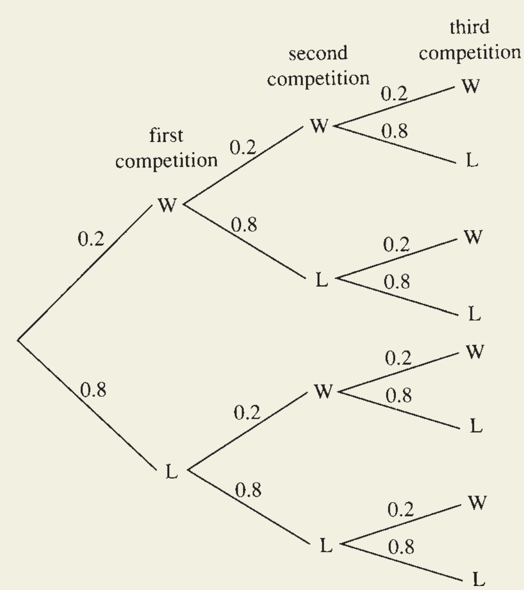
CONTINUED

$$P(\text{head and tail}) = P(HT) + P(TH)$$
$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right)$$
$$= \frac{1}{4} + \frac{1}{4}$$
$$= \frac{1}{2}$$

2. A person has probability of 0.2 of winning a prize in a competition. If he enters 3 competitions, find the probability of his winning

(a) 2 competitions
(b) at least 1 competition.

Solution



Probability of losing is 1 – 0.2.

(a) Probability of losing is 0.8

$$P(2W) = P(WWL) + P(WLW) + P(LWW)$$
$$= (0.2 \times 0.2 \times 0.8) + (0.2 \times 0.8 \times 0.2) + (0.8 \times 0.2 \times 0.2)$$
$$= 0.032 + 0.032 + 0.032$$
$$= 0.096$$

(b) $P(\text{at least one } W) = 1 - P(LLL)$
$$= 1 - (0.8 \times 0.8 \times 0.8)$$
$$= 1 - 0.512$$
$$= 0.488$$

3. A bag contains 3 red, 4 white and 7 blue marbles. Two marbles are drawn at random from the bag

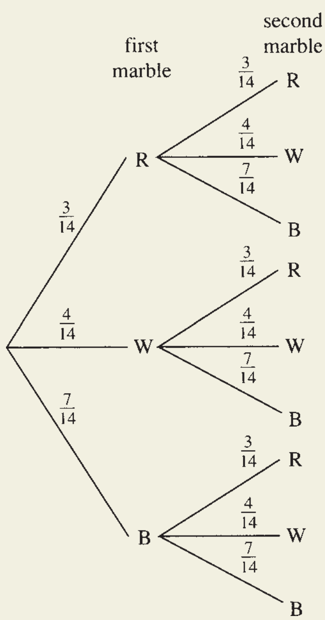
(a) replacing the first before the second is drawn

(b) without replacement

Find the probability of drawing out a red and a white marble in these cases.

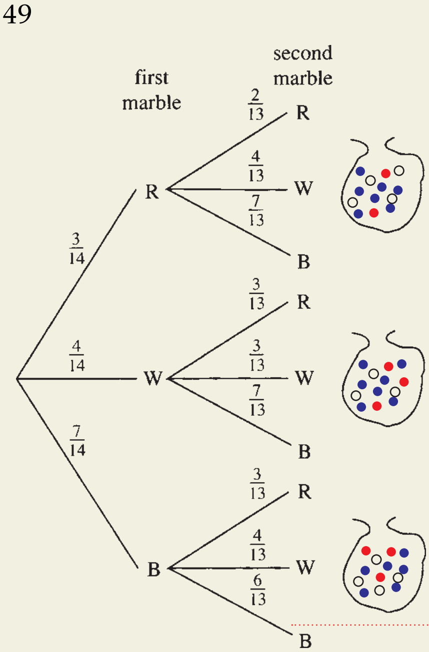
Solution

(a)



$$P(R \text{ and } W) = P(RW) + P(WR)$$
$$= \left(\frac{3}{14} \times \frac{4}{14}\right) + \left(\frac{4}{14} \times \frac{3}{14}\right)$$
$$= \frac{12}{196} + \frac{12}{196}$$
$$= \frac{6}{49}$$

(b)



The probabilities in each set of branches must add up to 1.

