1

Geometry 2

TERMINOLOGY

Congruent: Two figures are congruent if they have the same size and shape. They are identical in every way

Polygon: A polygon is a closed plane figure with straight sides

Similar: Two figures are similar if they have the same shape but a different size. Corresponding angles are equal and corresponding sides are in the same ratio

Vertex: A vertex is a corner of a figure (vertices is plural, meaning more than one vertex)

INTRODUCTION

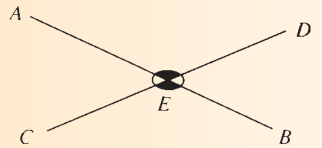
YOU STUDIED GEOMETRY IN the Preliminary Course. In this chapter, you will revise this work and extend it to include some more general applications of geometrical properties involving polygons.

You will also use the Preliminary topic on straight-line graphs to explore coordinate methods in geometry.

Plane Figure Geometry

Here is a summary of the geometry you studied in the Preliminary Course.

Vertically opposite angles

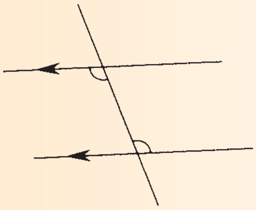


$\angle AEC$ and $\angle DEB$ are called **vertically opposite angles**. $\angle AED$ and $\angle CEB$ are also vertically opposite angles.

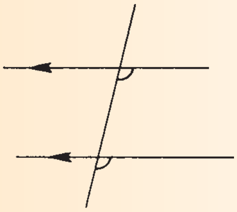
Vertically opposite angles are equal.

Parallel lines

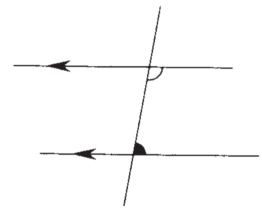
If the lines are parallel, then **alternate angles** are equal.



If the lines are parallel, then **corresponding angles** are equal.



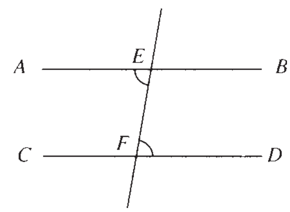
If the lines are parallel, **cointerior angles** are supplementary (i.e. their sum is 180°).



TESTS FOR PARALLEL LINES

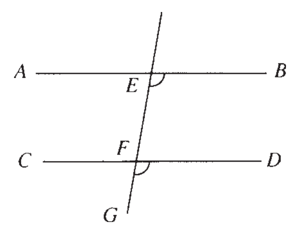
If **alternate angles** are equal, then the lines are parallel.

If $\angle AEF = \angle EFD$, then $AB \parallel CD$.



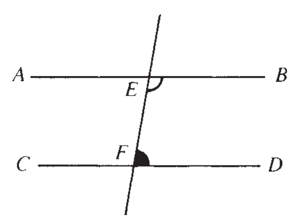
If **corresponding angles** are equal, then the lines are parallel.

If $\angle BEF = \angle DFG$, then $AB \parallel CD$.

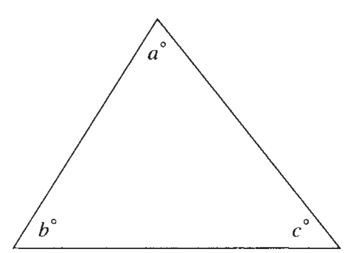


If **cointerior angles** are supplementary, then the lines are parallel.

If $\angle BEF + \angle DFE = 180°$, then $AB \parallel CD$.

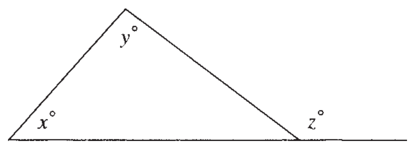


Angle sum of a triangle



The sum of the interior angles in any triangle is 180°, that is, $a + b + c = 180$

Exterior angle of a triangle



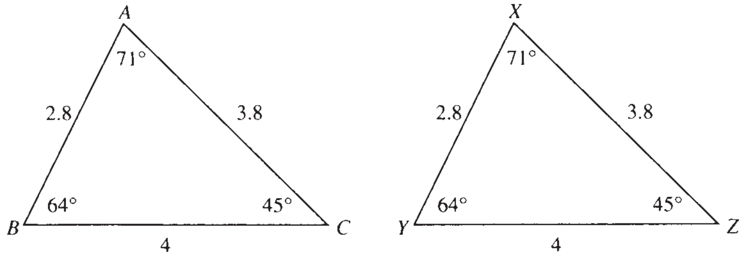
The exterior angle in any triangle is equal to the sum of the two opposite interior angles. That is,

$$x + y = z$$

Congruent triangles

Two triangles are **congruent** if they are the same shape and size. All pairs of corresponding sides and angles are equal.

For example:



We write $\Delta ABC \equiv \Delta XYZ$.

TESTS

To prove that two triangles are congruent, we only need to prove that certain combinations of sides or angles are equal.

Two triangles are congruent if

- **SSS**: all three pairs of corresponding sides are equal
- **SAS**: two pairs of corresponding sides and their included angles are equal
- **AAS**: two pairs of angles and one pair of corresponding sides are equal
- **RHS**: both have a right angle, their hypotenuses are equal and one other pair of corresponding sides are equal

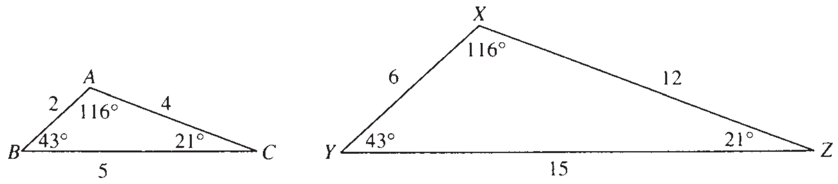
The included angle is the angle between the 2 sides.

Similar triangles

Triangles, for example *ABC* and *XYZ*, are similar if they are the same **shape** but different **sizes**.

As in the example, all three pairs of **corresponding angles** are equal.

All three pairs of **corresponding sides** are in proportion (in the same ratio).



