Practice Assessment Task
SET 3

1. Find the 5th term of $4 + 2\frac{2}{5} + 1\frac{11}{25} + \ldots$

2. The probability that a seed will produce a plant with a white flower is $\frac{1}{7}$. If 4 seeds are planted, find the probability that
   (a) no plants will have white flowers
   (b) exactly 1 plant will have white flowers
   (c) at least 1 plant will have white flowers.

3. By writing the recurring decimal $0.444\ldots$ as an infinite geometric series or otherwise, express it as a rational number.

4. Find in index form the 10th term of $3 + \frac{3}{4} + \frac{3}{16} + \ldots$

5. George deposits \$400 at the beginning of each year in an investment account that pays 13% interest per annum. How much will there be in the account after 5 years?

6. The second term of a geometric series is 52 and the 4th term is 13. Find two series that satisfy these requirements.

7. Find which term –370 is in the series $17 + 8 - 1 - \ldots$

8. A man promises his son a sum of money on his 18th birthday, made up of \$10 for his first year of life, \$15 for his second year, \$20 for his third year and so on, up to 18 years. How much will his son receive?

9. Two dice are thrown and the total of the dice is noted. Find the probability of throwing
   (a) double 1's
   (b) any double
   (c) at least one 3
   (d) a total of 6
   (e) a total of at least 8.

10. Find the sum of the first 9 terms of $a + \frac{a}{3} + \frac{a}{9} + \ldots$

11. A plant grows so that it increases its height each month by 0.2 of its previous month's height. If it grows to 3 m, find its height in the first month.

12. The population of an insect colony is given by the formula $P = t^3 - t + 3\,000\,000$, where t is time in days, Find
   (a) the initial population.
   (b) the population after a week.
   (c) the rate at which the population will be increasing after a week.

13. Find the stationary point on the curve $y = (x - 2)^3$ and determine its nature. Hence sketch the curve.

14. Each card in a set of 100 cards has a different number on it, ranging from 1 to 100. If one card is chosen at random, find the probability of getting
   (a) an even number less than 30
   (b) an odd number or a number divisible by 9.

15. Two families travelling on holidays drive along roads that intersect at right angles. One family is initially 230 km from the intersection and drives towards the intersection at an average of 65 kmh^{-1}. The other family is initially 125 km from the intersection and travels towards it at an average of 80 kmh^{-1}.
(a) Show that their distance apart after t hours is given by $d^2 = 10\,625t^2 - 49\,000t + 68\,525$.
(b) Hence find how long it will take them to reach their minimum distance apart.
(c) Find their minimum distance apart.

16. Find the 50th term of $3 + 7 + 11 + \ldots$ and calculate the sum of 50 terms.

17. If I buy 10 tickets, find the probability of my winning both first and second prizes in a raffle in which 100 tickets are sold.

18. The nth term of a series is given by $7n - 3$
(a) Find the first 3 terms and the 12th term.
(b) Evaluate $\sum_{n=1}^{20} (7n - 3)$.
(c) Which term is 200?

19. Find the exact value of $\sin\left(\pi + \frac{\pi}{4}\right)$

20. Sketch the graph of
(a) $y = x^2 - 5x + 6$
(b) $y = 2\sin 4x$ for $0 \leq x \leq \pi$
(c) $y = e^{-x}$

21. The 4th term of an arithmetic series is 18 and the 8th term is 62. Find the series.

22. A bag contains 7 red and 5 white marbles. Two are drawn out at random. Find the probability of drawing out
(a) 2 white marbles
(b) 1 white and 1 red marble.

23. I put \$2000 in the bank, where it earns interest at the rate of 12% p.a., paid quarterly. How much will there be in the account after 3 years?

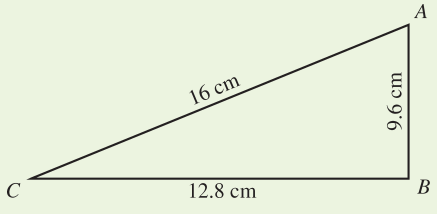
24. The displacement of a pendulum is given by $x = 3\cos 4t$ cm after time t seconds.
(a) Find the equation for the velocity of the pendulum.
(b) Find the equation for its acceleration.
(c) Find the initial displacement.
(d) Find the times when the pendulum will be at rest.
(e) What is the displacement at these times?
(f) When will there be zero displacement?
(g) Show that acceleration $a = -16x$.

25. Express $0.1\dot{7}$ as a rational number.

26. A coin is tossed and a die thrown. Find the probability of getting
(a) a head and a six
(b) a tail and an odd number.

27. I borrow \$5000 at 18% interest p.a. and make equal monthly payments over 3 years, at the end of which the loan is fully paid out. Find the amount of each monthly payment.

28. Prove that $\triangle ABC$ is right angled.



29. The geometric series $x + x^2 + x^3 + \ldots$ has a sum to infinity of 5. Find the value of x.

30. Prove that if one pair of adjacent sides is equal in a rhombus, then all sides are equal.

31. Josie buys 5 lottery tickets in which 200 000 tickets are sold. What is the probability that she wins 1st prize?

32. The mass (in grams) of a radioactive substance is given by $M = M_0 e^{-kt}$, where t is time in years. Find
(a) k if its mass is halved after 28 years
(b) when the mass will be 35% of the original amount
(c) the time taken to reduce the mass by 40%.

