Answers

Chapter 1: Geometry 2

Exercises 1.1

1. $\angle ABE = 180° - \angle ABD$ (straight angle 180°)
$\angle CBE = 180° - \angle CBD$ (straight angle)
$= 180° - \angle ABD$ ($\angle ABD = \angle CBD$—given)
$= \angle ABE$

2. $\angle DFB = 180° - (180 - x)°$ ($\angle AFB$ is a straight angle)
$= x$
$\therefore \angle AFC = x$ (vertically opposite angles)
$\angle CFE = 180° - (x + 180° - 2x)$
($\angle AFB$ is a straight angle)
$= x$
$\therefore \angle AFC = \angle CFE$
$\therefore CD$ bisects $\angle AFE$

3. $\angle WBC + \angle BCY = 2x + 115 + 65 - 2x$
$= 180°$
These are supplementary cointerior angles.
$\therefore VW \parallel XY$

4. $x + y = 180°$ (given)
$\therefore \angle A + \angle D = 180°$
These are supplementary cointerior angles.
$\therefore AB \parallel DC$
Also $\angle A = \angle B$ (similarly)
These are supplementary cointerior angles.
$\therefore AD \parallel BC$
$\therefore ABCD$ is a parallelogram.

5. $\angle ADB = \angle CDB = 110°$ (given)
$\angle ABD = \angle CBD$ (BD bisects $\angle ABC$)
BD is common
$\therefore$ by AAS,
$\Delta ABD \equiv \Delta CBD$

6. (a) $AB = AE$ (given)
$\angle B = \angle E$ (base angles of isosceles Δ)
$BC = DE$ (given)
$\therefore$ by SAS, $\Delta ABC \equiv \Delta AED$
(b) $\angle BCA = \angle EDA$
(corresponding angles in congruent Δs)
$\angle ACD = 180° - \angle BCA$ (BCD is a straight angle)
$= 180° - \angle EDA$
$= \angle ADC$
$\therefore$ since base angles are equal, ΔACD is isosceles

7. $DC = BC$ (given)
$\angle B = \angle D = 90°$ (given)
$DM = \frac{1}{2}AD$ (given)
and $BN = \frac{1}{2}AB$
$\therefore DM = BN$
$\therefore$ by SAS, $\Delta MDC \equiv \Delta NBC$
$\therefore MC = NC$
(corresponding sides in congruent Δs)

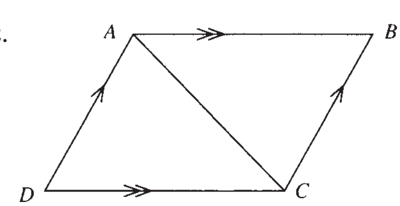
8. $\angle OCA = \angle OCB = 90°$ (given)
$OA = OB$ (equal radii)
OC is common
$\therefore$ by RHS, $\Delta OAC \equiv \Delta OBC$
$\therefore AC = BC$
(corresponding sides in congruent Δs)
$\therefore OC$ bisects AB

9. $\angle CDB = \angle BEC = 90°$ (altitudes given)
$\angle ACB = \angle ABC$ (base angles of isosceles Δ)
CB is common
$\therefore$ by AAS, $\Delta CDB \equiv \Delta BEC$
$\therefore CE = BD$
(corresponding sides in congruent Δs)

10. $AB = AD$ (given)
$BC = DC$ (given)
AC is common
$\therefore$ by SSS, $\Delta ABC \equiv \Delta ADC$
$\therefore \angle DAC = \angle BAC$
(corresponding angles in congruent Δs)
So AC bisects $\angle DAB$
Also $\angle BCA = \angle DCA$
(corresponding angles in congruent Δs)
$\therefore AC$ bisects $\angle DCB$

11. (a) $\angle NMO = \angle MOP$ (alternate angles, $MN \parallel PO$)
$\angle PMO = \angle MON$ (alternate angles, $PM \parallel ON$)
MO is common
$\therefore$ by AAS,
$\Delta MNO \equiv \Delta MPO$
(b) $\angle PMO = \angle MON$ (alternate angles, $PM \parallel ON$)
$MN = NO$ (given)
$\angle MON = \angle NMO$ (base angles of isosceles Δ)
$\therefore \angle PMO = \angle NMO$
i.e. $\angle PMQ = \angle NMQ$
(c) $MN = NO$ (given)
$PM = NO$
(corresponding sides in congruent Δs)
$\therefore PM = MN$
$\angle PMQ = \angle NMQ$ (from (b))
MQ is common
$\therefore$ by SAS, $\Delta PMQ \equiv \Delta NMQ$
(d) $\angle MQN = \angle MQP$
(corresponding angles in congruent Δs)
But $\angle MQN + \angle MQP = 180°$ ($\angle PQN$ straight angle)
$\therefore \angle MQN = \angle MQP = 90°$

12.



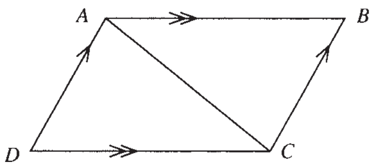
Let $ABCD$ be a parallelogram with diagonal AC.

$\angle DAC = \angle ACB$ (alternate angles, $AD \parallel BC$)

$\angle BAC = \angle ACD$ (alternate angles, $AB \parallel DC$)

AC is common

$\therefore$ by AAS, $\Delta ADC \equiv \Delta ABC$

13. 

$\Delta ADC \equiv \Delta ABC$ (see question 12)

$\therefore$ $\angle ADC = \angle ABC$ (corresponding angles in congruent Δs)

Similarly, by using diagonal BD we can prove $\angle A = \angle C$

$\therefore$ opposite angles are equal

14. $AB = DC$ (opposite sides in $\parallel$ gram)

$BM = DN$ (given)

$\therefore$ $AB - BM = DC - DN$

i.e. $AM = NC$

Also $AM \parallel NC$ ($ABCD$ is a $\parallel$ gram)

Since one pair of sides is both parallel and equal, $AMCN$ is a parallelogram.

15. $AD = BC$ (opposite sides in $\parallel$ gram)

$BC = FE$ (similarly)

$\therefore$ $AD = FE$

Also $AD \parallel BC$ ($ABCD$ is a $\parallel$ gram)

and $BC \parallel FE$ ($BCEF$ is a $\parallel$ gram)

$\therefore$ $AD \parallel FE$

Since one pair of sides is both parallel and equal, $AFED$ is a parallelogram.

16. $\angle DEC = \angle DCE$ (base angles of isosceles Δ)

Also, $\angle DEC = \angle ECB$ (alternate angles, $AD \parallel BC$)

$\therefore$ $\angle DCE = \angle ECB$

$\therefore$ CE bisects $\angle BCD$

17. $AB = CD$ (given)

$\angle BAC = \angle DCA$ (given)

AC is common

$\therefore$ by SAS, $\Delta ABC \equiv \Delta ADC$

$\therefore$ $AD = BC$

(corresponding sides in congruent Δs)

Since two pairs of opposite sides are equal, $ABCD$ is a parallelogram.

18. (a) $AE = EC$

(diagonals bisect each other in $\parallel$ gram)

$\angle AEB = \angle CEB = 90°$ (given)

EB is common

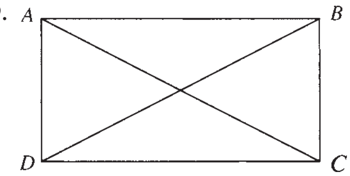
$\therefore$ by SAS, $\Delta ABE \equiv \Delta CBE$

$\therefore$ $AB = BC$

(corresponding sides in congruent Δs)

(b) $\angle ABE = \angle CBE$

(corresponding angles in congruent Δs)

19. 

Let $ABCD$ be a rectangle

$AD = BC$ (opposite sides in $\parallel$ gram)

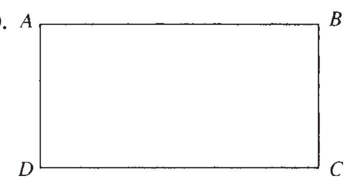
$\angle D = \angle C = 90°$

DC is common

$\therefore$ by SAS, $\Delta ADC \equiv \Delta BCD$

$\therefore$ $AC = DB$

(corresponding sides in congruent Δs)

20. 

Let $ABCD$ be a rectangle with $\angle D = 90°$

$\therefore$ $\angle C = 180° - 90°$

($\angle D$ and $\angle C$ cointerior angles, $AD \parallel BC$)

$= 90°$

$\angle B = 180° - 90°$

($\angle B$ and $\angle C$ cointerior angles, $AB \parallel DC$)

$= 90°$

$\angle A = 180° - 90°$

($\angle A$ and $\angle B$ cointerior angles, $AD \parallel BC$)

$= 90°$

$\therefore$ all angles are right angles

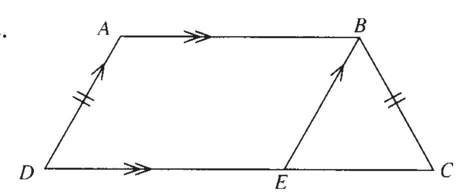
21. $AD = CD$ (given)

$AD = BC$ (opposite sides of $\parallel$ gram)

Also $AB = CD$ (similarly)

$\therefore$ $AB = AD = BC = CD$

$\therefore$ all sides of the rhombus are equal

22. 

Construct $BE \parallel AD$

Then $AD = BE$ (opposite sides of $\parallel$ gram)

But $AD = BC$ (given)

Then $BE = BC$

$\therefore$ $\angle BCD = \angle BEC$ (base angles of isosceles Δ)

Also, $\angle ADC = \angle BEC$ (corresponding angles, $AD \parallel BE$)

$\therefore$ $\angle ADC = \angle BCD$

23. $AD = AB$ (given)

$DC = BC$ (given)

AC is common

$\therefore$ by SSS, $\Delta ADC \equiv \Delta ABC$

$\therefore$ $\angle ADC = \angle ABC$

(corresponding angles in congruent Δs)

24. $AD = BC$ (opposite sides of $\|$ gram)
$\angle D = \angle C = 90°$ (given)
$DE = EC$ (E is the midpoint—given)
$\therefore$ by SAS, $\Delta ADE \equiv \Delta BCE$
$\therefore$ $AE = BE$
(corresponding sides in congruent Δs)

25. (a) $AD = BC$ (opposite sides of $\|$ gram)
$AB = DC$ (similarly)
DB is common
$\therefore$ by SSS, $\Delta ADB \equiv \Delta BCD$

(b) $\angle ABE = \angle CBE$
(corresponding angles in congruent Δs)

(c) $AB = BC$ (adjacent sides in rhombus)
$\angle ABE = \angle CBE$ (found)
BE is common
$\therefore$ by SAS, $\Delta ABE \equiv \Delta CBE$

(d) $\angle AEB = \angle BEC$
(corresponding angles in congruent Δs)
But $\angle AEB + \angle BEC = 180°$ (AEC is a straight angle)
$\therefore$ $\angle AEB = \angle BEC = 90°$

Exercises 1.2

1. (a) 14 452 mm^2 (b) 67 200 mm^3 **2.** $\sqrt[3]{90}$ m

3. $V = 2x^3 + 3x^2 - 2x$

4.
$$V = \pi r^2 h$$
$$250 = \pi r^2 h$$
$$\frac{250}{\pi h} = r^2$$
$$\sqrt{\frac{250}{\pi h}} = r$$

5. $V = 5\pi r^3$ **6.** $A = \frac{3b^2}{2}$ **7.** $V = (x + 2)^3$
$= x^3 + 6x^2 + 12x + 8$

8. $S = 24\pi h^2$

9. $V = x(3 - 2x)^2$
$= 4x^3 - 12x^2 + 9x$

10. 262 cm^3 **11.** $V = 3h^2 + 2h$

12. $V = 2h^2 + 5h$

13. $V = \frac{1}{3}(6x^3 - 5x^2 - 34x - 15)$

14. (a) $V = 18x^3 - 12x^2 + 2x$ (b) $S = 54x^2 - 30x + 4$

15. $l = \sqrt{h^2 + r^2}$

16. $V = \frac{x^2 y}{4\pi}$ **17.** $h = \frac{400}{\pi r^2}$ **18.** $h = \frac{750 - \pi r^2}{\pi r}$

19. $l = \frac{850 - \pi r^2}{\pi r}$

20. $y = \sqrt{810\,000 - x^2}$

Exercises 1.3

1. Show $m_{AB} = m_{CD} = 4$ and
$m_{AD} = m_{BC} = -\frac{4}{7}$

2. Show $m_{AC} \times m_{BC} = \frac{7}{4} \times -\frac{4}{7} = -1$;
$\therefore$ right-angled triangle with $\angle C = 90°$

3. (a) $AB = AC = \sqrt{73}$, $BC = 6$ (b) 8 units (c) 24 units^2

4. Show $m_{XY} = m_{YZ} = \frac{1}{5}$

5. (a) Show $AB = AD = \sqrt{26}$,
$BC = CD = \sqrt{80}$
(b) Show $m_{AC} \times m_{BD} = -1$
(c) $E = (-1, 2)$, $CE = \sqrt{72} = 6\sqrt{2}$,
$AE = \sqrt{18} = 3\sqrt{2}$

6. $r = 1$

7. (a) $2x - 3y + 13 = 0$
(b) Substitute (7, 9) into the equation
(c) Isosceles

8. $\angle AOB = \angle COD = 90°$
$$\frac{OD}{OB} = \frac{4}{2} = 2$$
$$\frac{OC}{OA} = \frac{14}{7} = 2$$
$$\therefore \frac{OD}{OB} = \frac{OC}{OA}$$
Since 2 pairs of sides are in proportion and their included angles are equal, $\Delta OAB \| \Delta OCD$.

9. (a) OB is common
$OA = BC = 5$
$AB = OC = \sqrt{20}$
$\therefore$ by SSS $\Delta OAB \equiv \Delta OCB$

(b) Show $m_{OA} = m_{BC} = 1\frac{1}{3}$ and $m_{AB} = m_{OC} = -2$

10. $\angle ABC = 90°$ and $AB = BC = 2$
So ABC is isosceles
$\therefore$ $\angle CAB = \angle ACB$
But $\angle CAB + \angle ACB = 90°$ (angle sum of triangle)
$\therefore$ $\angle CAB = \angle ACB = 45$
Similarly, other angles are 45°.

11. $PR = QS = \sqrt{145}$
Since diagonals are equal, $PQRS$ is a rectangle.

12. (a) $X = (-2, 2)$, $Y = (-1, 0)$
(b) $m_{XY} = m_{BC} = -2$
So $XY \| BC$
(c) $XY = \sqrt{5}$, $BC = \sqrt{20} = 2\sqrt{5}$
So $BC = 2XY$

13. $m_{AC} \times m_{BD} = 1 \times -1 = -1$
So AC and BD are perpendicular.
Midpoint AC = midpoint $BD = \left(-\frac{a}{2}, \frac{a}{2}\right)$
So AC and BD bisect each other.
So AC and BD are perpendicular bisectors.

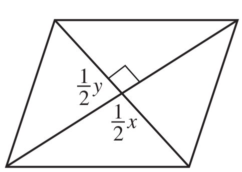
14. (a) Distance from X = distance from Y = 1 unit
(b) $Z = \left(\frac{1}{4}, 0\right)$
(c) $1\frac{1}{4}$ units^2

15. Midpoint AB: $W = \left(2, -1\frac{1}{2}\right)$
Midpoint BC: $X = (-2, -3)$
Midpoint CD: $Y = \left(-4\frac{1}{2}, \frac{1}{2}\right)$
Midpoint AD: $Z = \left(-\frac{1}{2}, 2\right)$
$m_{WX} = m_{ZY} = \frac{3}{8}$
So $WX \parallel ZY$
$m_{XY} = m_{WZ} = -\frac{7}{5}$
So $XY \parallel WZ$
$WXYZ$ is a parallelogram.

Test yourself 1

1. (a) $AB = AC$ (given)
So $BD = EC$ (midpoints)
$\angle DBC = \angle ECB$ (base $\angle$s in isosceles Δ)
BC is common
$\therefore \Delta BEC \equiv \Delta BDC$ (SAS)
(b) $\therefore BE = DC$ (corresponding sides in $\equiv \Delta$s)

2.



$$c^2 = a^2 + b^2$$
$$= \left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2$$
$$= \frac{x^2}{4} + \frac{y^2}{4}$$
$$= \frac{x^2 + y^2}{4}$$
$$c = \sqrt{\frac{x^2 + y^2}{4}}$$
$$= \frac{\sqrt{x^2 + y^2}}{2}$$

3. $AB = \sqrt{(7-4)^2 + (-5 - -1)^2}$
$= 5$
$BC = \sqrt{(7-1)^2 + (-5-3)^2}$
$= 10$
$AC = \sqrt{(4-1)^2 + (-1-3)^2}$
$= 5$
Since $AB = AC \neq BC$, ΔABC is isosceles.

4. $h = \frac{50 - \pi r^2}{\pi r}$

5. (a) $\angle ADE = 45°$ (corresponding $\angle$s $BC \parallel AD$)
$\angle EAD = 90° - 45°$ ($\angle$ sum of Δ)
$= 45°$
So ΔADE is isosceles.
(b) $AE = DE = y$ (isoceles Δ)
$CD = y$ ($CD = DE$, given)
$AB = y$ (opposite sides in $\parallel$ gram)
$AB \parallel CE$ (given)
So $ABCE$ is a trapezium.
$$A = \frac{1}{2}h(a + b)$$
$$= \frac{1}{2} \times y \times (y + 2y)$$
$$= \frac{1}{2} \times y \times 3y$$
$$= \frac{3y^2}{2}$$

6. (a) $\frac{CB}{BA} = \frac{CG}{GF}$ (equal ratio of intercepts)
$\frac{CG}{GF} = \frac{CD}{DE}$ (similarly)
$\therefore \frac{CB}{BA} = \frac{CD}{DE}$
(b) 20.4 cm

7. $m_{BC} = \frac{-4 - -4}{-2 - 5} = 0$
$m_{CD} = \frac{-4 - 3}{5 - 6} = 7$
$m_{AD} = \frac{3 - 3}{-1 - 6} = 0$
$m_{AB} = \frac{-4 - 3}{-2 - -1} = 7$
$BC \parallel AD$, $CD \parallel AB$
So $ABCD$ is a parallelogram.

8. $DC^2 + BC^2 = 12^2 + 5^2$
$= 144 + 25$
$= 169$
$= 13^2$
$= DB^2$
$\therefore \angle C = 90°$ (Pythagoras)
So $ABCD$ is a rectangle.

9. $\frac{9.18}{3.4} = \frac{5.13}{1.9} = 2.7$
$\angle Y = \angle P = 39°$ (given)
$\therefore \Delta PQR \parallel\!\parallel \Delta WXY$
(two pairs of sides in proportion, with included $\angle$s equal)

10. Let $\angle BAC = x$
Then $\angle DAC = x$ (given—AC bisects $\angle BAD$)
$\angle DCA = x$ (base $\angle$s of isosceles ΔDAC)
$\therefore \angle BAC = \angle DCA$

These are equal alternate angles.

$\therefore AB \parallel EC$

$\angle EDA = 2x$ (exterior $\angle$ of ΔDAC)

$\angle DEA = \angle DAE$ (base $\angle$s of isosceles ΔAED)

$\angle DAE = \dfrac{180° - 2x}{2}$ ($\angle$ sum of isosceles Δ)

$= 90° - x$

$\angle EAC = 90° - x + x$

$= 90°$

$\therefore \angle EAC = \angle ACB$

These are equal alternate angles.

$\therefore AE \parallel CB$

So $ABCE$ is a parallelogram.

11. $(-1, 0)$

12. (a) $4x - 3y - 1 = 0$
(b) 2.4 units
(c) 12 units^2

13. (a) $AB = AD$ (given)
$BC = DC$ (given)
AC is common
$\therefore \Delta ABC \equiv \Delta ADC$ (SSS)
(b) $AB = AD$ (given)
$\angle BAE = \angle DAE$ (corresponding $\angle$s in $\equiv$ Δs)
AE is common
$\therefore \Delta ABE \equiv \Delta ADE$ (SAS)

(c) $BE = DE$ (corresponding sides in $\equiv$ Δs)
$\therefore$ AC bisects BD
$\angle BEA = \angle DEA$ (coresponding $\angle$s in $\equiv$ Δs)
But $\angle BEA + \angle DEA = 180°$ (straight $\angle$)
$\therefore \angle BEA = \angle DEA = 90°$
So AC is perpendicular to BD.

14. (a) AB: $m_1 = \dfrac{1}{2}$, BC: $m_2 = -2$

$m_1 m_2 = \dfrac{1}{2} \times -2 = -1$

So AB and BC are perpendicular.

(b) $(3, -2)$

(c) $(3, \frac{1}{2})$

(d) 5 units

15. (a)
$$500 = 4x^2 + 6xh$$
$$500 - 4x^2 = 6xh$$
$$250 - 2x^2 = 3xh$$
$$\frac{250 - 2x^2}{3x} = h$$

(b)
$$V = 2x^2 h$$
$$= 2x^2\left(\frac{250 - 2x^2}{3x}\right)$$
$$= 2x\left(\frac{250 - 2x^2}{3}\right)$$
$$= \frac{500x - 4x^3}{3}$$

Challenge exercise 1

1. BD is common
$\angle ADB = \angle CDB = 90°$ (given)
$AD = DC$ (BD bisects AC—given)
$\therefore$ by SAS, $\Delta ABD \equiv \Delta CBD$
$\therefore$ $AB = BC$ (corresponding sides in congruent Δs)
So ΔABC is isosceles.

2. (a) $AD = \dfrac{1}{2}AB$

$\therefore \dfrac{AD}{AB} = \dfrac{1}{2}$

$AE = \dfrac{1}{2}AC$

$\therefore \dfrac{AE}{AC} = \dfrac{1}{2}$

$\therefore \dfrac{AD}{AB} = \dfrac{AE}{AC}$

$\angle A$ is common

Since two pairs of sides are in proportion and their included angles are equal,

$\Delta ADE \parallel\!\!| \; \Delta ABC$

$\therefore \angle ADE = \angle ABC$

These are equal corresponding angles

$\therefore DE \parallel BC$

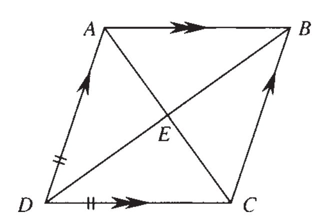
(b) Since $\Delta ADE \parallel\!\!| \; \Delta ABC$,

$\dfrac{AD}{AB} = \dfrac{DE}{BC} = \dfrac{1}{2}$

$\therefore \quad \dfrac{DE}{BC} = \dfrac{1}{2}$

$\therefore \quad DE = \dfrac{1}{2}BC$

3.



Let $ABCD$ be a rhombus with $AD = DC$
To prove:
$\angle ADE = \angle CDE$

Proof

$AD = DC$ (given)
$\therefore \Delta ADC$ is isosceles
$\therefore$ $\angle DAE = \angle DCE$
$AE = EC$ (diagonals bisect each other)
$\therefore$ by SAS, $\Delta ADE \equiv \Delta CDE$
$\therefore$ $\angle ADE = \angle CDE$
(corresponding angles in congruent Δs)

(Note: We can prove other pairs of angles equal similarly.)

4. 1189 mm $\times$ 841 mm

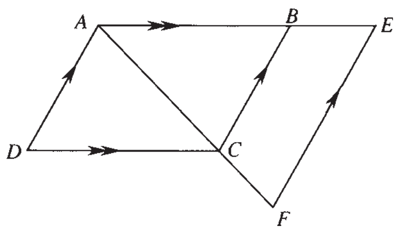
5. $S = 2x^2 + \dfrac{4000}{x}$

6. Each angle $= \dfrac{(n-2)\times 180°}{n}$

$= \dfrac{(180n - 360)°}{n}$

$= \left(180 - \dfrac{360}{n}\right)°$

7.

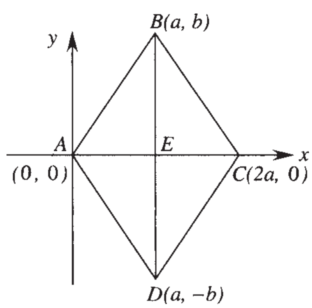


$\angle EAF = \angle ACD$ (alternate angles, $AB \parallel DC$)
$\angle ADC = \angle ABC$ (opposite angles in $\parallel$ gram)
$\angle ABC = \angle AEF$ (corresponding angles, $BC \parallel EF$)
$\therefore \angle ADC = \angle AEF$

Since 2 pairs of angles equal, third is equal by angle sum of Δ

$\therefore \Delta AEF \parallel\!\mid \Delta ADC$

8.



(a) Midpoint of BD: $\left(\dfrac{a+a}{2}, \dfrac{b+-b}{2}\right) = (a, 0)$

Midpoint of AC: $\left(\dfrac{0+2a}{2}, \dfrac{0+0}{2}\right) = (a, 0)$

$\therefore$ diagonals BD and AC bisect each other at $E(a, 0)$
BD is vertical and AC is horizontal
$\therefore$ the diagonals are perpendicular

(b) $AB = \sqrt{(a-0)^2 + (b-0)^2}$
$= \sqrt{a^2 + b^2}$
$BC = \sqrt{(2a-a)^2 + (0-b)^2}$
$= \sqrt{a^2 + b^2}$
$CD = \sqrt{(2a-a)^2 + (0+b)^2}$
$= \sqrt{a^2 + b^2}$
$AD = \sqrt{(a-0)^2 + (-b-0)^2}$
$= \sqrt{a^2 + b^2}$
$\therefore$ all sides are equal

(c) $BC = CD$ (from (a))
$BE = ED$ (from (a))
CE is common
$\therefore$ by SSS $\Delta CBE \equiv \Delta CDE$
$\therefore \angle BCE = \angle DCE$ (corresponding $\angle$s in congruent Δs)
So AC bisects $\angle BCD$

9. $DE = BE$ (digonals bisect in $\parallel$ gram)
$\angle AEB = \angle AED = 90°$ (given)
AE is common
$\therefore$ by SAS, $\Delta ADE \equiv \Delta ABE$
$\therefore AB = AD$ (corresponding sides in congruent Δs)

10. $\dfrac{CP}{PA} = \dfrac{CR}{RB} = \dfrac{1}{1}$ (P and R are midpoints)

$\therefore PR \parallel AB$ (equal ratios on $\parallel$ lines)
Similarly $PQ \parallel CB$ and $QR \parallel AC$
$\angle QPR = \angle PRC$ (alternate $\angle$s, $PQ \parallel CB$)
$\angle CPR = \angle PRQ$ (alternate $\angle$s, $AC \parallel RQ$)
PR is common
$\therefore$ by AAS, $\Delta PQR \equiv \Delta CPR$

11. 188 mm

12. (a) $\dfrac{DP}{DA} = \dfrac{DS}{DC} = \dfrac{1}{2}$ (P, S are midpoints)
$\angle D$ is common
Since 2 pairs of sides are in proportion and the included angles are equal,
$\Delta DPS \parallel\!\mid \Delta DAC$.

(b) $\dfrac{DP}{PA} = \dfrac{DS}{SC} = \dfrac{1}{1}$
$\therefore PS \parallel AC$ (equal rations on $\parallel$ lines)

Similarly, $\dfrac{BQ}{QA} = \dfrac{BR}{RC}$
$\therefore QR \parallel AC$
$\therefore PS \parallel QR$

(c) $\dfrac{AP}{PD} = \dfrac{AQ}{QB} = \dfrac{1}{1}$ (P, Q are midpoints)
$\therefore PQ \parallel DB$ (equal ratios on $\parallel$ lines)
Similarly $SR \parallel DB$
$\therefore PQ \parallel SR$
Since $PS \parallel QR$, $PQ \parallel SR$.
$PQRS$ is a parallelogram.

13. 70 cm

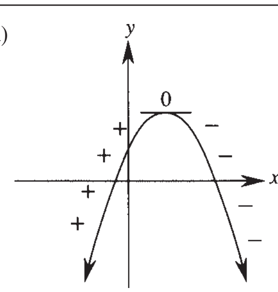
Chapter 2: Geometrical applications of calculus

Problem

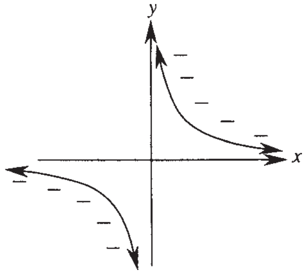
$(-0.25, -1.125)$

Exercises 2.1

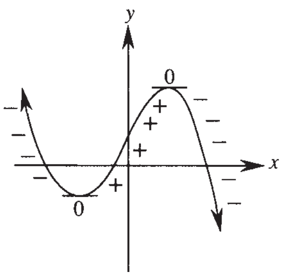
1. (a)



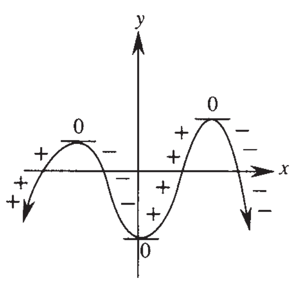
(b)



(c)



(d)



2. $x < \frac{1}{4}$ **3.** $x < 0$

4. (a) $x < 1.5$ (b) $x > 1.5$ (c) $x = 1.5$

5. $f'(x) = -2 < 0$ for all x

6. $y' = 3x^2 > 0$ for all $x \neq 0$

7. (0, 0) **8.** $x = -3, 2$ **9.** (a) $(1, -4)$ (b) $(0, 9)$
(c) $(1, 1)$ and $(2, 0)$ (d) $(0, 1)$, $(1, 0)$ and $(-1, 0)$

10. (2, 0) **11.** $-1 < x < 1$ **12.** $x < -5$, $x > -3$

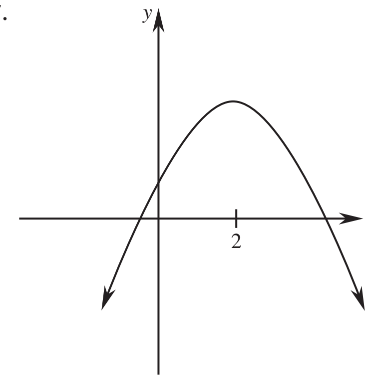
13. (a) $x = 2, 5$ (b) $2 < x < 5$ (c) $x < 2$, $x > 5$

14. $p = -12$

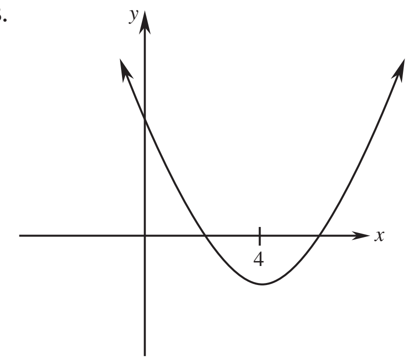
15. $a = 1\frac{1}{2}$, $b = -6$

16. (a) $\frac{dy}{dx} = 3x^2 - 6x + 27$
(b) The quadratic function has $a > 0$
$b^2 - 4ac = -288 < 0$
So $3x^2 - 6x + 27 > 0$ for all x
The function is monotonic increasing for all x.

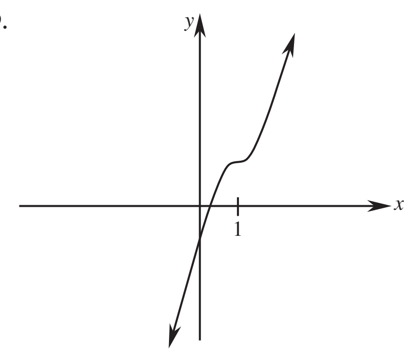
17.



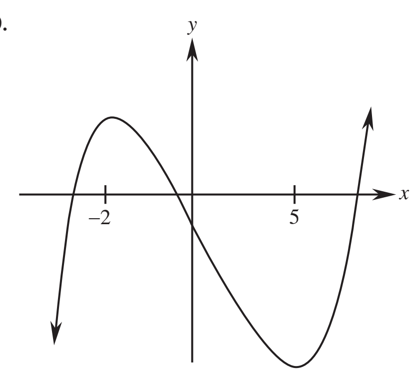
18.



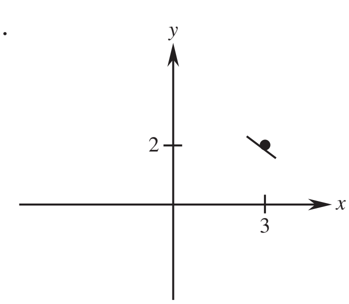
19.



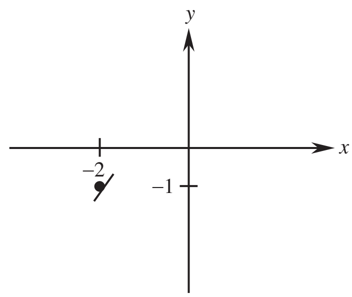
20.



21.



22.



23. $(2, 0)$ and $\left(\frac{2}{3}, 3\frac{13}{81}\right)$

24. $\frac{x}{2\sqrt{x+1}} + \sqrt{x+1} = \frac{3x+2}{2\sqrt{x+1}}; \left(-\frac{2}{3}, \frac{-2\sqrt{3}}{9}\right)$

25. $a = -1.75$

26. $\frac{1}{2\sqrt{x}} \neq 0$ 27. $\frac{-3}{x^4} \neq 0$

Exercises 2.2

1. $(0, -1)$; $y' < 0$ on LHS, $y' > 0$ on RHS

2. $(0, 0)$ minimum 3. $(0, 2)$ inflexion

4. $(-2, 11)$; show $f'(x) > 0$ on LHS and $f'(x) < 0$ on RHS.

