Practice Assessment Task
SET 2

1. The population of a city over t years is given by the formula $P = 100\,000e^{0.71t}$. After how many years, to 1 decimal place, will the population become 1 million?

2. Evaluate $\int_1^3 \frac{dx}{x}$ correct to 3 decimal places.

3. The acceleration of a particle is given by $a = 6t$ ms^{-2} and the particle is initially at rest at the origin. Find where it will be after 3 seconds.

4. Find $\log_5 \frac{1}{25}$.

5. Write $\log_e x$ as an equation with x in terms of y. Hence find the value of x, to 3 significant figures, when $y = 1.23$.

6. Differentiate $x^3 + e^{2x}$.

7. Solve $\log_x \frac{1}{16} = 4$.

8. If 100 g of a substance decays to 80 g after 3 years, find, to 1 decimal place,
(a) its mass after 10 years
(b) the rate of decay after 10 years
(c) when it will decay to 50 g.

9. A particle has displacement x in centimetres over time t in seconds according to the formula $x = 2 \sin 3t$.
(a) Find its velocity after $\frac{\pi}{2}$ s.
(b) Show that the acceleration is given by $\ddot{x} = -9x$.

10. Water is flowing out of a pool at the rate given by $R = 20t$ litres per minute. If the volume of water in the pool is initially 8000 L, find
(a) the volume after 5 minutes
(b) how long it will take to empty the pool.

11. Find the volume, to 1 decimal place, of the solid formed by rotating $y = e^x$ about the x-axis from $x = 1$ to $x = 3$.

12. Find the derivative of $\log_e (4x + 3)^3$.

13. Find $\int \frac{2x + 1}{3x^2 + 3x - 2}\, dx$.

14. The volume in litres of a rectangular container that is leaking over time t minutes is given by $V = -t^2 + 4t + 100$. Find
(a) the initial volume
(b) the volume after 10 minutes
(c) the rate of change in volume after 10 minutes
(d) how long it will take, to 1 decimal place, until the container is empty.

15. Find $\int (6x^2 - 2x + 4x^{-1})\, dx$.

16. Find the exact value of $\int_1^7 \frac{3}{x + 5}\, dx$.

17. The velocity v cms^{-1} of a particle over time t is given by $v = \frac{2}{t + 1}$. If the particle is initially 3 cm from the origin, find
(a) its displacement after 10 s, to 2 significant figures
(b) its acceleration after 5 s, to 1 significant figure.

18. Find $\int (e^{4x} + 1)\, dx$.

19. Differentiate $\frac{x}{e^{2x}}$.

20. The rate at which an epidemic of measles in a certain city is spreading is proportional to the number of people with measles. That is, $\frac{dP}{dt} = kP$. If 40 people initially have measles and after 10 days, 110 have measles, find

(a) the value of k to 3 significant figures
(b) how many people will have measles after 6 weeks
(c) after how long, to the nearest day, 300 people will have measles
(d) the rate at which the disease will be spreading after
 (i) 10 days
 (ii) 6 weeks.

21. (a) Find the value of $\log_3 7$ by changing the base to e.
(b) Differentiate $\log_3 x$ by changing the base to e.

22. Find the exact volume of the solid of revolution formed when the curve $y = e^x$ is rotated about the x-axis from $x = 1$ to $x = 3$.

23. A particle is moving such that its displacement after t seconds is given by $x = 3 \sin 2t$ metres.
(a) Find the initial velocity and acceleration.
(b) Find the maximum displacement.
(c) Find the times when the particle will be at rest.
(d) Prove that the acceleration is given by $a = -4x$.

24. Find the area enclosed between the curve $y = \log_e x$, the y-axis and the lines $y = 1$ and $y = 2$, correct to 3 significant figures.

25. The acceleration of a particle is given by $6t - 12$ ms^{-2}. If the particle is initially at rest 2 m to the left of the origin, find its displacement after 5 seconds.

26. Solve $7^{2x} = 3$.

27. Find the equation of the tangent to $y = e^{x+1}$ at the point where $x = -1$.

28. Find the stationary point on the curve $y = xe^{2x}$ and determine its nature.

29. The length of an arc in a circle of radius 2 cm is 1.6 cm. Find the area, correct to 2 decimal places, of the
(a) sector
(b) minor segment cut off by this arc.

30. A certain chemical treatment of blue-green algae in a river causes it to decrease at a rate proportional to the amount of blue-green algae in the river. If 250 kg of blue-green algae reduces to 150 kg after 3 months, find how long it will take, to the nearest month, to reduce the blue-green algae to 20 kg.

31. An angle of 30° is subtended at the centre of a circle with radius 5 cm. Find the exact
(a) arc length
(b) area of the sector.

32. Sketch $y = 2 \cos \dfrac{x}{2}$ for $0 \le x \le 2\pi$.

33. Find $\displaystyle\int_0^{\frac{\pi}{2}} \sin(2x)\,dx$.

34. Differentiate $\log_e(\sin x)$.

35. Find the derivative of $\tan(e^{5x} + 1)$.

36. Evaluate $\displaystyle\int_0^{\frac{\pi}{3}} \sec^2 x\,dx$, giving the exact value.

37. (a) Sketch $y = \sin 2x$ for $0 \le x \le 2\pi$.
(b) Find the area bounded by the curve $y = \sin 2x$, the x-axis and the lines $x = 0$ and $x = \pi$.

38. Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \tan x\,dx$ correct to 3 decimal places by using Simpson's rule with three ordinates.

39. Differentiate
(a) $e^x \sin x$
(b) $\tan^3 x$
(c) $2\cos\left(3x - \dfrac{\pi}{2}\right)$

40. A particle moves so that its displacement after time t seconds is given by $x = 5e^{2t}$ m.
(a) Find its velocity after 2 s, to 3 significant figures.
(b) Show that its acceleration is given by $a = 4x$.
(c) Find the initial acceleration.

41. Evaluate $\int_0^5 \frac{dx}{x+3}$, giving the exact value.

42. A gardener wishes to make a rectangular garden bed using a wall as one of the sides. She has 24 m of edging strip to place around the garden. What dimensions will give the maximum garden area?

43. Find the volume of the solid formed when the curve $y = \sqrt{\cos x}$ is rotated about the x-axis from $x = 0$ to $x = \frac{\pi}{3}$.

44. Sketch the curve $y = 3x^4 - 4x^3 - 12x^2 + 1$, showing any stationary points.

45. Find the equation of the tangent to the curve $y = \tan 3x$ at the point where $x = \frac{\pi}{4}$.

46. Find the exact area enclosed between the curve $y = \sin x$ and the line $y = \frac{1}{2}$ for the domain $0 \le x \le 2\pi$.

47. Differentiate $\sin^3(e^x)$.

48. Find the exact area bounded by the curve $y = \log_e(x+4)$, the y-axis and the lines $y = 0$ and $y = 1$.

49. Find the exact value of
(a) $\cos\frac{7\pi}{4}$
(b) $\sin\left(2\pi - \frac{\pi}{3}\right)$.

50. The length of an arc in a circle of radius 6 cm is 7π cm. Find the area of the
(a) sector
(b) minor segment cut off by this arc.

51. Differentiate $\tan(\log_e x + 1)$.

52. Find $\int \frac{3x^2 - 2x + 5}{x^2}\,dx$.

53. Find the volume of the solid formed, correct to 2 decimal places, when the curve $y = \log_e x$ is rotated about the x-axis from $x = 1$ to $x = 4$, by using the trapezoidal rule with 3 subintervals.

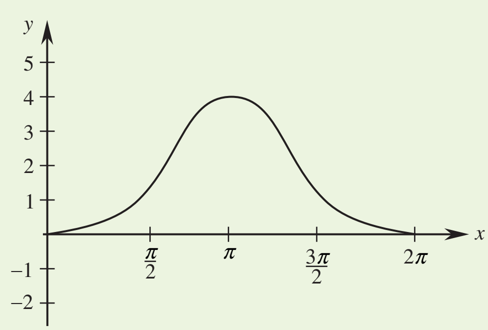
54. Find $\int (e^{5x} - \sin \pi x)\,dx$.

55. Solve graphically $\sin x = x - 1$ for $0 \le x \le 2\pi$.

56. Find the exact area enclosed between the curve $y = e^x$, the x-axis, the y-axis and the line $x = 2$.

57. Evaluate $\int_{\frac{\pi}{3}}^{\pi} \cos\left(\frac{x}{2} + \pi\right) dx$.

58.



The graph has the equation
(a) $y = 4 - 2\cos x$
(b) $y = 4 + 2\cos x$
(c) $y = 2 - 2\cos x$
(d) $y = 2 - 4\cos x$.

59. A radioactive substance decays by 60% after 200 years. This information can be shown by the equation of its mass
(a) $0.6M = Me^{200k}$
(b) $0.6M = Me^{-200k}$
(c) $0.4M = Me^{200k}$
(d) $0.4M = Me^{-200k}$

60. The value of $\sin\left(-\frac{\pi}{3}\right)$ is
(a) $-\frac{\sqrt{3}}{2}$
(b) $\frac{\sqrt{3}}{2}$
(c) $-\frac{1}{2}$
(d) $\frac{1}{2}$.

61. A particle has a displacement of $x = 4\cos 7t$. Its acceleration can be written as
(a) $\ddot{x} = 49x$
(b) $\ddot{x} = -196x$
(c) $\ddot{x} = 196x$
(d) $\ddot{x} = -49x$.

62. Evaluate $\int \frac{3x}{2x^2 - 5}\,dx$
(a) $3\ln(2x^2 - 5) + C$
(b) $\frac{3\ln(2x^2 - 5)}{4} + C$
(c) $\frac{\ln(2x^2 - 5)}{12} + C$
(d) $\frac{4\ln(2x^2 - 5)}{3} + C$

63. The rate at which a waterfall is flowing over a cliff is $R = 4t + 3t^2$ m^{3}s^{-1}. Find the amount of water flowing after a minute if the amount of water is 10 970 m^3 after 20 seconds.
(a) 223 220 m^3
(b) 8800 m^3
(c) 225 370 m^3
(d) 226 250 m^3