We write: $\Delta ABC \parallel\!| \Delta XYZ$

ΔXYZ is three times larger than ΔABC

$$\frac{XY}{AB} = \frac{6}{2} = 3$$

$$\frac{XZ}{AC} = \frac{12}{4} = 3$$

$$\frac{YZ}{BC} = \frac{15}{5} = 3$$

$$\therefore \frac{XY}{AB} = \frac{XZ}{AC} = \frac{YZ}{BC}$$

This shows that all 3 pairs of sides are in proportion.

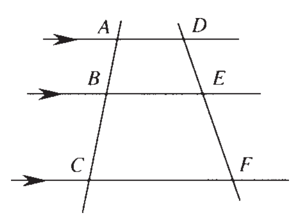
TESTS

There are three tests for similar triangles.

If 2 pairs of angles are equal then the third pair must also be equal.

Two triangles are similar if:
- three pairs of **corresponding angles** are equal
- three pairs of **corresponding sides** are in proportion
- two pairs of **sides** are **in proportion** and their **included angles** are **equal**

Ratios of intercepts



When two (or more) transversals cut a series of parallel lines, the ratios of their intercepts are equal.

That is, $AB:BC = DE:EF$

or $$\frac{AB}{BC} = \frac{DE}{EF}$$

Pythagoras' theorem

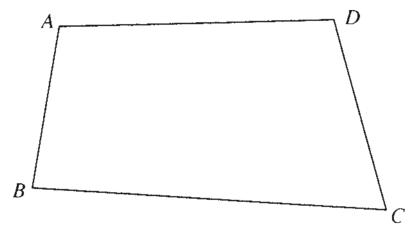
The square on the hypotenuse in any right angled triangle is equal to the sum of the squares on the other two sides.

That is, $c^2 = a^2 + b^2$

or $$c = \sqrt{a^2 + b^2}$$

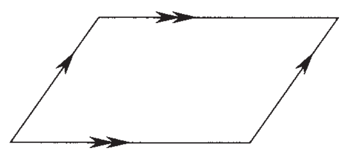
Quadrilaterals

A quadrilateral is any **four-sided** figure



In any quadrilateral the sum of the interior angles is 360°

PARALLELOGRAM



A parallelogram is a quadrilateral with opposite sides parallel

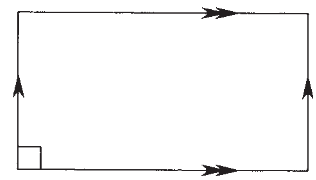
Properties of a parallelogram:
- **opposite sides** of a parallelogram are equal
- **opposite angles** of a parallelogram are equal
- **diagonals** in a parallelogram bisect each other
- each diagonal bisects the parallelogram into two **congruent triangles**

These properties can all be proved.

TESTS

A **quadrilateral** is a **parallelogram** if:
- both pairs of **opposite sides** are equal
- both pairs of **opposite angles** are equal
- **one pair of sides** is both equal and parallel
- the **diagonals** bisect each other

RECTANGLE



If one angle is a right angle, then you can prove all angles are right angles.

A rectangle is a parallelogram with one angle a right angle

Properties of a rectangle:
- the same as for a parallelogram, and also
- diagonals are equal

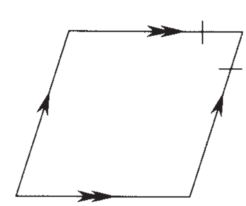
TEST

A quadrilateral is a rectangle if its diagonals are equal

Application

Builders use the property of equal diagonals to check if a rectangle is accurate. For example, a timber frame may look rectangular, but may be slightly slanting. Checking the diagonals makes sure that a building does not end up like the Leaning Tower of Pisa!

RHOMBUS



It can be proved that all sides are equal.

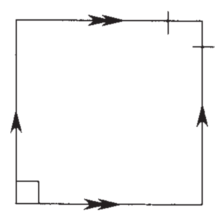
A rhombus is a parallelogram with a pair of adjacent sides equal

Properties of a rhombus:
- the same as for parallelogram, and also
- diagonals bisect at right angles
- diagonals bisect the angles of the rhombus

TESTS

A quadrilateral is a rhombus if:
- all sides are equal
- diagonals bisect each other at right angles

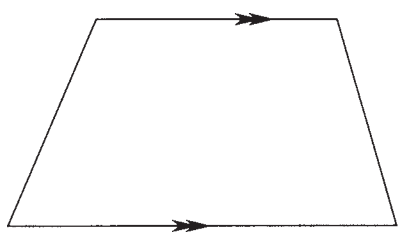
SQUARE



A square is a rectangle with a pair of adjacent sides equal

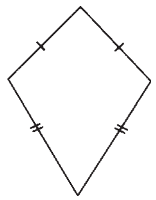
Properties of a square:
- the same as for rectangle, and also
- diagonals are perpendicular
- diagonals make angles of 45° with the sides

TRAPEZIUM



A trapezium is a quadrilateral with one pair of sides parallel

KITE



A kite is a quadrilateral with two pairs of adjacent sides equal

Polygons

A polygon is a plane figure with straight sides

A regular polygon has all sides and all interior angles equal

The sum of the interior angles of an n-sided polygon is given by

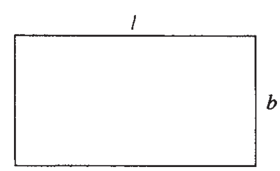
$$S = (n - 2) \times 180°$$

The sum of the exterior angles of any polygon is 360°

Areas

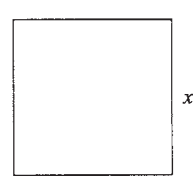
Most areas of plane figures come from the area of a rectangle.

RECTANGLE



$$A = lb$$

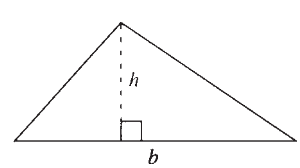
SQUARE



A square is a special rectangle.

$$A = x^2$$

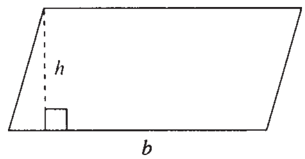
TRIANGLE



The area of a triangle is half the area of a rectangle.

$$A = \frac{1}{2}bh$$

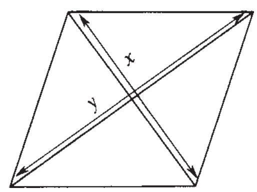
PARALLELOGRAM



$$A = bh$$

The area of a parallelogram is the same as the area of two triangles.