5. $(-1, -2)$ minimum 6. $(4, 0)$ minimum

7. $(0, 5)$ maximum, $(4, -27)$ minimum

8. $f'(0) = 0$, $f'(x) > 0$ on LHS and RHS

9. $(0, 5)$ maximum, $(2, 1)$ minimum

10. $(0, -3)$ maximum, $(1, -4)$ minimum, $(-1, -4)$ minimum

11. $(1, 0)$ minimum, $(-1, 4)$ maximum

12. $m = -6\frac{1}{12}$

13. $x = -3$ minimum 14. $x = 0$ minimum, $x = -1$ maximum

15. $x = 1$ inflexion, $x = 2$ minimum

16. (a) $\frac{dP}{dx} = 2 - \frac{50}{x^2}$
(b) $(5, 20)$ minimum, $(-5, -20)$ maximum

17. $\left(1, \frac{1}{2}\right)$ minimum

18. $(2.06, 54.94)$ maximum, $(-20.6, -54.94)$ minimum

19. $(4.37, 54.92)$ minimum, $(-4.37, -54.92)$ maximum

20. (a) $\frac{dA}{dx} = \sqrt{3600 - x^2} - \frac{x^2}{\sqrt{3600 - x^2}}$
$= \frac{3600 - 2x^2}{\sqrt{3600 - x^2}}$
(b) $(42.4, 1800)$ maximum, $(-42.4, -1800)$ minimum

Exercises 2.3

1. $7x^6 - 10x^4 + 4x^3 - 1$; $42x^5 - 40x^3 + 12x^2$;
$210x^4 - 120x^2 + 24x$; $840x^3 - 240x + 24$

2. $f''(x) = 72x^7$ 3. $f'(x) = 10x^4 - 3x^2$, $f''(x) = 40x^3 - 6x$

4. $f'(1) = 11$, $f''(-2) = 168$

5. $7x^6 - 12x^5 + 16x^3$; $42x^5 - 60x^4 + 48x^2$; $210x^4 - 240x^3 + 96x$

6. $\frac{dy}{dx} = 4x - 3$, $\frac{d^2y}{dx^2} = 4$

7. $f'(-1) = -16$, $f''(2) = 40$ 8. $-4x^{-5}, 20x^{-6}$

9. $g''(4) = -\frac{1}{32}$ 10. $\frac{d^2h}{dt^2} = 26$ when $t = 1$

11. $x = \frac{7}{18}$ 12. $x > \frac{1}{3}$

13. $20(4x-3)^4$; $320(4x-3)^3$

14. $f'(x) = -\frac{1}{2\sqrt{2-x}}$;
$f''(x) = -\frac{1}{4\sqrt{(2-x)^3}}$

15. $f'(x) = -\frac{16}{(3x-1)^2}$; $f''(x) = \frac{96}{(3x-1)^3}$

16. $\frac{d^2v}{dt^2} = 24t + 16$ 17. $b = \frac{2}{3}$ 18. $f''(2) = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

19. $f''(1) = 196$ 20. $b = -2.7$

Exercises 2.4

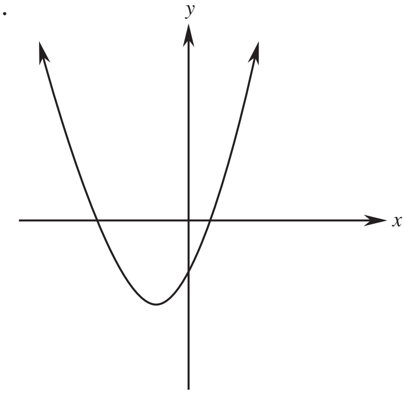
1. $x > -\frac{1}{3}$ 2. $x < 3$ 3. $y'' = -8 < 0$ 4. $y'' = 2 > 0$

5. $x < 2\frac{1}{3}$ 6. $(1, 9)$ 7. $(1, -17)$ and $(-1, -41)$

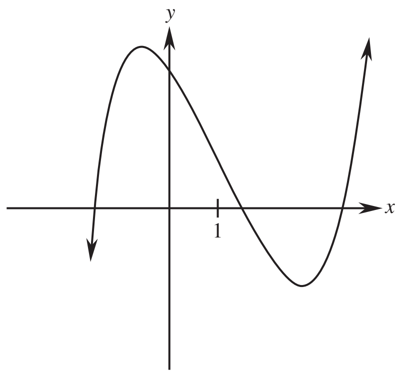
8. $(0, -2)$; $y'' < 0$ on LHS, $y'' > 0$ on RHS 9. $-2 < x < 1$

10. (a) No—minimum at $(0, 0)$
(b) Yes—inflexion at $(0, 0)$
(c) Yes—inflexion at $(0, 0)$
(d) Yes—inflexion at $(0, 0)$
(e) No—minimum at $(0, 0)$

11.



12.



13. None: (2, 31) is not an inflexion since concavity does not change.

14. $f''(x) = \dfrac{12}{x^4}$

$x^4 > 0$ for all $x \neq 0$

So $\dfrac{12}{x^4} > 0$ for all $x \neq 0$

So the function is concave upward for all $x \neq 0$.

15. (a) (0, 7), (1, 0) and (−1, 14) (b) (0, 7)

16. (a) $\dfrac{d^2y}{dx^2} = 12x^2 + 24$

$x^2 \geq 0$ for all x

So $12x^2 \geq 0$ for all x

$12x^2 + 24 \geq 24$

So $12x^2 + 24 \neq 0$ and there are no points of inflexion.

(b) The curve is always concave upwards.

17. $a = 2$ **18.** $p = 4$ **19.** $a = 3$, $b = -3$

20. (a) $(0, -8), (2, 2)$

(b) $\dfrac{dy}{dx} = 6x^5 - 15x^4 + 21$

At $(0, -8)$: $\dfrac{dy}{dx} = 6(0)^5 - 15(0)^4 + 21$

$= 21$

$\neq 0$

At $(2, 2)$: $\dfrac{dy}{dx} = 6(2)^5 - 15(2)^4 + 21$

$= -27$

$\neq 0$

So these points are not horizontal points of inflexion.

Exercises 2.5

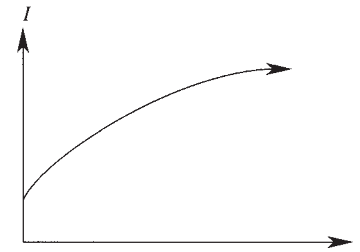
1. (a) $\dfrac{dy}{dx} > 0, \dfrac{d^2y}{dx^2} > 0$ (b) $\dfrac{dy}{dx} < 0, \dfrac{d^2y}{dx^2} < 0$

(c) $\dfrac{dy}{dx} > 0, \dfrac{d^2y}{dx^2} < 0$ (d) $\dfrac{dy}{dx} < 0, \dfrac{d^2y}{dx^2} > 0$

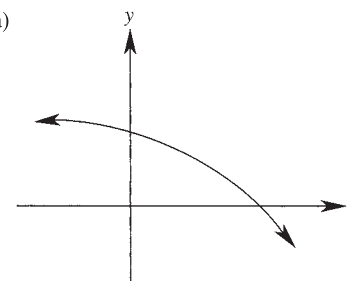
(e) $\dfrac{dy}{dx} > 0, \dfrac{d^2y}{dx^2} > 0$

2. (a) $\dfrac{dP}{dt} > 0, \dfrac{d^2P}{dt^2} < 0$ (b) The rate is decreasing.

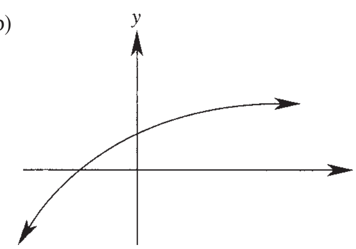
3.



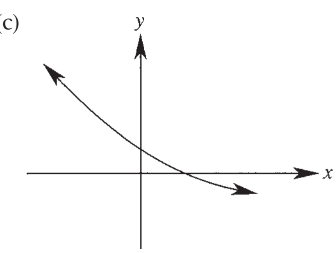
4. (a)



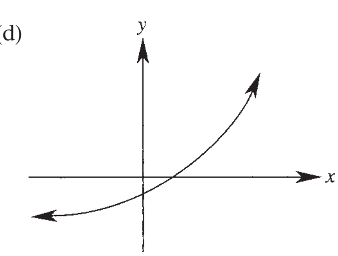
(b)



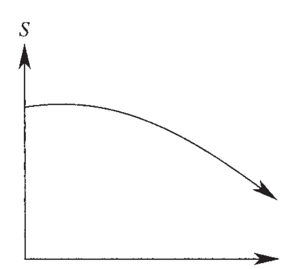
(c)



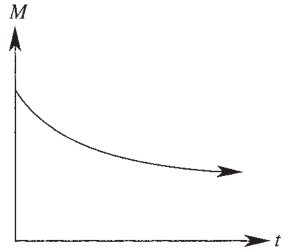
(d)



5.

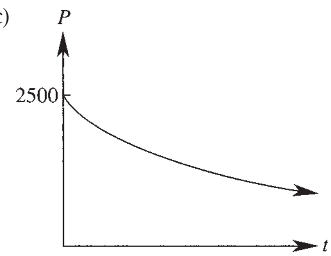


6.



7. $\frac{dM}{dt} < 0, \frac{d^2M}{dt^2} > 0$

8. (a) The number of fish is decreasing.
(b) The population rate is increasing.
(c)



9. The level of education is increasing, but the rate is slowing down.

10. The population is decreasing, and the population rate is decreasing.

Exercises 2.6

1. (1, 0) minimum **2.** (0,1) minimum

3. (2, −5), $y'' = 6 > 0$ **4.** (0.5, 0.25), $y'' < 0$ so maximum

5. (0, −5); $f''(x) = 0$ at (0, −5), $f''(x) < 0$ on LHS, $f''(x) > 0$ RHS

6. Yes—inflexion at (0, 3)

7. (−2, −78) minimum, (−3, −77) maximum

8. (0, 1) maximum, (−1, −4) minimum, (2, −31) minimum

9. (0, 1) maximum, (0.5, 0) minimum, (−0.5, 0) minimum

10. (a) (4, 176) maximum, (5, 175) minimum
(b) (4.5, 175.5)

11. (3.67, 0.38) maximum

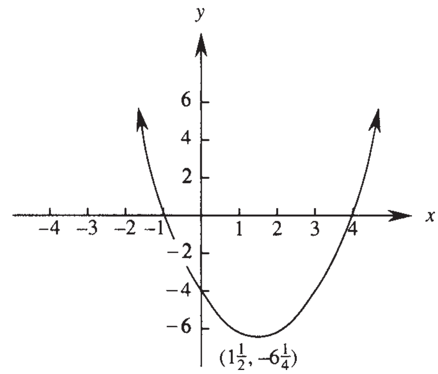
12. (0, −1) minimum, (−2, 15) maximum, (−4, −1) minimum

13. (a) $a = -\frac{2}{3}$ (b) maximum, as $y'' < 0$

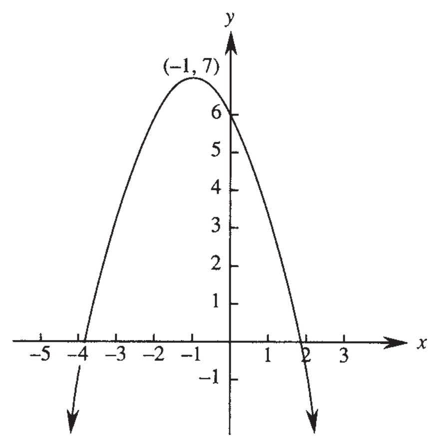
14. $m = -5\frac{1}{2}$ **15.** $a = 3$, $b = -9$

Exercises 2.7

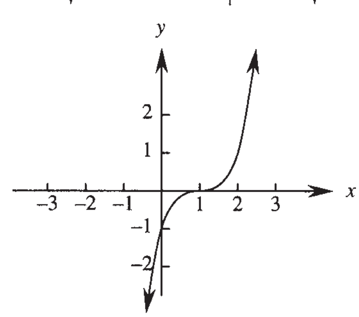
1.



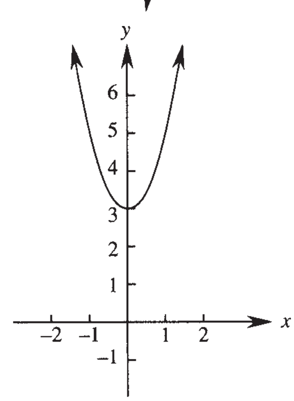
2.



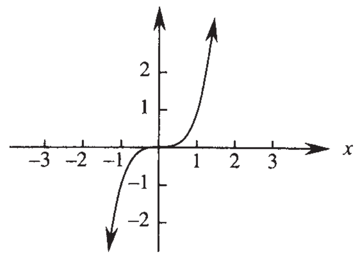
3.



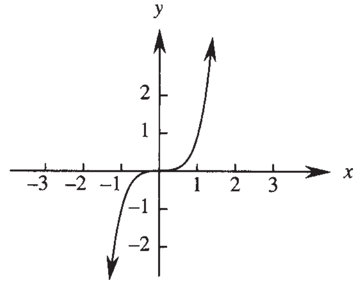
4.



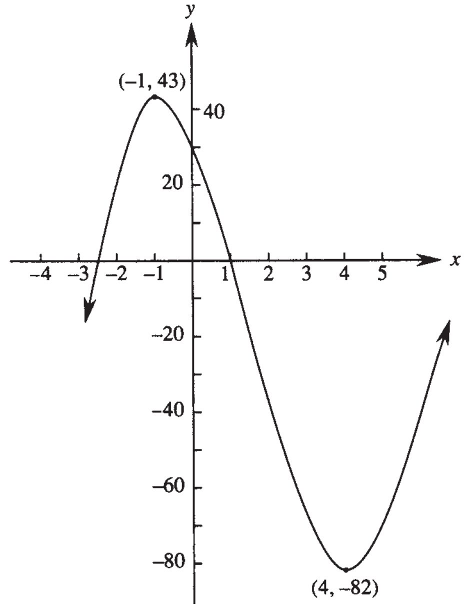
5.



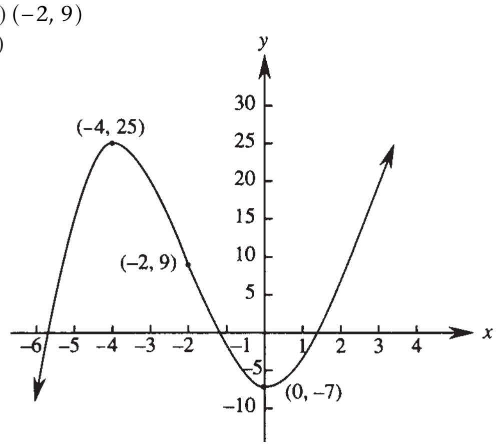
6.



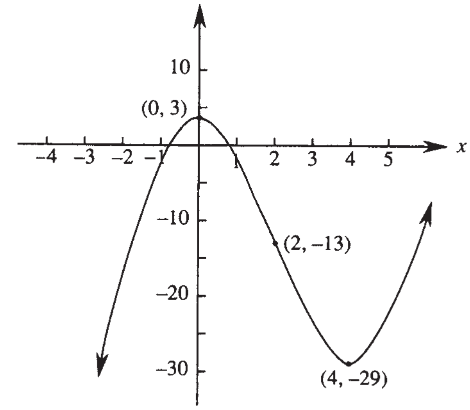
7.



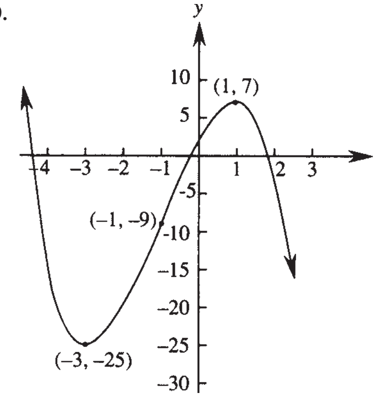
8. (a) $(0, -7)$ minimum, $(-4, 25)$ maximum
(b) $(-2, 9)$
(c)



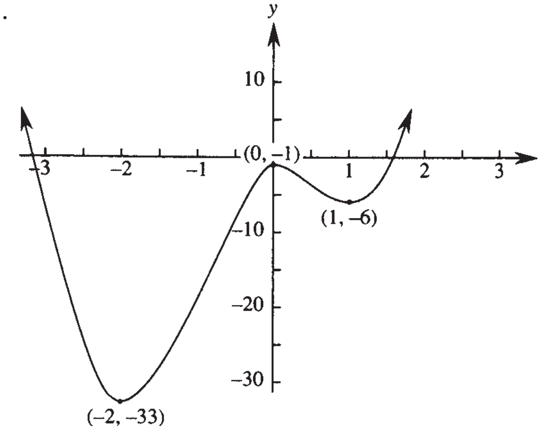
9.



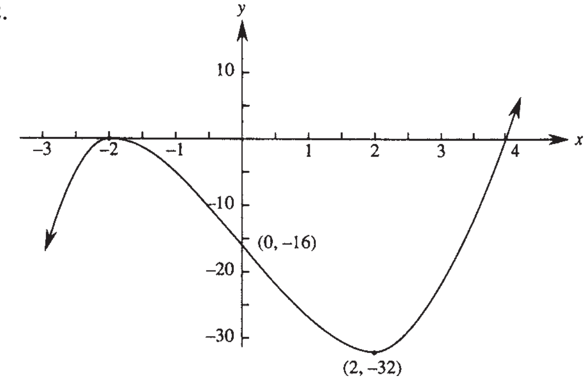
10.

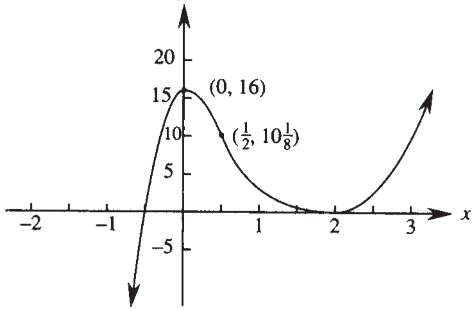


11.

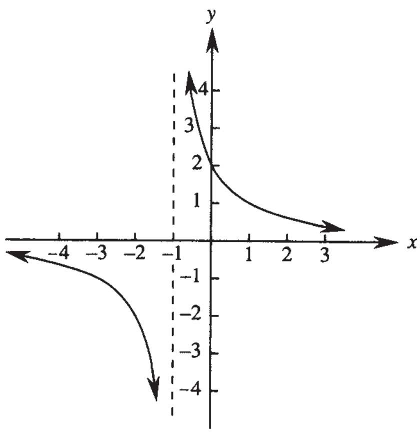


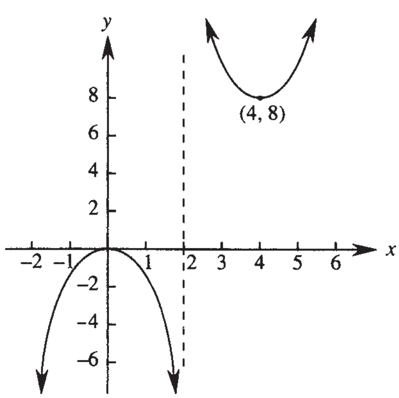
12.



13. 

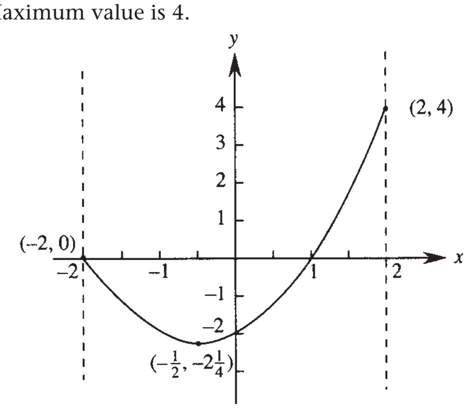
14. $\frac{dy}{dx} = \frac{-2}{(1+x)^2} \neq 0$



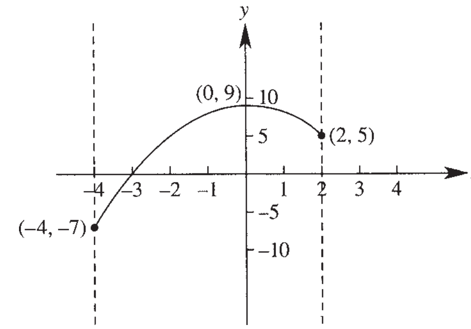
15. 

Exercises 2.8

1. Maximum value is 4.

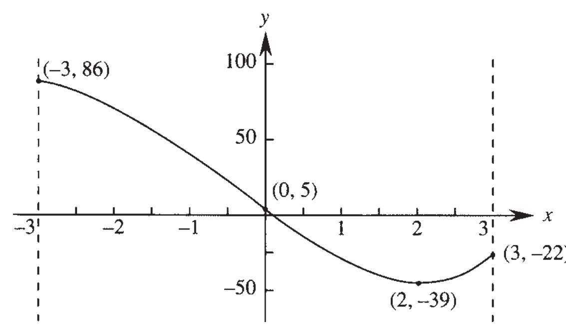


2. Maximum value is 9, minimum value is −7.



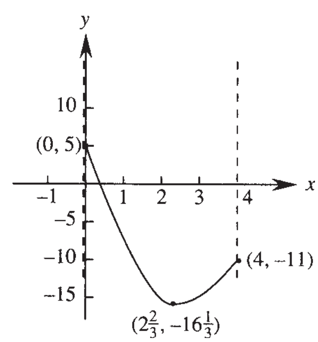
3. Maximum value is 25.

4. Maximum value is 86, minimum value is −39.



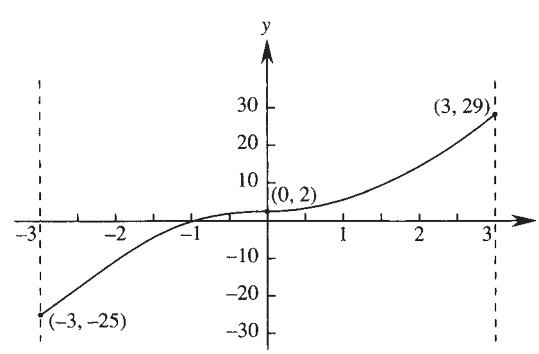
5. Maximum value is −2.

6. Maximum value is 5, minimum value is $-16\frac{1}{3}$.

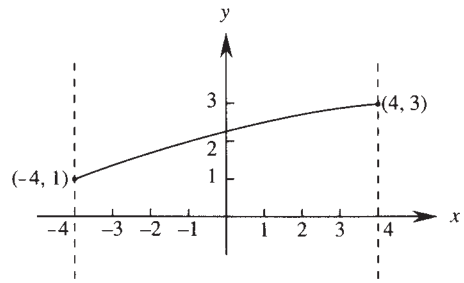


7. Absolute maximum 29, relative maximum −3, absolute minimum −35, relative minimum −35, −8

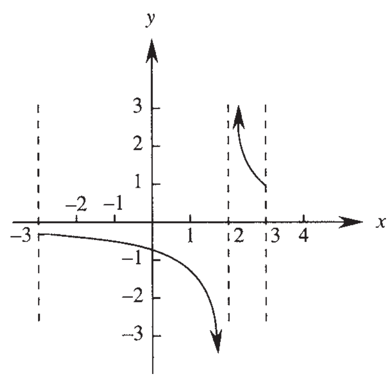
8. Minimum −25, maximum 29



9. Maximum 3, minimum 1



10. Maximum ∞, minimum −∞



Problem

The disc has radius $\frac{30}{7}$ cm. (This result uses Stewart's theorem—check this by research.)

Exercises 2.9

1.
$$A = xy$$
$$50 = xy$$
$$\therefore \frac{50}{x} = y$$
$$P = 2x + 2y$$
$$= 2x + 2 \times \frac{50}{x}$$
$$= 2x + \frac{100}{x}$$

2.
$$2x + 2y = 120$$
$$2y = 120 - 2x$$
$$y = 60 - x$$
$$A = xy$$
$$= x(60 - x)$$
$$= 60x - x^2$$

3.
$$xy = 20$$
$$y = \frac{20}{x}$$
$$S = x + y$$
$$= x + \frac{20}{x}$$

4.
$$V = \pi r^2 h$$
$$400 = \pi r^2 h$$
$$\frac{400}{\pi r^2} = h$$
$$S = 2\pi r^2 + 2\pi rh$$
$$= 2\pi r^2 + 2\pi r\left(\frac{400}{\pi r^2}\right)$$
$$= 2\pi r^2 + \frac{800}{r}$$

5. (a) $x + y = 30$

$\therefore \quad y = 30 - x$

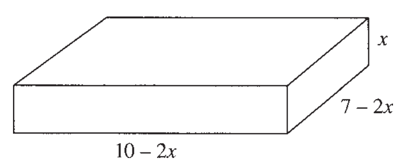
(b) The perimeter of one square is x, so its side is $\frac{1}{4}x$. The other square has side $\frac{1}{4}y$.

$$A = \left(\frac{1}{4}x\right)^2 + \left(\frac{1}{4}y\right)^2$$
$$= \frac{x^2}{16} + \frac{y^2}{16}$$
$$= \frac{x^2 + y^2}{16}$$
$$= \frac{x^2 + (30 - x)^2}{16}$$
$$= \frac{x^2 + 900 - 60x + x^2}{16}$$
$$= \frac{2x^2 - 60x + 900}{16}$$
$$= \frac{2(x^2 - 30x + 450)}{16}$$
$$= \frac{x^2 - 30x + 450}{8}$$

6. (a)
$$x^2 + y^2 = 280^2$$
$$= 78\,400$$
$$y^2 = 78\,400 - x^2$$
$$y = \sqrt{78\,400 - x^2}$$

(b)
$$A = xy$$
$$= x\sqrt{78\,400 - x^2}$$

7.

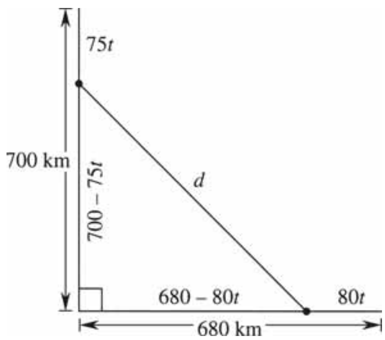


$$V = x(10 - 2x)(7 - 2x)$$
$$= x(70 - 20x - 14x + 4x^2)$$
$$= x(70 - 34x + 4x^2)$$
$$= 70x - 34x^2 + 4x^3$$

8. Profit per person = Cost − Expenses
$$= (900 - 100x) - (200 + 400x)$$
$$= 900 - 100x - 200 - 400x$$
$$= 700 - 500x$$

For x people,
$$P = x(700 - 500x)$$
$$= 700x - 500x^2$$

9.



After t hours, Joel has travelled $75t$ km. He is $700 - 75t$ km from the town.
After t hours, Nick has travelled $80t$ km. He is $680 - 80t$ km from the town.

$$\begin{aligned} d &= \sqrt{(700-75t)^2 + (680-80t)^2} \\ &= \sqrt{490\,000 - 105\,000t + 5625t^2 + 462\,400 - 108\,800t + 6400t^2} \\ &= \sqrt{952\,400 - 213\,800t + 12\,025t^2} \end{aligned}$$

10. The river is 500 m, or 0.5 km, wide Distance AB:

$$\begin{aligned} d &= \sqrt{x^2 + 0.5^2} \\ &= \sqrt{x^2 + 0.25} \end{aligned}$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\therefore \quad \text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$t = \frac{\sqrt{x^2 + 0.25}}{5}$$

Distance BC:

$$d = 7 - x$$

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$t = \frac{7-x}{4}$$

So total time taken is:

$$t = \frac{\sqrt{x^2 + 0.25}}{5} + \frac{7-x}{4}$$

Exercises 2.10

1. 2 s, 16 m **2.** 7.5 km

3.
$$\begin{aligned} 2x + 2y &= 60 \\ 2y &= 60 - 2x \\ y &= 30 - x \qquad (1) \\ A &= xy \\ &= x(30 - x) \qquad \text{from } (1) \\ &= 30x - x^2 \end{aligned}$$

Max. area 225 m^2

4. (a) $A = xy = 4000$

$$\begin{aligned} \therefore \quad y &= \frac{4000}{x} \qquad (1) \\ P &= 2x + 2y \\ &= 2x + 2\left(\frac{4000}{x}\right) \qquad \text{from } (1) \\ &= 2x + \frac{8000}{x} \end{aligned}$$

(b) 63.2 m by 63.2 m
(c) \$12 332.89

5. 4 m by 4 m **6.** 14 and 14 **7.** –2.5 and 2.5

8. $x = 1.25$ m, $y = 1.25$ m

9. (a)
$$\begin{aligned} V &= x(30-2x)(80-2x) \\ &= x(2400 - 220x + 4x^2) \\ &= 2400x - 220x^2 + 4x^3 \end{aligned}$$

(b) $x = 6\frac{2}{3}$ cm

(c) 7407.4 cm^3

10.
$$\begin{aligned} V &= \pi r^2 h = 54\pi \\ h &= \frac{54\pi}{\pi r^2} \\ &= \frac{54}{r^2} \\ S &= 2\pi r(r + h) \\ &= 2\pi r\left(r + \frac{54}{r^2}\right) \\ &= 2\pi r^2 + \frac{108\pi}{r} \end{aligned}$$

Radius is 3 m.

11. (a) $S = 2\pi r^2 + \frac{17\,200}{r}$ (b) 2323.7 m^2

12. 72 cm^2

13. (a)
$$\begin{aligned} xy &= 400 \\ \therefore \quad y &= \frac{400}{x} \\ A &= (y-10)(x-10) \\ &= xy - 10y - 10x + 100 \\ &= x\left(\frac{400}{x}\right) - 10\left(\frac{400}{x}\right) - 10x + 100 \\ &= 400 - \frac{4000}{x} - 10x + 100 \\ &= 500 - 10x - \frac{4000}{x} \end{aligned}$$

(b) 100 cm^2

14. 20 cm by 20 cm by 20 cm **15.** 1.12 m^2

16. (a) 7.5 m by 7.5 m (b) 2.4 m

17. 301 cm^2 **18.** $160\frac{1}{6}$ cm^2 **19.** 1.68 cm, 1.32 cm

20.
$$\begin{aligned} d^2 &= (200 - 80t)^2 + (120 - 60t)^2 \\ &= 40\,000 - 32\,000t + 6400t^2 + 14\,400 - 14\,400t + 3600t^2 \\ &= 10\,000t^2 - 46\,400t + 54\,400 \end{aligned}$$
24 km

21. (a)
$$\begin{aligned} d &= (x^2 - 2x + 5) - (4x - x^2) \\ &= x^2 - 2x + 5 - 4x + x^2 \\ &= 2x^2 - 6x + 5 \end{aligned}$$
(b) $\frac{1}{2}$ unit

22. (a)

$$\text{Perimeter} = 2x + y + \frac{1}{2}(2\pi r) \text{ where } r = \frac{y}{2}$$

$$1200 = 2x + y + \frac{1}{2}\left(2\pi \times \frac{y}{2}\right)$$

$$= 2x + y + \frac{\pi y}{2}$$

$$1200 - y - \frac{\pi y}{2} = 2x$$

$$600 - \frac{y}{2} - \frac{\pi y}{4} = x$$

$$\frac{2400 - 2y - \pi y}{4} = x$$

(b) $A = xy + \frac{1}{2}\pi r^2$

$$= \left(\frac{2400 - 2y - \pi y}{4}\right)y + \frac{1}{2}\pi\left(\frac{y}{2}\right)^2$$

$$= \frac{2400y - 2y^2 - \pi y^2}{4} + \frac{\pi y^2}{8}$$

$$= \frac{4800y - 4y^2 - 2\pi y^2}{8} + \frac{\pi y^2}{8}$$

$$= \frac{4800y - 4y^2 - \pi y^2}{8}$$

(c) $x = 168$ m, $y = 336$ m

23. (a) Equation AB:

$$y = mx + b$$

$$= \frac{b}{a}x + b$$

Substitute $(-1, 2)$

$$2 = \frac{b}{a}(-1) + b$$

$$= -\frac{b}{a} + b$$

$$2a = -b + ab$$

$$= b(-1 + a)$$

$$= b(a - 1)$$

$$\frac{2a}{a - 1} = b$$

(b) $a = 2$, $b = 4$

24. 26 m

25. (a) $s = \frac{d}{t}$

So $t = \frac{d}{s}$

$$= \frac{1500}{s}$$

Cost of trip taking t hours:

$$C = (s^2 + 9000)t$$

$$= (s^2 + 9000)\frac{1500}{s}$$

$$= 1500s + \frac{9000 \times 1500}{s}$$

$$= 1500\left(s + \frac{9000}{s}\right)$$

(b) 95 km/h

(c) \$2846

Exercises 2.11

1. (a) $x^2 - 3x + C$ (b) $\frac{x^3}{3} + 4x^2 + x + C$
(c) $\frac{x^6}{6} - x^4 + C$ (d) $\frac{x^3}{3} - x^2 + x + C$
(e) $6x + C$

2. (a) $f(x) = 2x^3 - \frac{x^2}{2} + C$ (b) $f(x) = \frac{x^5}{5} - x^3 + 7x + C$
(c) $f(x) = \frac{x^2}{2} - 2x + C$ (d) $f(x) = \frac{x^3}{3} - x^2 - 3x + C$
(e) $f(x) = \frac{2x^{\frac{3}{2}}}{3} + C$

3. (a) $y = x^5 - 9x + C$ (b) $y = -\frac{x^{-3}}{3} + 2x^{-1} + C$
(c) $y = \frac{x^4}{20} - \frac{x^3}{3} + C$ (d) $y = -\frac{2}{x} + C$
(e) $y = \frac{x^4}{4} - \frac{x^2}{3} + x + C$

4. (a) $\frac{2\sqrt{x^3}}{3} + C$ (b) $-\frac{x^{-2}}{2} + C$
(c) $-\frac{1}{7x^7} + C$ (d) $2x^{\frac{1}{2}} + 6x^{\frac{1}{3}} + C$
(e) $-\frac{x^{-6}}{6} + 2x^{-1} + C$

5. $y = \frac{x^4}{4} - x^3 + 5x - \frac{1}{4}$ **6.** $f(x) = 2x^2 - 7x + 11$

7. $f(1) = 8$ **8.** $y = 2x - 3x^2 + 19$ **9.** $x = 16\frac{1}{3}$

10. $y = 4x^2 - 8x + 7$ **11.** $y = 2x^3 + 3x^2 + x - 2$

12. $f(x) = x^3 - x^2 - x + 5$ **13.** $f(2) = 20.5$

14. $y = \frac{x^3}{3} + \frac{x^2}{2} - 12x + 24\frac{1}{2}$ **15.** $y = \frac{4x^3}{3} - 15x - 14\frac{1}{3}$

