Sample Examination Papers

This chapter contains two sample Mathematics HSC papers. They are designed to give you some practice in working through an examination. These papers contain work from the Year 11 Preliminary Course, as up to 20% of questions are allowed to contain this work. You may need to revise this work before you try these papers.

If you can, set yourself a time limit and work under examination conditions. Give yourself 3 hours to do each Mathematics paper. Try not to look up any notes while working through these papers.

MATHEMATICS—PAPER 1

Time allowed—Three hours
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES

- *All* questions may be attempted.
- *All* questions are of equal value.
- All necessary working should be shown in every question, as marks are awarded for this. Badly arranged or careless work may not receive marks.

QUESTION 1

(a) Find, correct to 2 decimal places, the value of $\dfrac{28.3}{15.7 \times 2.4}$.

(b) Factorise $3x^2 - 11x + 6$.

(c) Solve the equation $\dfrac{x}{2} - \dfrac{x-1}{3} = 5$.

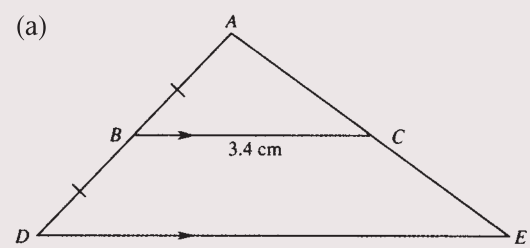
(d) The volume of a cone is given by $V = \dfrac{1}{3}\pi r^2$. If a cone has volume 12 m^3, find its radius correct to 2 decimal places.

(e) Two cities are 3295 km apart. Write this number correct to 2 significant figures.

(f) Solve $|x + 3| < 7$.

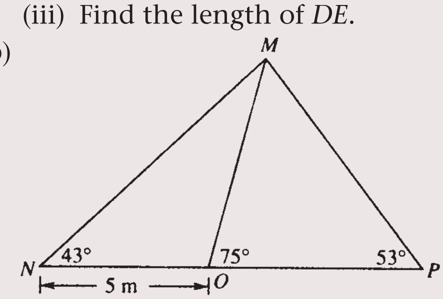
QUESTION 2

(a)



(i) Copy the diagram into your examination booklet.
(ii) Show that $AC = CE$.
(iii) Find the length of DE.

(b)



(i) Copy the diagram into your examination booklet.
(ii) Use the sine rule to calculate MO correct to 1 decimal place.
(iii) Find MP to the nearest metre.

QUESTION 3

(a) Differentiate
(i) $\sqrt{x} + 5x^3 + 1$
(ii) $3 \ln x + \dfrac{1}{x}$
(iii) $(2x + 3)^5$

(b) Find
(i) $\int (x - e^{-x})\, dx$
(ii) $\int_0^{\pi} (\sin\theta + 1)\, d\theta$

(c) (i) Rationalise the denominator of $\frac{5}{\sqrt{2}-1}$.

(ii) Find integers a and b such that $\frac{5}{\sqrt{2}-1} = a + \sqrt{b}$.

QUESTION 4

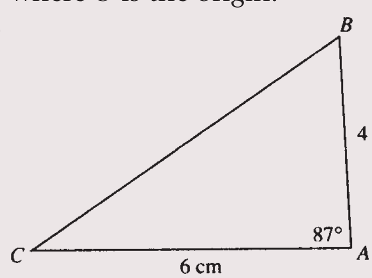
(a) (i) Plot points $A(3, 2)$ and $B(-1, 5)$ on a number plane.

(ii) Show that line AB has equation $3x + 4y - 17 = 0$.

(iii) Find the perpendicular distance from the origin to line AB.

(iv) Find the area of triangle OAB where O is the origin.

(b)



Find the length of side BC using the cosine rule and give your answer correct to the nearest centimetre.

QUESTION 5

(a) Kate earns \$24 600 p.a. In the following year she receives a pay increase of \$800. Each year after that, she receives a pay increase of \$800.

(i) What percentage increase did Kate receive in the first year?

(ii) What will be Kate's salary after 12 years?

(iii) What will Kate's total earnings be over the 12 years?

(b) A function is given by $y = x^3 - 3x^2 - 9x + 2$.

(i) Find the coordinates of the stationary points and determine their nature.

(ii) Find the point of inflexion.

(iii) Draw a sketch of the function.

QUESTION 6

(a) A plant has a probability of $\frac{2}{7}$ of producing white flowers. If 3 plants are chosen at random, find the probability that

(i) 2 plants will produce white flowers

(ii) at least 1 plant will produce white flowers.

(b) Solve $\sin x = \frac{\sqrt{3}}{2}$ for $0° \le x \le 360°$.