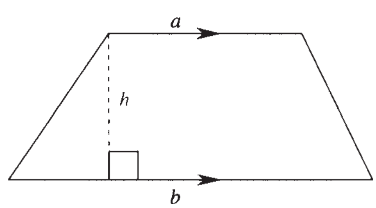
RHOMBUS



$$A = \frac{1}{2}xy$$

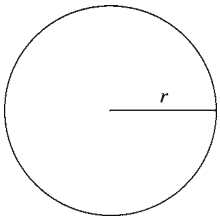
(x and y are lengths of diagonals)

TRAPEZIUM



$$A = \frac{1}{2}h(a + b)$$

CIRCLE



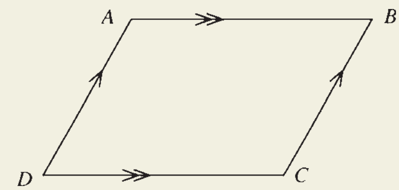
You will study the circle in more detail. See Chapter 5.

$$A = \pi r^2$$

The following examples and exercises use these results to prove properties of plane figures.

EXAMPLES

1. Prove $\angle A = \angle C$ in parallelogram $ABCD$.



Solution

Let $\angle A = x°$

Then $\angle B = 180° - x°$ ($\angle A$, $\angle B$ cointerior angles, $AD \parallel BC$)

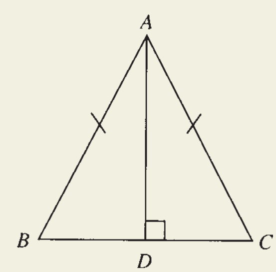
$\angle C = 180° - (180° - x°)$ ($\angle B$, $\angle C$ cointerior angles, $AB \parallel DC$)

$= 180° - 180° + x°$

$= x°$

$\therefore \quad \angle A = \angle C$

2. Triangle ABC below is isosceles with $AB = AC$. Prove that the base angles of ΔABC are equal by showing that ΔABD and ΔACD are congruent.



Solution

$\angle ADB = \angle ADC = 90°$ (given)

$AB = AC$ (given)

AD is common

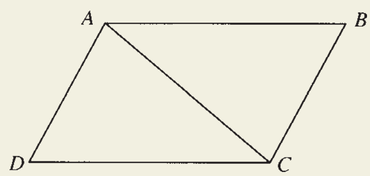
$\therefore \Delta ABD \equiv \Delta ACD$ (RHS)

So $\angle ABD = \angle ACD$ (corresponding angles in congruent Δs)

$\therefore$ base angles are equal

3. Prove that opposite sides in a parallelogram are equal.

Solution



Let $ABCD$ be any parallelogram and draw in diagonal AC.

$\angle DAC = \angle ACB$ (alternate $\angle$s, $AD \parallel BC$)

$\angle BAC = \angle ACD$ (alternate $\angle$s, $AB \parallel DC$)

AC is common.

$\therefore \Delta ABC \equiv \Delta ADC$ (AAS)

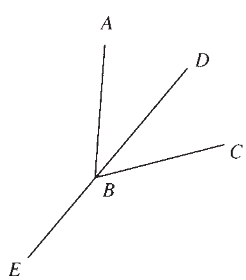
$\therefore$ $AB = DC$ (corresponding sides in congruent Δs)

$AD = BC$ (similarly)

$\therefore$ opposite sides in a parallelogram are equal

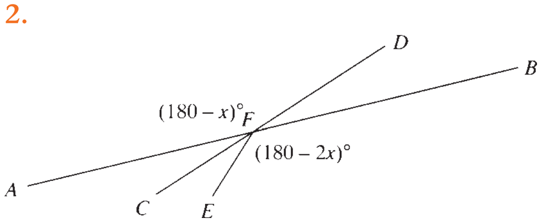
1.1 Exercises

1.



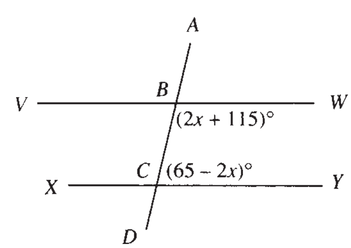
DE bisects acute angle $\angle ABC$ so that $\angle ABD = \angle CBD$. Prove that DE also bisects reflex angle $\angle ABC$. That is, prove $\angle ABE = \angle CBE$.

2.

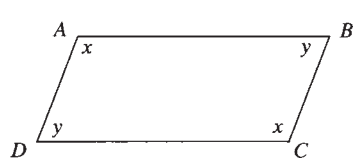


Prove that CD bisects $\angle AFE$.

3. Prove $VW \parallel XY$.



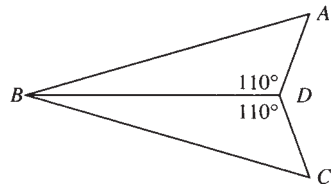
4.



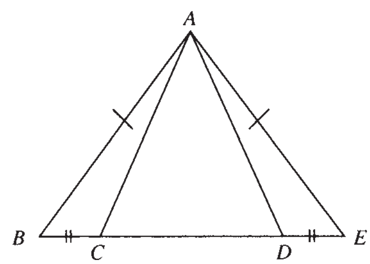
Given $x + y = 180°$, prove that $ABCD$ is a parallelogram.

The **altitude** is perpendicular to the other side of the triangle

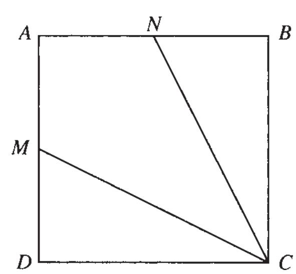
5. BD bisects $\angle ABC$. Prove that $\Delta ABD \equiv \Delta CBD$.



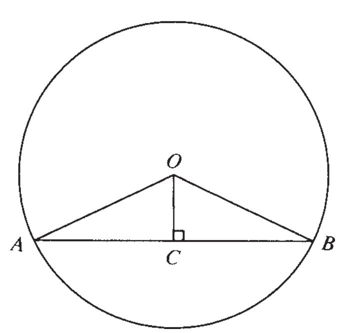
6. (a) Show that $\Delta ABC \equiv \Delta AED$.
   (b) Hence prove that ΔACD is isosceles.