16. $y = \frac{x^3}{3} - 2x^2 + 3x - 4\frac{2}{3}$ **17.** $f(x) = x^4 - x^3 + 2x^2 + 4x - 2$

18. $y = 3x^2 + 8x + 8$ **19.** $f(-2) = 77$ **20.** $y = 0$

Test yourself 2

1. $(-3, -11)$ maximum, $(-1, -15)$ minimum

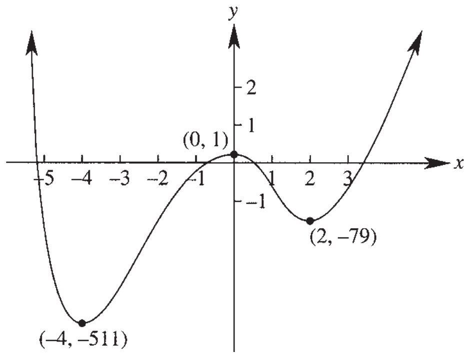
2. $x > 1\frac{1}{6}$ **3.** $y = 2x^3 + 6x^2 - 5x - 33$

4. (a) -8 (b) 26 (c) -90 **5.** 50 m

6. (0, 0) minimum **7.** $x > -1$

8. (a) (0, 1) maximum, (−4, −511) minimum, (2, −79) minimum

(b)



9. $\left(\frac{1}{2}, -1\right)$ **10.** $f(x) = \frac{5x^3}{2} + 6x^2 - 49x + 59$

11. (a)

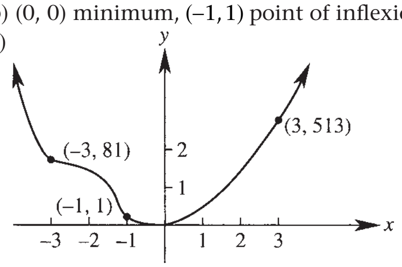
$$V = \pi r^2 h$$
$$375 = \pi r^2 h$$
$$\frac{375}{\pi r^2} = h$$
$$S = 2\pi r^2 + 2\pi r h$$
$$= 2\pi r^2 + 2\pi r\left(\frac{375}{\pi r^2}\right)$$
$$= 2\pi r^2 + \frac{750}{r}$$

(b) 3.9 cm

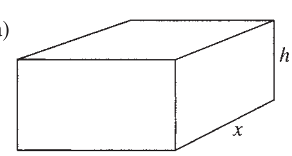
12. (a) (0, 0) and (−1, 1)

(b) (0, 0) minimum, (−1, 1) point of inflexion

(c)



13. (a)



$$S = 2x^2 + 4xh$$
$$250 = 2x^2 + 4xh$$
$$250 - 2x^2 = 4xh$$
$$\frac{250 - 2x^2}{4x} = h$$
$$\frac{2(125 - x^2)}{4x} = h$$
$$\frac{125 - x^2}{2x} = h$$
$$V = x^2 h$$
$$= x^2\left(\frac{125 - x^2}{2x}\right)$$
$$= \frac{x(125 - x^2)}{2}$$
$$= \frac{125x - x^3}{2}$$

(b) 6.45 cm × 6.45 cm × 6.45 cm

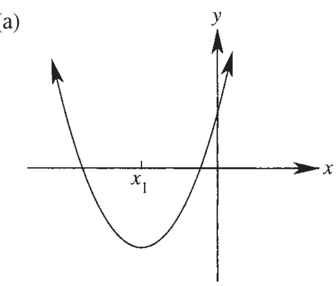
14. $x < 3$ **15.** $y = x^3 + 3x^2 + 3$ **16.** 150 products

17. For decreasing curve, $\frac{dy}{dx} < 0$

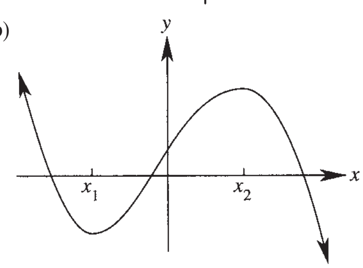
$$\frac{dy}{dx} = -3x^2$$
$$< 0 \quad (\text{since } x^2 > 0 \text{ for all } x \neq 0)$$

$\therefore$ monotonic decreasing function

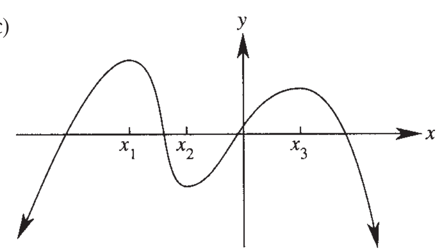
18. (a)



(b)



(c)



19. (a)

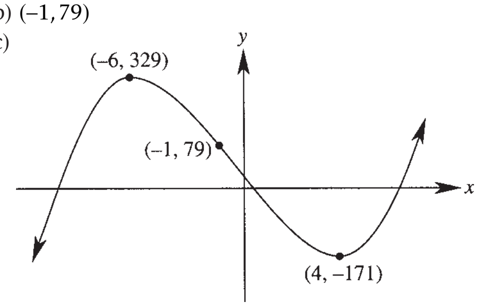
$$x^2 + y^2 = 5^2 = 25$$
$$y^2 = 25 - x^2$$
$$y = \sqrt{25 - x^2}$$
$$A = \frac{1}{2}xy$$
$$= \frac{1}{2}x\sqrt{25 - x^2}$$

(b) 6.25 m^2

20. (a) (4, −171) minimum, (−6, 329) maximum

(b) (−1, 79)

(c)



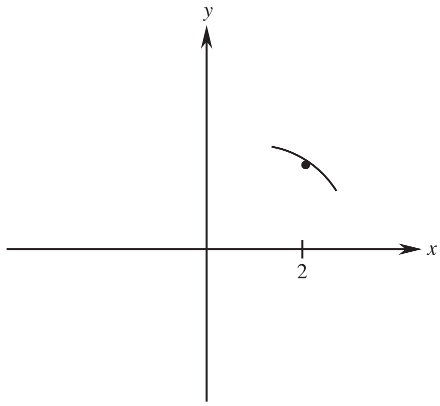
21. $x < 1$

22. $f(x) = 2x^3 - 3x^2 - 31x + 68$

23. (0, 1) and (3, −74)

24. 179

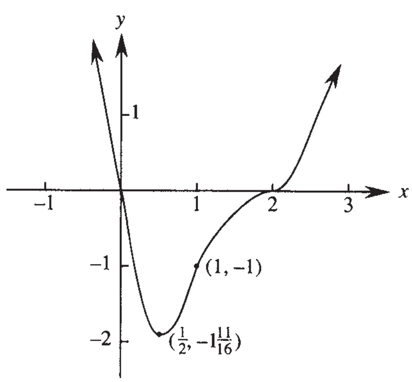
25.



Challenge exercise 2

1. $\dfrac{20x^2 - 120x - 1}{(4x^2+1)^4}$; $\dfrac{-8(60x^3 - 420x^2 - 9x + 15)}{(4x^2+1)^5}$

2.



3. $x < -\dfrac{1}{2}, x > 4$

4. 16 m^2

5. 27; –20.25

6. $f'(0.6) = f''(0.6) = 0$ and concavity changes

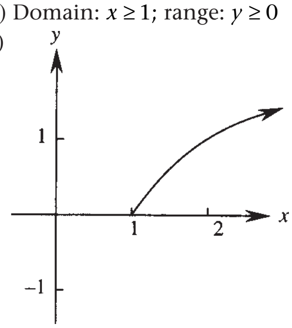
7. Show sum of areas is least when $r = s = 12.5$

8. $25\dfrac{5}{6}$

9. (a) $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x-1}} \neq 0$

(b) Domain: $x \geq 1$; range: $y \geq 0$

(c)



10. $r = 3.17$ cm, $h = 6.34$ cm

11. $y = \dfrac{x^3}{3} - x^2 - 15x - 1$

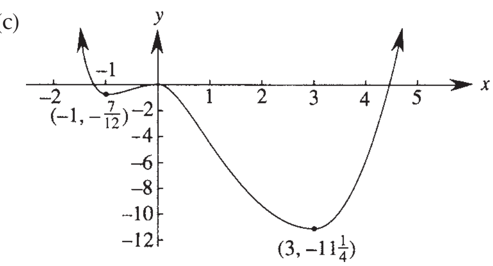
12. 110 km/h

13. $y = x^2 + 2x + 3$ (There may be other solution.)

14. (a) $f(x) = \dfrac{x^4}{4} - \dfrac{2x^3}{3} - \dfrac{3x^2}{2}$

(b) $(0, 0), \left(3, -11\dfrac{1}{4}\right), \left(-1, -\dfrac{7}{12}\right)$

(c)



15. 4 m × 4 m × 4 m

16. $f(3) = -22\dfrac{1}{6}$ **17.** (a) –2 (b) –1

18. $y' = 0$ at (0, 0);
(a) $y'' > 0$ on LHS and RHS
(b) $y'' < 0$ on LHS, $y'' > 0$ on RHS

19. $21\dfrac{1}{3}$ cm^3 **20.** (a) (0, 1) (b) $k = 2, 4, 6, 8, \ldots$

21. minimum –1; maximum $-\dfrac{1}{5}$

22. 87 kmh^{-1}

Chapter 3: Integration

Exercises 3.1

1. 2.5 **2.** 10 **3.** 2.4 **4.** 0.225 **5.** (a) 28 (b) 22 **6.** 0.39 **7.** 0.41 **8.** 1.08 **9.** 0.75 **10.** 0.65 **11.** 0.94 **12.** 0.92 **13.** 75.1 **14.** 16.5 **15.** 650.2

Exercises 3.2

1. 48.7 **2.** 30.7 **3.** 1.1 **4.** 0.41 **5.** (a) 3.4475 (b) 3.4477 **6.** 2.75 **7.** 0.693 **8.** 1.93 **9.** 72 **10.** 5.25 **11.** 0.558 **12.** 0.347 **13.** 3.63 **14.** 7.87 **15.** 175.8

Exercises 3.3

1. 8 **2.** 10 **3.** 125 **4.** –1 **5.** 10 **6.** 54 **7.** $3\frac{1}{3}$

8. 16 **9.** 50 **10.** $52\frac{2}{3}$ **11.** $\frac{2}{3}$ **12.** $21\frac{1}{4}$ **13.** 0

14. $4\frac{2}{3}$ **15.** $1\frac{1}{4}$ **16.** $4\frac{1}{3}$ **17.** 0 **18.** $2\frac{1}{3}$ **19.** 0

20. $6\frac{2}{9}$ **21.** $101\frac{1}{4}$ **22.** $-12\frac{3}{4}$ **23.** $22\frac{2}{3}$ **24.** $2\frac{1}{3}$

25. 0.0126

Exercises 3.4

1. $\frac{x^3}{3}+C$ 2. $\frac{x^6}{2}+C$ 3. $\frac{2x^5}{5}+C$ 4. $\frac{m^2}{2}+m+C$

5. $\frac{t^3}{3}-7t+C$ 6. $\frac{h^8}{8}+5h+C$ 7. $\frac{y^2}{2}-3y+C$

8. x^2+4x+C 9. $\frac{b^3}{3}+\frac{b^2}{2}+C$ 10. $\frac{a^4}{4}-\frac{a^2}{2}-a+C$

11. $\frac{x^3}{3}+x^2+5x+C$ 12. $x^4-x^3+4x^2-x+C$

13. $x^6+\frac{x^5}{5}+\frac{x^4}{2}+C$ 14. $\frac{x^8}{8}-\frac{3x^7}{7}-9x+C$

15. $\frac{x^4}{2}+\frac{x^3}{3}-\frac{x^2}{2}-2x+C$ 16. $\frac{x^6}{6}+\frac{x^4}{4}+4x+C$

17. $\frac{4x^3}{3}-\frac{5x^2}{2}-8x+C$ 18. $\frac{3x^5}{5}-\frac{x^4}{2}+\frac{x^2}{2}+C$

19. $\frac{3x^4}{2}+\frac{5x^3}{3}-4x+C$ 20. $-x^{-3}-\frac{x^{-2}}{2}-2x^{-1}+C$

21. $-\frac{1}{7x^7}+C$ 22. $\frac{3x^{\frac{4}{3}}}{4}+C$ 23. $\frac{x^4}{4}-x^3+x^2+C$

24. $x-2x^2+\frac{4x^3}{3}+C$ 25. $\frac{x^3}{3}+\frac{3x^2}{2}-10x+C$ 26. $-\frac{3}{x}+C$

27. $-\frac{1}{2x^2}+C$ 28. $-\frac{4}{x}-x+\frac{3}{2x^2}-\frac{7}{4x^4}+C$

29. $\frac{y^3}{3}+\frac{y^{-6}}{6}+5y+C$ 30. $\frac{t^4}{4}-\frac{t^3}{3}-2t^2+4t+C$

31. $\frac{2\sqrt{x^3}}{3}+C$ 32. $-\frac{1}{2t^4}+C$ 33. $\frac{3\sqrt[3]{x^4}}{4}+C$

34. $\frac{2\sqrt{x^5}}{5}+C$ 35. $\frac{2\sqrt{x^3}}{3}+x+C$

Exercises 3.5

1. (a) (i) $3x^3-12x^2+16x+C$ (ii) $\frac{(3x-4)^3}{9}+C$
   (b) $\frac{(x+1)^5}{5}+C$ (c) $\frac{(5x-1)^{10}}{50}+C$
   (d) $\frac{(3y-2)^8}{24}+C$ (e) $\frac{(4+3x)^5}{15}+C$
   (f) $\frac{(7x+8)^{13}}{91}+C$ (g) $-\frac{(1-x)^7}{7}+C$
   (h) $\frac{\sqrt{(2x-5)^3}}{3}+C$ (i) $-\frac{2(3x+1)^{-3}}{9}+C$
   (j) $-3(x+7)^{-1}+C$ (k) $-\frac{1}{16(4x-5)^2}+C$
   (l) $\frac{3\sqrt[3]{(4x+3)^4}}{16}+C$ (m) $-2(2-x)^{\frac{1}{2}}+C$
   (n) $\frac{2\sqrt{(t+3)^5}}{5}+C$ (o) $\frac{2\sqrt{(5x+2)^7}}{35}+C$

2. (a) 288.2 (b) $-1\frac{1}{4}$ (c) $-\frac{1}{8}$ (d) $60\frac{2}{3}$ (e) $\frac{1}{6}$ (f) $\frac{1}{7}$
   (g) $4\frac{2}{3}$ (h) $-\frac{1}{8}$ (i) $1\frac{1}{5}$ (j) $\frac{3}{5}$

Exercises 3.6

1. $1\frac{1}{3}$ units^2 2. 36 units^2 3. 4.5 units^2 4. $10\frac{2}{3}$ units^2

5. $\frac{1}{6}$ units^2 6. 14.3 units^2 7. 4 units^2 8. 0.4 units^2

9. 8 units^2 10. 24.25 units^2 11. 2 units^2 12. $9\frac{1}{3}$ units^2

13. $11\frac{2}{3}$ units^2 14. $\frac{1}{6}$ units^2 15. $\frac{2}{3}$ units^2 16. $\frac{1}{3}$ units^2

17. $5\frac{1}{3}$ units^2 18. 18 units^2 19. $\pi = 3.14$ units^2

20. $\frac{a^4}{2}$ units^2

Exercises 3.7

1. $21\frac{1}{3}$ units^2 2. 20 units^2 3. $4\frac{2}{3}$ units^2

4. 1.5 units^2 5. $1\frac{1}{4}$ units^2 6. $2\frac{1}{3}$ units^2

7. $10\frac{2}{3}$ units^2 8. $\frac{1}{6}$ units^2 9. $3\frac{7}{9}$ units^2

10. 2 units^2 11. $11\frac{1}{4}$ units^2 12. 60 units^2

13. 4.5 units^2 14. $1\frac{1}{3}$ units^2 15. 1.9 units^2

Exercises 3.8

1. $1\frac{1}{3}$ units^2 2. $1\frac{1}{3}$ units^2 3. $\frac{1}{6}$ units^2

4. $10\frac{2}{3}$ units^2 5. $20\frac{5}{6}$ units^2 6. 8 units^2

7. $\frac{2}{3}$ units^2 8. $166\frac{2}{3}$ units^2 9. 0.42 units^2

10. $\frac{2}{3}$ units^2 11. $\frac{1}{12}$ units^2 12. $\frac{1}{3}$ units^2

13. 36 units^2 14. $2\frac{2}{3}$ units^2 15. $(\pi-2)$ units^2

Problem

$\frac{206\pi}{15}$ units^3

Exercises 3.9

1. $\frac{243\pi}{5}$ units³ 2. $\frac{485\pi}{3}$ units³

3. $\frac{376\pi}{15}$ units³ 4. $\frac{\pi}{7}$ units³ 5. $\frac{39\pi}{2}$ units³ 6. $\frac{758\pi}{3}$ units³

7. $\frac{2\pi}{3}$ units³ 8. $\frac{992\pi}{5}$ units³ 9. $\frac{5\pi}{3}$ units³ 10. $\frac{9\pi}{2}$ units³

11. $\frac{27\pi}{2}$ units³ 12. $\frac{64\pi}{3}$ units³ 13. $\frac{16\,385\pi}{7}$ units³

14. $\frac{25\pi}{2}$ units³ 15. $\frac{65\pi}{2}$ units³ 16. $\frac{1023\pi}{5}$ units³

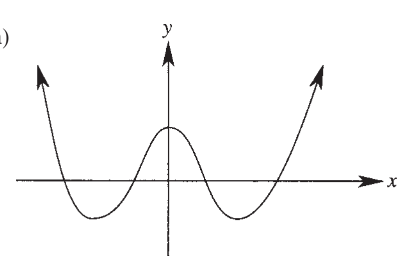
17. $\frac{5\pi}{3}$ units³ 18. 13π units³ 19. $\frac{344\pi}{27}$ units³

20. $\frac{3\pi}{5}$ units³ 21. $\frac{2\pi}{5}$ units³ 22. $\frac{72\pi}{5}$ units³

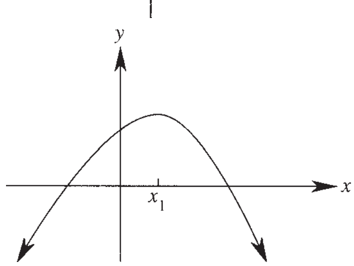
23.
$$
\begin{aligned}
y &= \sqrt{r^2 - x^2}\\
\therefore\ y^2 &= r^2 - x^2\\
V &= \pi\int_a^b y^2\,dx\\
&= \pi\int_{-r}^{r} (r^2 - x^2)\,dx\\
&= \pi\left[r^2x - \frac{x^3}{3}\right]_{-r}^{r}\\
&= \pi\left[\left(r^3 - \frac{r^3}{3}\right) - \left(-r^3 - \frac{(-r)^3}{3}\right)\right]\\
&= \pi\left(\frac{2r^3}{3} - \frac{-2r^3}{3}\right)\\
&= \frac{4\pi r^3}{3}\text{ units}^3
\end{aligned}
$$

Test yourself 3

1. (a) 0.535 (b) 0.5
2. (a) $\frac{3x^2}{2} + x + C$ (b) $\frac{5x^2}{2} - x + C$ (c) $\frac{2\sqrt{x^3}}{3} + C$
   (d) $\frac{(2x+5)^8}{16} + C$
3. 14.83 4. (a) 2 (b) 0 (c) $2\frac{1}{5}$
5. (a)



(b)



6. 3 units² 7. 1.1 units²

8. $2\frac{2}{3}$ units² 9. 9π units³ 10. $4\frac{1}{2}$ 11. $\frac{3}{4}$ units²

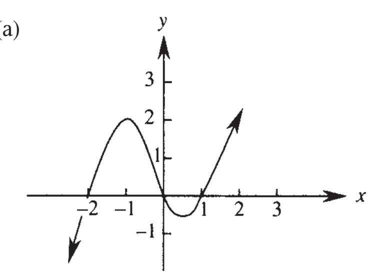
12. $\frac{(7x+3)^{12}}{84} + C$ 13. 3 units² 14. (a) $\frac{206\pi}{15}$ units³
    (b) $\frac{\pi}{2}$ units³ 15. (a) $x = \pm\sqrt{y} - 3$ (b) $3\frac{2}{3}$ units²
    (c) $\frac{5\pi}{2}$ units³

16. 36 17. $85\frac{1}{3}$ units² 18. $\frac{3\pi}{5}$ units³

19. (a) $\frac{(2x-1)^5}{2} + C$ (b) $\frac{x^6}{8} + C$

Challenge exercise 3

1. (a) $\frac{1}{12}$ units² (b) $\frac{2\pi}{35}$ units³
2. (a) Show $f(-x) = -f(x)$ (b) 0 (c) 12 units²
3. 27.2 units³ 4. 9 units² 5. (a) $36x^3(x^4 - 1)^8$
   (b) $\frac{(x^4-1)^9}{36} + C$
6. (a) $\frac{-22x}{(3x^2-4)^2}$ (b) $\frac{1}{8}$ 7. 7.35 units² 8. $\frac{2\pi}{3}$ units³
9. $f(0) = \frac{1}{0} = \infty$
10. (a)

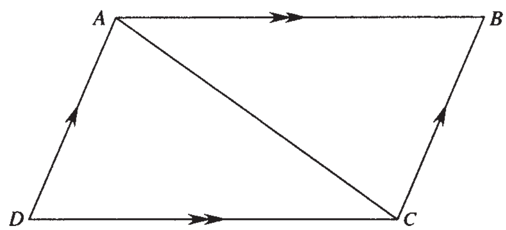


(b) 3.08 units²

11. $\frac{17\sqrt{17}}{6}$ units² 12. $\frac{215\pi}{6}$ units³
13. (a) $\frac{3x+6}{2\sqrt{x+3}} = \frac{3(x+2)}{2\sqrt{x+3}}$ (b) $\frac{2x\sqrt{x+3}}{3} + C$
14. (a) $6\frac{2}{3}$ (b) $6\frac{2}{3}$
15. $\frac{5}{12}$ units²
16. (a) $\frac{8a^2}{3}$ units²
    (b) $2\pi a^3$ units³

Practice assessment task set 1

1.



Let $ABCD$ be a parallelogram with diagonal AC.

$\angle ACD = \angle BAC$ (alternate $\angle$s, $AB \parallel DC$)

$\angle DAC = \angle BCA$ (alternate $\angle$s, $AD \parallel BC$)

AC is common

$\therefore$ by AAS $\Delta ACD \equiv \Delta ACB$

$\therefore AB = DC$ and $AD = BC$

(corresponding sides in congruent Δs)

$\therefore$ opposite sides are equal

2. $x < \frac{1}{2}$ **3.** $x^3 - x^2 + x + C$ **4.** 24 **5.** 8 m

6. $AC = FD$ (opposite sides of $\parallel$gram equal)

$BC = FE$ (given)

$\therefore AB = AC - BC$

$= FD - FE$

$= ED$

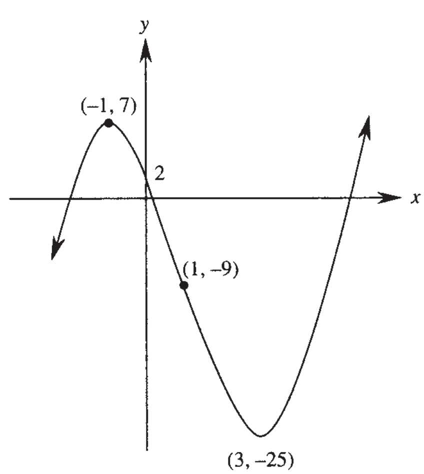
Also $AB \parallel ED$ (since $ACDF$ is $\parallel$ gram)

$\therefore$ since $AB = ED$ and $AB \parallel ED$,

$ABDE$ is a parallelogram

7. $\frac{x^9}{3} + 2x^2 + C$

8.



9. (a) $\frac{198\pi}{7}$ units^3 (b) $\frac{96\pi}{5}$ units^3

10. $1\frac{1}{3}$ units^2 **11.** $f'(3) = 20$, $f''(-2) = -16$ **12.** 68

13. $AB = AC$ (given)

$BD = CD$ (given)

AD is common.

$\therefore$ by SSS $\Delta ABD \equiv \Delta ACD$

$\therefore \quad \angle ADB = \angle ADC$

(corresponding $\angle$s in congruent Δs)

But $\angle ADB + \angle ADC = 180°$

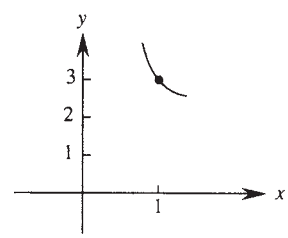
($\angle BDC$ is a straight $\angle$)

$\therefore \angle ADB = \angle ADC = 90°$

$\therefore AD \perp BC$

14. (a) 78.7 units^3 (b) 1.57 units^3 **15.** $x > -\frac{7}{9}$

16. $f(1) = 3$, $f'(1) = -2$, $f''(1) = 18$; curve is decreasing and concave upwards at (1, 3)



17. $P = 8x + 4y = 4$

$4y = 4 - 8x$

$y = 1 - 2x$

$A = 3x^2 + y^2$

$= 3x^2 + (1 - 2x)^2$

$= 3x^2 + 1 - 4x + 4x^2$

$= 7x^2 - 4x + 1$

Rectangle $\frac{2}{7}$ m $\times$ $\frac{6}{7}$ m, square with sides $\frac{3}{7}$ m

18. $f(-1) = 0$ **19.** 12 **20.** 0.837

21. $AB^2 = 24^2$

$= 576$

$BC^2 = 32^2$

$= 1024$

$AC^2 = 40^2$

$= 1600$

$AB^2 + BC^2 = 576 + 1024$

$= 1600$

$= AC^2$

$\therefore \Delta ABC$ is right angled at $\angle B$ (Pythagoras' theorem)

22. $\frac{(3x+5)^8}{24} + C$ **23.** $-5\frac{1}{3}$ **24.** (1, 1)

25. (a) 1.11 (b) 1.17

26. (0, 3) maximum, (1, 2) minimum, (−1, 2) minimum

27. $2\frac{8}{15}$ **28.** (a) 1.58 units^2 (b) $\frac{5\pi}{2}$ units^3

29. $12\frac{2}{3}$ **30.** $10\frac{2}{3}$ units^2

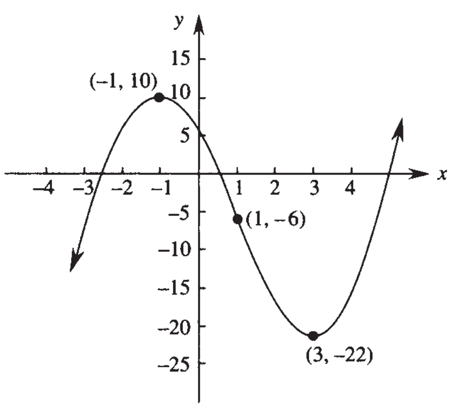
31. $\angle B$ is common
$\angle BDC = \angle ACB = 90°$ (given)
$\therefore \Delta ABC \parallel\!\!| \Delta CBD$ (AAA)

32. Show $f'(x) = f''(x) = 0$ and $f''(x) > 0$ on both LHS and RHS of (0, 0)

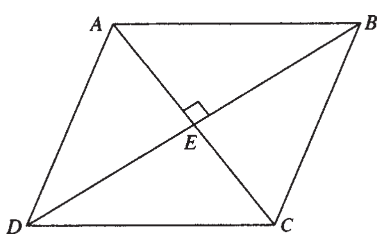
33. $2\frac{2}{3}$ m^3 **34.** $f(2) = -16$ **35.** 9 units^2

36. 119.3 m^2 **37.** $f(x) = x^3 - 4x^2 - 3x + 20$

38.



39.



Let $ABCD$ be a rhombus with $AC = x$ and $BD = y$.
$\angle AEB = 90°$
(diagonals perpendicular in rhombus)

$DE = BE = \frac{1}{2}y$
(diagonals bisect each other)

ΔACB has area $\frac{1}{2} \times x \times \frac{1}{2}y = \frac{1}{4}xy$

ΔADC has area $\frac{1}{2} \times x \times \frac{1}{2}y = \frac{1}{4}xy$

$\therefore ABCD$ has area $\frac{1}{4}xy + \frac{1}{4}xy = \frac{1}{2}xy$

40. $\frac{AB}{AC} = \frac{AG}{AD}$ (equal ratios of intercepts, $BG \parallel CD$)

$\frac{AG}{AD} = \frac{AF}{AE}$ (equal ratios of intercepts, $GF \parallel DE$)

$\therefore \frac{AB}{AC} = \frac{AF}{AE}$

41. $f(2) = 1\frac{2}{3}$

42. (a) $\frac{x^{n+1}}{n+1} + C$

(b) Since $\frac{d}{dx}(C) = 0$, the primitive function could include C.

43. (c), (d) **44.** (a), (b) **45.** (c)

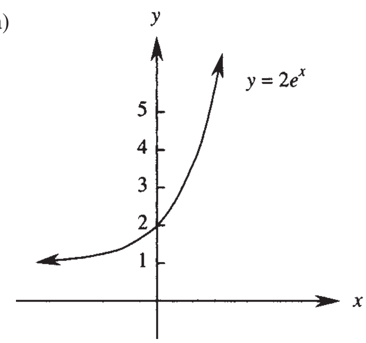
46. (d) **47.** (b) **48.** (a) **49.** (b) **50.** (d)

Chapter 4: Exponential and logarithmic functions

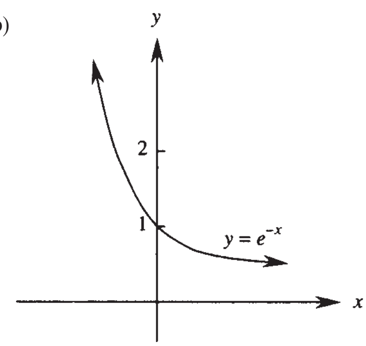
Exercises 4.1

1. (a) 4.48 (b) 0.14 (c) 2.70 (d) 0.05 (e) −0.14

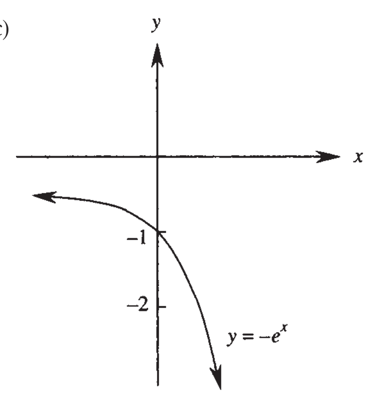
2. (a)



(b)



(c)

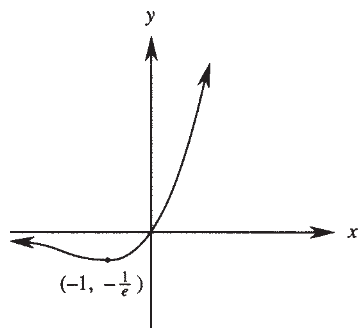


3. (a) $9e^x$ (b) $-e^x$ (c) $e^x + 2x$ (d) $6x^2 - 6x + 5 - e^x$
(e) $3e^x(e^x + 1)^2$ (f) $7e^x(e^x + 5)^6$ (g) $4e^x(2e^x - 3)$
(h) $e^x(x + 1)$ (i) $\frac{e^x(x-1)}{x^2}$ (j) $xe^x(x + 2)$
(k) $(2x + 1)e^x + 2e^x = e^x(2x + 3)$ (l) $\frac{e^x(7x - 10)}{(7x - 3)^2}$
(m) $\frac{5e^x - 5xe^x}{e^{2x}} = \frac{5(1 - x)}{e^x}$

4. $f'(1) = 6 - e$; $f''(1) = 6 - e$ **5.** e **6.** $-e^{-5} = -\frac{1}{e^5}$

7. 19.81 **8.** $ex + y = 0$ **9.** $x + e^3y - 3 - e^6 = 0$

10. $\left(-1, -\frac{1}{e}\right)$ min



11. $\frac{dy}{dx} = 7e^x; \frac{d^2y}{dx^2} = 7e^x = y$

12. $\frac{dy}{dx} = 2e^x; \frac{d^2y}{dx^2} = 2e^x$

$y = 2e^x + 1$

$\therefore\ y - 1 = 2e^x$

$\therefore\ \frac{d^2y}{dx^2} = y - 1$

Exercises 4.2

1. (a) $7e^{7x}$ (b) $-e^{-x}$ (c) $6e^{6x-2}$ (d) $2xe^{x^2+1}$
(e) $(3x^2+5)e^{x^3+5x+7}$ (f) $5e^{5x}$ (g) $-2e^{-2x}$ (h) $10e^{10x}$
(i) $2e^{2x}+1$ (j) $2x+2-e^{1-x}$ (k) $5(1+4e^{4x})(x+e^{4x})^4$
(l) $e^{2x}(2x+1)$ (m) $\frac{e^{3x}(3x-2)}{x^3}$ (n) $x^2e^{5x}(5x+3)$
(o) $\frac{4e^{2x+1}(x+2)}{(2x+5)^2}$

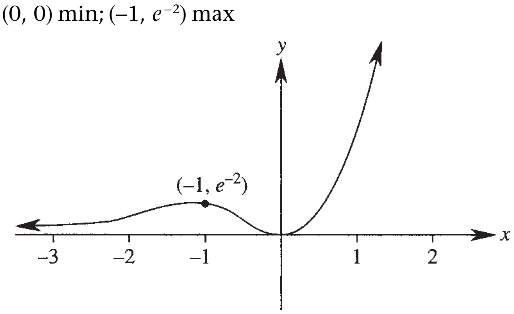
2. $28e^{2x}(e^{2x}+1)^5(7e^{2x}+1)$

3. $f(1) = 3e; f''(0) = 9e^{-2}$ 4. 5

5. $x + y - 1 = 0$ 6. $-\frac{1}{3e^3}$

7. $y = 2ex - e$ 8. $f''(-1) = -18 - 4e^2$

9. $(0, 0)$ min; $(-1, e^{-2})$ max



10. $\frac{dy}{dx} = 4e^{4x} - 4e^{-4x}$

$\frac{d^2y}{dx^2} = 16e^{4x} + 16e^{-4x}$

$= 16(e^{4x} + e^{-4x})$

$= 16y$

11. $y = 3e^{2x}$

$\frac{dy}{dx} = 6e^{2x}$

$\frac{d^2y}{dx^2} = 12e^{2x}$

$$\text{LHS} = \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y$$
$$= 12e^{2x} - 3(6e^{2x}) + 2(3e^{2x})$$
$$= 12e^{2x} - 18e^{2x} + 6e^{2x}$$
$$= 0$$
$$= \text{RHS}$$

$\therefore\ \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

12. $y = ae^{bx}$

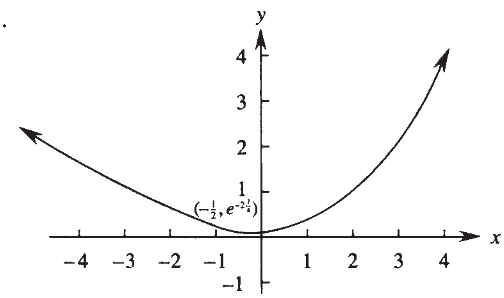
$\frac{dy}{dx} = bae^{bx}$

$\frac{d^2y}{dx^2} = b^2ae^{bx}$

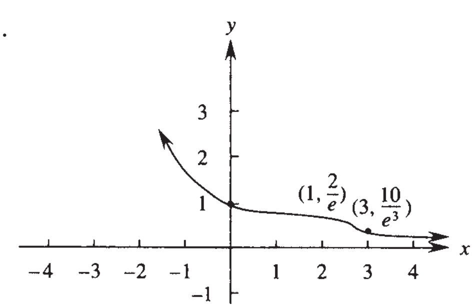
$= b^2y$

13. $n = -15$

14.