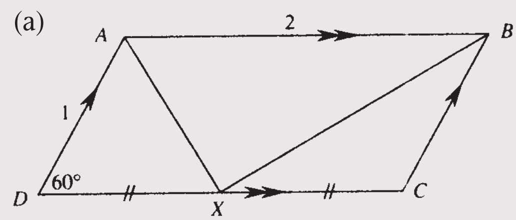
(c) A particle P moves so that it is initially at rest at the origin, and its acceleration is given by $a = 6t + 4$ cms^{-2}.

(i) Find the velocity of P when $t = 5$.

(ii) Find the displacement of P when $t = 2$.

QUESTION 7

(a)



$ABCD$ is a parallelogram with $AD = 1$, $AB = 2$ and $\angle ADX = 60°$. AX is drawn so that it bisects DC.

(i) Copy the diagram into your examination booklet.

(ii) Show that ADX is an equilateral triangle.

(iii) Show that AXB is a right-angled triangle.

(iv) Find the exact length of side BX.

(b) On a number plane, shade in the region given by the two conditions $y \ge x^2$ and $x + y \le 3$.

(c) The number of people P with chickenpox is increasing but the rate at which the disease is spreading is slowing down over t weeks.

(i) Sketch a graph showing this information.

(ii) Describe $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$ for this information.

QUESTION 8

(a) Consider the function given by $y = \log_{10} x$.

(i) Copy and complete the table, to 3 decimal places, in your examination booklet.

x	1	2	3	4	5
y	0				

(ii) Apply the trapezoidal rule with 4 subintervals to find an approximation, to 2 decimal places, of $\int_1^5 \log_{10} x \, dx$.

(b) A population of mice at time t in weeks is given by $P = P_0 e^{kt}$, where k is a constant and P_0 is the population when $t = 0$.

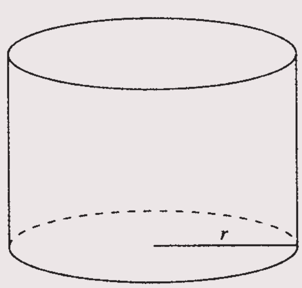
(i) Given that 20 mice increase to 100 after 6 weeks, calculate the value of k, to 3 decimal places.

(ii) How many mice will there be after 10 weeks?

(iii) After how many weeks will there be 500 mice?

QUESTION 9

(a)



The surface area of a cylinder is given by the formula $S = 2\pi r(r + h)$.
A cylinder is to have a surface area of 160 cm².

(i) Show that the volume is given by $V = 80r - \pi r^3$.

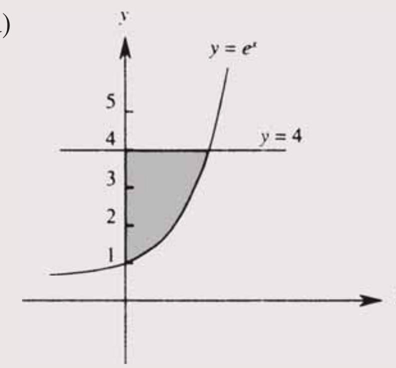
(ii) Find the value of r, to 2 decimal places, that gives the maximum volume.

(iii) Find the maximum volume, to 1 decimal place.

(b) Kim invests \$500 at the beginning of each year in a superannuation fund. The money earns 12% interest per annum. If she starts the fund at the beginning of 1996, what will the fund be worth at the end of 2025?

QUESTION 10

(a)



(i) Find the exact point of intersection of the curve $y = e^x$ and the line $y = 4$.

(ii) Find the exact shaded area enclosed between the curve and the line.

(b) (i) The quadratic equation $x^2 + (k - 1)x + k = 0$ has real and equal roots. Find the exact values of k in simplest surd form.

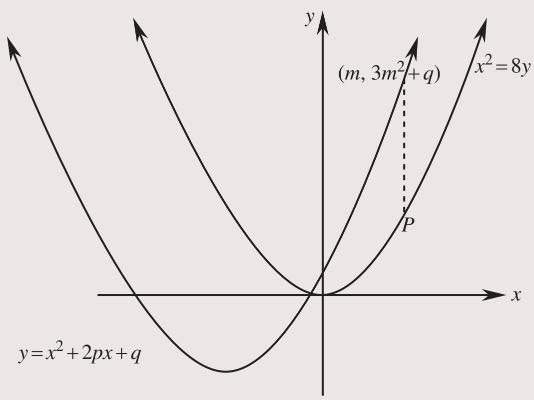
(ii) If $k = 5$, show that $x^2 + (k - 1)x + k > 0$ for all x.

(c) A parabola has equation $y = x^2 + 2px + q$.

(i) Show that the coordinates of its vertex are $(-p, q - p^2)$.