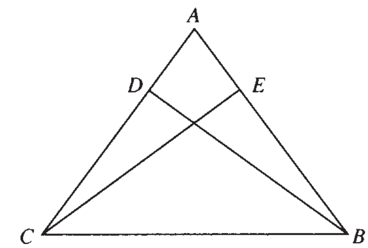
7. $ABCD$ is a square. Lines are drawn from C to M and N, the midpoints of AD and AB respectively. Prove that $MC = NC$.



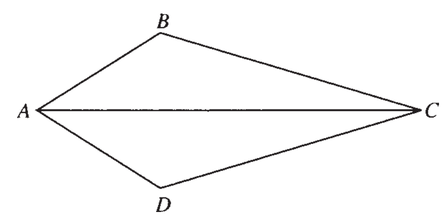
8. OC is drawn so that it is perpendicular to chord AB and O is the centre of the circle. Prove that ΔOAC and ΔOBC are congruent, and hence that OC bisects AB.



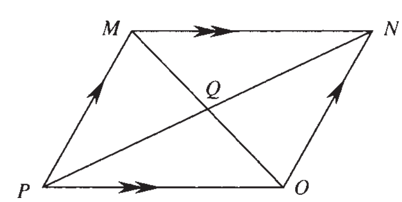
9. CE and BD are altitudes of ΔABC, and ΔABC is isosceles $(AB = AC)$. Prove that $CE = BD$.



10. $ABCD$ is a kite where $AB = AD$ and $BC = DC$. Prove that diagonal AC bisects both $\angle DAB$ and $\angle DCB$.



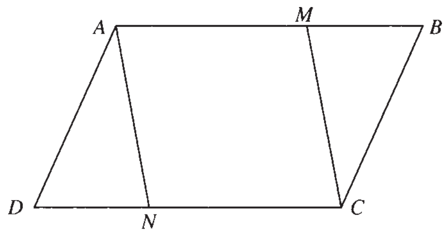
11. $MNOP$ is a rhombus with $MN = NO$.
    Show that
    (a) ΔMNO is congruent to ΔMPO
    (b) $\angle PMQ = \angle NMQ$
    (c) ΔPMQ is congruent to ΔNMQ
    (d) $\angle MQN = 90°$



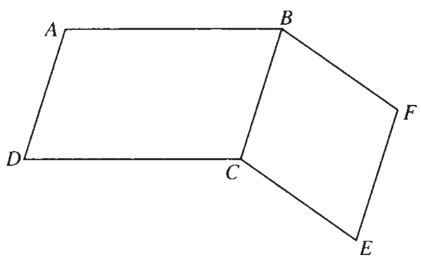
12. Show that a diagonal cuts a parallelogram into two congruent triangles.

13. Prove that opposite angles are equal in any parallelogram.

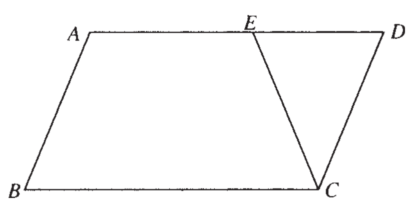
14. *ABCD* is a parallelogram with $BM = DN$. Prove that *AMCN* is also a parallelogram.



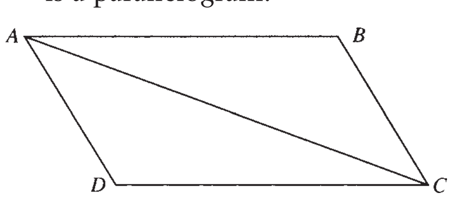
15. *ABCD* and *BCEF* are parallelograms. Show that *AFED* is a parallelogram.



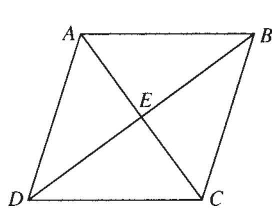
16. *ABCD* is a parallelogram with $DE = DC$. Prove that *CE* bisects $\angle BCD$.



17. In quadrilateral *ABCD*, $AB = CD$ and $\angle BAC = \angle DCA$. Prove *ABCD* is a parallelogram.



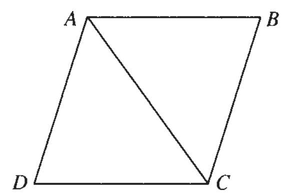
18. *ABCD* is a parallelogram with $\angle AEB = 90°$. Prove
(a) $AB = BC$
(b) $\angle ABE = \angle CBE$



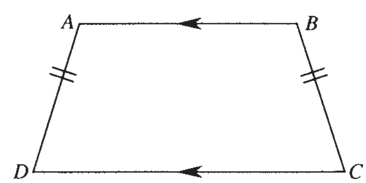
19. Prove that the diagonals in any rectangle are equal.

20. Prove that if one angle in a rectangle is 90° then all the angles are 90°.

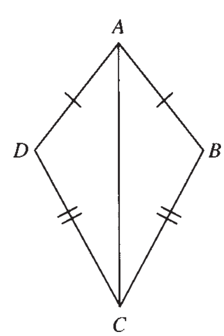
21. *ABCD* is a rhombus with $AD = CD$. Prove that all sides of the rhombus are equal.



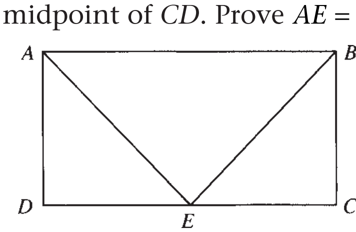
22. *ABCD* is an isosceles trapezium. Prove the base angles $\angle ADC$ and $\angle BCD$ are equal.



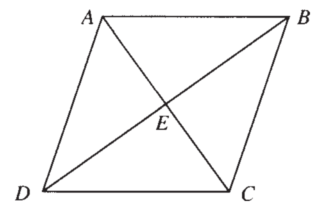
23. Prove that $\angle ADC = \angle ABC$ in kite *ABCD*.