15.



Exercises 4.3

1. (a) $\frac{1}{2}e^{2x} + C$ (b) $\frac{1}{4}e^{4x} + C$ (c) $-e^{-x} + C$ (d) $\frac{1}{5}e^{5x} + C$
(e) $-\frac{1}{2}e^{-2x} + C$ (f) $\frac{1}{4}e^{4x+1} + C$ (g) $-\frac{3}{5}e^{5x} + C$
(h) $\frac{1}{2}e^{2t} + C$ (i) $\frac{1}{7}e^{7x} - 2x + C$ (j) $e^{x-3} + \frac{x^2}{2} + C$

2. (a) $\frac{1}{5}(e^5 - 1)$ (b) $e^{-2} - 1 = \frac{1}{e^2} - 1$ (c) $\frac{2}{3}e^7(e^9 - 1)$
(d) $19 - \frac{1}{2}e^4(e^2 - 1)$ (e) $\frac{1}{2}e^4 + 1\frac{1}{2}$ (f) $e^2 - e - 1\frac{1}{2}$
(g) $\frac{1}{2}e^6 + e^{-3} - 1\frac{1}{2}$

3. (a) 0.32 (b) 268.29 (c) 37 855.68 (d) 346.85 (e) 755.19

4. $e^4 - e^2 = e^2(e^2 - 1)$ units^2 5. $\frac{1}{4}(e - e^{-3})$ units^2

6. 2.86 units^2 7. 29.5 units^2 8. $\frac{\pi}{2}(e^6 - 1)$ units^3

9. 4.8 units^3 10. 7.4 11. (a) $x(2 + x)e^x$ (b) $x^2 e^x + C$

12. πe units^3

13. $\frac{1}{2}(e^4 - 5)$ units^2

Exercises 4.4

1. (a) 4 (b) 2 (c) 3 (d) 1 (e) 2 (f) 1 (g) 0 (h) 7

2. (a) 9 (b) 3 (c) –1 (d) 12 (e) 8 (f) 4 (g) 14 (h) 14 (i) 1 (j) 2

3. (a) –1 (b) $\frac{1}{2}$ (c) $\frac{1}{2}$ (d) –2 (e) $\frac{1}{4}$ (f) $-\frac{1}{3}$ (g) $-\frac{1}{2}$ (h) $\frac{1}{3}$ (i) $1\frac{1}{2}$ (j) $-1\frac{1}{2}$

4. (a) 3.08 (b) 2.94 (c) 3.22 (d) 4.94 (e) 10.40 (f) 7.04 (g) 0.59 (h) 3.51 (i) 0.43 (j) 2.21

5. (a) $\log_3 y = x$ (b) $\log_5 z = x$ (c) $\log_x y = 2$ (d) $\log_2 a = b$ (e) $\log_b d = 3$ (f) $\log_8 y = x$ (g) $\log_6 y = x$ (h) $\log_e y = x$ (i) $\log_a y = x$ (j) $\log_e Q = x$

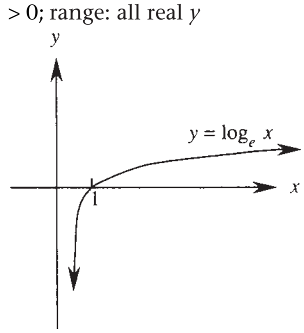
6. (a) $3^x = 5$ (b) $a^x = 7$ (c) $3^b = a$ (d) $x^9 = y$ (e) $a^y = b$ (f) $2^y = 6$ (g) $3^y = x$ (h) $10^y = 9$ (i) $e^y = 4$ (j) $7^y = x$

7. (a) $x = 1\,000\,000$ (b) $x = 243$ (c) $x = 7$ (d) $x = 2$ (e) $x = -1$ (f) $x = 3$ (g) $x = 44.7$ (h) $x = 10\,000$ (i) $x = 8$ (j) $x = 64$

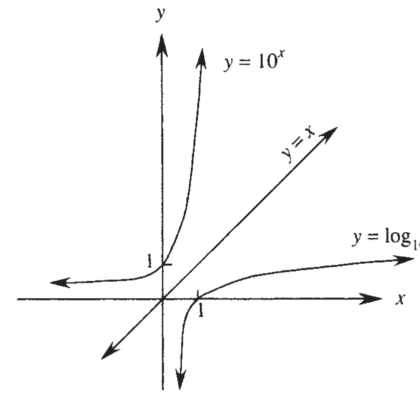
8. $y = 5$ 9. 44.7 10. 2.44 11. 0 12. 1

13. (a) 1 (b) (i) 3 (ii) 2 (iii) 5 (iv) $\frac{1}{2}$ (v) –1 (vi) 2 (vii) 3 (viii) 5 (ix) 7 (x) 1 (xi) e

14. Domain: $x > 0$; range: all real y



15. Curves are symmetrical about the line $y = x$.



16. $x = e^y$

Exercises 4.5

1. (a) $\log_a 4y$ (b) $\log_a 20$ (c) $\log_a 4$ (d) $\log_a \frac{b}{5}$ (e) $\log_x y^3 z$ (f) $\log_k 9y^3$ (g) $\log_a \frac{x^5}{y^2}$ (h) $\log_a \frac{xy}{z}$ (i) $\log_{10} ab^4c^3$ (j) $\log_3 \frac{p^3 q}{r^2}$

2. (a) 1.19 (b) –0.47 (c) 1.55 (d) 1.66 (e) 1.08 (f) 1.36 (g) 2.02 (h) 1.83 (i) 2.36 (j) 2.19

3. (a) 2 (b) 6 (c) 2 (d) 3 (e) 1 (f) 3 (g) 7 (h) $\frac{1}{2}$ (i) –2 (j) 4

4. (a) $x + y$ (b) $x - y$ (c) $3x$ (d) $2y$ (e) $2x$ (f) $x + 2y$ (g) $x + 1$ (h) $1 - y$ (i) $2x + 1$ (j) $3y - 1$

5. (a) $p + q$ (b) $3q$ (c) $q - p$ (d) $2p$ (e) $p + 5q$ (f) $2p - q$ (g) $p + 1$ (h) $1 - 2q$ (i) $3 + q$ (j) $p - 1 - q$

6. (a) 1.3 (b) 12.8 (c) 16.2 (d) 9.1 (e) 6.7 (f) 23.8 (g) –3.7 (h) 3 (i) 22.2 (j) 23

7. (a) $x = 4$ (b) $y = 28$ (c) $x = 48$ (d) $x = 3$ (e) $k = 6$

Exercises 4.6

1. (a) 1.58 (b) 1.80 (c) 2.41 (d) 3.58 (e) 2.85 (f) 2.66 (g) 1.40 (h) 4.55 (i) 4.59 (j) 7.29

2. (a) $x = 1.6$ (b) $x = 1.5$ (c) $x = 1.4$ (d) $x = 3.9$ (e) $x = 2.2$ (f) $x = 2.3$ (g) $x = 6.2$ (h) $x = 2.8$ (i) $x = 2.9$ (j) $x = 2.4$

3. (a) $x = 2.58$ (b) $y = 1.68$ (c) $x = 2.73$ (d) $m = 1.78$ (e) $k = 2.82$ (f) $t = 1.26$ (g) $x = 1.15$ (h) $p = 5.83$ (i) $x = 3.17$ (j) $n = 2.58$

4. (a) $x = 0.9$ (b) $n = 0.9$ (c) $x = 6.6$ (d) $n = 1.2$ (e) $x = -0.2$ (f) $n = 2.2$ (g) $x = 2.2$ (h) $k = 0.9$ (i) $x = 3.6$ (j) $y = 0.6$

5. (a) $x = 5.30$ (b) $t = 0.536$ (c) $t = 3.62$ (d) $x = 3.81$ (e) $n = 3.40$ (f) $t = 0.536$ (g) $t = 24.6$ (h) $k = 67.2$ (i) $t = 54.9$ (j) $k = -43.3$

Exercises 4.7

1. (a) $1+\frac{1}{x}$ (b) $-\frac{1}{x}$ (c) $\frac{3}{3x+1}$ (d) $\frac{2x}{x^2-4}$

(e) $\frac{15x^2+3}{5x^3+3x-9}$ (f) $\frac{5}{5x+1}+2x=\frac{10x^2+2x+5}{5x+1}$

(g) $6x+5+\frac{1}{x}$ (h) $\frac{8}{8x-9}$ (i) $\frac{6x+5}{(x+2)(3x-1)}$

(j) $\frac{4}{4x+1}-\frac{2}{2x-7}=\frac{-30}{(4x+1)(2x-7)}$

(k) $\frac{5}{x}(1+\log_e x)^4$ (l) $9\left(\frac{1}{x}-1\right)(\ln x-x)^8$

(m) $\frac{4}{x}(\log_e x)^3$ (n) $6\left(2x+\frac{1}{x}\right)(x^2+\log_e x)^5$

(o) $1+\log_e x$ (p) $\frac{1-\log_e x}{x^2}$

(q) $\frac{2x+1}{x}+2\log_e x$ (r) $\frac{x^3}{x+1}+3x^2\log_e(x+1)$

(s) $\frac{1}{x\log_e x}$ (t) $\frac{x-2-x\log_e x}{x(x-2)^2}$

(u) $\frac{e^{2x}(2x\log_e x-1)}{x(\log_e x)^2}$

(v) $e^x\left(\frac{1}{x}+\log_e x\right)$

(w) $\frac{10\log_e x}{x}$

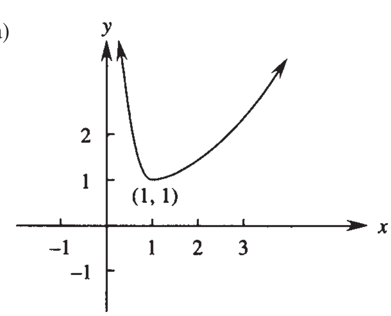
2. $f'(1)=-\frac{1}{2}$ 3. $\frac{1}{x\log_e 10}$ 4. $x-2y-2+2\log_e 2=0$

5. $y=x-2$ 6. $-\frac{2}{5}$ 7. $5x+y-\log_e 5-25=0$

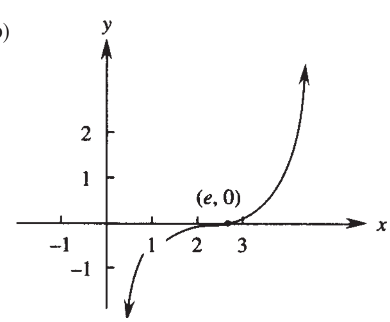
8. $5x-19y+19\log_e 19-15=0$ 9. $\left(\frac{1}{2},\frac{1}{2}\log_e\frac{1}{2}-\frac{1}{4}\right)$

10. $\left(e,\frac{1}{e}\right)$ maximum

11. (a)



(b)



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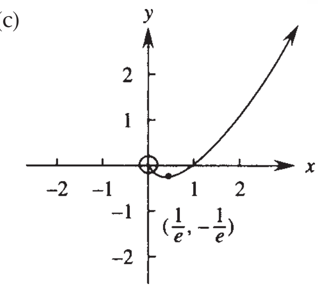


12. $\frac{2}{(2x+5)\log_e 3}$ 13. (a) $3^x\ln 3$ (b) $10^x\ln 10$
(c) $3\ln 2\times 2^{3x-4}$

14. $4\ln 4\cdot x-y+4=0$ 15. $3\log_e 3\cdot x+y-1-9\log_e 3=0$

Exercises 4.8

1. (a) $\log_e(2x+5)+C$ (b) $\log_e(2x^2+1)+C$

(c) $\ln(x^5-2)+C$ (d) $\frac{1}{2}\log_e x+C$ or $\frac{1}{2}\log_e 2x+C$

(e) $2\ln x+C$ (f) $\frac{5}{3}\log_e x+C$ (g) $\log_e(x^2-3x)+C$

(h) $\frac{1}{2}\ln(x^2+2)+C$ (i) $\frac{3}{2}\log_e(x^2+7)+C$

(j) $\frac{1}{2}\log_e(x^2+2x-5)+C$

2. (a) $\ln(4x-1)+C$ (b) $\log_e(x+3)+C$

(c) $\frac{1}{6}\ln(2x^3-7)+C$ (d) $\frac{1}{12}\log_e(2x^6+5)+C$

(e) $\frac{1}{2}\log_e(x^2+6x+2)+C$

3. (a) 0.5 (b) 0.7 (c) 1.6 (d) 3. 1 (e) 0.5

4. $\log_e 3-\log_e 2=\log_e 1.5$ units^2 5. $\log_e 2$ units^2

6. $(0.5+\log_e 2)$ units^2 7. 0.61 units^2

8. $\pi\log_e 3$ units^3 9. $2\pi\log_e 9$ units^3

10. 47.2 units^2 11. $\frac{\pi}{2}e^2(e^4-1)$ units^3

12. (a)
$$\begin{aligned}\text{RHS} &= \frac{1}{x+3}+\frac{2}{x-3}\\ &= \frac{1(x-3)}{(x+3)(x-3)}\\ &\quad+\frac{2(x+3)}{(x+3)(x-3)}\\ &= \frac{x-3+2x+6}{x^2-9}\\ &= \frac{3x+3}{x^2-9}\\ &= \text{LHS}\end{aligned}$$
$$\therefore \frac{3x+3}{x^2-9}=\frac{1}{x+3}+\frac{2}{x-3}$$

(b) $\log_e(x+3)+2\log_e(x-3)+C$

13. (a) RHS $= 1 - \dfrac{5}{x-1}$

$= \dfrac{x-1}{x-1} - \dfrac{5}{x-1}$

$= \dfrac{x-6}{x-1}$

$=$ LHS

$\therefore \dfrac{x-6}{x-1} = 1 - \dfrac{5}{x-1}$

(b) $x - 5\log_e(x-1) + C$

14. $\dfrac{3^{2x-1}}{2\log_e 3} + C$ **15.** 1.86 units^2

Test yourself 4

1. (a) 6.39 (b) 1.98 (c) 3.26 (d) 1.40 (e) 0.792 (f) 3.91 (g) 5.72 (h) 72.4 (i) 6 (j) 2

2. (a) $5e^{5x}$ (b) $-2e^{1-x}$ (c) $\dfrac{1}{x}$ (d) $\dfrac{4}{4x+5}$ (e) $e^x(x+1)$

(f) $\dfrac{1-\ln x}{x^2}$ (g) $10e^x(e^x+1)^9$

3. (a) $\dfrac{1}{4}e^{4x} + C$ (b) $\dfrac{1}{2}\ln(x^2-9) + C$ (c) $-e^{-x} + C$

(d) $\ln(x+4) + C$

4. $3x - y + 3 = 0$ **5.** $-\dfrac{e^2}{e^2+1}$ **6.** $\dfrac{1}{2}e^4(e^6-1)$ units^2

7. $\dfrac{\pi}{6}e^6(e^6-1)$ units^3

8. (a) 0.92 (b) 1.08 (c) 0.2 (d) 1.36 (e) 0.64

9. $e(e^2-1)$ units^2

10. (a) 2.16 units^2 (b) $x = e^y$ (c) 2.16 units^2

11. (a) $x = 1.9$
(b) $x = 1.9$
(c) $x = 3$
(d) $x = 36$
(e) $t = 18.2$

12. (a) $\dfrac{3}{2}(e^2-1)$

(b) $\dfrac{1}{3}\ln 10$

(c) $8\dfrac{1}{6} + 3\ln 2$

13. $e^4x - y - 3e^4 = 0$

14. 0.9

15. (a) $e(e-1)$ units^2

(b) $\dfrac{\pi}{2}e^2(e^2-1)$ units^3

16. (a) $\log_a x^5y^3$

(b) $\log_x \dfrac{k^2p}{3}$

17. $2x + y - \ln 2 - 4 = 0$

18. (0, 0) point of inflexion, $(-3, -27e^{-3})$ minimum

19. 5.36 units^2

20. (a) 0.65
(b) 1.3

Challenge exercise 4

1. $\dfrac{(e^{2x}+x)\frac{1}{x} - (2e^{2x}+1)\log_e x}{(e^{2x}+x)^2}$ **2.** $2e$

3. (a) 2.8 (b) 1.8 (c) 2.6

4. $9\left(4e^{4x} + \dfrac{1}{x}\right)(e^{4x} + \log_e x)^8$ **5.** 0.42 units^2

6. $-\dfrac{2}{2x-3}$ **7.** 12 units^3 **8.** $5^x\log_e 5$

9. $\dfrac{d}{dx}(x^2\log_e x) = x(1 + 2\log_e x)$; $18\log_e 3$ **10.** $\dfrac{3^x}{\log_e 3} + C$

11. (a) (1, 0) (b) $x - y - 1 = 0$; $x - \log_e 10 \cdot y - 1 = 0$

(c) $\left(1 - \dfrac{1}{\log_e 10}\right)$ units **12.** 0.645 units^2

13. $\dfrac{e^x(1+\log_e x) - xe^x\log_e x}{e^{2x}}$

$= \dfrac{1 + \log_e x - x\log_e x}{e^x}$

14. $y = e^x + e^{-x}$

$\dfrac{dy}{dx} = e^x - e^{-x}$

$\dfrac{d^2y}{dx^2} = e^x - (-e^{-x})$

$= e^x + e^{-x}$

$= y$

15. $y = 3e^{5x} - 2$

$\dfrac{dy}{dx} = 15e^{5x}$

$\dfrac{d^2y}{dx^2} = 75e^{5x}$

LHS $= \dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} - 5y - 10$

$= 75e^{5x} - 4(15e^{5x}) - 5(3e^{5x} - 2) - 10$

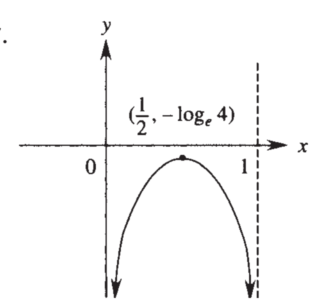
$= 75e^{5x} - 60e^{5x} - 15e^{5x} + 10 - 10$

$= 0$

$=$ RHS

16. $f(x) = 3e^{2x} - 6x$

17.



Chapter 5: Trigonometric functions

Exercises 5.1

1. (a) 36° (b) 120° (c) 225° (d) 210° (e) 540° (f) 140° (g) 240° (h) 420° (i) 20° (j) 50°

2. (a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{4\pi}{3}$ (e) $\frac{5\pi}{3}$ (f) $\frac{7\pi}{20}$ (g) $\frac{\pi}{12}$ (h) $\frac{5\pi}{2}$ (i) $\frac{5\pi}{4}$ (j) $\frac{2\pi}{3}$

3. (a) 0.98 (b) 1.19 (c) 1.78 (d) 1.54 (e) 0.88

4. (a) 0.32 (b) 0.61 (c) 1.78 (d) 1.54 (e) 0.88

5. (a) 62° 27′ (b) 44° 0′ (c) 66° 28′ (d) 56° 43′ (e) 18° 20′ (f) 183° 21′ (g) 154° 42′ (h) 246° 57′ (i) 320° 51′ (j) 6° 18′

6. (a) 0.34 (b) 0.07 (c) 0.06 (d) 0.83 (e) −1.14 (f) 0.33 (g) −1.50 (h) 0.06 (i) −0.73 (j) 0.16

Exercises 5.2

1.

	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
sin	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
cos	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
tan	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$
cosec	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2
sec	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$
cot	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

2. (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3\sqrt{3}}{8}$ (d) $\frac{4\sqrt{3}}{3}$ (e) 0 (f) $\frac{2\sqrt{3}+1}{2}$ (g) $\sqrt{2}-\sqrt{3}$ (h) $\frac{\sqrt{2}+2}{2}$ (i) $\frac{3-2\sqrt{2}}{2}$ (j) $\frac{2+\sqrt{3}}{2}$

3. (a) $1\frac{1}{4}$ (b) $\frac{\sqrt{6}-\sqrt{2}}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1 (e) $4\frac{1}{4}$

4. (a) $\frac{\sqrt{6}+\sqrt{2}}{4}$ (b) $\sqrt{3}-2$

5. $\text{LHS} = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{1}{2}\times\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$\text{RHS} = \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\times\frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

LHS = RHS

So $\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4} = \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

6. (a) $\frac{3\pi}{4} = \frac{4\pi}{4} - \frac{\pi}{4} = \pi - \frac{\pi}{4}$ (b) 2nd (c) $-\frac{1}{\sqrt{2}}$

7. (a) $\frac{5\pi}{6} = \frac{6\pi}{6} - \frac{\pi}{6} = \pi - \frac{\pi}{6}$ (b) 2nd (c) $\frac{1}{2}$

8. (a) $\frac{7\pi}{4} = \frac{8\pi}{4} - \frac{\pi}{4} = 2\pi - \frac{\pi}{4}$ (b) 4th (c) -1

9. (a) $\frac{4\pi}{3} = \frac{3\pi}{3} + \frac{\pi}{3} = \pi + \frac{\pi}{3}$ (b) 3rd (c) $-\frac{1}{2}$

10. (a) $\frac{5\pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = 2\pi - \frac{\pi}{3}$ (b) 4th (c) $-\frac{\sqrt{3}}{2}$

11. (a) -1 (b) $\frac{\sqrt{3}}{2}$ (c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{2}}$ (e) $\frac{1}{\sqrt{3}}$

12. (a) (i) $\frac{13\pi}{6} = \frac{12\pi}{6} + \frac{\pi}{6} = 2\pi + \frac{\pi}{6}$ (ii) 1st (iii) $\frac{\sqrt{3}}{2}$

(b) (i) $\frac{1}{\sqrt{2}}$ (ii) $\sqrt{3}$ (iii) $-\frac{1}{\sqrt{2}}$ (iv) $\frac{1}{\sqrt{3}}$ (v) $-\frac{\sqrt{3}}{2}$

13. (a) $\frac{\pi}{3}, \frac{5\pi}{3}$ (b) $\frac{5\pi}{4}, \frac{7\pi}{4}$ (c) $\frac{\pi}{4}, \frac{5\pi}{4}$ (d) $\frac{\pi}{3}, \frac{4\pi}{3}$ (e) $\frac{5\pi}{6}, \frac{7\pi}{6}$

14. (a) $\sin\theta$ (b) $-\tan x$ (c) $-\cos\alpha$ (d) $\sin x$ (e) $\cot\theta$

15. $\sin^2\theta$

16. $\cos x = -\frac{4}{5}$; $\sin x = \frac{3}{5}$

17. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Exercises 5.3

1. (a) 4π cm (b) π m (c) $\frac{25\pi}{3}$ cm (d) $\frac{\pi}{2}$ cm (e) $\frac{7\pi}{4}$ mm

2. (a) 0.65 m (b) 3.92 cm (c) 6.91 mm (d) 2.39 cm (e) 3.03 m

3. 1.8 m **4.** 7.5 m **5.** $\frac{2\pi}{21}$ **6.** 25 mm **7.** 1.83

8. $13\frac{7}{9}$ mm **9.** 25.3 mm **10.** $SA = \frac{175\pi}{36}$ cm², $V = \frac{125\sqrt{35}\,\pi}{648}$ cm³

Exercises 5.4

1. (a) 8π cm² (b) $\frac{3\pi}{2}$ m² (c) $\frac{125\pi}{3}$ cm² (d) $\frac{3\pi}{4}$ cm² (e) $\frac{49\pi}{8}$ mm²

2. (a) 0.48 m² (b) 6.29 cm² (c) 24.88 mm² (d) 7.05 cm² (e) 3.18 m²

3. 16.6 m² **4.** $\theta = 4\frac{4}{9}$ **5.** 6 m **6.** (a) $\frac{7\pi}{6}$ cm (b) $\frac{49\pi}{12}$ cm²

7. $\frac{6845}{8\pi}$ mm² **8.** 75 cm² **9.** 11.97 cm²

10. $\theta = \frac{\pi}{15}$, $r = 3$ cm

Exercises 5.5

1. (a) 8π cm² (b) $\frac{6\pi - 9\sqrt{3}}{4}$ m² (c) $\frac{125\pi - 75}{3}$ cm² (d) $\frac{3(\pi - 3)}{4}$ cm² (e) $\frac{49(\pi - 2\sqrt{2})}{8}$ mm²

2. (a) 0.01 m² (b) 1.45 cm² (c) 3.65 mm² (d) 0.19 cm² (e) 0.99 m²

3. 0.22 cm² **4.** (a) $\frac{3\pi}{7}$ cm (b) $\frac{9\pi}{14}$ cm² (c) 0.07 cm²

5. 134.4 cm **6.** (a) 2.6 cm (b) $\frac{5\pi}{6}$ cm (c) 0.29 cm²

7. (a) 10.5 mm (b) 4.3 mm²

8. (a) $\frac{25\pi}{4}$ cm² (b) 0.5 cm²

9. (a) 77° 22′ (b) 70.3 cm² (c) 26.96 cm² (d) 425.43 cm²

10. 9.4 cm²

11. (a) $\frac{11\pi}{9}$ cm (b) $22 - \frac{121\pi}{18} = \frac{396 - 121\pi}{18}$ cm² (c) $22 + \frac{11\pi}{9} = \frac{11(18 + \pi)}{9}$ cm

12. (a) 5 cm² (b) 0.3% (c) 15.6 cm

13. (a) 10π cm (b) 24π cm²

14. (a) $8 + \frac{20\pi}{9} = \frac{4(18 + 5\pi)}{9}$ cm (b) 3:7

15. (a) $\frac{225\pi}{2}$ cm³ (b) $\frac{105\pi}{2} + 180 = \frac{15(7\pi + 24)}{2}$ cm²

Exercises 5.6

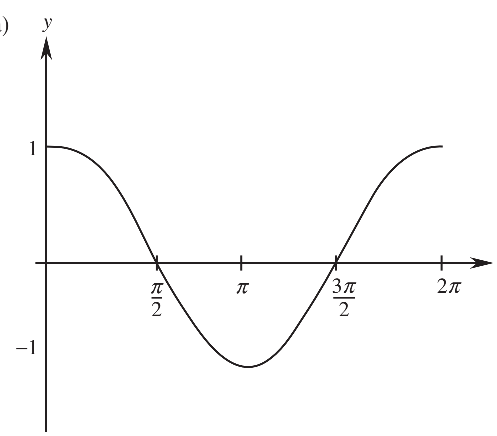
1. (a) 0.045 (b) 0.003 (c) 0.999 (d) 0.065 (e) 0.005

2. $\frac{1}{4}$ **3.** $\frac{1}{3}$

4. 1 343 622 km **5.** 7367 m

Exercises 5.7

1. (a)



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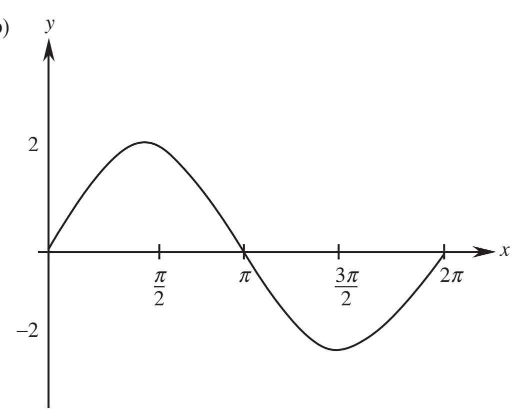

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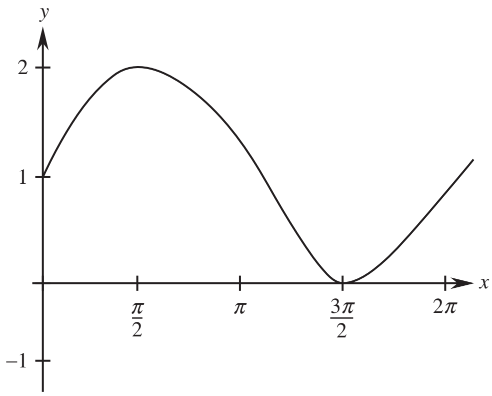
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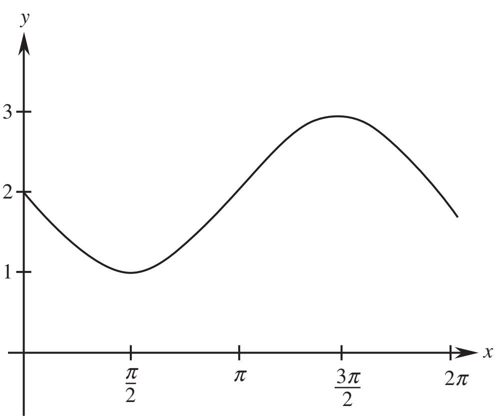
x
π/2
π
3π/2
2π

(d)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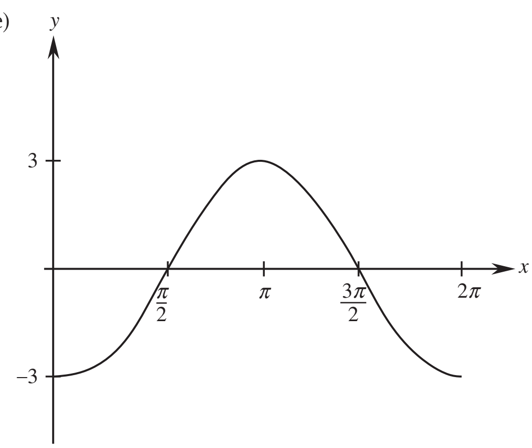
(e)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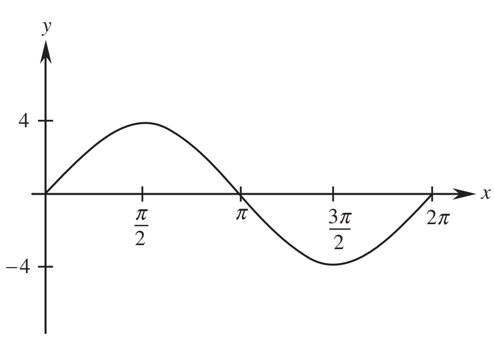
(f)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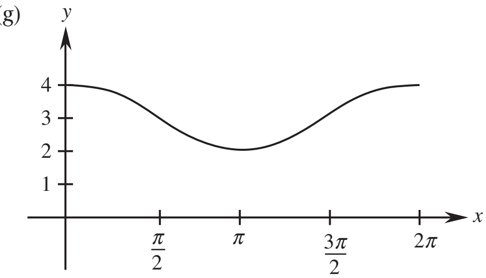
(g)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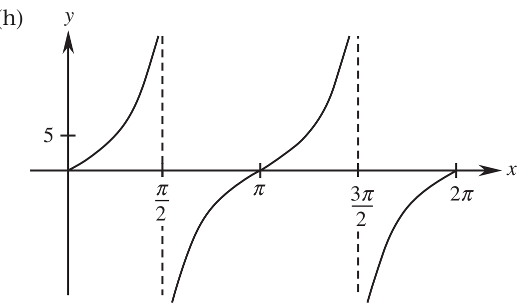
(h)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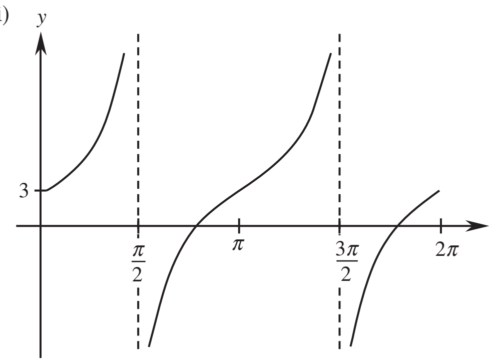
2π