(ii) Find the coordinates of its focus.

(iii) Find the distance between the point $(m, 3m^2 + q)$ and point P vertically below it on the parabola $x^2 = 8y$ when $q > 0$.



(iv) Find the minimum distance between these two points when $m + q = 5$.

MATHEMATICS—PAPER 2

Time allowed—Three hours
(Plus 5 minutes' reading time)

QUESTION 1

(a) Solve $|x - 3| = 5$.
(b) If $f(x) = 5 - x^2$, find x when $f(x) = -4$.
(c) Find the exact value of $\sin \frac{5\pi}{6}$.
(d) Factorise fully $a^3 - 2a^2 - 4a + 8$.
(e) Simplify $2\sqrt{24} - \sqrt{150}$.
(f) Evaluate $\log_a 50$ if $\log_a 5 = 1.3$ and $\log_a 2 = 0.43$.
(g) Find the midpoint of $(-3, 4)$ and $(0, -2)$.

QUESTION 2

Plot points $A\left(5, -1\frac{1}{2}\right)$, $B\left(1, 1\frac{1}{2}\right)$ and $C(-4, -1)$ on a number plane.
(a) Show that the equation of line AB is given by $3x + 4y - 9 = 0$.
(b) Find the equation of the straight line l through C that is perpendicular to AB.
(c) Find the point of intersection P of the two lines, AB and l.
(d) Find the area of triangle ABC.
(e) Find the coordinates of D such that $ADCB$ is a rectangle.

QUESTION 3

(a) Differentiate
(i) $x \cos x$
(ii) e^{5x}
(iii) $\log_e(2x^2 - 1)$
(b) Find the indefinite integral (primitive function) of
(i) $(3x - 2)^4$
(ii) $3 \sin 2x$
(c) Evaluate $\int_0^3 (e^x - e^{-x})\, dx$.
(d) For a certain curve, $\frac{d^2y}{dx^2} = 18x - 6$. If there is a stationary point at $(2, -1)$, find the equation of the curve.

QUESTION 4

(a) Mark plays a game of chance that has a probability of $\frac{2}{5}$ of winning the game.
Hong plays a different game in which there is a probability of $\frac{3}{8}$ of winning.
Find the probability that
(i) both Mark and Hong win their games
(ii) one of them wins the game
(iii) at least one of them wins the game.
(b) (i) On a number plane, shade the region where $y \geq 0$, $x \leq 3$ and $y \leq x + 2$.
(ii) Find the area of this shaded region.
(iii) This area is rotated about the x-axis. Find the volume of the solid formed.
(c) (i) Sketch the graph of $y = |2x - 1|$ on a number plane.
(ii) Hence solve $|2x - 1| < 3$.

QUESTION 5

(a) For the curve $y = 2x^3 - 9x^2 + 12x - 7$
(i) find any stationary points on the curve and determine their nature
(ii) find any points of inflexion
(iii) sketch the curve in the domain $-3 \leq x \leq 3$.

(b) A plane leaves Bankstown airport and flies for 850 km on a bearing of 100°. It then turns and flies for 1200 km on a bearing of 040°.

(i) Draw a diagram showing this information.

(ii) How far from the airport is the plane, to the nearest km?

(c) Simplify $(\operatorname{cosec} \theta + \cot \theta)(\operatorname{cosec} \theta - \cot \theta)$.

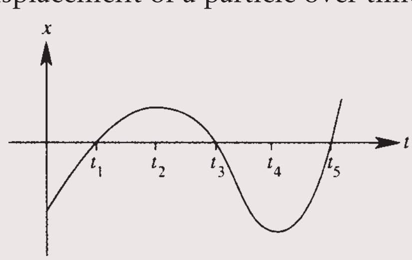
QUESTION 6

(a) (i) Find the equation of the normal to the curve $y = x^2$ at point $P(-2, 4)$.

(ii) This normal cuts the parabola again at point Q. Find the coordinates of Q.

(iii) Find the shaded area enclosed between the parabola and the normal, to 3 significant figures.

(b) The graph below shows the displacement of a particle over time t.



(i) When is the particle at the origin?

(ii) When is the particle at rest?

(c) The temperature T of a metal is cooling exponentially over t minutes. It cools down from 97°C to 84°C after 5 minutes. Find

(i) the temperature after 15 minutes

(ii) when it cools down to 20°C.

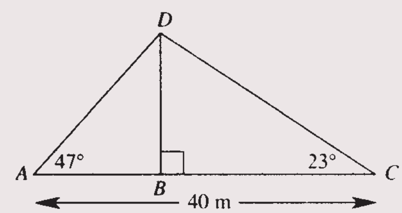
QUESTION 7

(a) (i) Find the area bounded by the curve $y = \tan x$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{4}$, by using Simpson's rule with 5 function values (to 2 decimal places).