24. In rectangle *ABCD*, *E* is the midpoint of *CD*. Prove $AE = BE$.



25. *ABCD* is a rhombus.
   (a) Prove ΔADB and ΔBCD are congruent.
   (b) Hence show $\angle ABE = \angle CBE$.
   (c) Prove ΔABE and ΔCBE are congruent.
   (d) Prove $\angle AEB = 90°$.

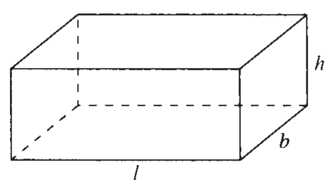


Surface Areas and Volumes

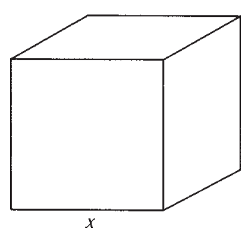
You will need some of these formulae when you study maxima and minima problems in Chapter 2.

Areas are used in finding the surface area and volume of solids. Here is a summary of some of the most common ones.

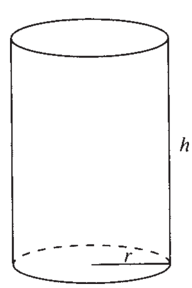
SURFACE AREA | **VOLUME**



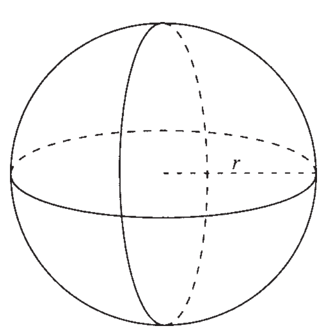
$S = 2(lb + bh + lh)$ | $V = lbh$



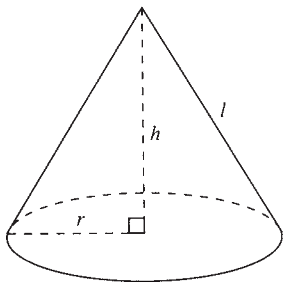
$S = 6x^2$ | $V = x^3$



$S = 2\pi r(r + h)$ | $V = \pi r^2 h$



$S = 4\pi r^2$ | $V = \frac{4}{3}\pi r^3$

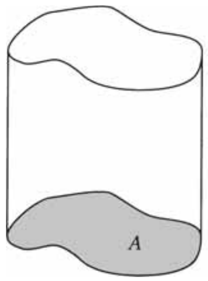


$$S = \pi r (r + l) \qquad V = \frac{1}{3}\pi r^2 h$$

In general, the volume of any prism is given by

$$V = Ah$$

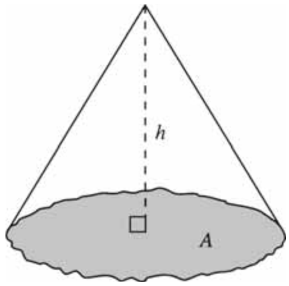
where A is the area of the base and h is its height



In general, the volume of any pyramid is given by

$$V = \frac{1}{3}Ah$$

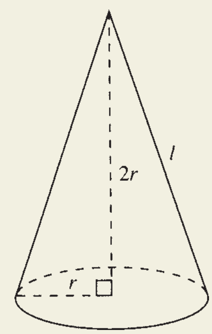
Where A is the area of the base and h is its height



EXAMPLE

Find the surface area of a cone whose height is twice the radius, in terms of r.

Solution



$$h = 2r$$
$$l^2 = r^2 + h^2$$
$$= r^2 + (2r)^2$$
$$= r^2 + 4r^2$$
$$= 5r^2$$
$$\therefore \quad l = \sqrt{5r^2}$$
$$= \sqrt{5}\, r$$

Surface area $S = \pi r(r + l)$ where l is slant height

$$= \pi r(r + \sqrt{5}\, r)$$

While surface area and volume is not a part of the geometry in the HSC syllabus, the topic in Chapter 2 uses calculus to find maximum or minimum areas, perimeters, surface areas or volumes. So you will need to know these formulae in order to answer the questions in the next chapter. Here are some questions to get you started.

1.2 Exercises

1. A rectangular prism has dimensions 12.5 mm, 84 mm and 64 mm. Find its
   (a) surface area and
   (b) volume.

2. A sphere has a volume of 120π m^3. Find the exact value of r.

3. A rectangular prism has dimensions x, $x + 2$ and $2x - 1$. Find its volume in terms of x.

4. A cylinder has a volume of 250 cm^3. If its base has radius r and its height is h, show that $r = \sqrt{\dfrac{250}{\pi h}}$.

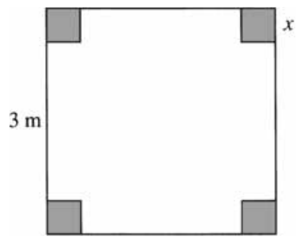
5. Find the volume of a cylinder in terms of r if its height is five times the size of its radius.

6. The ratio of the length to the breadth of a certain rectangle is 3:2. If the breadth of this rectangle is b units, find a formula for the area of the rectangle in terms of b.

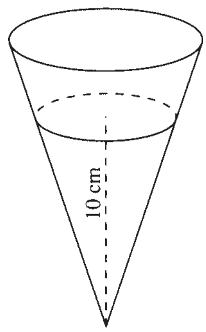
7. Find the volume of a cube with sides $(x + 2)$ cm.

8. What would the surface area of a cylinder be in terms of h if its height is a third of its radius?

9. A square piece of metal with sides 3 m has a square of side x cut out of each corner. The metal is then folded up to form a rectangular prism. Find its volume in terms of x.



10. A cone-shaped vessel has a height of twice its radius. If I fill the vessel with water to a depth of 10 cm, find the volume of water to the nearest cm^3.



11. The area of the base of a prism is given by $3h + 2$, where h is the height of the prism. Find a formula for the volume of the prism.

12. The area of the base of a pyramid is $6h + 15$ where h is the height of the pyramid. Find the volume of the pyramid in terms of h.

13. A rectangular pyramid has base dimensions $x - 3$ and $3x + 5$, and a height of $2x + 1$. Write a formula for the volume of the pyramid in terms of x.

14. The height of a rectangular prism is twice the length of its base. If the width of the base is x and the length is $3x - 1$, find an expression for the
(a) volume and
(b) surface area of the prism.