(i)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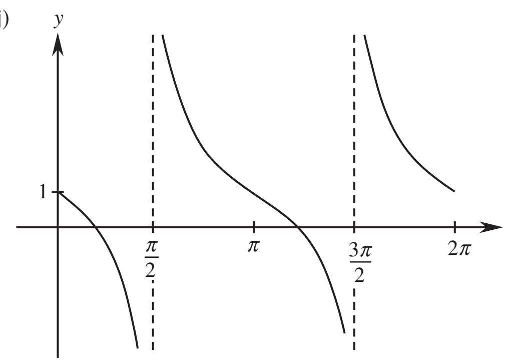
(j)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x
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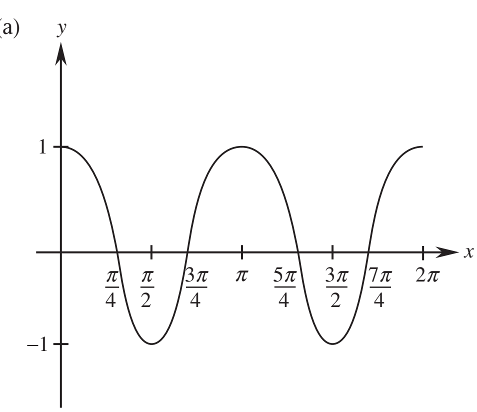
2π

2. (a)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x
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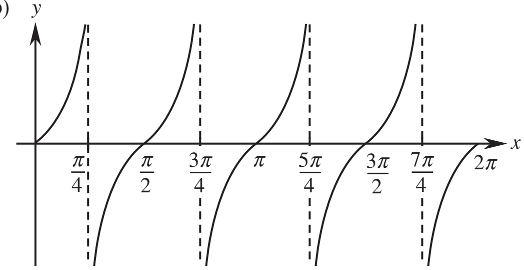
π
5π/4
3π/2
7π/4
2π

(b)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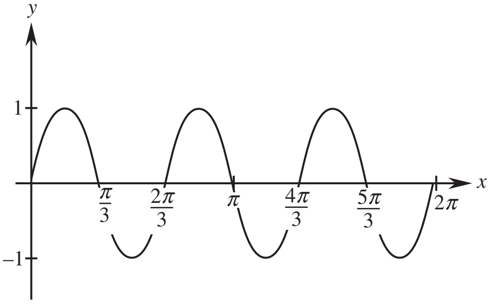
π
$\frac{5\pi}{4}$
$\frac{3\pi}{2}$
$\frac{7\pi}{4}$
2π

(c)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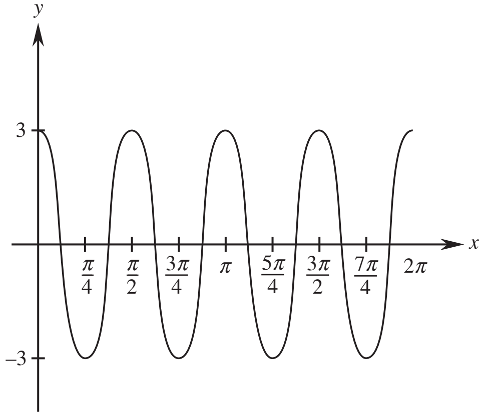
2π

(d)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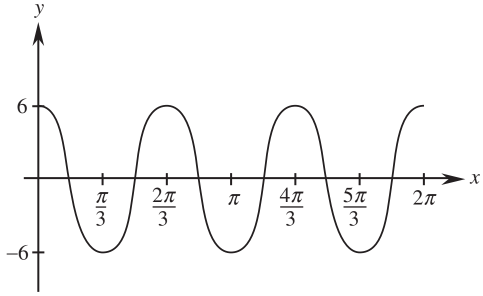
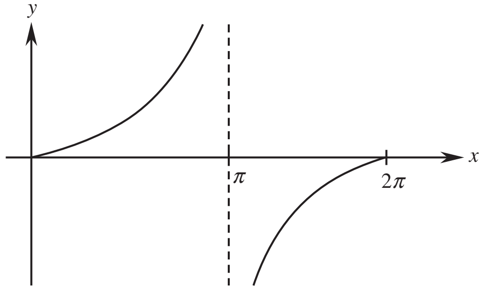
$\frac{7\pi}{4}$
2π

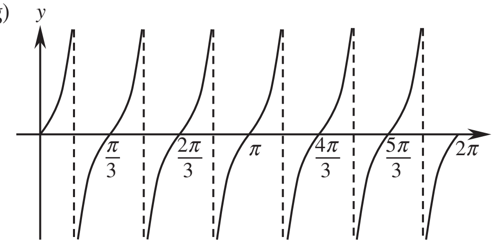
(e)


y
6
−6
x
$\frac{\pi}{3}$
$\frac{2\pi}{3}$
π
$\frac{4\pi}{3}$
$\frac{5\pi}{3}$
2π

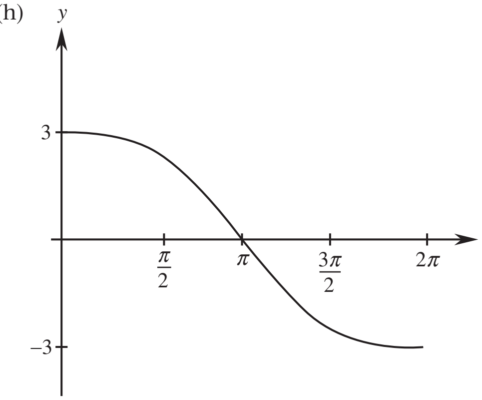
(f)


y
x
π
2π

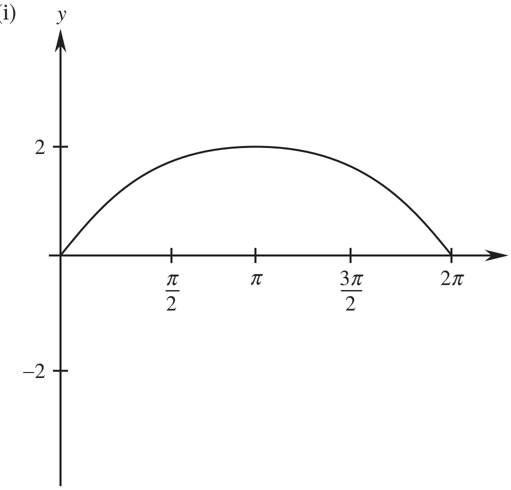
(g)


y
x
$\frac{\pi}{3}$
$\frac{2\pi}{3}$
π
$\frac{4\pi}{3}$
$\frac{5\pi}{3}$
2π

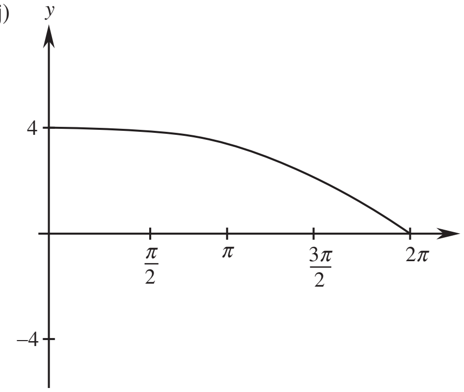
(h)


y
3
−3
x
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π

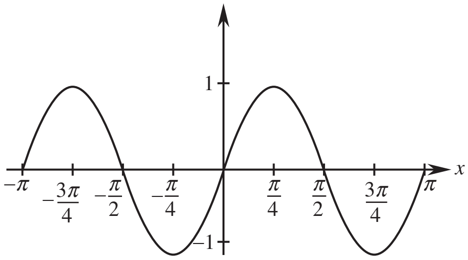
(i)


y
2
−2
x
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π

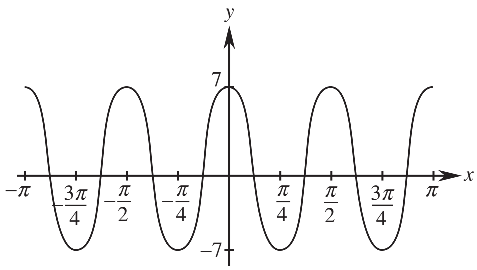
(j)


y
4
−4
x
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π

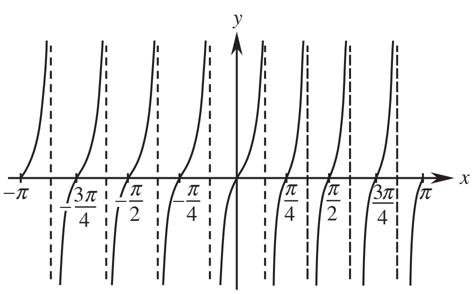
3. (a)



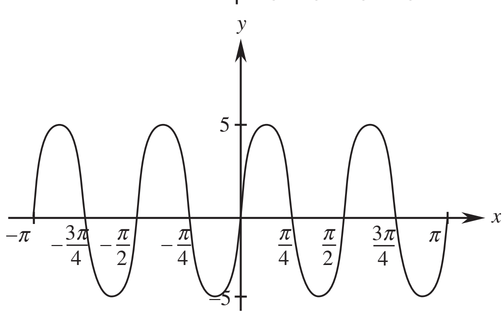
(b)



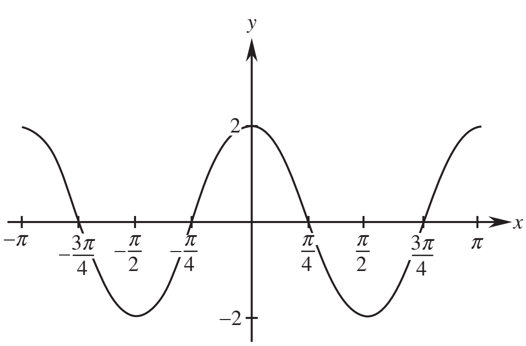
(c)



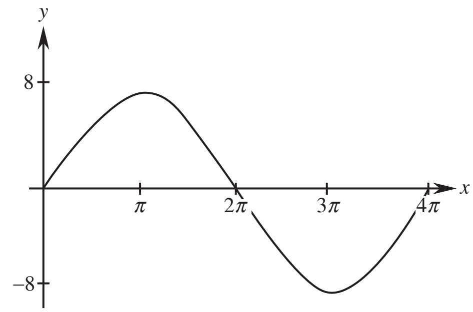
(d)



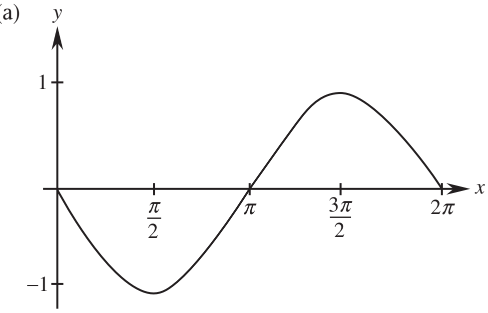
(e)



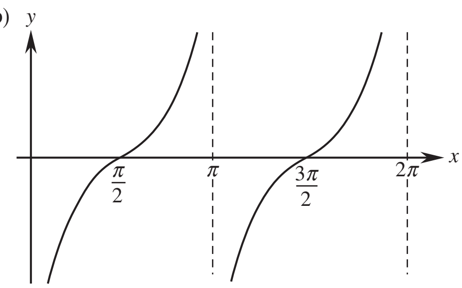
4.



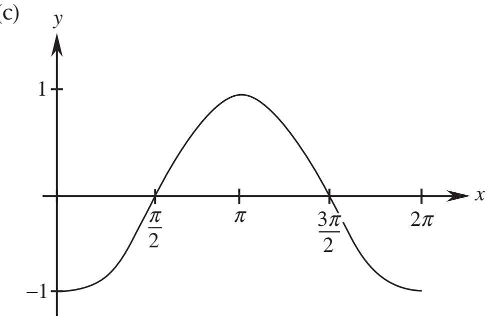
5. (a)



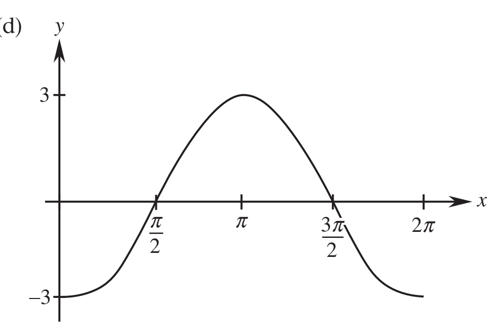
(b)



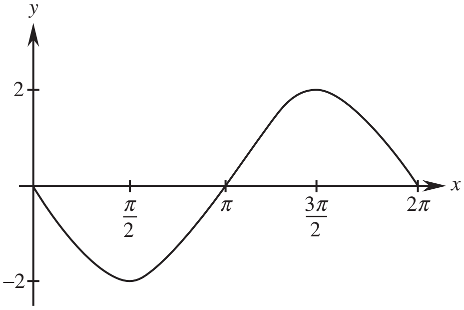
(c)



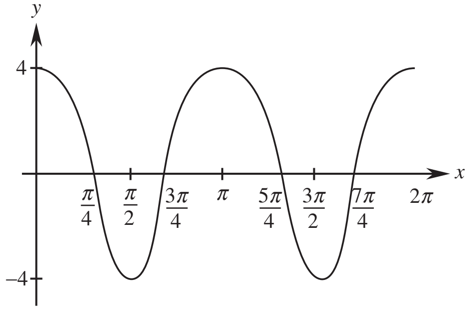
(d)



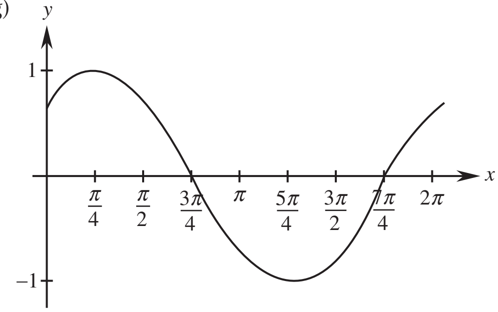
(e)



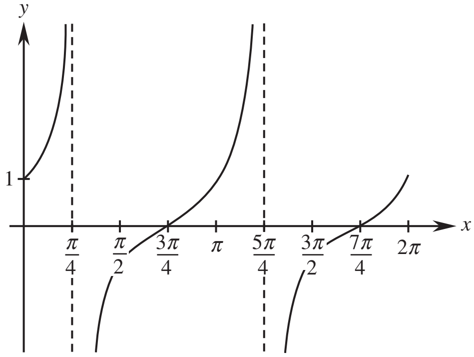
(f)



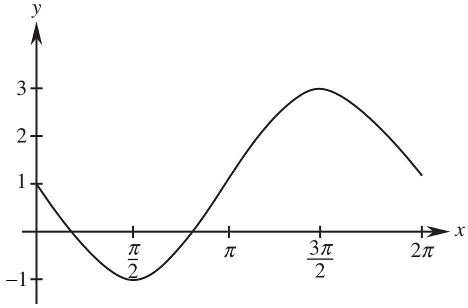
(g)



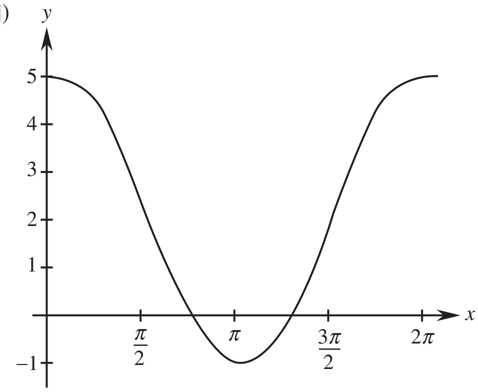
(h)



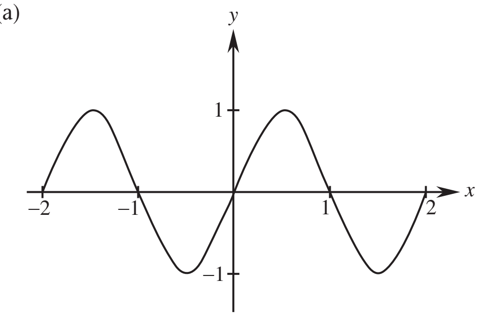
(i)



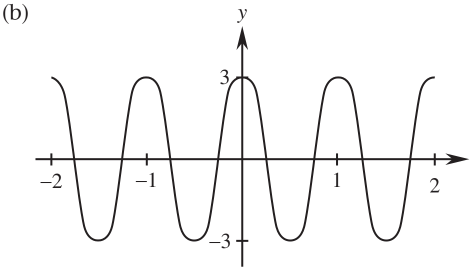
(j)



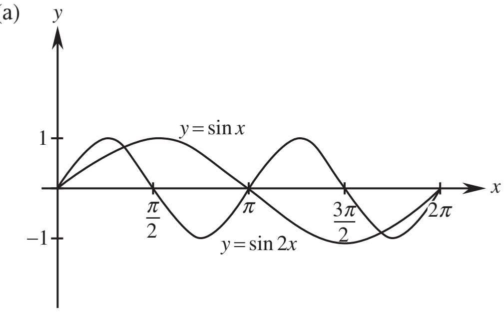
6. (a)



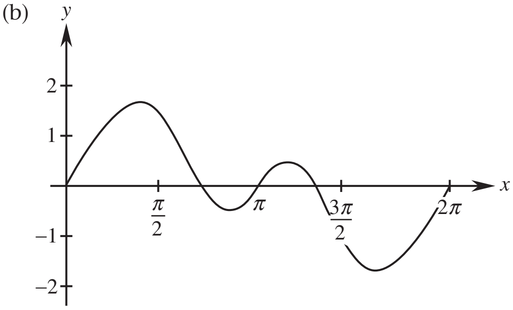
(b)



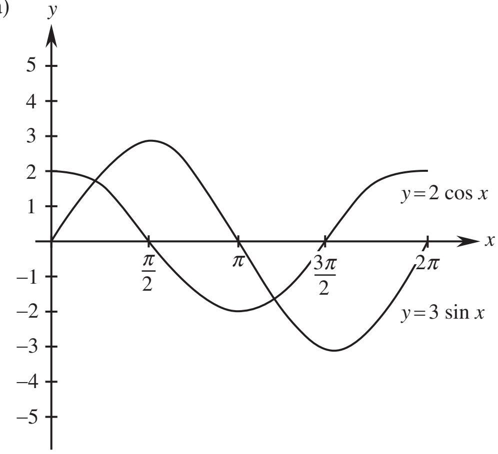
7. (a)



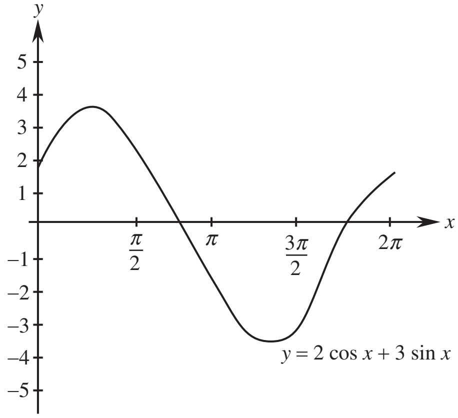
(b)



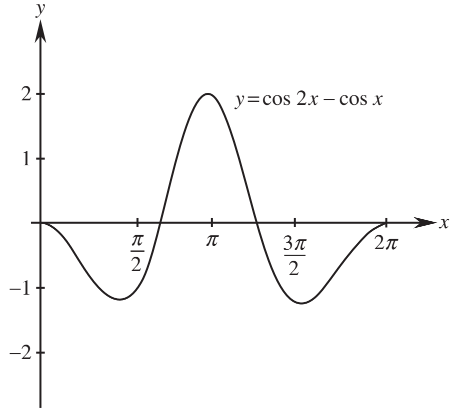
8. (a)


y
5
4
3
2
1
−1
−2
−3
−4
−5
x
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$y = 2\cos x$
$y = 3\sin x$

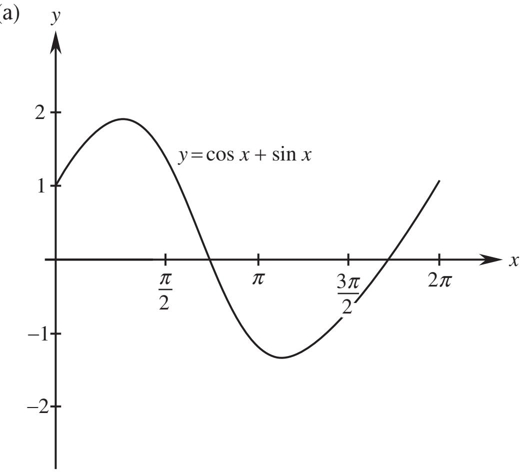
(b)


y
5
4
3
2
1
−1
−2
−3
−4
−5
x
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$y = 2\cos x + 3\sin x$

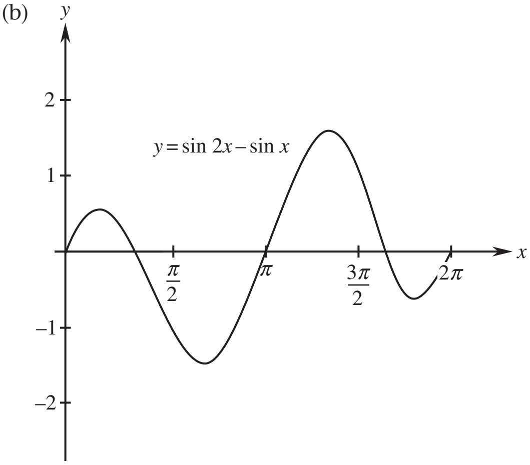
9.


y
2
1
−1
−2
x
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$y = \cos 2x - \cos x$

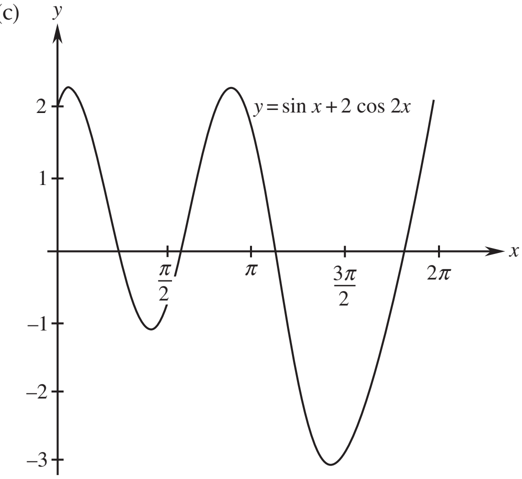
10. (a)


y
2
1
−1
−2
x
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$y = \cos x + \sin x$

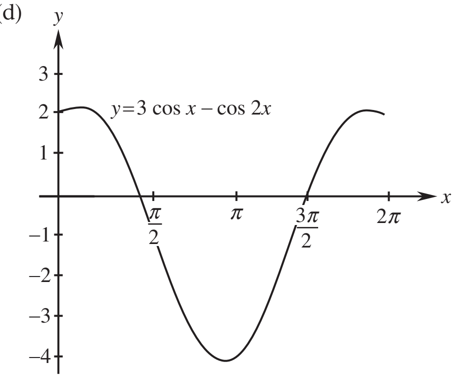
(b)


y
2
1
−1
−2
x
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$y = \sin 2x - \sin x$

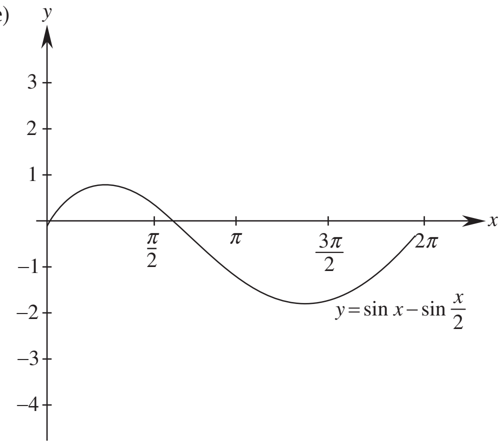
(c)


y
2
1
−1
−2
−3
x
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$y = \sin x + 2\cos 2x$

(d)

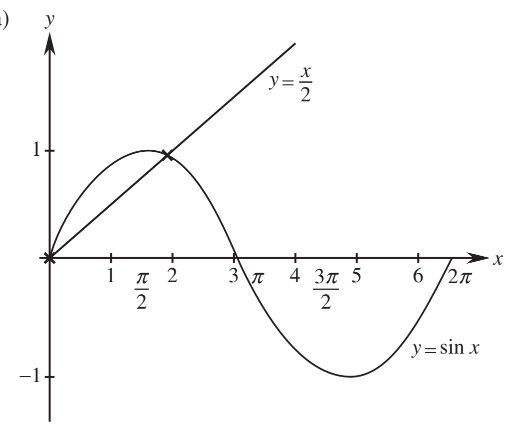

y
3
2
1
−1
−2
−3
−4
x
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$y = 3\cos x - \cos 2x$

(e)



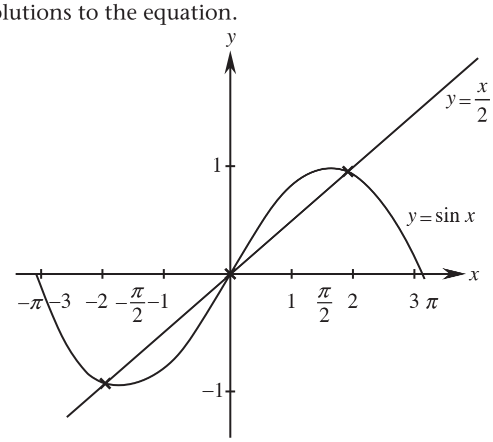
Exercises 5.8

1. (a)



There are 2 points of intersection, so there are 2 solutions to the equation.

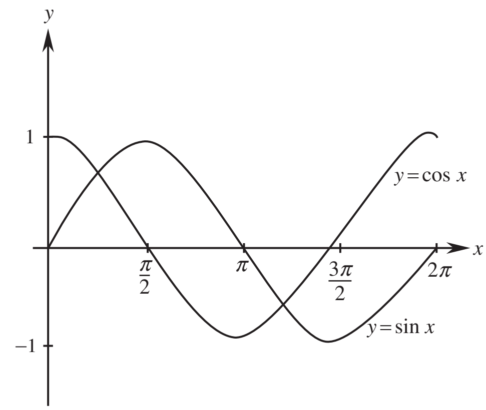
(b)



There are 3 points of intersection, so there are 3 solutions to the equation.

2. $x = 0$ **3.** $x = 1.5$ **4.** $x = 0, 4.5$ **5.** $x = 0, 1$

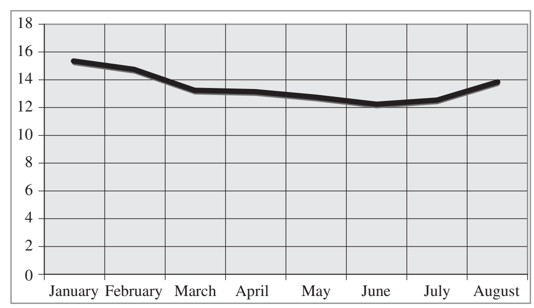
6. $x = 0.8, 4$



7. (a) Period 12 months, amplitude 1.5 (b) 5.30 p.m.

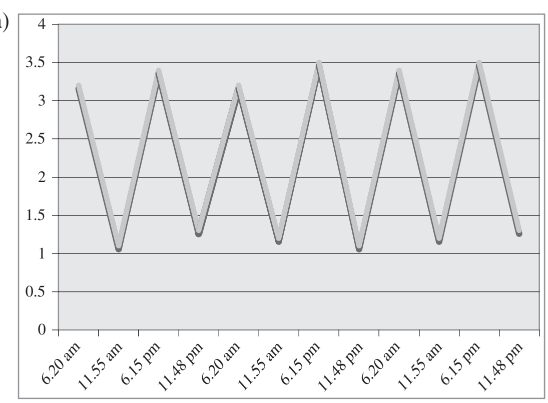
8. (a) 1300 (b) (i) 1600 (ii) 1010
(c) Amplitude 300, period 10 years

9. (a)



(b) It may be periodic – hard to tell from this data. Period would be about 10 months.
(c) Amplitude is 1.5

10. (a)



(b) Period 24 hours, amplitude 1.25 (c) 2.5 m

Exercises 5.9

1. (a) $4 \cos 4x$ (b) $-3 \sin 3x$ (c) $5 \sec^2 5x$
(d) $3 \sec^2 (3x + 1)$ (e) $\sin (-x)$ (f) $3 \cos x$
(g) $-20 \sin (5x - 3)$ (h) $-6x^2 \sin (x^3)$
(i) $14x \sec^2 (x^2 + 5)$ (j) $3 \cos 3x - 8 \sin 8x$

(k) $\sec^2(\pi + x) + 2x$ (l) $x\sec^2 x + \tan x$

(m) $3\sin 2x \sec^2 3x + 2\tan 3x \cos 2x$

(n) $\dfrac{2x\cos x - 2\sin x}{4x^2} = \dfrac{x\cos x - \sin x}{2x^2}$

(o) $\dfrac{3\sin 5x - 5(3x+4)\cos 5x}{\sin^2 5x}$

(p) $9(2 + 7\sec^2 7x)(2x + \tan 7x)^8$

(q) $2\sin x\cos x$ (r) $-45\sin 5x\cos^2 5x$

(s) $e^x + 2\sin 2x$ (t) $-\dfrac{1}{x}\cos(1 - \log_e x)$

(u) $(e^x + 1)\cos(e^x + x)$ (v) $\dfrac{\cos x}{\sin x} = \cot x$

(w) $-2e^{3x}\sin 2x + 3e^{3x}\cos 2x$
$= e^{3x}(3\cos 2x - 2\sin 2x)$

(x) $\dfrac{2e^{2x}\tan 7x - 7e^{2x}\sec^2 7x}{\tan^2 7x}$
$= \dfrac{e^{2x}(2\tan 7x - 7\sec^2 7x)}{\tan^2 7x}$

2. $4\cos^2 x\sin^3 x - \sin^5 x$
$= \sin^3 x(4\cos^2 x - \sin^2 x)$

3. 12 **4.** $6\sqrt{3}x - 12y + 6 - \pi\sqrt{3} = 0$

5. $\dfrac{-\sin x}{\cos x} = -\tan x$ **6.** $-\dfrac{2}{3\sqrt{3}} = -\dfrac{2\sqrt{3}}{9}$

7. $\sec^2 x e^{\tan x}$ **8.** $8\sqrt{2}x + 48y - 72\sqrt{2} - \pi\sqrt{2} = 0$

9.
$$\begin{aligned} y &= 2\cos 5x \\ \frac{dy}{dx} &= -10\sin 5x \\ \frac{d^2y}{dx^2} &= -50\cos 5x \\ &= -25(2\cos 5x) \\ &= -25y \end{aligned}$$

10.
$$\begin{aligned} f(x) &= -2\sin x \\ f'(x) &= -2\cos x \\ f''(x) &= 2\sin x \\ &= -f(x) \end{aligned}$$

11.
$$\begin{aligned} \text{LHS} &= \frac{d}{dx}[\log_e(\tan x)] \\ &= \frac{\sec^2 x}{\tan x} \\ &= \frac{\tan^2 x + 1}{\tan x} \\ &= \frac{\tan^2 x}{\tan x} + \frac{1}{\tan x} \\ &= \tan x + \cot x \\ &= \text{RHS} \end{aligned}$$

$\therefore \dfrac{d}{dx}[\log_e(\tan x)] = \tan x + \cot x$

12. $\left(\dfrac{\pi}{3}, \sqrt{3} - \dfrac{\pi}{3}\right)$ maximum,
$\left(\dfrac{5\pi}{3}, -\sqrt{3} - \dfrac{5\pi}{3}\right)$ minimum

13. (a) $\dfrac{\pi}{180}\sec^2 x°$ (b) $\dfrac{-\pi}{60}\sin x°$ (c) $\dfrac{\pi}{900}\cos x°$

14.
$$\begin{aligned} y &= 2\sin 3x - 5\cos 3x \\ \frac{dy}{dx} &= 6\cos 3x + 15\sin 3x \\ \frac{d^2y}{dx^2} &= -18\sin 3x + 45\cos 3x \\ &= -9(2\sin 3x - 5\cos 3x) \\ &= -9y \end{aligned}$$

15. $a = -7$, $b = -24$

Exercises 5.10

1. (a) $\sin x + C$ (b) $-\cos x + C$ (c) $\tan x + C$
(d) $\dfrac{-45}{\pi}\cos x° + C$ (e) $-\dfrac{1}{3}\cos 3x + C$ (f) $\dfrac{1}{7}\cos 7x + C$
(g) $\dfrac{1}{5}\tan 5x + C$ (h) $\sin(x + 1) + C$
(i) $-\dfrac{1}{2}\cos(2x - 3) + C$ (j) $\dfrac{1}{2}\sin(2x - 1) + C$
(k) $\cos(\pi - x) + C$ (l) $\sin(x + \pi) + C$
(m) $\dfrac{2}{7}\tan 7x + C$ (n) $-8\cos\dfrac{x}{2} + C$
(o) $9\tan\dfrac{x}{3} + C$ (p) $-\cos(3 - x) + C$

2. (a) 1 (b) $\sqrt{3} - \dfrac{1}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$ (c) $\dfrac{2}{\sqrt{2}} = \sqrt{2}$ (d) $-\dfrac{1}{3}$
(e) $\dfrac{1}{\pi}$ (f) $\dfrac{1}{2}$ (g) $\dfrac{3}{4}$ (h) $-\dfrac{1}{5}$

3. 4 units^2 **4.** $\dfrac{1}{3\sqrt{2}} = \dfrac{\sqrt{2}}{6}$ units^2 **5.** 0.86 units^2

6. 0.51 units^3 **7.** $\dfrac{\pi}{4}$ units^3

8. $\sqrt{3} - \dfrac{\pi}{3} = \dfrac{3\sqrt{3} - \pi}{3}$ units^2 **9.** $2\sqrt{2}$ units^2

10. (a)
$$\begin{aligned} V &= \pi\int_a^b y^2\,dx \\ &= \pi\int_0^{\frac{\pi}{2}}\cos x\,dx \\ &= \pi\Big[\sin x\Big]_0^{\frac{\pi}{2}} \\ &= \pi\left(\sin\frac{\pi}{2} - \sin 0\right) \\ &= \pi(1 - 0) \\ &= \pi \text{ units}^3 \end{aligned}$$