(ii) Differentiate $\ln(\cos x)$.

(iii) Hence evaluate $\int_0^{\frac{\pi}{4}} \tan x \, dx$ to 2 decimal places.

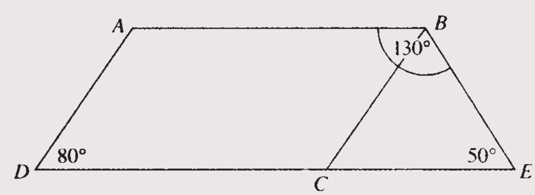
(b) A bridge is 40 metres long, held by wires at A and C with angles of elevation of 47° and 23° as shown.



(i) Find the length of AD to 3 significant figures.

(ii) Find the height of the bridge BD to 1 decimal place.

(c) Triangle BEC is isosceles with $BC = CE$. Also $\angle BEC = 50°$, $\angle ABE = 130°$, and $\angle ADC = 80°$.



Prove that $ABCD$ is a parallelogram.

QUESTION 8

(a) (i) Sketch the curve $y = 3 \sin 2x$ for $0 \le x \le 2\pi$.

(ii) On the same set of axes, sketch $y = \frac{x}{2}$.

(iii) How many roots does the equation $3 \sin 2x = \frac{x}{2}$ have in this domain?

(b) Solve $2 \sin x - 1 = 0$ for $0° \le x \le 360°$.

(c) If $\log_x 3 = p$ and $\log_x 2 = q$, write in terms of p and q

(i) $\log_x 12$

(ii) $\log_x 2x$

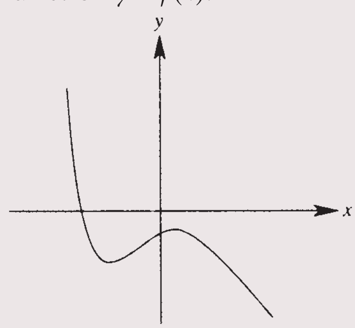
(d) A stack of oranges has 1 orange at the top, 3 in the next row down, then each row has 2 more oranges than the previous one.

(i) How many oranges are in the 12th row?

(ii) If there are 289 oranges stacked altogether, how many rows are there?

QUESTION 9

(a) The diagram below shows the graph of a function $y = f(x)$.



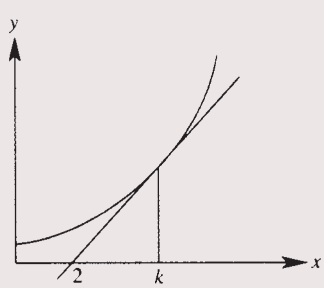
(i) Copy this diagram into your writing booklet.

(ii) On the same set of axes, draw a sketch of the derivative $f'(x)$ of the function.

(b) 'A bag contains yellow, blue and white marbles. Therefore if I choose one marble at random from the bag, the probability that it is blue is $\frac{1}{3}$.'
Is this statement true or false? Explain why in no more than one sentence.

(c) Solve the equation $2 \ln x = \ln (2x + 3)$.

(d)



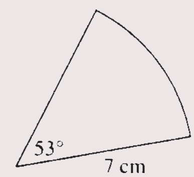
The diagram shows the graph of $y = e^x$ and the tangent to the curve at $x = k$.

(i) Find the gradient of the tangent at $x = k$.

(ii) Find the equation of the tangent at $x = k$.

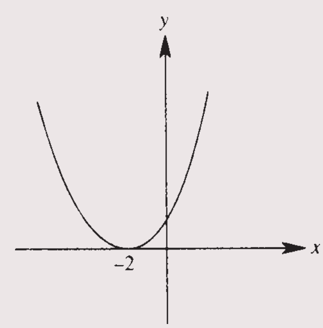
(iii) Find the value of k if the tangent has an x-intercept of 2.

(e) Find the area of the sector to 1 decimal place.



QUESTION 10

(a) The graph of $y = (x + 2)^2$ is drawn below. Copy the graph into your writing booklet.



(i) Shade the region bounded by the curve, the x-axis and the line $x = 1$.

(ii) This area is rotated about the x-axis. Find the volume of the solid of revolution formed.

(b) The accountant at Acme Business Solutions calculated that the hourly cost of running a business car is $s^2 + 7500$ cents where s is the average speed of the car. The car travels on a 3000 km journey.

(i) Show that the cost of the journey is given by $C = 3000\left(s + \frac{7500}{s}\right)$.

(ii) Find the speed that minimises the cost of the journey.

(iii) Find the cost of the trip to the nearest dollar.