15. Find a formula for the slant height of a cone in terms of its radius r and height h.

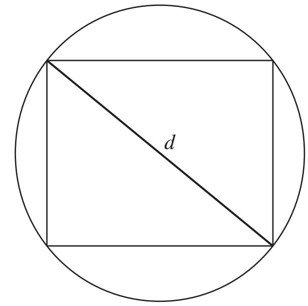
16. A page measuring x by y is curved around to make an open cylinder with height y. Find the volume of the cylinder in terms of x and y.

17. The volume of a cylinder is 400 cm^3. Find the height of the cylinder in terms of its radius r.

18. A cylinder has a surface area of 1500 cm^2. Find a formula for its height h in terms of r.

19. The surface area of a cone is given by $S = \pi r(r + l)$ where l is the slant height. Find a formula for the slant height of a cone with surface area 850 cm^2 in terms of r.

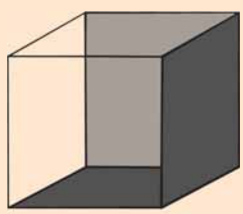
20. A rectangular timber post is cut out of a log with diameter d as shown. If the post has length x and breadth y, write y in terms of x when $d = 900$ mm.



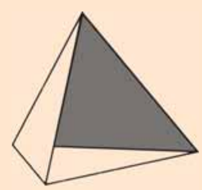
DID YOU KNOW?

REGULAR SOLIDS

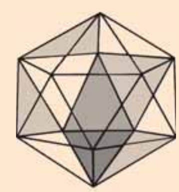
There are only five solids with each face the same size and shape. These are called platonic solids. Research these on the internet.



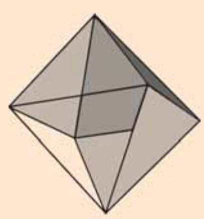
cube (hexahedron)



tetrahedron

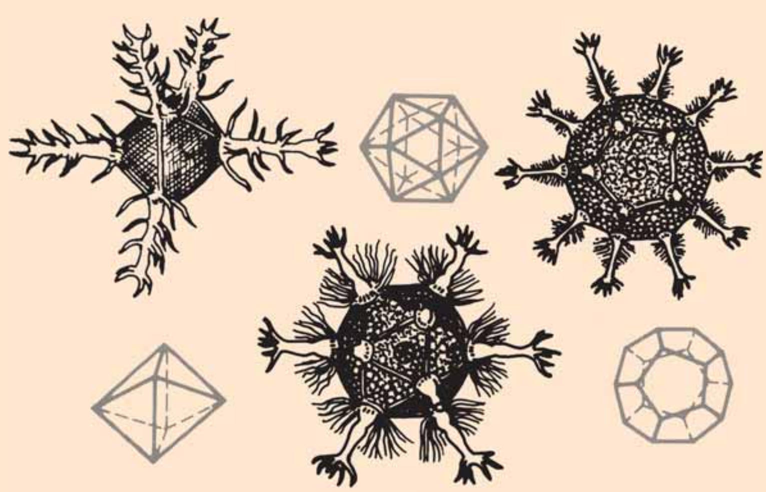


icosahedron



octahedron

dodecahedron



radiolarians

The skeletons opposite are those of radiolarians. These are tiny sea animals, with their skeletons shaped like regular solids.

A salt crystal is a cube. A diamond crystal is an octahedron.

Diamond crystal

Coordinate Methods in Geometry

Problems in plane geometry can be solved by using the number plane.

You studied straight-line graphs in the Preliminary Course. Some of the main results that you learned will be used in this section. You may need to revise that work before studying this section.

Here is a summary of the main formulae.

Distance

The distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint

The midpoint of two points (x_1, y_1) and (x_2, y_2) is given by

$$P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Gradient

The gradient of the line between (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The gradient of a straight line is given by

$$m = \tan\theta$$

where θ is the angle the line makes with the x-axis in the positive direction.

The gradient of the line $ax + by + c = 0$ is given by

$$m = -\frac{a}{b}$$

Equation of a straight line

The equation of a straight line is given by

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) lies on the line with gradient m.

Parallel lines

If two lines are parallel, then they have the same gradient. That is,

$$m_1 = m_2$$

Perpendicular lines

If two lines with gradients m_1 and m_2 respectively are perpendicular, then

$$m_1 m_2 = -1 \text{ or } m_2 = -\frac{1}{m_1}$$

Perpendicular distance

The perpendicular distance from (x_1, y_1) to the line $ax + by + c = 0$ is given by

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

EXAMPLES

1. Show that triangle ABC is right angled, where $A = (3, 4)$, $B = (-1, -1)$ and $C = (-2, 8)$.

Solution

Method 1:

$$\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
AB &= \sqrt{(-1 - 3)^2 + (-1 - 4)^2} \\
&= \sqrt{(-4)^2 + (-5)^2} \\
&= \sqrt{16 + 25} \\
&= \sqrt{41} \\
AC &= \sqrt{(-2 - 3)^2 + (8 - 4)^2} \\
&= \sqrt{(-5)^2 + 4^2} \\
&= \sqrt{25 + 16} \\
&= \sqrt{41} \\
BC &= \sqrt{(-1 - (-2))^2 + (-1 - 8)^2} \\
&= \sqrt{1^2 + (-9)^2} \\
&= \sqrt{1 + 81} \\
&= \sqrt{82}
\end{aligned}$$

$$AB^2 + AC^2 = 41 + 41$$
$$= 82$$
$$= BC^2$$

Since Pythagoras' theorem is true, the triangle ABC is right angled.

Method 2:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_{AB} = \frac{-1 - 4}{-1 - 3}$$
$$= \frac{-5}{-4}$$
$$= \frac{5}{4}$$
$$m_{AC} = \frac{8 - 4}{-2 - 3}$$
$$= \frac{4}{-5}$$
$$= -\frac{4}{5}$$
$$m_{AB}m_{AC} = \frac{5}{4} \times -\frac{4}{5}$$
$$= -1$$

So AB and AC are perpendicular

So triangle ABC is right angled at A.