(b) 3.1 units^3

11. $y = -2\sin 3x$

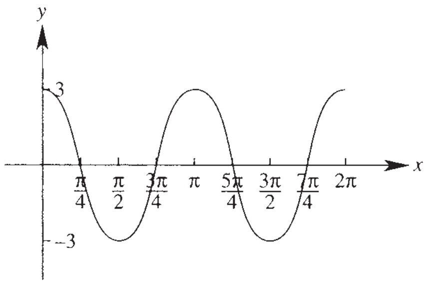
Test yourself 5

1. (a) $\frac{5\pi}{6}$ cm (b) $\frac{25\pi}{12}$ cm^2 (c) 0.295 cm^2

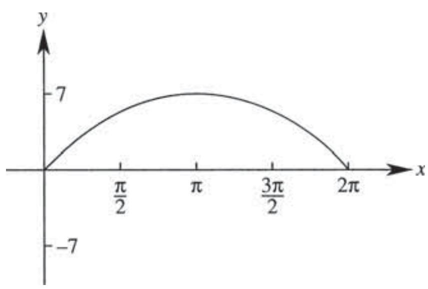
2. (a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $-\frac{1}{\sqrt{2}}$

3. (a) $x = \frac{3\pi}{4}, \frac{7\pi}{4}$ (b) $x = \frac{\pi}{6}, \frac{5\pi}{6}$

4. (a)



(b)



5. (a) $-\sin x$ (b) $2\cos x$ (c) $\sec^2 x$ (d) $x\cos x + \sin x$

(e) $\frac{x\sec^2 x - \tan x}{x^2}$ (f) $-3\sin 3x$ (g) $5\sec^2 5x$

6. (a) $-\frac{1}{2}\cos 2x + C$ (b) $3\sin x + C$ (c) $\frac{1}{5}\tan 5x + C$

(d) $x - \cos x + C$

7. (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{2\sqrt{3}}{3}$

8. $3x + \sqrt{2}y - 1 - \frac{3\pi}{4} = 0$

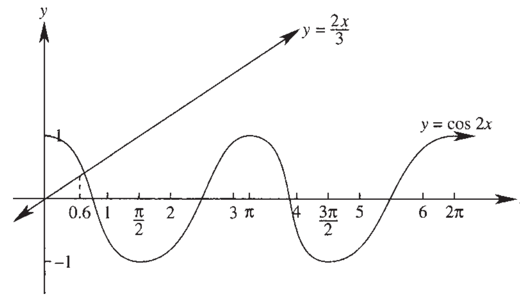
9.

$$x = \cos 2t$$
$$\frac{dx}{dt} = -2\sin 2t$$
$$\frac{d^2x}{dt^2} = -4\cos 2t$$
$$= -4x$$

10. $\frac{1}{\sqrt{2}}$ units^2 **11.** $\frac{\pi}{\sqrt{3}}$ units^3 **12.** (a) 5 (b) 2

13. $-3\sqrt{3}$ **14.** (a) $\frac{8\pi}{7}$ cm^2 (b) 0.12 cm^2

15. (a)

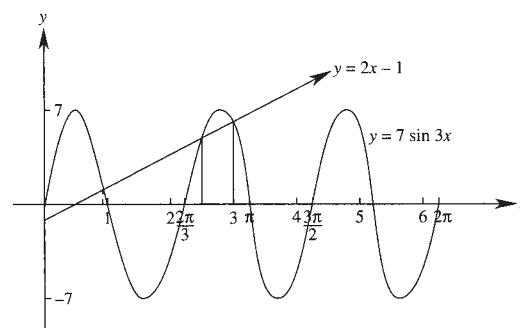


(b) $x = 0.6$

16. $\frac{\sqrt{3} - \sqrt{2}}{2}$ units^2

17. 2 units^2 **18.** $4x + 8y - 8 - \pi = 0$ **19.** $y = -3\cos 2x$

20. (a)



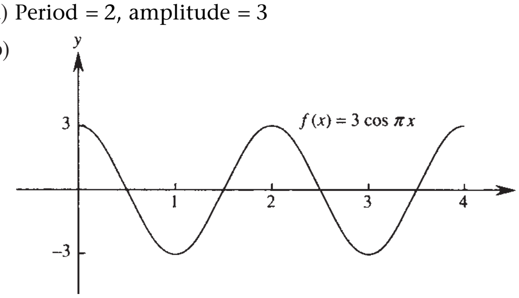
(b) $x = 0.9, 2.3, 3$

Challenge exercise 5

1. 0.27 **2.** $\frac{1}{2}\left(1 - \frac{1}{\sqrt{3}}\right) = \frac{3 - \sqrt{3}}{6}$ **3.** $r = 64$ units, $\theta = \frac{\pi}{512}$

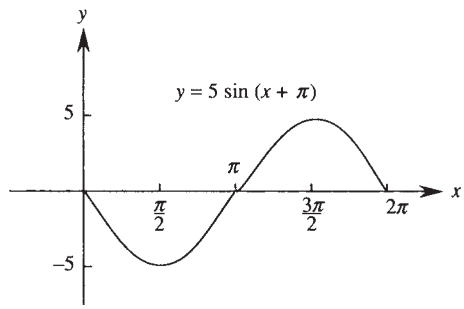
4. (a) Period = 2, amplitude = 3

(b)



5. (a) $y = -\sin 3x$

(b) LHS $= \frac{d^2y}{dx^2} + 9y$
$= 9\sin 3x + 9(-\sin 3x)$
$= 9\sin 3x - 9\sin 3x$
$= 0$
$=$ RHS

6. 

7. $\dfrac{\pi}{180}\sec^2 x°$

8. (a)

$$\begin{aligned} \text{RHS} &= \frac{\sec^2 x}{\tan x} \\ &= \frac{\dfrac{1}{\cos^2 x}}{\dfrac{\sin x}{\cos x}} \\ &= \frac{1}{\cos^2 x} \times \frac{\cos x}{\sin x} \\ &= \frac{1}{\cos x} \times \frac{1}{\sin x} \\ &= \sec x \operatorname{cosec} x \\ &= \text{LHS} \end{aligned}$$

$\therefore \sec x \operatorname{cosec} x = \dfrac{\sec^2 x}{\tan x}$

(b) $\log_e \sqrt{3} = \dfrac{1}{2}\log_e 3$

9. $(2x\cos 2x + \sin 2x)e^{x\sin 2x}$ **10.** (a) $\left(\dfrac{\pi}{4}, 4\right)$ and $\left(\dfrac{3\pi}{4}, 2\right)$

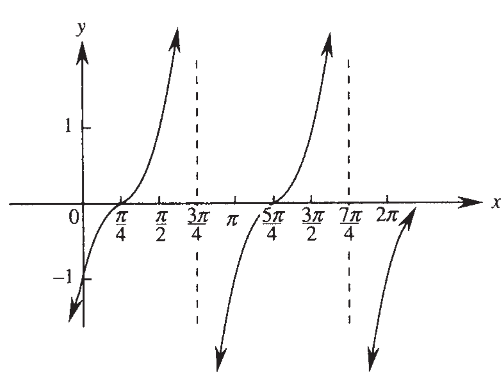
(b) Maximum = 4 (c) Amplitude = 1

11. $8\left(\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2}\right)\text{cm}^2 = \dfrac{4(2\pi - 3\sqrt{3})}{3}\ \text{cm}^2$

12. $-\dfrac{180}{\pi}\cos x° + C$ **13.** $-\dfrac{1}{2}$

14. 0.204 units³ **15.** $\dfrac{\cos x - \sin x}{\sin x + \cos x}$

16. $\dfrac{9}{2}\left(\dfrac{\pi}{2} - 1\right) = \dfrac{9(\pi - 2)}{4}\ \text{cm}^2$

17. 

18. $\left(\dfrac{\pi}{8}, 0\right), \left(\dfrac{5\pi}{8}, 0\right), \left(\dfrac{9\pi}{8}, 0\right), \left(\dfrac{13\pi}{8}, 0\right)$

19. $\dfrac{1}{\sqrt{2}} - \dfrac{1}{2} = \dfrac{\sqrt{2}-1}{2}$ units²

20.
$$\begin{aligned} f(x) &= 2\cos 3x \\ f'(x) &= -6\sin 3x \\ f''(x) &= -18\cos 3x \\ &= -9(2\cos 3x) \\ &= -9f(x) \end{aligned}$$

Chapter 6: Applications of calculus to the physical world

Exercises 6.1

1. (a) $R = 20 - 8t$ (b) $R = 15t^2 + 4t$ (c) $R = 16 - 4x$
(d) $R = 15t^4 - 4t^3 + 2$ (e) $R = e^t$ (f) $R = -15\sin 5\theta$
(g) $R = 2\pi - \dfrac{100}{r^3}$ (h) $R = \dfrac{x}{\sqrt{x^2 - 4}}$ (i) $R = 800 - \dfrac{400}{r^2}$
(j) $R = 4\pi r^2$

2. (a) $h = 2t^2 - 4t^3 + C$ (b) $A = 2x^4 + x + C$
(c) $V = \dfrac{4}{3}\pi r^3 + C$ (d) $d = -7\cos t + C$
(e) $s = 4e^{2t} - 3t + C$

3. 20 **4.** 1 **5.** $6e^{12}$ **6.** 13 **7.** 900 **8.** $2e^3 + 5$

9. $y = x^3 - x^2 + x + 6$ **10.** $R = \dfrac{dM}{dt} = 1 - 4t$; $R = -19$
[i.e. melting at the rate of 19 g per minute (g min⁻¹)]

11. −11 079.25 cm per second (cms^{-1}) **12.** 21 000 L

13. 165 cm² per second (cm^2s^{-1}) **14.** −0.25

15. 41 cm² per minute ($\text{cm}^2\ \text{min}^{-1}$) **16.** 31 cm³

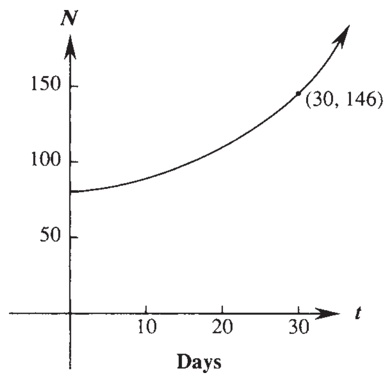
17. 108 731 people per year **18.** (a) 27 g
(b) −2.7 (i.e. decaying at a rate of 2.7 g per year)

19.
$$\begin{aligned} y &= e^{4x} \\ \frac{dy}{dx} &= 4e^{4x} \\ &= 4y \end{aligned}$$

20.
$$\begin{aligned} S &= 2e^{2t} + 3 \\ \frac{dS}{dt} &= 2(2e^{2t}) \\ &= 2(2e^{2t} + 3 - 3) \\ &= 2(S - 3) \end{aligned}$$

Exercises 6.2

1. (a) 80 (b) 146 (c) 92 days
(d)



2. (a) 99 061 (b) 7 hours

3. (a) When $t = 0$, $M = 100$

$$\begin{aligned} \therefore \quad M &= 100e^{-kt} \\ \text{When} \quad t &= 5,\ M = 95 \\ \therefore \quad 95 &= 100e^{-5k} \\ 0.95 &= e^{-5k} \\ \ln 0.95 &= -5k \\ 0.01 &= k \\ \text{So} \quad M &= 100e^{-0.01t} \end{aligned}$$

(b) 90.25 kg
(c) 67.6 years

4. (a) 35.6 L (b) 26.7 minutes

5. (a) $P_0 = 5000$ (b) $k = 0.157$ (c) 12 800 units
(d) 8.8 years

6. 2.3 million m^2 **7.** (a) $P = 50\,000e^{0.069t}$ (b) 70 599
(c) 4871 people per year (d) 2040

8. (a) 65.61° C (b) 1 hour 44 minutes

9. (a) 92 kg (b) Reducing at the rate of 5.6 kg per hour
(c) 18 hours

10. (a) $M_0 = 200$; $k = 0.00253$ (b) 192.5 g
(c) Reducing by 0.49 g per year (d) 273.8 years

11. (a) $B = 15\,000e^{0.073t}$ (b) 36 008 (c) 79.6 hours

12. 11.4 years **13.** (a) 19% (b) 3200 years

14. (a)

$$\begin{aligned} P(t) &= P(t_0)e^{-kt} \\ \frac{dP(t)}{dt} &= -kP(t_0)e^{-kt} \\ &= -kP(t) \end{aligned}$$

(b) 23% (c) 2% decline per year (d) 8.5 years

15. 12.6 minutes **16.** 12.8 years

17. (a) 76.8 mg/dL (b) 9 hours **18.** 15.8 s **19.** 8.5 years

20. (a)

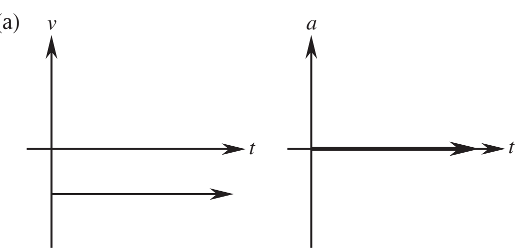
$$\begin{aligned} Q &= Ae^{kt} \\ \frac{dQ}{dt} &= kAe^{kt} \\ &= kQ \end{aligned}$$

(b)

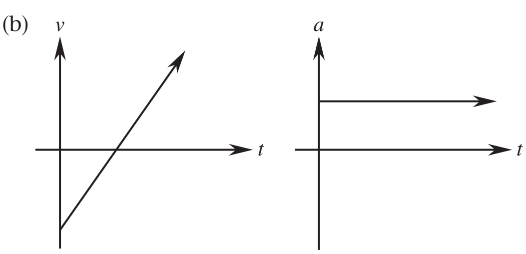
$$\begin{aligned} \frac{dQ}{dt} &= kQ \\ \text{So } \frac{dt}{dQ} &= \frac{1}{kQ} \\ t &= \int \frac{1}{kQ}\, dQ \\ &= \frac{1}{k}\int \frac{1}{Q}\, dQ \\ &= \frac{1}{k}\ln Q + C \\ kt &= \ln Q + C_1 \\ kt - C_1 &= \ln Q \\ e^{kt - C_1} &= Q \\ e^{kt} \times e^{-C_1} &= Q \\ Ae^{kt} &= Q \end{aligned}$$

Exercises 6.3

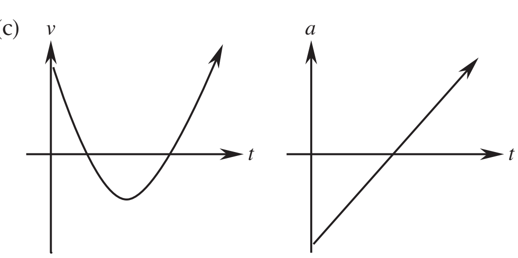
1. (a)



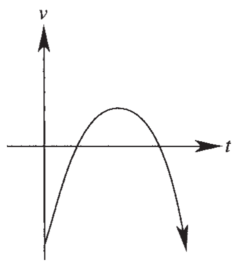
(b)

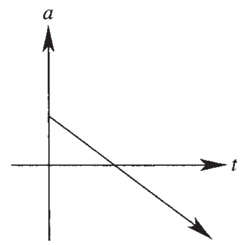


(c)

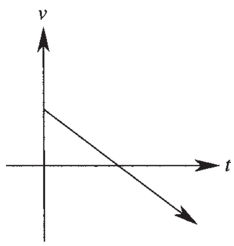


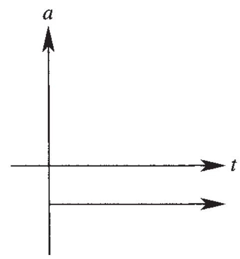
(d)





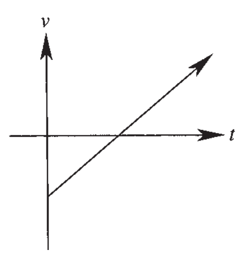
(e)



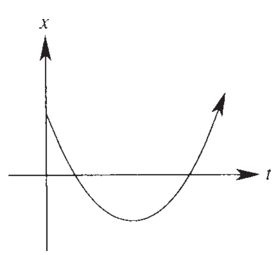


2. (a) t_2, t_4, t_6
(b) 0 to t_1, t_3, t_5
(c) 0 to t_1
(d) t_5

3. (a)



(b)



4. (a) O, t_2, t_4, t_6 (b) t_1, t_3, t_5 (c) t_5
(d) (i) At rest, accelerating to the left.
(ii) Moving to the left with zero acceleration.

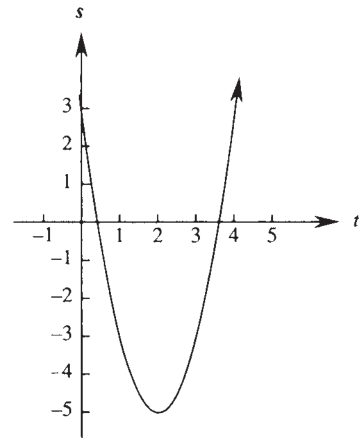
5. (a) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \ldots$ (b) $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

6. (a) At the origin, with positive velocity and positive constant acceleration (moving to the right and speeding up).
(b) To the right of the origin, at rest with negative constant acceleration.
(c) To the left of the origin, with negative velocity and positive acceleration (moving to the left and slowing down).
(d) To the right of the origin, with negative velocity and acceleration (moving to the left and speeding up).
(e) To the left of the origin, at rest with positive acceleration.

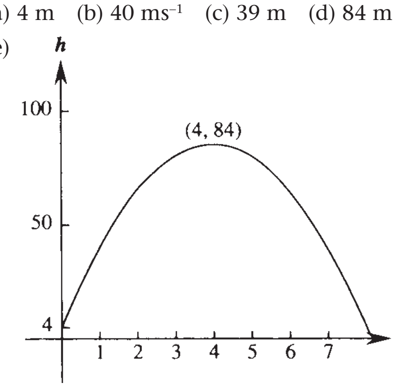
Exercises 6.4

1. (a) 18 cms^{-1} (b) 12 cms^{-2} (c) When $t = 0$, $x = 0$; after 3 s
(d) After 5 s

2. (a) -8 ms^{-1} (b) $a = 4$; constant acceleration of 4 ms^{-2}
(c) 13 m (d) after 2 s (e) -5 m
(f)

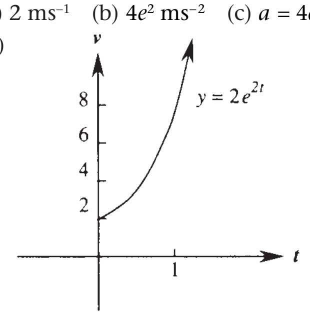


3. (a) 4 m (b) 40 ms^{-1} (c) 39 m (d) 84 m
(e)



4. (a) 2 cm (b) After 1 s (c) -4 cm (d) 6 cm
(e) -7 cms^{-1}

5. (a) 2 ms^{-1} (b) $4e^2$ ms^{-2} (c) $a = 4e^{2t} = 2(2e^{2t}) = 2v$
(d)



6. (a) $v = -2\sin 2t$ (b) $a = -4\cos 2t$ (c) 1 cm
(d) $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$ s (e) ± 1 cm (f) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \ldots$ s
(g) $a = -4\cos 2t = -4x$

7. (a) $v = 3t^2 + 12t - 2$; $a = 6t + 12$ (b) 266 m (c) 133 ms^{-1}
(d) 42 ms^{-1}

8. (a) $\dot{x} = 20(4t - 3)^4$, $\ddot{x} = 320(4t - 3)^3$
(b) $x = 1$, cm, $\dot{x} = 20$ cms^{-1}, $\ddot{x} = 320$ cms^{-2}
(c) The particle is on the RHS of the origin, travelling to the right and accelerating.

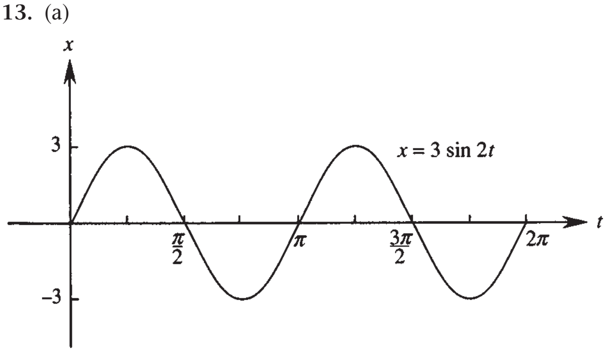
9. (a) $v = 5 - 10t$ (b) -95 ms^{-1} (c) $a = -10 = g$

10. $v = \dfrac{17}{(3t+1)^2}$, $a = \dfrac{-102}{(3t+1)^3}$

11. (a) At the origin (b) $\dfrac{1}{6}$ cms^{-1} (c) $-\dfrac{1}{36}$ cms^{-2}
(d) The particle is moving to the right but decelerating
(e) $(e^3 - 1)$ s

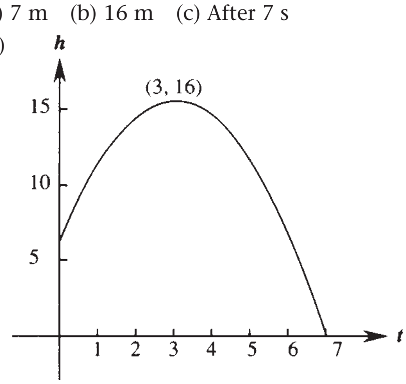
12. (a) 3 ms^{-1} (b) When $t = 0$ s, 1 s, 3 s (c) 10 ms^{-2}

13. (a)



(b) $\dot{x} = 6\cos 2t$, $\ddot{x} = -12\sin 2t$
(c) $-6\sqrt{3}$ cms^{-2}
(d) $\ddot{x} = -12\sin 2t = -4(3\sin 2t) = -4x$

14. (a) 7 m (b) 16 m (c) After 7 s
(d)



(e) 10 m

15. (a) 18.75 m (b) -15 ms^{-1} (c) 5 s

16. (a) At the origin: $x = 0$

$$2t^3 - 3t^2 + 42t = 0$$
$$t(2t^2 - 3t + 42) = 0$$
$$t = 0,\ 2t^2 - 3t + 42 = 0$$

Since $t = 0$, the particle is initially at the origin.

$$2t^2 - 3t + 42 = 0$$
$$\begin{aligned} b^2 - 4ac &= (-3)^2 - 4(2)(42) \\ &= -327 \\ &< 0 \end{aligned}$$

So the quadratic equation has no real roots.
So the particle is never again at the origin.

(b) $\dfrac{dx}{dt} = 6t^2 - 6t + 42$

At rest: $\dfrac{dx}{dt} = 0$

$$\begin{aligned} 6t^2 - 6t + 42 &= 0 \\ t^2 - t + 7 &= 0 \\ b^2 - 4ac &= (-1)^2 - 4(1)(7) \\ &= -27 \\ &< 0 \end{aligned}$$

So the quadratic equation has no real roots.
So the particle is never at rest.

17. (a) 0 cm (at the origin) (b) $\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \ldots$ s (c) ± 12 cm

18. (a) $8e^{16}$ cms^{-1} (b) 0 s (initially) (c) 1 cm

19. (a) 7 s (b) $\dfrac{7}{\sqrt{2}}$ or $\dfrac{7\sqrt{2}}{2}$ s (c) 49 cm

Exercises 6.5

1. 12 cm **2.** 28 m **3.** -42.5 cm

4. (a) 570 cms^{-2} (b) 135 cm (c) After 0.5 s

5. $(e^3 + 1)$ cm **6.** 163 m **7.** (a) 95 cms^{-1} (b) 175 cm

8. $h = -4.9t^2 + 4t + 2$ **9.** 262 m **10.** $(e^5 - 3)$ m

11. -744 cm **12.** $(2\pi - 3)$ cm **13.** 1.77 m **14.** 893 m

15. (a) $(\sqrt{3} + 3)$ m (b) $-4\sqrt{3}$ ms^{-2}

16. (a) $\dfrac{4}{15}$ ms^{-1} (b) $x = \dfrac{2n}{3} - 2\ln(n+3)$ m

(c)
$$\begin{aligned} v &= \frac{2}{3} - \frac{2}{t+3} \\ &= \frac{2(t+3) - 6}{3(t+3)} \\ &= \frac{2t + 6 - 6}{3(t+3)} \\ &= \frac{2t}{3t+9} \end{aligned}$$
$$t \geq 0$$

When $t = 0$: $v = 0$
When $t > 0$: $v > 0$
Also $2t > 0$ and $3t + 9 > 0$ when $t > 0$
So $2t < 3t + 9$

$$\frac{2t}{3t+9} < 1$$

$\therefore\ 0 \leq v < 1$

17. (a) $5e^{45}$ ms^{-1} (b) e^{30} m (c) $\ddot{x} = 25e^{5t}$
$= 25x$ (d) 50 ms^{-2}

Test yourself 6

1. (a) 0 m, 0 ms^{-1}, 8 ms^{-2} (b) 0, 0.8 s (c) 0.38 m

2. -76 m, -66 ms^{-2} **3.** 39.6 years

4. (a) 6 cms^{-1} (b) 145 855.5 cms^{-2}
(c) $x = 2e^{3t}$
$\dot{x} = 6e^{3t}$
$\ddot{x} = 18e^{3t}$
$= 9(2e^{3t})$
$= 9x$

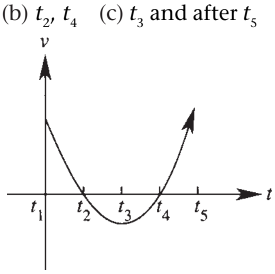
5. 1 m

6. $x = 2\sin 3t$
$\dot{x} = 6\cos 3t$
$\ddot{x} = -18\sin 3t$
$= -9(2\sin 3t)$
$= -9x$

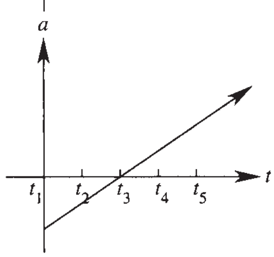
7. (a) 2, 6 s
(b) (i) 16 cm
(ii) 15 cms^{-1}
(iii) -18 cms^{-2}
(c) Particle is 16 cm to the right of the origin, travelling at 16 cms^{-1} to the right. Acceleration is -18 cms^{-2} (to the left), so the particle is slowing down.

8. (a) (i) 18 ms^{-2}
(ii) 15 ms^{-1}
(iii) -28 m
(b) Particle is 28 m to the left of the origin, travelling at 15 ms^{-1} to the right, with 18 ms^{-2} acceleration (to the right), so the particle is speeding up.

9. (a) t_1, t_3, t_5 (b) t_2, t_4 (c) t_3 and after t_5
(d) (i)



(ii)



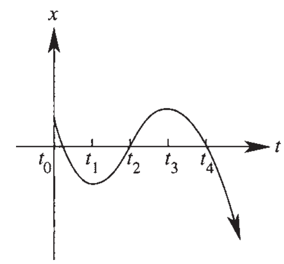
10. (a) 48.2% (b) 1052.6 years

11. (a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \ldots$ s (b) $\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \ldots$ s
(c) $-\frac{1}{\sqrt{2}}$ ms^{-2}

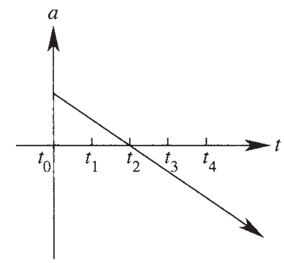
12. (a) 15 m (b) 20 m (c) 4 s

13. (a) 16 941 (b) 1168 birds/year
(c) 18.3 years

14. (a) (i)



(ii)



(b) t_1, t_3

15. 55 033 m

Challenge exercise 6

1. (a) $x = -\frac{2}{3}\cos 3t + \frac{8}{3} = \frac{2(4-\cos 3t)}{3}$
(b) $\ddot{x} = -9\left(x - \frac{8}{3}\right)$

2. (a) 1 m, 0 ms^{-1} (b) 3.26×10^7 ms^{-2}
(c) Show $(t^3+1)^6 = 0$ has no solution for $t \geq 0$

3. (a) $x = \cos 4t$ (b) $\pm 2\sqrt{3}$ cms^{-1}

4. (a) $v = 5\cos 5t$
$x = \int (5\cos 5t)\,dt$
$= \sin 5t + C$
When $t = 0, x = 0$
$0 = \sin 0 + C$
$= C$
$\therefore$ $x = \sin 5t$
$a = \frac{d}{dt}(5\cos 5t)$
$= -25\sin 5t$
$= -25x$

(b) 25 ms^{-2} (c) -7.5 ms^{-2}

5. (a) 19.9 years (b) 16%

6. $$N = \frac{kN_0}{bN_0 + (k - bN_0)e^{-kt}}$$

$$\frac{dN}{dt} = \frac{-kN_0\left[-k(k - bN_0)e^{-kt}\right]}{\left[bN_0 + (k - bN_0)e^{-kt}\right]^2}$$

$$= \frac{k^2 N_0 (k - bN_0)e^{-kt}}{\left[bN_0 + (k - bN_0)e^{-kt}\right]^2}$$

$$= \frac{k^2 N_0\left[bN_0 + (k - bN_0)e^{-kt} - bN_0\right]}{\left[bN_0 + (k - bN_0)e^{-kt}\right]^2}$$

$$= \frac{k^2 N_0\left[bN_0 + (k - bN_0)e^{-kt}\right]}{\left[bN_0 + (k - bN_0)e^{-kt}\right]^2} - \frac{bk^2 N_0^2}{\left[bN_0 + (k - bN_0)e^{-kt}\right]^2}$$

$$= \frac{k^2 N_0}{bN_0 + (k - bN_0)e^{-kt}} - b\left(\frac{kN_0}{bN_0 + (k - bN_0)e^{-kt}}\right)^2$$

$$= kN - bN^2$$

7. $3e$ cms^{-2}

Practice assessment task set 2

1. 3.2 years **2.** 1.099 **3.** 27 m **4.** -2

5. $x = e^y$, $x = 3.42$ **6.** $3x^2 + 2e^{2x}$ **7.** $x = \pm\frac{1}{2}$

8. (a) 47.5 g (b) 3.5 g/year (c) After 9.3 years

9. (a) 0 cms^{-1} (b) $a = -18\sin 3t = -9x$

10. (a) 7750 L (b) 28 minutes

11. 622.1 units^3 **12.** $\frac{12}{4x + 3}$

13. $\frac{1}{3}\log_e(3x^2 + 3x - 2) + C$

14. (a) 100 L (b) 40 L
(c) -16 L per minute, i.e. leaking at the rate of 16 L per minute
(d) 12.2 minutes

15. $2x^3 - x^2 + 4\log_e x + C$ **16.** $3\log_e 2$

17. (a) 7.8 cm (b) -0.06 cms^{-2}

18. $\frac{1}{4}e^{4x} + x + C$ **19.** $\frac{1 - 2x}{e^{2x}}$

20. (a) $k = 0.101$ (b) 2801 (c) 20 days
(d) (i) 11 people per day
(ii) 283 people per day

21. (a) 1.77 (b) $\frac{1}{x\log_e 3}$

22. $\frac{e^2\pi}{2}(e^4 - 1)$ units^3

23. (a) $v = 6$ ms^{-1}, $a = 0$ ms^{-2} (b) 3 m
(c) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \ldots$ seconds
(d) $a = -12\sin 2t$
$= -4(3\sin 2t)$
$= -4x$

24. 4.67 units^2 **25.** -27 m **26.** $x = 0.28$

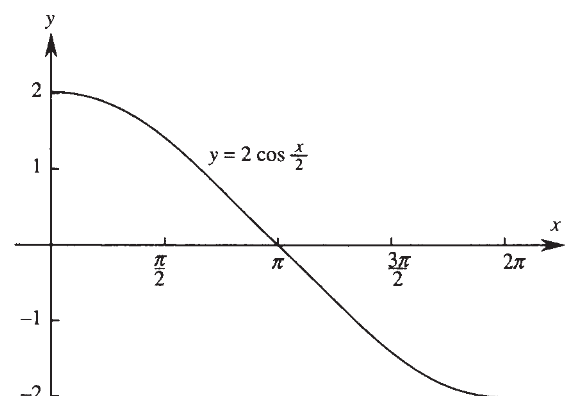
27. $x - y + 2 = 0$ **28.** $\left(-\frac{1}{2}, -\frac{1}{2e}\right)$ minimum

29. (a) 1.60 cm^2 (b) 0.17 cm^2

30. 15 months

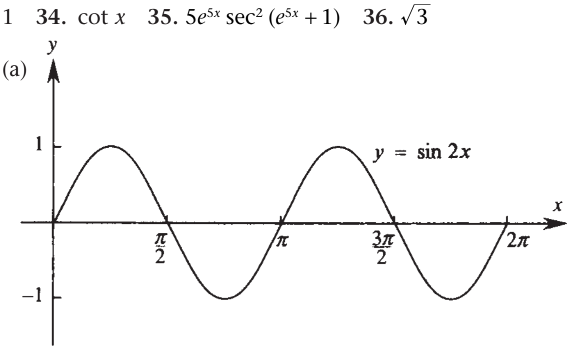
31. (a) $\frac{5\pi}{6}$ cm (b) $\frac{25\pi}{12}$ cm^2

32.



33. 1 **34.** $\cot x$ **35.** $5e^{5x}\sec^2(e^{5x} + 1)$ **36.** $\sqrt{3}$

37. (a)



(b) 2 units^2

38. 0.348

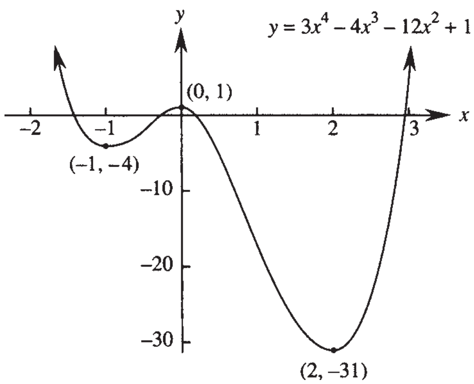
39. (a) $e^x(\sin x + \cos x)$ (b) $3\tan^2 x\sec^2 x$
(c) $-6\sin\left(3x - \frac{\pi}{2}\right)$

40. (a) 546 ms^{-1} (b) $a = 20e^{2t}$ (c) 20 ms^{-2}
$= 4(5e^{2t})$
$= 4x$

41. $\ln 8 - \ln 3 = \ln\frac{8}{3}$ **42.** 6 m × 12m

43. $\frac{\sqrt{3}\pi}{2}$ units^3

44.