CONTINUED

$$\begin{aligned} P(R \text{ and } W) &= P(RW) + P(WR) \\ &= \left(\frac{3}{14} \times \frac{4}{13}\right) + \left(\frac{4}{14} \times \frac{3}{13}\right) \\ &= \frac{12}{182} + \frac{12}{182} \\ &= \frac{12}{91} \end{aligned}$$

8.5 Exercises

1. Two coins are tossed. Find the probability of getting
   (a) 2 heads
   (b) a head followed by a tail
   (c) a head and a tail.

2. Three coins are tossed. Find the probability of getting
   (a) 3 tails
   (b) 2 heads and 1 tail
   (c) at least 1 head.

3. In a set of 30 cards, each one has a number on it from 1 to 30. If 1 card is drawn out, then replaced and another drawn out, find the probability of getting
   (a) two 8s
   (b) a 3 on the first card and an 18 on the second card
   (c) a 3 on one card and an 18 on the other card.

4. Five cards are labelled A, B, C, D and E. If 2 are selected at random, with replacement, find the probability that they will be
   (a) both As
   (b) an A and a D.

5. A bag contains 5 red marbles and 8 blue marbles. If 2 marbles are chosen at random, with the first replaced before the second is drawn out, find the probability of getting
   (a) 2 red marbles
   (b) a red and a blue marble.

6. A certain breed of cat has a 35% probability of producing a white kitten. If a cat has 3 kittens, find the probability that she will produce
   (a) no white kittens
   (b) 2 white kittens
   (c) at least 1 white kitten.

7. The probability of a certain type of photocopier in a school needing a service on any one day is 0.3. Find the probability that a school with 2 of these photocopiers will need to service, on a particular day,
   (a) 1 machine
   (b) both machines
   (c) neither of them

8. The probability of rain in May each year is given by $\frac{3}{10}$. A school holds a fete in May for three years running. Find the probability that it will rain at
   (a) 2 of the fetes
   (b) 1 fete
   (c) at least 1 fete

9. A certain type of plant has a probability of 0.85 of producing a variegated leaf. If I grow 3 of these plants, find the probability of getting a variegated leaf in
(a) 2 of the plants
(b) none of the plants
(c) at least 1 plant.

10. A bag contains 6 white balls and 5 green balls. If 2 balls are chosen at random, find the probability of getting a white and a green ball
(a) with replacement
(b) without replacement.

11. A bag contains 3 yellow balls, 4 pink balls and 2 black balls. If 2 balls are chosen at random, find the probability of getting a yellow and a black ball
(a) with replacement
(b) without replacement.

12. Alan buys 4 tickets in a raffle in which 100 tickets are sold altogether. There are two prizes in the raffle. Find the probability that Alan will win
(a) first prize
(b) both prizes
(c) 1 prize
(d) no prizes
(e) at least 1 prize.

13. Mary buys 20 tickets in a lottery that has 5000 tickets altogether. Find the probability that Mary will win
(a) first and second prize
(b) second prize only
(c) neither first nor second prize.

14. Two musicians are selected at random to lead their band. One person is chosen from Band A, which has 8 females and 7 males, and the other is chosen from Band B, which has 6 females and 9 males. Find the probability of choosing
(a) 2 females
(b) 1 female and 1 male.

15. The two machines in a workshop each have a probability of $\frac{1}{45}$ of breaking down. Find the probability that at any one time
(a) neither machine will be broken down
(b) 1 machine will be broken down.

16. Two tennis players are said to have a probability of $\frac{2}{5}$ and $\frac{3}{4}$ respectively of winning a tournament. Find the probability that
(a) 1 of them will win
(b) neither one will win.

17. If 4 dice are thrown, find the probability that the dice will have
(a) four 6's
(b) only one 6
(c) at least one 6.

18. In a batch of 100 cars, past experience would suggest that 3 could be faulty. If 3 cars are selected at random, find the probability that
(a) 1 is faulty
(b) none are faulty
(c) all 3 cars are faulty.

19. In a certain poll, 46% of people surveyed liked the current government, 42% liked the opposition and 12% had no preference. If 2 people from the survey are selected at random, find the probability that
(a) both will prefer the opposition
(b) 1 will prefer the government and the other will have no preference
(c) both will prefer the government.