33. Evaluate $7 + 12 + 17 + \ldots + 872$.

34. A rectangle is cut from a circular disc of radius 15 cm. Find the area of the largest rectangle that can be produced.

35. The equation for displacement of a particle is given by $x = 3e^{4t} + 2$ metres after time t seconds.
(a) Find the initial velocity.
(b) Find the exact acceleration after 1 s.
(c) Show that the acceleration is always 4 times greater than the velocity.
(d) Sketch the graph of velocity over time t.

36. Find the first value of n for which the sum of the series $20 + 4 + \frac{4}{5} + \ldots$ is greater than 24.85.

37. If $y = \sin 7x$, show that $\frac{d^2y}{dx^2} = -49y$.

38. Evaluate $\sum_{r=2}^{\infty} \frac{2}{5^r}$.

39. A farmer wants to fence a rectangular paddock and a square paddock with a combined area of 5000 m^2. If the length of the rectangular paddock is to be 3 times the size of its breadth
(a) find the dimensions of the paddocks that give the largest perimeter
(b) calculate the cost of fencing the paddocks, at \$19.95 per metre.

40. Two dice are rolled, and the numbers of the dice are totalled. Find the probability of rolling
(a) a total of 8
(b) a total less than 7
(c) a total greater than 9
(d) a total of 4 or 5
(e) an odd total.

41. Sketch the function $f(x) = 3x^4 + 4x^3 - 12x^2$ showing all stationary points.

42. The area of a circle is 5π and an arc 3 cm long cuts off a sector with an angle of θ subtended at the centre. Find
(a) θ in degrees and minutes
(b) the area of the minor segment cut off by the arc, correct to 2 decimal places.

43. Ian pays \$50 into a superannuation fund at the beginning of each month, where it earns interest of 4% p.a. How much will be in the fund at the end of 20 years?

44. (a) Show that $\log 3 + \log 9 + \log 27 + \ldots$ is an arithmetic series.
(b) Find the exact sum of 20 terms of the series.

45. Sketch the curve $y = 2x^3 + 3x^2 - 36x + 1$, showing any stationary points and points of inflexion.

46. A chemical reaction causes the amount of hydrogen to be reduced at a rate proportional to the amount of hydrogen present at any one time. If the amount of hydrogen is given by the formula $A = A_0 e^{-kt}$, and 800 mL reduces to 650 mL after 3 minutes, find
(a) the amount of hydrogen after 1 hour
(b) how long it will take for the hydrogen to reduce to 100 mL.

47. Find the domain over which the curve $y = x^3 + 3x^2 - 24x + 7$ is concave downwards.
(a) $x < -1$
(b) $x > -1$
(c) $-4 < x < 2$
(d) $x < -4,\ x > 2$

48. I borrow \$10 000 over 5 years at 1.85% monthly interest. How much do I need to pay each month?

49. Find the probability of drawing out a blue and a white ball from a bag containing 7 blue and 5 white balls if the first ball is not replaced before taking out the second.
(a) $\frac{70}{121}$
(b) $\frac{70}{144}$
(c) $\frac{1225}{17\,424}$
(d) $\frac{70}{132}$

50. The limiting sum of a geometric series exists when
(a) $|r| > 1$
(b) $|r| < 1$
(c) $|r| \geq 1$
(d) $|r| \leq 1$

51. Find the limiting sum of the series $\frac{2}{3} + \frac{1}{2} + \frac{3}{8} + \ldots$
(a) 2
(b) $1\frac{1}{3}$
(c) $2\frac{2}{3}$
(d) $\frac{8}{9}$

52. Which statement is the same as $3^x = 7$? There may be more than one answer.
(a) $x = \log \frac{7}{3}$
(b) $\log_3 x = 7$
(c) $\log_3 7 = x$
(d) $x = \frac{\log 7}{\log 3}$

53. The nth term of the series $7 + 49 + 343 + \ldots$ is
(a) $7n$
(b) 7^{n-1}
(c) $7n - 1$
(d) 7^n

54. An amount of \$6000 is invested at 3.5% p.a. with interest paid quarterly. Find the balance after 10 years.
(a) \$8501.45
(b) \$8463.59
(c) \$6546.16
(d) \$6899.96

55. The limiting sum of $\frac{3}{5} + \frac{2}{5} + \frac{4}{15} + \ldots$ is
(a) $\frac{1}{5}$
(b) $\frac{9}{10}$
(c) $1\frac{4}{5}$
(d) $\frac{6}{15}$

56. In a group of 25 students, 19 catch a train to school and 21 catch a bus. If one of these students is chosen at random, find the probability that the student only catches a bus to school.

(a) $\frac{6}{25}$

(b) $\frac{21}{25}$

(c) $\frac{3}{5}$

(d) $\frac{3}{20}$

57. The formula for the sum $1 + 1.03 + 1.03^2 + \ldots + 1.03^{n-1}$ is

(a) $S = \frac{1.03(1.03^{n-1} - 1)}{1.03 - 1}$

(b) $S = \frac{1.03(1.03^{n} - 1)}{1.03 - 1}$

(c) $S = \frac{1.03^{n-1} - 1}{1.03 - 1}$

(d) $S = \frac{1.03^{n} - 1}{1.03 - 1}$