45. $6x - y - 1 - \frac{3\pi}{2} = 0$

46. $\sqrt{3} - \frac{\pi}{3} = \frac{3\sqrt{3} - \pi}{3}$ units²

47. $3e^x \sin^2(e^x)\cos(e^x)$

48. $(5 - e)$ units²

49. (a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{\sqrt{3}}{2}$

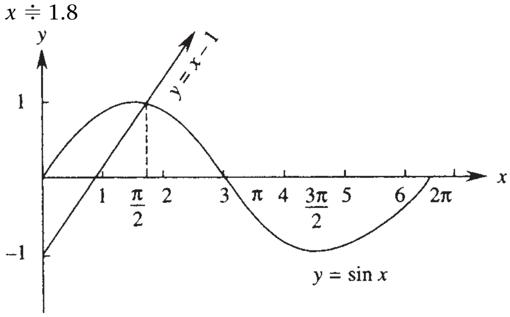
50. (a) 21π cm² (b) $(21\pi + 9)$ cm²

51. $\frac{\sec^2(\log_e x + 1)}{x}$

52. $3x - 2\ln x - \frac{5}{x} + C$

53. 8.32 units³ 54. $\frac{1}{5}e^{5x} + \frac{1}{\pi}\cos \pi x + C$

55. $x \doteqdot 1.8$



56. $(e^2 - 1)$ units² 57. −1

58. (c) 59. (d) 60. (a)

61. (d) 62. (b) 63. (c)

Chapter 7: Series

Exercises 7.1

1. 14, 17, 20 2. 23, 28, 33 3. 44, 55, 66

4. 85, 80, 75 5. 1, −1, −3 6. 87, 83, 79

7. 2, $2\frac{1}{2}$, 3 8. 3.1, 3.7, 4.3 9. 16, 32, 64

10. 108, 324, 972 11. 16, −32, 64 12. 48, −96, 192

13. $\frac{1}{16}, \frac{1}{32}, \frac{1}{64}$ 14. $\frac{16}{135}, \frac{32}{405}, \frac{64}{1215}$

15. 36, 49, 64 16. 125, 216, 343 17. 35, 48, 63

18. 38, 51, 66 19. 126, 217, 344 20. 21, 34, 55

Exercises 7.2

1. (a) $T_1 = 3, T_2 = 11, T_3 = 19$ (b) $T_1 = 5, T_2 = 7, T_3 = 9$
(c) $u_1 = 5, u_2 = 11, u_3 = 17$ (d) $T_1 = 3, T_2 = -2, T_3 = -7$
(e) $t_1 = 19, t_2 = 18, T_3 = 17$ (f) $u_1 = 3, u_2 = 9, u_3 = 27$
(g) $Q_1 = 9, Q_2 = 11, Q_3 = 15$ (h) $t_1 = 2, t_2 = 12, t_3 = 58$
(i) $T_1 = 8, T_2 = 31, T_3 = 70$ (j) $T_1 = 2, T_2 = 10, T_3 = 30$

2. (a) $T_1 = 1, T_2 = 4, T_3 = 7$ (b) $t_1 = 4, t_2 = 16, t_3 = 64$
(c) $T_1 = 2, T_2 = 6, T_3 = 12$

3. (a) 349 (b) 105
(c) 248 (d) −110 (e) −342

4. (a) 1029 (b) −59 039 (c) 1014
(d) 53 (e) 1002

5. (a), (b), (d) 6. (b), (d) 7. 16th term 8. Yes

9. 7th term 10. 23rd term 11. (a) 1728 (b) 25th term

12. (a) −572 (b) 17th term

13. $n = 33$ 14. $n = 9$ 15. $n = 88, 89, 90, \ldots$

16. $n = 41, 42, 43$ 17. $n = 501$ 18. $n = 151$

19. (a) $n = 14$ (b) −4 20. −6

Exercises 7.3

1. (a) 128 (b) 54 (c) 70 (d) 175 (e) 220
(f) $\frac{47}{60}$ (g) 40 (h) 21 (i) 126 (j) 1024

2. (a) 65 (b) 99 (c) 76 (d) 200 (e) 11
(f) 39 (g) 97 (h) 66 (i) 75 (j) 45

3. (a) $\sum_{n=1}^{6} 2n - 1$ (b) $\sum_{n=1}^{10} 7n$ (c) $\sum_{n=1}^{5} n^3$ (d) $\sum_{k=1}^{n} 6k - 4$
(e) $\sum_{k=3}^{n} k^2$ (f) $\sum_{n=1}^{50} (-n)$ (g) $\sum_{k=0}^{n} 3.2^k$ (h) $\sum_{n=0}^{9} \frac{1}{2^n}$
(i) $\sum_{k=1}^{n} a + (k-1)d$ (j) $\sum_{k=1}^{n} ar^{k-1}$

Exercises 7.4

1. (a) $y = 13$ (b) $x = -4$ (c) $x = 72$ (d) $b = 11$ (e) $x = 7$
(f) $x = 42\frac{1}{2}$ (g) $k = 4\frac{1}{2}$ (h) $x = 1$ (i) $t = -2$ (j) $t = 3$

2. (a) 46 (b) 78 (c) 94 (d) –6 (e) 67

3. (a) 590 (b) –850 (c) 414 (d) 1610 (e) –397

4. (a) –110 (b) 12.4 (c) –8.3 (d) 37 (e) $15\frac{4}{5}$

5. $T_n = 2n + 1$

6. (a) $T_n = 8n + 1$ (b) $T_n = 2n + 98$ (c) $T_n = 3n + 3$
(d) $T_n = 6n + 74$ (e) $T_n = 4n - 25$ (f) $T_n = 20 - 5n$
(g) $T_n = \frac{n+6}{8}$ (h) $T_n = -2n - 28$ (i) $T_n = 1.2n + 2$
(j) $T_n = \frac{3n-1}{4}$

7. 28th term **8.** 54th term **9.** 30th term

10. 15th term **11.** Yes **12.** No

13. Yes **14.** $n = 13$ **15.** $n = 30, 31, 32 \ldots$

16. –2 **17.** 103 **18.** 785

19. (a) $d = 8$ (b) 87

20. $d = 9$ **21.** $a = 12$, $d = 7$

22. 173 **23.** $a = 5$

24. 280 **25.** 1133

26. (a)
$$\begin{aligned} T_2 - T_1 &= \log_5 x^2 - \log_5 x \\ &= 2\log_5 x - \log_5 x \\ &= \log_5 x \\ T_3 - T_2 &= \log_5 x^3 - \log_5 x^2 \\ &= 3\log_5 x - 2\log_5 x \\ &= \log_5 x \end{aligned}$$
Since $T_2 - T_1 = T_3 - T_2$ it is an arithmetic series with $d = \log_5 x$.
(b) $80 \log_5 x$ or $\log_5 x^{80}$
(c) 8.6

27. (a)
$$\begin{aligned} T_2 - T_1 &= \sqrt{12} - \sqrt{3} \\ &= \sqrt{4} \times \sqrt{3} - \sqrt{3} \\ &= 2\sqrt{3} - \sqrt{3} \\ &= \sqrt{3} \\ T_3 - T_2 &= \sqrt{27} - \sqrt{12} \\ &= \sqrt{9} \times \sqrt{3} - \sqrt{4} \times \sqrt{3} \\ &= 3\sqrt{3} - 2\sqrt{3} \\ &= \sqrt{3} \end{aligned}$$
Since $T_2 - T_1 = T_3 - T_2$ it is an arithmetic series with $d = \sqrt{3}$.
(b) $50\sqrt{3}$

28. 26 **29.** $122b$ **30.** 38th term

Exercises 7.5

1. (a) 375 (b) 555 (c) 480

2. (a) 2640 (b) 4365 (c) 240

3. (a) 2050 (b) –2575

4. (a) –4850 (b) 4225

5. (a) 28 875 (b) 3276 (c) –1419 (d) 6426
(e) 6604 (f) 598 (g) –2700
(h) 11 704 (i) –290 (j) 1284

6. (a) 700 (b) –285 (c) –1170 (18 terms)
(d) 6525 (e) –2286

7. 21 **8.** 8 **9.** 11 **10.** $a = 14$, $d = 4$

11. $a = -3$, $d = 5$ **12.** 2025 **13.** 3420

14. 8 and 13 terms **15.** 1010

16. (a)
$$\begin{aligned} &(2x + 4) - (x + 1) \\ &= (3x + 7) - (2x + 4) \\ &= x + 3 \end{aligned}$$
(b) $25(51x + 149)$

17. 1290 **18.** 16

19.
$$\begin{aligned} S_n &= S_{n-1} + T_n \\ \therefore S_n - S_{n-1} &= T_n \end{aligned}$$

20. 4234

Exercises 7.6

1. (a) No (b) Yes, $r = -\frac{3}{4}$ (c) Yes, $r = \frac{2}{7}$
(d) No (e) No (f) No (g) Yes, $r = 0.3$
(h) Yes, $r = -\frac{3}{5}$ (i) No (j) Yes, $r = -8$

2. (a) $x = 196$ (b) $y = -48$ (c) $a = \pm 12$
(d) $y = \frac{2}{3}$ (e) $x = 2$ (f) $p = \pm 10$
(g) $y = \pm 21$ (h) $m = \pm 6$ (i) $x = 4 \pm 3\sqrt{5}$
(j) $k = 1 \pm 3\sqrt{7}$ (k) $t = \pm\frac{1}{6}$ (l) $t = \pm\frac{2}{3}$

3. (a) $T_n = 5^{n-1}$ (b) $T_n = 1.02^{n-1}$ (c) $T_n = 9^{n-1}$
(d) $T_n = 2 \cdot 5^{n-1}$ (e) $T_n = 6 \cdot 3^{n-1}$ (f) $T_n = 8 \cdot 2^{n-1} = 2^{n+2}$
(g) $T_n = \frac{1}{4} \cdot 4^{n-1} = 4^{n-2}$ (h) $T_n = 1000(-10)^{n-1} = -(-10)^{n+2}$
(i) $T_n = -3(-3)^{n-1} = (-3)^n$ (j) $T_n = \frac{1}{3}\left(\frac{2}{5}\right)^{n-1}$

4. (a) 1944 (b) 9216 (c) –8192
(d) 3125 (e) $\frac{64}{729}$

5. (a) 256 (b) 26 244 (c) 1.369
(d) –768 (e) $\frac{3}{1024}$

6. (a) 234 375 (b) 268.8 (c) –81 920
(d) $\frac{2187}{156\,250}$ (e) 27

7. (a) 3×2^{19} (b) 7^{19} (c) 1.04^{20}

(d) $\frac{1}{4}\left(\frac{1}{2}\right)^{19} = \frac{1}{2^{21}}$ (e) $\left(\frac{3}{4}\right)^{20}$

8. 11^{49} **9.** 6th term

10. 5th term **11.** No

12. 7th term **13.** 11th term

14. 9th term **15.** $n = 5$ **16.** $r = 3$

17. (a) $r = -6$
(b) -18

18. $a = \frac{1}{10}, r = \pm 2$ **19.** $n = 7$ **20.** $208\frac{2}{7}$

Exercises 7.7

1. (a) 2 097 150 (b) 7 324 218

2. (a) 720 600 (b) 26 240

3. (a) 131 068 (b) $\frac{32\,769}{65\,536}$

4. (a) 7812 (b) $35\frac{55}{64}$
(c) 8403 (d) 273 (e) 255

5. (a) 255 (b) $\frac{364}{729}$ (c) 97 656.2
(d) $1\frac{127}{128}$ (e) 87 376

6. (a) 1792 (b) 3577

7. 148.58 **8.** 133.33

9. $n = 9$ **10.** 10 terms

11. $a = 9$ **12.** 10 terms

13. (a) \$33 502.39 (b) \$178 550.21

14. (a) $\sum_{k=1}^{n} 2(-5)^{k-1}$ (b) $S_n = -\frac{(-5)^n - 1}{3} = \frac{1 - (-5)^n}{3}$

15. 2146

Puzzles

1. Choice 1 gives \$465.00. Choice 2 gives \$10 737 418.23!
2. 382 apples

Exercises 7.8

1. (a) Yes LS $= 13\frac{1}{2}$ (b) No (c) Yes LS $= 12\frac{4}{5}$ (d) No
(e) Yes LS = 3 (f) Yes LS $= \frac{25}{32}$ (g) No
(h) Yes LS $= -1\frac{5}{22}$ (i) No (j) Yes LS $= 1\frac{3}{7}$

2. (a) 80 (b) $426\frac{2}{3}$ (c) $66\frac{2}{3}$ (d) 12 (e) $\frac{7}{10}$ (f) 54
(g) $-10\frac{2}{7}$ (h) $\frac{9}{20}$ (i) 48 (j) $-\frac{16}{39}$

3. (a) $\frac{7}{12}$ (b) $\frac{4}{27}$ (c) $\frac{1}{12500}$ (d) $\frac{1}{64}$ (e) $\frac{3645}{4096}$

4. (a) $1\frac{1}{4}$ (b) $\frac{2}{5}$ (c) $\frac{1}{48}$ (d) $2\frac{1}{2}$ (e) 3 (f) 5 (g) $\frac{2}{5}$
(h) $-5\frac{1}{3}$ (i) $1\frac{4}{5}$ (j) $\frac{5}{6}$

5. $a = 4$ **6.** $r = \frac{2}{5}$ **7.** $a = 5\frac{3}{5}$ **8.** $r = \frac{7}{8}$ **9.** $r = -\frac{1}{4}$

10. $r = -\frac{2}{3}$ **11.** $a = 3, r = \frac{2}{3}$ and $a = 6, r = \frac{1}{3}$

12. $a = 192, r = -\frac{1}{4}$, LS $= 153\frac{3}{5}$

13. $a = 1, r = \frac{2}{3}$, LS $= 3$, $a = -1, r = -\frac{2}{3}$, LS $= -\frac{3}{4}$

14. $a = 150, r = \frac{3}{5}$, LS $= 375$

15. $a = \frac{2}{5}, r = \frac{2}{3}$, LS $= 1\frac{1}{5}$ **16.** $a = 3,\ r = \frac{2}{5}$ and $a = 2,\ r = \frac{3}{5}$

17. $x = \frac{21}{32}$ **18.** (a) $-1 < k < 1$ (b) $-\frac{2}{5}$ (c) $k = \frac{3}{4}$

19. (a) $-\frac{1}{2} < p < \frac{1}{2}$ (b) $\frac{5}{7}$ (c) $p = \frac{1}{14}$

20.
$$\begin{aligned} \text{LS} - S_n &= \frac{a}{1-r} - \frac{a(1-r^n)}{1-r} \\ &= \frac{a - a(1-r^n)}{1-r} \\ &= \frac{a - a + ar^n}{1-r} \\ &= \frac{ar^n}{1-r} \end{aligned}$$

Exercises 7.9

1. (a) 210 (b) 13th (c) 57

2. (a) 39 (b) 29th (c) 32

3. (a) $3n + 3$
(b)
$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ &= \frac{1}{2}n[2 \times 6 + (n-1) \times 3] \\ &= \frac{1}{2}n(12 + 3n - 3) \\ &= \frac{1}{2}n(3n + 9) \\ &= \frac{3}{2}n(n + 3) \end{aligned}$$

4. (a) (i) \$23 200 (ii) \$26 912 (iii) \$31 217.92
(b) \$102 345.29 (c) 6.2 years

5. (a) (i) 93% (ii) 86.49% (iii) 80.44%
(b) 33.67% (c) 19 weeks

6. (a) 0.01 m (b) 91.5 m

7. (a) 49 (b) 4 mm

8. (a) $3k$ m (b) $k(3k + 3)$ m (c) 9

9. (a) 96.04% (b) 34 (c) 68.6

10. (a) 77.4% (b) 13.5 (c) 31.4

11. (a) $\frac{4}{9}$ (b) $\frac{7}{9}$ (c) $1\frac{2}{9}$
(d) $\frac{25}{99}$ (e) $2\frac{9}{11}$ (f) $\frac{7}{30}$

(g) $1\frac{43}{90}$ (h) $1\frac{7}{450}$ (i) $\frac{131}{990}$ (j) $2\frac{361}{999}$

12. 0.625 m **13.** 15 m **14.** 20 cm **15.** 3 m

16. (a) 74.7 cm (b) 75 m

17. (a) 4.84 m (b) After 3 years

18. 300 cm **19.** 3.5 m **20.** 32 m

21. (a) 1, 8, 64, … (b) 16 777 216 people
(c) 19 173 961 people

Exercises 7.10

1. (a) \$740.12 (b) \$14 753.64 (c) \$17 271.40
(d) \$9385.69 (e) \$5298.19

2. (a) \$2007.34 (b) \$2015.87 (c) \$2020.28

3. (a) \$4930.86 (b) \$4941.03

4. \$408.24 **5.** \$971.40

6. \$1733.99 **7.** \$3097.06

8. \$22 800.81 **9.** \$691.41

10. \$1776.58 **11.** \$14 549.76 **12.** \$1 301 694.62

13. (a) \$4113.51 (b) \$555.32
(c) \$9872.43 (d) \$238.17 (e) \$10 530.59

14. \$4543.28 **15.** 4 years **16.** 8 years

17. (a) $x = 7$ (b) $x = 5$ (c) $x = 8$
(d) $x = 6.5$ (e) $x = 8.5$

18. \$7.68 **19.** Kate \$224.37

20. Account A \$844.94

Exercises 7.11

1. \$27 882.27 **2.** \$83 712.95

3. \$50 402.00 **4.** \$163 907.81

5. \$40 728.17 **6.** \$29 439.16

7. \$67 596.72 **8.** \$62 873.34

9. \$164 155.56 (28 years) **10.** \$106 379.70

11. \$3383.22 **12.** \$65 903.97

13. \$2846.82 **14.** \$13 601.02

15. \$6181.13 **16.** \$4646.71 **17.** \$20 405.74

18. (a) \$26 361.59 (b) \$46 551.94

19. \$45 599.17

20. (a) \$7335.93 (b) \$1467.18

21. \$500 for 30 years **22.** Yes, \$259.80 over

23. No, shortfall of \$2013.75

24. (a) \$14 281.87 (b) \$9571.96
(c) No, they will only have \$23 853.83.

25. \$813.16

Exercises 7.12

1. \$1047.62 **2.** \$394.46 **3.** \$139.15

4. (a) \$966.45 (b) \$1265.79

5. \$2519.59

6. (a) \$592.00 (b) \$39 319.89

7. (a) \$77.81 (b) \$2645.42

8. \$78 700

9. (a) Get Rich \$949.61, Capital Bank \$491.27
(b) \$33 427.80 more through Capital Bank

10. \$43 778.80 **11.** \$61 292.20

12. NSW Bank \$175.49 a month (\$5791.25 altogether)
Sydney Bank \$154.39 a month (\$5557.88 altogether)
Sydney Bank is better

13. (a) \$249.69
(b) \$13 485.12

14. (a) \$13 251.13 (b) \$374.07 (c) \$20 199.77

15. (a) \$1835.68 (b) \$9178.41

Test yourself 7

1. (a) $T_n = 4n + 5$ (b) $T_n = 14 - 7n$
(c) $T_n = 2 \cdot 3^{n-1}$ (d) $T_n = 200\left(\frac{1}{4}\right)^{n-1}$
(e) $T_n = (-2)^n$

2. (a) 2 (b) 1185 (c) 1183
(d) $T_{15} = S_{15} - S_{14}$
$S_{15} = S_{14} + T_{15}$
(e) $n = 16$

3. (a) 11 125 (b) $1\frac{13}{140}$
(c) 3 985 785 (d) 34 750
(e) $\frac{1}{2}$

4. (a) Each slat rises 3 mm so the bottom one rises up 30×3 mm or 90 mm.
(b) 87 mm
(c) 90, 87, 84, … which is an arithmetic sequence with $a = 90$, $d = -3$
(d) 42 mm (e) 1395 mm

5. \$3400.01

6. (a) (i) (b) (ii) (c) (i) (d) (iii) (e) (i) (f) (ii)
(g) (ii) (h) (i) (i) (i) (j) (i)

7. $n = 108$

8. (a) \$24 050 (b) \$220 250

9. $a = -33$, $d = 13$

10. (a) 59 (b) 80 (c) 18th term

11. (a) $x = 25$ (b) $x = \pm 15$

12. (a) $\frac{4}{9}$ (b) $\frac{13}{18}$ (c) $1\frac{19}{33}$

13. $x = 3$

14. (a) 136 (b) 44 (c) 6

15. $121\frac{1}{2}$ 16. \$8066.42

17. (a) $T_n = 4n + 1$ (b) $T_n = 1.07^{n-1}$

18. (a) $-1 < x < 1$ (b) $2\frac{1}{2}$ (c) $x = \frac{1}{3}$

19. $d = 5$

20. (a) 39 words/min (b) 15 weeks

21. (a) \$59 000 (b) \$15 988.89

22. 4.8 m 23. $x = -\frac{2}{17}, 2$

24. (a) \$2385.04 (b) \$2392.03

25. 1300

26. (a) 735 (b) 4315

27. (a) \$1432.86 (b) \$343 886.91

28. $n = 20$

29. $n = 11$

Challenge exercise 7

1. (a) 8.1 (b) 19th term

2. (a) $\frac{\pi}{4}$ (b) $\frac{9\pi}{4}$ (c) $\frac{33\pi}{4}$

3. (a) 2 097 170 (b) −698 775

4. (a) \$40 (b) \$2880

5. 6th term 6. 17 823

7. 5 terms 8. $n = 1, 2, 3$

9. −56 10. \$1799.79

11. $x = \frac{3}{8}$ 12. \$8522.53 13. $k = 20$

14. (a) \$10 100 (b) \$11 268.25
(c) \$4212.41 (d) \$2637.23

15. (a) $\operatorname{cosec}^2 x$
(b) $-1 \le \cos x \le 1$
So $0 \le \cos^2 x \le 1$
$|\cos^2 x| \le 1$
So the limiting sum exists.

16. \$240 652.62.

Chapter 8: Probability

Exercises 8.1

1. $\frac{1}{30}$ 2. $\frac{1}{52}$ 3. $\frac{1}{6}$ 4. $\frac{1}{40}$ 5. $\frac{1}{20\,000}$

6. (a) $\frac{4}{7}$ (b) $\frac{3}{7}$ 7. $\frac{3}{37}$ 8. $\frac{1}{12}$ 9. (a) $\frac{11}{20}$ (b) $\frac{3}{4}$

10. (a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$

11. (a) $\frac{1}{62}$ (b) $\frac{3}{31}$ (c) $\frac{1}{2}$ (d) $\frac{99}{124}$

12. (a) $\frac{8}{15}$ (b) $\frac{7}{15}$ (c) $\frac{1}{15}$ 13. $\frac{1}{50}$ 14. $\frac{1}{2}, 1$ 15. $\frac{23}{44}$

16. (a) $\frac{7}{31}$ (b) $\frac{7}{31}$ (c) $\frac{12}{31}$ 17. $\frac{1}{175}$ 18. 8 19. $\frac{25}{43}$

20. 34 21. $\frac{1}{3}$ 22. (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$

23. (a) False: outcomes are not equally likely. Each horse and rider has different skills.
(b) False: outcomes are not equally likely. Each golfer has different skills.
(c) False: outcomes are not dependent on the one before. Each time the coin is tossed, the probability is the same.
(d) False: outcomes are not dependent on the one before. Each birth has the same probability of producing a girl or boy.
(e) False: outcomes are not equally likely. Each car and driver has different skills.

Exercises 8.2

1. $\frac{5}{11}$ 2. $\frac{2}{9}$ 3. 99.8% 4. 0.73 5. 38%

6. 98.5% 7. $\frac{22}{23}$ 8. $\frac{5}{18}$ 9. 0.21 10. 91.9%

11. $\frac{7}{8}$ 12. $\frac{46}{49}$ 13. (a) $\frac{2}{15}$ (b) $\frac{13}{15}$ 14. $\frac{7}{11}$ 15. $\frac{15}{16}$

Exercises 8.3

1. (a) $\frac{3}{10}$ (b) $\frac{3}{5}$ (c) $\frac{11}{20}$ (d) $\frac{7}{10}$

2. (a) $\frac{1}{5}$ (b) $\frac{1}{2}$ (c) $\frac{3}{5}$ (d) $\frac{3}{5}$ (e) $\frac{19}{50}$

3. (a) $\frac{5}{26}$ (b) $\frac{9}{26}$ (c) $\frac{12}{13}$ **4.** (a) $\frac{29}{100}$ (b) $\frac{13}{20}$ (c) $\frac{9}{25}$

5. (a) $\frac{27}{45}$ (b) $\frac{4}{9}$ (c) $\frac{2}{3}$ **6.** (a) $\frac{3}{14}$ (b) $\frac{13}{28}$ (c) $\frac{9}{28}$

7. (a) $\frac{21}{80}$ (b) $\frac{17}{80}$ (c) $\frac{21}{40}$ **8.** (a) $\frac{1}{10}$ (b) $\frac{11}{20}$ (c) $\frac{7}{20}$

9. (a) $\frac{7}{25}$ (b) $\frac{2}{15}$ (c) $\frac{44}{75}$ **10.** (a) $\frac{3}{10}$ (b) $\frac{2}{5}$ (c) $\frac{3}{10}$

Exercises 8.4

1. $\frac{1}{36}$ **2.** $\frac{1}{4}$ **3.** $\frac{1}{8}$ **4.** $\frac{1}{4}$ **5.** $\frac{25}{121}$

6. (a) 0.0441 (b) 0.6241 **7.** 80.4% **8.** 32.9%

9. (a) $\frac{9}{49}$ (b) $\frac{15}{91}$ **10.** $\frac{3}{2075}$ **11.** $\frac{19}{99}$ **12.** $\frac{1}{16\,170}$

13. (a) $\frac{29\,791}{35\,937}$ (b) $\frac{8}{35\,937}$ (c) $\frac{35\,929}{35\,937}$

14. (a) $\frac{1}{2400}$ (b) $\frac{1}{5\,760\,000}$ (c) $\frac{5\,755\,201}{5\,760\,000}$

15. (a) $\frac{1}{7776}$ (b) $\frac{3125}{7776}$ (c) $\frac{4651}{7776}$

16. (a) $\frac{9}{25\,000\,000}$ (b) $\frac{24\,970\,009}{25\,000\,000}$ (c) $\frac{29\,991}{25\,000\,000}$

17. (a) $\frac{1}{4}$ (b) $\frac{9}{100}$ (c) $\frac{9}{100}$

18. (a) $\frac{1}{22}$ (b) $\frac{1}{11}$ (c) $\frac{7}{22}$ (d) $\frac{15}{22}$

19. (a) 61.41% (b) 0.34% (c) 99.66%

20. (a) $\frac{1}{2^n}$ (b) $\frac{1}{2^n}$ (c) $1-\frac{1}{2^n}=\frac{2^n-1}{2^n}$

Exercises 8.5

1. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ **2.** (a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{7}{8}$

3. (a) $\frac{1}{900}$ (b) $\frac{1}{900}$ (c) $\frac{1}{450}$ **4.** (a) $\frac{1}{25}$ (b) $\frac{2}{25}$

5. (a) $\frac{25}{169}$ (b) $\frac{80}{169}$ **6.** (a) 27.5% (b) 23.9% (c) 72.5%

7. (a) 0.42 (b) 0.09 (c) 0.49 **8.** (a) $\frac{189}{1000}$ (b) $\frac{441}{1000}$ (c) $\frac{657}{1000}$ **9.** (a) 0.325 (b) 0.0034 (c) 0.997

10. (a) $\frac{60}{121}$ (b) $\frac{6}{11}$ **11.** (a) $\frac{4}{27}$ (b) $\frac{1}{6}$

12. (a) $\frac{1}{25}$ (b) $\frac{1}{825}$ (c) $\frac{64}{825}$ (d) $\frac{152}{165}$ (e) $\frac{13}{165}$

13. (a) $\frac{19}{1\,249\,750}$ (b) $\frac{498}{124\,975}$ (c) $\frac{1\,239\,771}{1\,249\,750}$

14. (a) $\frac{16}{75}$ (b) $\frac{38}{75}$ **15.** (a) $\frac{1936}{2025}$ (b) $\frac{88}{2025}$

16. (a) $\frac{11}{20}$ (b) $\frac{3}{20}$ **17.** (a) $\frac{1}{1296}$ (b) $\frac{125}{324}$ (c) $\frac{671}{1296}$

18. (a) $\frac{84\,681}{1\,000\,000}$ (b) $\frac{912\,673}{1\,000\,000}$ (c) $\frac{27}{1\,000\,000}$

19. (a) 17.6% (b) 11% (c) 21.2%

20. (a) $\frac{1488}{3025}$ (b) $\frac{1}{121}$ **21.** (a) $\frac{1}{19}$ (b) $\frac{6}{95}$ (c) $\frac{21}{190}$ (d) $\frac{17}{38}$

22. (a) $\frac{22}{425}$ (b) $\frac{368}{425}$ (c) $\frac{7}{425}$ **23.** (a) $\frac{17}{65}$ (b) $\frac{133}{715}$ (c) $\frac{496}{2145}$

24. (a) 0.23 (b) 0.42 (c) 0.995 **25.** (a) 33% (b) 94%

26. (a) $\frac{1}{216}$ (b) $\frac{5}{72}$ (c) $\frac{91}{216}$ **27.** (a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{2}{5}$

28. (a) $\frac{25}{81}$ (b) $\frac{40}{81}$ (c) $\frac{56}{81}$

29. (a) $\frac{343}{1331}$ (b) $\frac{336}{1331}$ (c) $\frac{988}{1331}$

30. (a) $\frac{1}{8000}$ (b) $\frac{6859}{8000}$ (c) $\frac{1141}{8000}$

Test yourself 8

1. (a) 80.4% (b) 1.4% (c) 99.97%

2. (a)

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

(b) (i) $\frac{1}{6}$ (ii) $\frac{1}{6}$ (iii) $\frac{1}{2}$ **3.** (a) (i) $\frac{1}{40}$ (ii) $\frac{39}{40}$ (b) $\frac{39}{796}$ **4.** (a) $\frac{4}{15}$ (b) $\frac{1}{10}$

5. False: the events are independent and there is the same chance next time $\left(\frac{1}{4}\right)$

6. (a) $\frac{1}{2}$ (b) $\frac{29}{100}$ (c) $\frac{1}{5}$ (d) $\frac{11}{25}$ (e) $\frac{16}{25}$

7. (a) $\frac{2}{5}$ (b) $\frac{7}{15}$ (c) $\frac{2}{15}$ **8.** (a) $\frac{35}{72}$ (b) $\frac{35}{66}$ **9.** $\frac{1}{56}$

10. (a) 0.009% (b) 12.9% **11.** (a) $\frac{1}{13}$ (b) $\frac{3}{13}$ (c) $\frac{5}{26}$

12. (a) $\frac{5}{12}$ (b) $\frac{1}{3}$

13. (a) $\frac{9}{40}$ (b) (i) $\frac{3}{10}$ (ii) $\frac{27}{160}$ (iii) $\frac{4}{25}$

14. (a) $\frac{1}{200}$ (b) $\frac{81}{200}$ (c) $\frac{11}{100}$ **15.** (a) $\frac{1}{15}$ (b) $\frac{4}{5}$

16. (a) $\frac{1}{50}$ (b) $\frac{147}{7450}$ (c) $\frac{1}{3725}$ (d) $\frac{3577}{3725}$

17. (a) $\frac{80}{361}$ (b) $\frac{40}{171}$ **18.** (a) $\frac{2}{9}$ (b) $\frac{1}{3}$

19. (a) $\frac{64}{243}$ (b) $\frac{728}{729}$ **20.** (a) $\frac{21}{50}$ (b) $\frac{3}{25}$ (c) $\frac{23}{50}$

Challenge exercise 8

1. (a) $\frac{1}{7}$ (b) $\frac{4}{7}$ **2.** (a) 0.04 (b) 0.75 (c) 0.25

3. (a) $\frac{1}{54\,145}$ (b) $\frac{33}{173\,264}$ **4.** (a) $\frac{4}{13}$ (b) $\frac{25}{52}$ (c) $\frac{4}{13}$

5. No—any combination of numbers is equally likely to win.

6. (a) 0 (b) $\frac{1}{10}$ (c) $\frac{3}{10}$ **7.** (a) $\frac{1}{7776}$ (b) $\frac{1}{1296}$

8. (a) $\frac{3}{10}$ (b) $\frac{12}{145}$ **9.** (a) $\frac{1}{144}$ (b) $\frac{5}{144}$ (c) $\frac{7}{144}$ (d) $\frac{3}{144}$

Practice assessment task set 3

1. $\frac{324}{625}$ **2.** (a) $\frac{1296}{2401}$ (b) $\frac{864}{2401}$ (c) $\frac{1105}{2401}$

3. $\frac{4}{9}$ **4.** $\frac{3}{4^9}$ **5.** \$2929.08 **6.** $\pm 104 + 52 \pm 26 + \ldots$

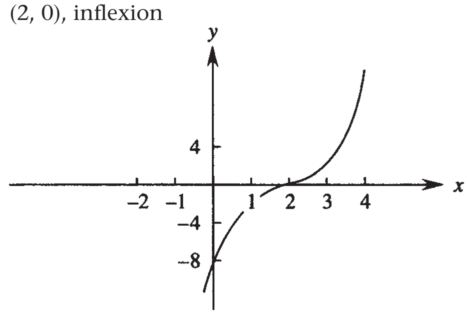
7. 44th term **8.** \$945

9. (a) $\frac{1}{36}$ (b) $\frac{1}{6}$ (c) $\frac{11}{36}$ (d) $\frac{5}{36}$ (e) $\frac{5}{12}$

10. $\frac{9841a}{6561}$ **11.** 2.4 m

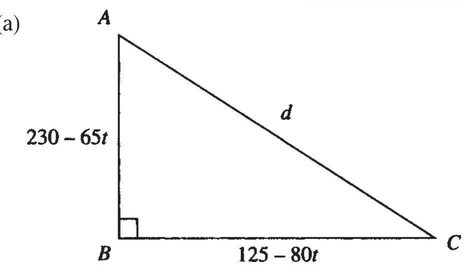
12. (a) 3 000 000 (b) 3 000 336 (c) 146 insects per day

13. (2, 0), inflexion



14. (a) $\frac{7}{50}$ (b) $\frac{11}{20}$

15. (a)



$d^2 = (230 - 65t)^2 + (125 - 80t)^2$ (Pythagoras)
$= 52\,900 - 29\,900t + 4225t^2 + 15\,625 - 20\,000t + 6400t^2$
$= 10\,625t^2 - 49\,900t + 68\,525$

(b) 2.3 h (c) 109.7 km

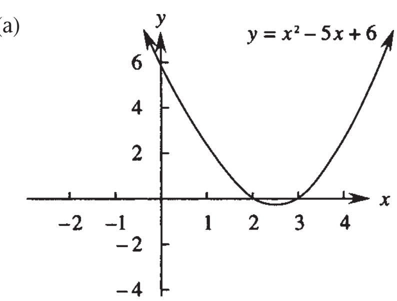
16. 199; 5050 **17.** $\frac{1}{110}$

18. (a) $T_1 = 4$, $T_2 = 11$, $T_3 = 18$, $T_{12} = 81$

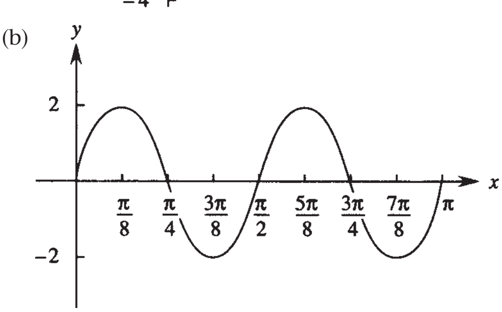
(b) 1410 (c) 29th term

19. $-\frac{1}{\sqrt{2}}$

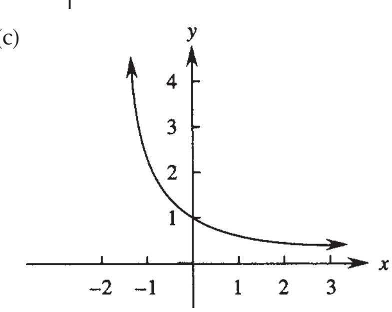
20. (a)



(b)



(c)



21. $-15 - 4 + 7 + \ldots$ **22.** (a) $\frac{5}{33}$ (b) $\frac{35}{66}$ **23.** \$2851.52

24. (a) $v = -12 \sin 4t$ (b) $a = -48 \cos 4t$ (c) 3 cm

(d) $t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \ldots$ s (e) ±3 cm (f) $t = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \ldots$ s

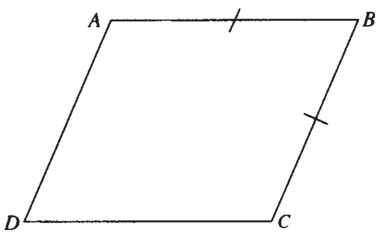
(g) $a = -48 \cos 4t$
$= -16(3 \cos 4t)$
$= -16x$

25. $\frac{8}{45}$ **26.** (a) $\frac{1}{12}$ (b) $\frac{1}{4}$ **27.** \$180.76

28.
$$AC^2 = 16^2 = 256$$
$$AB^2 + BC^2 = 9.6^2 + 12.8^2$$
$$= 256$$
Since $AC^2 = AB^2 + BC^2$,
ΔABC is right angled at $\angle B$.