20. A manufacturer of X brand of soft drink surveyed a city and found that 31 people liked X drinks best, 19 liked another brand better and 5 did not drink soft drink. If any 2 people are selected at random from that city, find the probability that
(a) one person would like the X brand of soft drink
(b) both people would not drink soft drink.

21. A bag contains 5 red, 6 blue, 2 white and 7 green balls. If 2 are selected at random (without replacement), find the probability of getting
(a) 2 red balls
(b) a blue and a white ball
(c) 2 green balls
(d) at least 1 red ball.

22. In a group of people, 32 are Australian born, 12 were born in Asia and 7 were born in Europe. If 2 of the people are selected at random, find the probability that
(a) they were both born in Asia
(b) at least 1 of them will be Australian born
(c) both were born in Europe.

23. There are 34 men and 32 women at a party. Of these, 13 men and 19 women are married. If 2 people are chosen at random, find the probability that
(a) both will be men
(b) 1 will be a married woman and the other an unmarried man
(c) both will be married.

24. In a certain city, the probability that the pollution level will be high is 0.27. If the pollution is monitored for 4 successive days, find the probability that the pollution levels will be
(a) high on 2 days
(b) high on 1 day
(c) low on at least 1 day.

25. At City Heights School it was found that 75% of students in year 12 study 13 units, 21% study 12 units and 4% study 11 units. If 2 students are selected at random from year 12, find the probability that
(a) 1 student will study 12 units
(b) at least 1 student will study 13 units.

26. Three dice are rolled. Find the probability of rolling
(a) 3 sixes
(b) 2 sixes
(c) at least 1 six.

27. A set of 5 cards, each labelled with the letters A, B, C, D and E, is placed in a hat and two selected at random without replacement. Find the probability of getting
(a) D and E
(b) Neither D nor E on either card
(c) At least one D.

28. The ratio of girls to boys at a school is four to five. Two students are surveyed at random from the school. Find the probability that the students are
(a) both boys
(b) a girl and a boy
(c) at least one girl.

29. The number of cats to dogs at a pet hotel is in the ratio of 4 to 7. If 3 pets are chosen at random, find the probability that
(a) they are all dogs
(b) just one is a dog
(c) at least one is a cat.

30. A set of 20 cards is numbered 1 to 20 and three are selected at random with replacement. Find the probability of selecting
(a) all three 10's
(b) no 10's
(c) at least one 10.

Test Yourself 8

1. The probability that a certain type of seed will germinate is 93%. If 3 of this type of seeds are planted, find the probability that
   (a) all will germinate
   (b) just 1 will germinate
   (c) at least 1 will germinate.
2. A game is played where the differences of the numbers on 2 dice are taken.
   (a) Draw a table showing the sample space (all possibilities).
   (b) Find the probability of getting a difference of
      (i) 3
      (ii) 0
      (iii) 1 or 2.
3. Mark buys 5 tickets in a raffle in which 200 are sold altogether.
   (a) What is the probability that he will
      (i) win
      (ii) not win the raffle?
   (b) If the raffle has 2 prizes, find the probability that Mark will win just 1 prize.
4. In a class of 30 students, 17 study history, 11 study geography and 5 study neither. One of these students is chosen at random. Find the probability that this student will
   (a) study geography but not history
   (b) study both history and geography.
5. 'In the casino, when tossing 2 coins, 2 tails came up 10 times in a row. So there is less chance that 2 tails will come up next time.' Is this statement true? Why?
6. A set of 100 cards numbered 1 to 100 is placed in a box and one drawn at random. Find the probability that the card chosen will be
   (a) odd
   (b) less than 30
   (c) a multiple of 5
   (d) less than 30 or a multiple of 5
   (e) odd or less than 30.
7. One game has a probability of $\frac{3}{5}$ of winning and a second game has a probability of $\frac{2}{3}$ of winning. If Jenny plays one of each game, find the probability that she wins
   (a) both games
   (b) one game
   (c) neither game.
8. A bag contains 5 black and 7 white marbles. Two are chosen at random from the bag (a) with replacement (b) without replacement. Find the probability of getting a black and a white marble.
9. There are 7 different colours and 8 different sizes of leather jackets in a shop. If Jean selects a jacket at random, find the probability that she will select one the same size and colour as her friend does.
10. Each of a certain type of machine in a factory has a probability of 4.5% of breaking down at any time. If the factory has 3 of these machines, find the probability that at any one time
    (a) all will be broken down
    (b) at least one will be broken down.