---

2. Prove that points $A(1, 1)$, $B(-2, -1)$ and $C(4, 3)$ are collinear.

Solution

Collinear points lie on the same straight line, so they will have the same gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_{AB} = \frac{-1 - 1}{-2 - 1}$$
$$= \frac{-2}{-3}$$
$$= \frac{2}{3}$$
$$m_{BC} = \frac{3 - (-1)}{4 - (-2)}$$
$$= \frac{4}{6}$$
$$= \frac{2}{3}$$
$$m_{AB} = m_{BC}$$

So the points are collinear.

1.3 Exercises

1. Show that points $A(-1, 0)$, $B(0, 4)$, $C(7, 0)$ and $D(6, -4)$ are the vertices of a parallelogram.

2. Prove that $A(1, 5)$, $B(4, -6)$ and $C(-3, -2)$ are vertices of a right angled triangle.

3. Given ΔABC with vertices $A(0, 8)$, $B(3, 0)$ and $C(-3, 0)$
   (a) show that ΔABC is isosceles
   (b) find the length of the altitude from A
   (c) find the area of the triangle.

4. Show that the points $X(3, 2)$, $Y(-2, 1)$ and $Z(8, 3)$ are collinear.

5. (a) Show that the points $A(2, 5)$, $B(1, 0)$, $C(-7, -4)$ and $D(-3, 4)$ are the vertices of a kite.
   (b) Prove that the diagonals of the kite are perpendicular.
   (c) Show that $CE = 2AE$ where E is the point of intersection of the diagonals.

6. Find the radius of the circle that has its centre at the origin and a tangent with equation given by $4x - 3y - 5 = 0$.

7. (a) Find the equation of the perpendicular bisector of the line joining $A(3, 2)$ and $B(-1, 8)$.
   (b) Show that the point $C(7, 9)$ lies on the perpendicular bisector.
   (c) What type of triangle is ΔABC?

8. Show that ΔOAB and ΔOCD are similar where $(0, 7), (2, 0), (0, -14)$ and $(-4, 0)$ are the points A, B, C and D respectively and O is the origin.

9. (a) Prove that ΔOAB and ΔOCB are congruent given $A(3, 4)$, $B(5, 0)$, $C(2, -4)$ and O the origin.
   (b) Show that $OABC$ is a parallelogram.

10. The points $A(0, 0)$, $B(2, 0)$, $C(2, 2)$ and $D(0, 2)$ are the vertices of a square. Prove that its diagonals make angles of 45° with the sides of the square.

11. Prove that $P(-2, 0)$, $Q(0, 5)$, $R(10, 1)$ and $S(8, -4)$ are the vertices of a rectangle.

12. The points $A(-5, 0)$, $B(1, 4)$ and $C(3, 0)$ form the vertices of a triangle.
    (a) Find X and Y, the midpoints of AB and AC respectively.
    (b) Show that XY and BC are parallel.
    (c) Show that $BC = 2XY$.

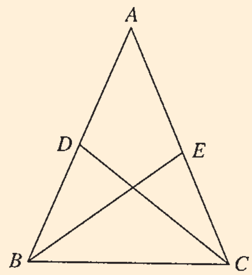
13. Show that the diagonals of a square are perpendicular bisectors, given the vertices of square $ABCD$ where $A = (-a, 0)$, $B = (-a, a)$, $C = (0, a)$ and $D = (0, 0)$.

14. (a) Show that points $X(3, 2)$ and $Y(-1, 0)$ are equal distances from the line $4x - 3y - 1 = 0$.
    (b) Find Z, the x-intercept of the line.
    (c) What is the area of triangle XYZ?

15. $ABCD$ is a quadrilateral with $A(3, 1)$, $B(1, -4)$, $C(-5, -2)$ and $D(-4, 3)$. Show that the midpoints of each side are the vertices of a parallelogram.

Test Yourself 1

1. Triangle ABC is isosceles, with $AB = AC$. D is the midpoint of AB and E is the midpoint of AC.



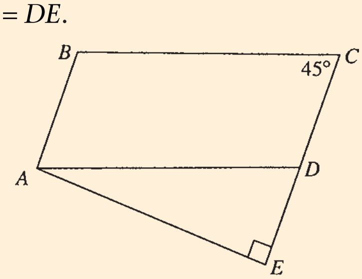
(a) Prove that ΔBEC is congruent to ΔBDC.
(b) Prove $BE = DC$.

2. If the diagonals of a rhombus are x and y, show that the length of its side is $\frac{\sqrt{x^2 + y^2}}{2}$.

3. If $A = (4, -1)$, $B = (7, -5)$ and $C = (1, 3)$, prove that triangle ABC is isosceles.

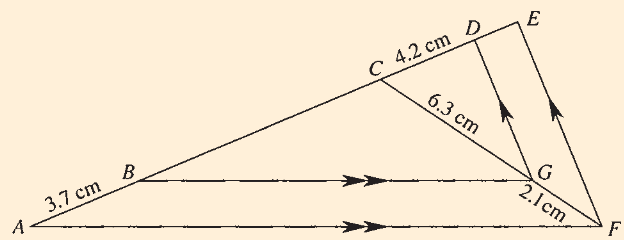
4. The surface area of a closed cylinder is 100 m². Write the height h of the cylinder in terms of its radius r.

5. $ABCD$ is a parallelogram with $\angle C = 45°$, AE perpendicular to ED and $CD = DE$.



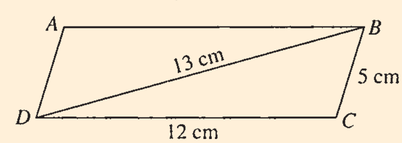
(a) Show that ΔADE is isosceles.
(b) If $AE = y$, show that the area of $ABCE$ is $\frac{3y^2}{2}$.

6. In the figure AEF
(a) prove $\frac{CB}{BA} = \frac{CD}{DE}$
(b) find the length of AE.



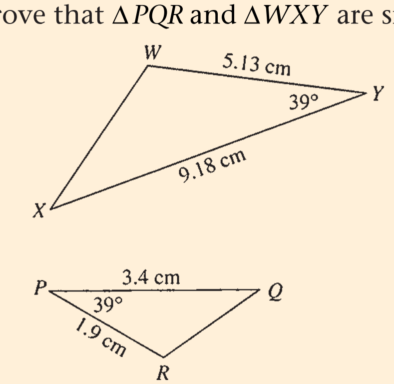
7. Given $A = (-1, 3)$, $B = (-2, -4)$, $C = (5, -4)$ and $D = (6, 3)$, prove $ABCD$ is a parallelogram.