11. A bag contains 4 yellow, 3 red and 6 blue balls. Two are chosen at random. Find the probability of choosing
(a) 2 yellow balls
(b) a red and a blue ball
(c) 2 blue balls.

12. In a group of 12 friends, 8 have seen the movie *Star Wars 20* and 9 have seen the movie *Mission Impossible 6*. Everyone in the group has seen at least one of these movies. If one of the friends is chosen at random, find the probability that this person has seen
(a) both movies
(b) only *Mission Impossible 6*.

13. A game of chance has a $\frac{2}{5}$ probability of a win or a $\frac{3}{8}$ probability of a draw.
(a) If I play one of these games, find the probability of losing.
(b) If I play 2 of these games, find the probability of
(i) a win and a draw
(ii) a loss and a draw
(iii) 2 wins.

14. A card is chosen at random from a set of 10 cards numbered 1 to 10. A second card is chosen from a set of 20 cards numbered 1 to 20. Find the probability that the combination number these cards make is
(a) 911
(b) less than 100
(c) between 300 and 500.

15. A loaded die has a $\frac{2}{3}$ probability of coming up 6. The other numbers have an equal probability of coming up. If the die is rolled, find the probability that it comes up
(a) 2
(b) even.

16. Amie buys 3 raffle tickets. If 150 tickets are sold altogether, find the probability that Amie wins
(a) 1st prize
(b) only 2nd prize
(c) 1st and 2nd prizes
(d) neither prize.

17. A bag contains 6 white, 8 red and 5 blue balls. If two balls are selected at random, find the probability of choosing a red and a blue ball
(a) with replacement
(b) without replacement.

18. A group of 9 friends go to the movies. If 5 buy popcorn and 7 buy ice creams, find the probability that one friend chosen at random will have
(a) popcorn but not ice cream
(b) both popcorn and ice cream.

19. The probability that an arrow will hit a target is $\frac{8}{9}$. If 3 arrows are fired, find the probability that
(a) 2 hit the target
(b) at least 1 hits the target.

20. The probability of winning Game A is $\frac{3}{5}$ and the probability of winning Game B is $\frac{7}{10}$. Find the probability of winning
(a) both games
(b) neither game
(c) one game.

Challenge Exercise 8

1. In a group of 35 students, 25 go to the movies and 15 go to the football. If all the students like at least one of these activities, find the probability that a student chosen at random will
(a) go to both the movies and the football
(b) only go to the movies.

2. A certain soccer team has a probability of 0.5 of winning a match and a probability of 0.2 of drawing. If the team plays 2 matches, find the probability that it will
(a) draw both matches
(b) win at least 1 match
(c) not win either match.

3. A game of poker uses a deck of 52 cards with 4 suits (hearts, diamonds, spades and clubs). Each suit has 13 cards, consisting of an ace, cards numbered from 2 to 10, a jack, queen and king. If a person is dealt 5 cards find the probability of getting
(a) four aces
(b) a flush (all cards the same suit).

4. If a card is drawn out at random from a set of playing cards find the probability that it will be
(a) an ace or a heart
(b) a diamond or an odd number
(c) a jack or a spade.

5. Bill does not select the numbers 1, 2, 3, 4, 5 and 6 for Lotto as he says this combination would never win. Is he correct?

6. In a set of 5 cards, each has one of the letters A, B, C, D and E on it. If two cards are selected at random without replacement, find the probability that
(a) both cards are A's
(b) one card is an A and the other is a D
(c) neither card is an A or D.

7. In the game of Yahtzee, 5 dice are rolled. Find the probability of rolling
(a) all 6's
(b) all the same number

8. Out of a class of 30 students, 19 play a musical instrument and 7 play both a musical instrument and a sport. Two students play neither.
(a) One student is selected from the class at random. Find the probability that this person plays a sport but not a musical instrument.
(b) Two people are selected at random from the class. Find the probability that both these people only play a sport.

9. A game involves tossing 2 coins and rolling 2 dice. The scoring is shown in the table.

Result	Score (points)
2 heads and double 6	5
2 heads and double (not 6)	3
2 tails and double 6	4
2 tails and double (not 6)	2

(a) Find the probability of getting 2 heads and a double 6.
(b) Find the probability of getting 2 tails and a double that is not 6.
(c) What is the probability that Andre will score 13 in three moves?
(d) What is the probability that Justin will beat Andre's score in three moves?