8. A parallelogram has sides 5 cm and 12 cm, with diagonal 13 cm.

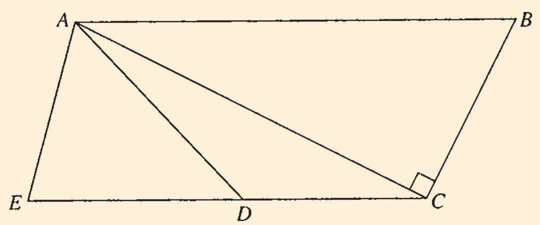


Show that the parallelogram is a rectangle.

9. Prove that ΔPQR and ΔWXY are similar.



10. In quadrilateral $ABCE$, $AD = ED = DC$ and $\angle ACB = 90°$. Also, AC bisects $\angle BAD$.

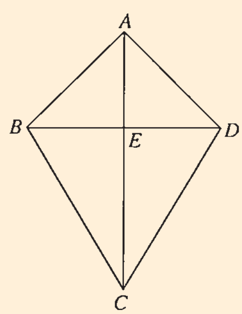


Prove $ABCE$ is a parallelogram.

11. If $A = (1, 5)$, $B = (4, 2)$ and $C = (2, -3)$, find the coordinates of D such that $ABCD$ is a parallelogram.

12. (a) Find the equation of AB if $A = (-2, -3)$ and $B = (4, 5)$.
(b) Find the perpendicular distance from $C(1, -3)$ to line AB.
(c) Find the area of ΔABC.

13. $ABCD$ is a kite.



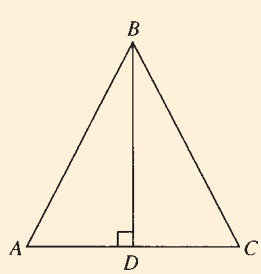
(a) Prove ΔABC and ΔADC are congruent.
(b) Prove ΔABE and ΔADE are congruent.
(c) Prove AC is the perpendicular bisector of BD.

14. $A(1, 2)$, $B(3, 3)$ and $C(5, -1)$ are points on a number plane.
(a) Show that AB is perpendicular to BC.
(b) Find the coordinates of D such that $ABCD$ is a rectangle.
(c) Find the point where the diagonals of the rectangle intersect.
(d) Calculate the length of the diagonal.

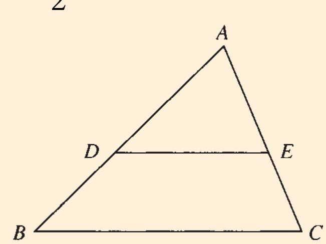
15. The surface area of a box is 500 cm^2. Its length is twice its breadth x.
(a) Show that the height h of the box is given by $h = \dfrac{250 - 2x^2}{3x}$.
(b) Show that the volume of the box is $V = \dfrac{500x - 4x^3}{3}$.

Challenge Exercise 1

1. In the figure, BD is the perpendicular bisector of AC. Prove that ΔABC is isosceles.



2. Given E and D are midpoints of AC and AB respectively, prove that
(a) DE is parallel to BC
(b) $DE = \dfrac{1}{2}BC$.



3. Prove that the diagonals in a rhombus bisect the angles they make with the sides.

4. Paper comes in different sizes, called A0, A1, A2, A3, A4 and so on. The largest size is A0, which has an area of one square metre. If the ratio of its length to breadth is $\sqrt{2}:1$, find the dimensions of its sides in millimetres, to the nearest millimetre.

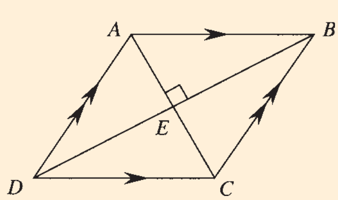
5. The volume of a prism with a square base of side x is $1000\ \text{cm}^3$. Find its surface area in terms of x.

6. Prove that in any regular n-sided polygon the size of each angle is $\left(180 - \dfrac{360}{n}\right)^\circ$.

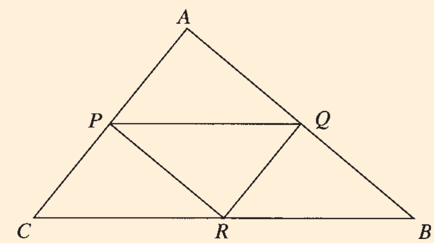
7. A parallelogram $ABCD$ has AB produced to E and diagonal AC produced to F so that $EF \parallel BC$. Prove that ΔAEF is similar to ΔADC.

8. $ABCD$ is a rhombus with $A(0, 0)$, $B(a, b)$, $C(2a, 0)$ and $D(a, -b)$. Show
(a) the diagonals bisect each other at right angles
(b) all sides are equal
(c) AC bisects $\angle BCD$.

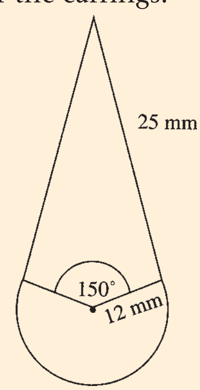
9. In the parallelogram $ABCD$, AC is perpendicular to BD. Prove that $AB = AD$.



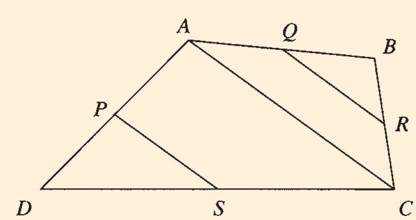
10. Triangle ABC has P, Q and R as midpoints of the sides, as shown in the diagram below. Prove that $\Delta PQR \equiv \Delta CPR$.



11. A pair of earrings is made with a wire surround holding a circular stone, as shown. Find the total length of wire needed for the earrings.



12. The sides of a quadrilateral $ABCD$ have midpoints P, Q, R and S, as shown below.



(a) Show that ΔDPS is similar to ΔDAC.
(b) Show $PS \parallel QR$.
(c) Show that $PQRS$ is a parallelogram.

13. A plastic frame for a pair of glasses is designed as below. Find the length of plastic needed for the frame, to the nearest centimetre.