29. $x = \frac{5}{6}$

30.



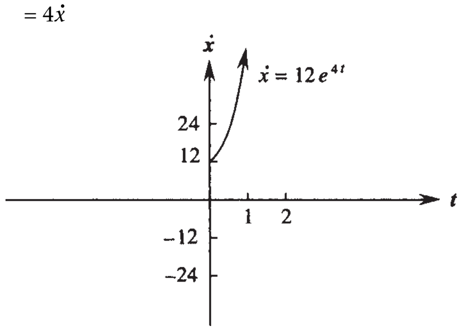
Let $ABCD$ be a rhombus with $AB = BC$.
$AB = DC$ and $AD = BC$
(opposite sides in a parallelogram)
$\therefore AB = BC = DC = AD$
$\therefore$ all sides are equal

31. $\frac{1}{40\,000}$

32. (a) $k \doteqdot 0.025$ (b) after 42.4 years (c) 20.6 years

33. 76 473 **34.** 450 cm^2

35. (a) 12 ms^{-1} (b) $48e^4$ ms^{-2}
(c) $x = 3e^{4t} + 2$
$$\dot{x} = 12e^{4t}$$
$$\ddot{x} = 48e^{4t}$$
$$= 4(12e^{4t})$$
$$= 4\dot{x}$$
(d)



36. $n = 4$

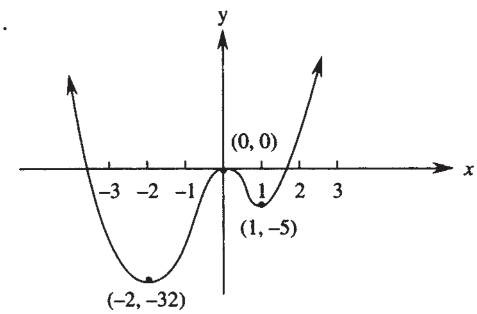
37.
$$y = \sin 7x$$
$$\frac{dy}{dx} = 7\cos 7x$$
$$\frac{d^2y}{dx^2} = 7(-7\sin 7x)$$
$$= -49\sin 7x$$
$$= -49y$$

38. $\frac{1}{10}$

39. (a) Square 46.3 m × 46.3 m, rectangle 30.9 m × 92.7 m
(b) \$8626.38

40. (a) $\frac{5}{36}$ (b) $\frac{5}{12}$ (c) $\frac{1}{6}$ (d) $\frac{7}{36}$ (e) $\frac{1}{2}$

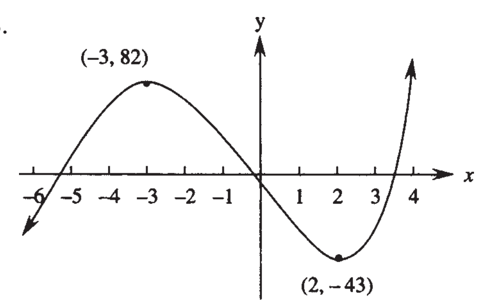
41.



42. (a) $\theta = 76°\,52'$ (b) 0.92 cm^2 **43.** \$18 399.86

44. (a) $\log 3 + \log 9 + \log 27 + \ldots$
$= \log 3 + \log 3^2 + \log 3^3 + \ldots$
$= \log 3 + 2\log 3 + 3\log 3 + \ldots$
Arithmetic series, since
$2\log 3 - \log 3 = 3\log 3 - 2\log 3$
$= \log 3$
(b) $210\log 3$

45.



46. (a) 12.6 mL (b) 30 minutes **47.** (a)

48. \$277.33 **49.** (d) **50.** (b) **51.** (c) **52.** (c), (d)

53. (d) **54.** (a) **55.** (c) **56.** (a) **57.** (d)

Sample examination papers

Mathematics—Paper 1

1. (a) 0.75
(b) $(3x - 2)(x - 3)$
(c) $\frac{6}{1}\left(\frac{x}{2}\right) - \frac{6}{1}\left(\frac{x-1}{3}\right) = 6(5)$
$$3x - 2(x - 1) = 30$$
$$3x - 2x + 2 = 30$$
$$x + 2 = 30$$
$$x = 28$$
(d)
$$12 = \frac{1}{3}\pi r^2$$
$$36 = \pi r^2$$
$$\frac{36}{\pi} = r^2$$
$$\sqrt{\frac{36}{\pi}} = r$$
$$3.39 = r$$

(e) 3300

(f) $x + 3 < 7$

$x < 4$

or $-(x + 3) < 7$

$x + 3 > -7$

$x > -10$

$\therefore \quad -10 < x < 4$

2. (a) (ii) Since $AB = BD$, $AB:AD = 1:2$

$\angle ABC = \angle ADE$ (corresponding $\angle$s, $BC \parallel DE$)

$\angle ACB = \angle AED$ (similarly)

$\angle A$ is common

$\therefore \ \Delta ABC \parallel\!\!| \ \Delta ADE$

$\therefore \ \dfrac{AB}{AD} = \dfrac{AC}{AE} = \dfrac{1}{2}$

$\therefore \ AE = 2AC$

$AC + CE = 2AC$

$\therefore \ CE = AC$

(iii) $\dfrac{AB}{AD} = \dfrac{BC}{DE} = \dfrac{1}{2}$

$\therefore \ \dfrac{3.4}{DE} = \dfrac{1}{2}$

$DE = 2 \times 3.4$

$= 6.8$ cm

(b) (ii) $\angle NOM = 105°$ (straight angle)

$\angle NMO = 32°$ (angle sum of Δ)

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\dfrac{MO}{\sin 43°} = \dfrac{5}{\sin 32°}$

$MO = \dfrac{5 \sin 43°}{\sin 32°}$

$\doteqdot 6.4$ m

(iii) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\dfrac{MP}{\sin 75°} = \dfrac{6.4}{\sin 53°}$

$MP = \dfrac{6.4 \sin 75°}{\sin 53°}$

$\doteqdot 8$ m

3. (a) (i) $\sqrt{x} + 5x^3 + 1 = x^{\frac{1}{2}} + 5x^3 + 1$

$\dfrac{dy}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}} + 15x^2 = \dfrac{1}{2\sqrt{x}} + 15x^2$

(ii) $3 \ln x + \dfrac{1}{x} = 3 \ln x + x^{-1}$

$\dfrac{dy}{dx} = 3 \times \dfrac{1}{x} - x^{-2}$

$= \dfrac{3}{x} - \dfrac{1}{x^2}$

$= \dfrac{3x - 1}{x^2}$

(iii) $\dfrac{dy}{dx} = 5(2x + 3)^4 \times 2$

$= 10(2x + 3)^4$

(b) (i) $\dfrac{x^2}{2} - \dfrac{1}{-1}e^{-x} + C = \dfrac{x^2}{2} + e^{-x} + C$

(ii) $\Big[-\cos\theta + \theta\Big]_0^{\pi} = (-\cos\pi + \pi) - (-\cos 0 + 0)$

$= -(-1) + \pi + 1$

$= 2 + \pi$

(c) (i) $\dfrac{5}{\sqrt{2} - 1} \times \dfrac{\sqrt{2} + 1}{\sqrt{2} + 1} = \dfrac{5\sqrt{2} + 5}{(\sqrt{2})^2 - 1^2}$

$= \dfrac{5\sqrt{2} + 5}{1}$

$= 5\sqrt{2} + 5$

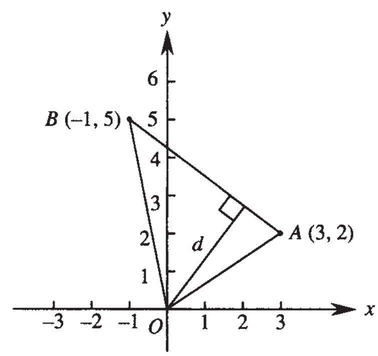
(ii) $5\sqrt{2} + 5 = 5 + 5\sqrt{2}$

$= 5 + \sqrt{25} \times \sqrt{2}$

$= 5 + \sqrt{50}$

$\therefore \quad a = 5, b = 50$

4. (a) (i)



(ii) Substitute $A(3, 2)$ into $3x + 4y - 17 = 0$

$3(3) + 4(2) - 17 = 0$

$9 + 8 - 17 = 0$

$0 = 0$ (true)

$\therefore$ A lies on the line

Substitute $B(-1, 5)$ into $3x + 4y - 17 = 0$

$3(-1) + 4(5) - 17 = 0$

$-3 + 20 - 17 = 0$

$0 = 0$ (true)

$\therefore$ B lies on the line

Since both A and B lie on the line $3x + 4y - 17 = 0$, this is the equation of AB

(iii) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \dfrac{|3(0) + 4(0) - 17|}{\sqrt{3^2 + 4^2}}$

$= \dfrac{|-17|}{\sqrt{9 + 16}}$

$= \dfrac{17}{\sqrt{25}}$

$= \dfrac{17}{5}$

$= 3.4$ units

(iv) Length AB: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[3 - (-1)]^2 + (2 - 5)^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$

Area ΔOAB: $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 3.4$

$= 8.5 \text{ units}^2$

(b) $c^2 = a^2 + b^2 - 2ab\cos C$

$BC^2 = 6^2 + 4^2 - 2(6)(4)\cos 87°$

$\doteqdot 49.49$

$BC = \sqrt{49.49}$

$\doteqdot 7 \text{ cm}$

5. (a) (i) $\frac{800}{24\,600} \times 100\% = 3.25\%$

(ii) Arithmetic series with $a = 24\,600$, $d = 800$ and $n = 12$

$T_n = a + (n-1)d$

$T_{12} = 24\,600 + (12-1)800$

$= 33\,400$

So Kate earns \$33 400 in the 12th year.

(iii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{12}{2}[2 \times 24\,600 + (12-1)800]$

$= 348\,000$

So Kate earns \$348 000 over the 12 years.

(b) (i) $y = x^3 - 3x^2 - 9x + 2$

$y' = 3x^2 - 6x - 9$

$y'' = 6x - 6$

For stationary points, $y' = 0$

i.e. $3x^2 - 6x - 9 = 0$

$3(x-3)(x+1) = 0$

$\therefore \quad x = 3, -1$

When $x = 3$, $y = 3^3 - 3(3)^2 - 9(3) + 2$

$= -25$

When $x = -1$, $y = (-1)^3 - 3(-1)^2 - 9(-1) + 2$

$= 7$

So $(3, -25)$ and $(-1, 7)$ are stationary points.

At $(3, -25)$, $y'' = 6(3) - 6$

> 0 (minimum point)

At $(-1, 7)$, $y'' = 6(-1) - 6$

< 0 (maximum point)

$\therefore (-1, 7)$ is a maximum, $(3, -25)$ minimum stationary point

(ii) For inflexions, $y'' = 0$

i.e. $6x - 6 = 0$

$6x = 6$

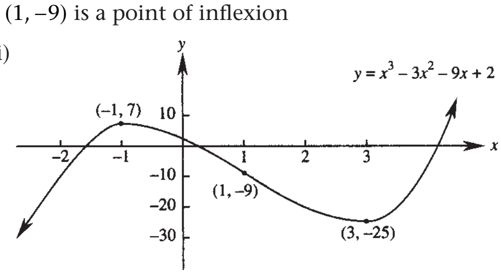
$x = 1$

When $x = 1$, $y = 1^3 - 3(1)^2 - 9(1) + 2$

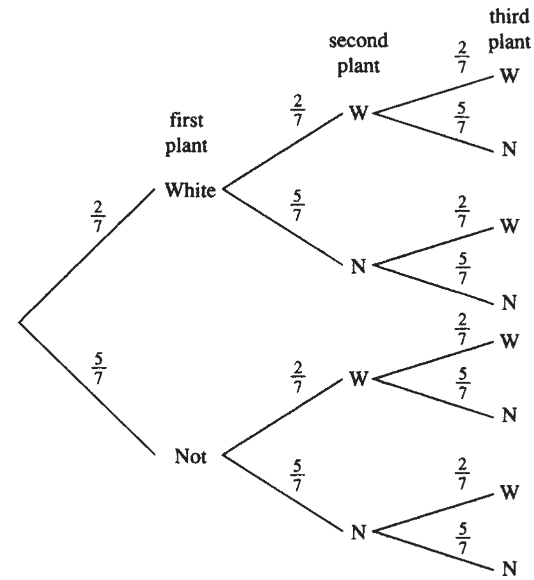
$= -9$

$\therefore (1, -9)$ is a point of inflexion

(iii)



6. (a)



(i) $P(2W, 1N) = P(WWN) + P(WNW) + P(NWW)$

$= \frac{2}{7} \times \frac{2}{7} \times \frac{5}{7} + \frac{2}{7} \times \frac{5}{7} \times \frac{2}{7} + \frac{5}{7} \times \frac{2}{7} \times \frac{2}{7}$

$= \frac{60}{343}$

(ii) $P(\text{at least one W}) = 1 - P(\text{NNN})$

$= 1 - \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7}$

$= \frac{218}{343}$

(b) $x = 60°, 180° - 60°$ (first, second quadrants)

$= 60°, 120°$

(c) (i) $v = \int (6t + 4)\, dt$

$= 3t^2 + 4t + C$

When $t = 0$, $v = 0$

$\therefore \quad 0 = 3(0)^2 + 4(0) + C$

$= C$

$\therefore \quad v = 3t^2 + 4t$

When $t = 5$,

$v = 3(5)^2 + 4(5)$

$= 95 \text{ cms}^{-1}$

(ii) $x = \int (3t^2 + 4t)\, dt$

$= t^3 + 2t^2 + C$

When $t = 0$, $x = 0$

$\therefore \quad 0 = (0)^3 + 2(0)^2 + C$

$= C$

$\therefore \quad x = t^3 + 2t^2$

When $t = 2$,

$\therefore \quad x = (2)^3 + 2(2)^2$

$= 16 \text{ cm}$

7. (a) (ii) $DC = AB = 2$ (opposite sides of $\parallel$ gram)

$\therefore DX = AD = 1 \quad \left(DX = \frac{1}{2}DC - \text{given}\right)$

$\therefore \Delta ADX$ is isosceles

$\angle DAX = \angle DXA = (180° - 60°) \div 2$

$= 60°$

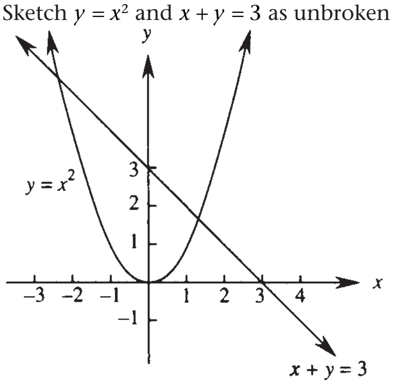
$\therefore ADX$ is equilateral

(iii) $\angle XCB = 180° - 60°$
($\angle ADC$, $\angle XCB$ cointerior $\angle$s, $AD \parallel BC$)
$= 120°$
$\Delta\, CXB$ is isosceles
[$XC = CB = 1$, similar to part (ii)]

$\therefore\ \angle CXB = \angle CBX = (180° - 120°) \div 2$
$= 30°$
$\angle AXB = 180° - (60° + 30°)$
($\angle DXC$ straight angle)
$= 90°$
$\therefore\ \Delta AXB$ is right angled

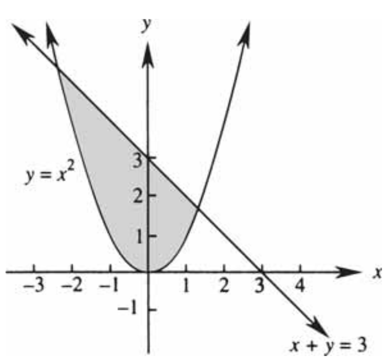
(iv) $AX = 1$ (ΔADX equilateral)
$\therefore\ c^2 = a^2 + b^2$
$2^2 = 1^2 + BX^2$
$4 = 1 + BX^2$
$3 = BX^2$
$\sqrt{3} = BX$

(b) Sketch $y = x^2$ and $x + y = 3$ as unbroken lines.

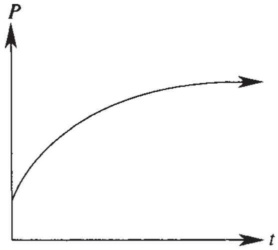


Substitute $(1, 0)$ into $y \geq x^2$
$0 \geq 1^2$ (false)

Substitute $(1, 0)$ into $x + y \leq 3$
$1 + 0 \leq 3$ (true)



(c) (i)



(ii) The curve is increasing so $\dfrac{dP}{dt} > 0$.
The curve is concave downwards so $\dfrac{d^2P}{dt^2} < 0$.

8. (a) (i)

x	1	2	3	4	5
y	0	0.301	0.477	0.602	0.699

(ii) $\displaystyle\int_a^b f(x)\,dx = \frac{1}{2}(b-a)[f(a)+f(b)]$

$$\int_1^5 \log_{10} x\,dx = \frac{1}{2}(2-1)[f(1)+f(2)] + \frac{1}{2}(3-2)[f(2)+f(3)] + \frac{1}{2}(4-3)[f(3)+f(4)] + \frac{1}{2}(5-4)[f(4)+f(5)]$$
$$= \frac{1}{2}(\log 1 + \log 2) + \frac{1}{2}(\log 2 + \log 3) + \frac{1}{2}(\log 3 + \log 4) + \frac{1}{2}(\log 4 + \log 5)$$
$$\doteqdot 1.73$$

(b) (i) When $t = 0$, $P = 20$
$20 = P_0 e^0$
$= P_0$
$\therefore\ P = 20e^{kt}$
When $t = 6$, $P = 100$
$100 = 20e^{k(6)}$
$\dfrac{100}{20} = e^{6k}$
$5 = e^{6k}$
$\ln 5 = \ln e^{6k}$
$= 6k \ln e$
$= 6k$
$\dfrac{\ln 5}{6} = k$
$0.268 \doteqdot k$

(ii) $P = 20e^{0.268t}$
When $t = 10$,
$P = 20e^{0.268(10)}$
$= 20e^{2.68}$
$= 292$ mice

(iii) When $P = 500$,
$500 = 20e^{0.268t}$
$\dfrac{500}{20} = e^{0.268t}$
$25 = e^{0.268t}$
$\ln 25 = \ln e^{0.268t}$
$= 0.268t \ln e$
$= 0.268t$
$\dfrac{\ln 25}{0.268} = t$
$12 = t$ (i.e. after 12 weeks)

9. (a) (i) $S = 160$
$\therefore\ 2\pi r(r + h) = 160$
$r + h = \dfrac{160}{2\pi r}$
$h = \dfrac{160}{2\pi r} - r$

$$= \frac{80}{\pi r} - r$$
$$V = \pi r^2 h$$
$$= \pi r^2 \left(\frac{80}{\pi r} - r\right)$$
$$= 80r - \pi r^3$$

(ii) $V' = 80 - 3\pi r^2$

For max./min. volume, $V' = 0$

i.e. $80 - 3\pi r^2 = 0$
$$80 = 3\pi r^2$$
$$\frac{80}{3\pi} = r^2$$
$$\pm\sqrt{\frac{80}{3\pi}} = r$$
$$\pm 2.91 \doteqdot r$$

$V'' = -6\pi r$

When $r = 2.91$, $V'' = -6\pi(2.91)$

< 0 (maximum)

$\therefore$ $r = 2.91$ cm

(iii) When $r = 2.91$, $V = 80(2.91) - \pi(2.91)^3$
$= 206.4\text{ cm}^3$

(b) 1996 to 2025 inclusive is 30 years.

$A = 500(1.12^{30}) + 500(1.12^{29}) + 500(1.12^{28}) + \ldots + 500(1.12^1)$

$= 500(1.12^{30} + 1.12^{29} + \ldots + 1.12^1)$

$= 500(1.12^1 + 1.12^2 + \ldots + 1.12^{30})$

$1.12^1 + 1.12^2 + \ldots + 1.12^{30}$ is a geometric series

$a = 1.12$, $r = 1.12$
$$S_n = \frac{a(r^n - 1)}{r - 1}$$
$$S_{30} = \frac{1.12(1.12^{30} - 1)}{1.12 - 1}$$
$$\doteqdot 270.29$$

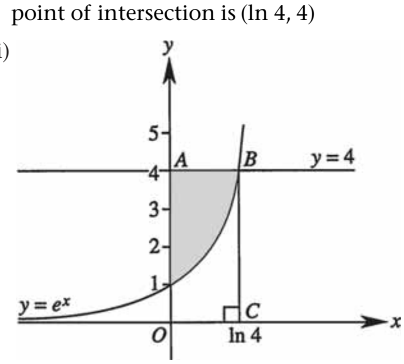
$\therefore A = 500(270.29)$

$= \$135\,146.30$

10. (a) (i) Substitute $y = 4$ into $y = e^x$
$$4 = e^x$$
$\therefore$ $\ln 4 = x$ (by definition of log)

$\therefore$ point of intersection is $(\ln 4, 4)$

(ii)



A = Area of rectangle $OABC - \int_0^{\ln 4} e^x\,dx$

$= 4\ln 4 - [e^x]_0^{\ln 4}$

$= 4\ln 4 - (e^{\ln 4} - e^0)$

$= 4\ln 4 - 4 + 1$

$= (4\ln 4 - 3)\text{ units}^2$

(b) (i) For real, equal roots, $\Delta = 0$

i.e. $b^2 - 4ac = 0$

$(k-1)^2 - 4(1)(k) = 0$

$k^2 - 2k + 1 - 4k = 0$

$k^2 - 6k + 1 = 0$
$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{6 \pm \sqrt{32}}{2}$$
$$= \frac{6 \pm 4\sqrt{2}}{2}$$
$$= 3 \pm 2\sqrt{2}$$

(ii) When $k = 5$,

$x^2 + (k-1)x + k = x^2 + 4x + 5$

$a > 0$

$\Delta = b^2 - 4ac$

$= 4^2 - 4(1)(5)$

$= -4$

$\therefore$ $\Delta < 0$

Since $a > 0$ and $\Delta < 0$, $x^2 + 4x + 5 > 0$ for all x

(c) (i) $y = x^2 + 2px + q$

$y - q = x^2 + 2px$

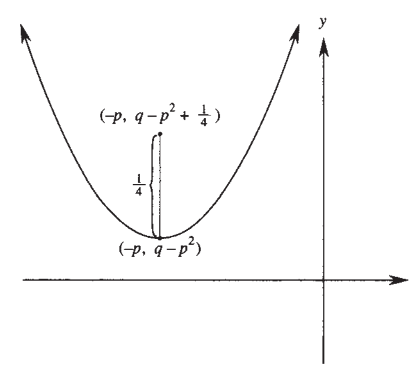
$y - q + p^2 = x^2 + 2px + p^2$

$y - (q - p^2) = (x + p)^2$

This is in the form $(x-h)^2 = 4a(y-k)$, where a is the focal length and (h, k) is the vertex. $h = -p$ and $k = q - p^2$

$\therefore$ vertex is $(-p, q - p^2)$

(ii) $4a = 1$
$$\therefore a = \frac{1}{4}$$



Count up $\frac{1}{4}$ units for the focus

$\therefore$ focus is $\left(-p, q - p^2 + \frac{1}{4}\right)$

(iii) For P: $x = m$ since it is vertically below $(m, 3m^2 + q)$

When $x = m$

$m^2 = 8y$
$$\frac{m^2}{8} = y$$

So $P = \left(m, \frac{m^2}{8}\right)$

$$\text{Distance} = \left|3m^2 + q - \frac{m^2}{8}\right|$$
$$= \left|\frac{23m^2}{8} + q\right|$$
$$= \frac{23m^2}{8} + q \qquad \text{since } m^2 \geq 0 \text{ and } q > 0$$

(iv) $m + q = 5$

$\therefore \quad q = 5 - m$

$$D = \frac{23m^2}{8} + q$$
$$= \frac{23m^2}{8} + 5 - m$$
$$\frac{dD}{dm} = \frac{46m}{8} - 1$$

For stationary points $\frac{dD}{dm} = 0$

$$\frac{46m}{8} - 1 = 0$$
$$\frac{46m}{8} = 1$$
$$46m = 8$$
$$m = \frac{8}{46}$$
$$= \frac{4}{23}$$

So there is a stationary point at $m = \frac{4}{23}$.

To determine its nature

$$\frac{d^2D}{dm^2} = \frac{46}{8}$$
$$> 0$$

So concave upwards.

$\therefore$ minimum turning point

When $m = \frac{4}{23}$

$$D = \frac{23m^2}{8} + 5 - m$$
$$= \frac{23\left(\frac{4}{23}\right)^2}{8} + 5 - \frac{4}{23}$$
$$= 4\frac{21}{23}$$

So minimum distance is $4\frac{21}{23}$ units.

Mathematics—Paper 2

1. (a) (i) $x - 3 = 5$

$x = 8$

(ii) $x - 3 = -5$

$x = -2$

(b) $5 - x^2 = -4$

$9 - x^2 = 0$

$9 = x^2$

$\pm 3 = x$

(c) $\sin\frac{5\pi}{6}$

$= \sin\left(\pi - \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{6} \qquad$ (2nd quadrant)

$= \frac{1}{2}$

(d) $a^2(a-2) - 4(a-2)$

$= (a-2)(a^2-4)$

$= (a-2)(a+2)(a-2)$

$= (a+2)(a-2)^2$

(e) $2 \times \sqrt{4} \times \sqrt{6} - \sqrt{25} \times \sqrt{6}$

$= 2 \times 2 \times \sqrt{6} - 5 \times \sqrt{6}$

$= 4\sqrt{6} - 5\sqrt{6}$

$= -\sqrt{6}$

(f) $\log_a 50 = \log_a (5^2 \times 2)$

$= \log_a 5^2 + \log_a 2$

$= 2\log_a 5 + \log_a 2$

$= 2 \times 1.3 + 0.43$

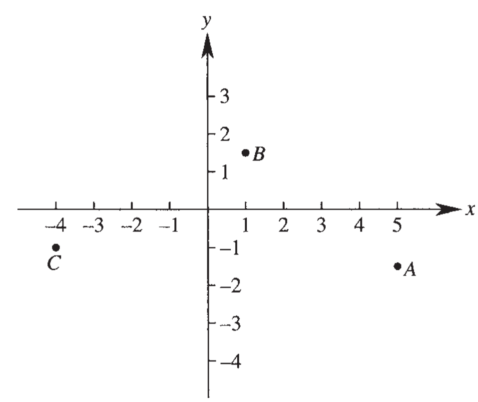
$= 3.03$

(g) $P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-3+0}{2}, \frac{4+-2}{2}\right)$

$= \left(-1\frac{1}{2}, 1\right)$

2.



(a) Substitute $A\left(5, -1\frac{1}{2}\right)$ into $3x + 4y - 9 = 0$

$\text{LHS} = 3 \times 5 + 4 \times -1\frac{1}{2} - 9$

$= 0$

$= \text{RHS}$

$\therefore$ A lies on the line

Substitute $B\left(1, 1\frac{1}{2}\right)$ into $3x + 4y - 9 = 0$

$\text{LHS} = 3 \times 1 + 4 \times 1\frac{1}{2} - 9$

$= 0$

$= \text{RHS}$

$\therefore$ B lies on the line

$\therefore$ AB has equation $3x + 4y - 9 = 0$

(b) $3x + 4y - 9 = 0$

$4y = -3x + 9$

$y = -\frac{3}{4}x + \frac{9}{4}$

$\therefore \quad m_1 = -\frac{3}{4}$

l is perpendicular to AB, so $m_1 m_2 = -1$

$$-\frac{3}{4}m_2 = -1$$
$$\therefore \quad m_2 = \frac{4}{3}$$

Equation of l:

$$y - y_1 = m(x - x_1)$$
$$y - -1 = \frac{4}{3}(x - -4)$$
$$3y + 3 = 4(x+4)$$
$$= 4x + 16$$
$$0 = 4x - 3y + 13$$

(c) $4x - 3y + 13 = 0$ (1)
$3x + 4y - 9 = 0$ (2)
(1) × 4: $16x - 12y + 52 = 0$ (3)
(2) × 3: $9x + 12y - 27 = 0$ (4)
(3) + (4): $25x + 25 = 0$
$25x = -25$
$x = -1$

Substitute $x = -1$ in (1):

$$4 \times -1 - 3y + 13 = 0$$
$$9 - 3y = 0$$
$$9 = 3y$$
$$3 = y$$

So point of intersection is $(-1, 3)$.

(d) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB:

$$d = \sqrt{(5-1)^2 + \left(-1\frac{1}{2} - 1\frac{1}{2}\right)^2}$$
$$= \sqrt{4^2 + (-3)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25}$$
$$= 5$$

CP:

$$d = \sqrt{(-1 - -4)^2 + (3 - -1)^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 5 \times 5$$
$$= 12.5 \text{ units}^2$$

(e) Midpoint $AC = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \left(\frac{-4+5}{2}, \frac{-1 + -1\frac{1}{2}}{2}\right)$$
$$= \left(\frac{1}{2}, -1\frac{1}{4}\right)$$

Midpoint AC = midpoint BD

where $D = (x, y)$

$$x = \frac{x_1 + x_2}{2}$$
$$\frac{1}{2} = \frac{x+1}{2}$$
$$1 = x + 1$$
$$0 = x$$
$$y = \frac{y_1 + y_2}{2}$$
$$-1\frac{1}{4} = \frac{y + 1\frac{1}{2}}{2}$$
$$-2\frac{1}{2} = y + 1\frac{1}{2}$$
$$-4 = y$$

(f) So $D = (0, -4)$

3. (a) (i) $\frac{dy}{dx} = u'v + v'u$

$$= 1 \cdot \cos x + (-\sin x)x$$
$$= \cos x - x\sin x$$

(ii) $\frac{dy}{dx} = 5e^{5x}$

(iii) $\frac{dy}{dx} = \frac{4x}{2x^2 - 1}$

(b) (i) $\frac{(3x-2)^5}{3 \times 5} + C$

$$= \frac{(3x-2)^5}{15} + C$$

(ii) $-3 \times \frac{1}{2}\cos 2x + C$

$$= -\frac{3}{2}\cos 2x + C$$

(c) $\left[e^x - \frac{1}{-1}e^{-x}\right]_0^3$

$$= [e^x + e^{-x}]_0^3$$
$$= [e^3 + e^{-3}] - [e^0 + e^{-0}]$$
$$= e^3 + e^{-3} - 1 - 1$$
$$= e^3 + e^{-3} - 2$$

(d) $\frac{dy}{dx} = \int (18x - 6)\, dx$

$$= 9x^2 - 6x + C$$

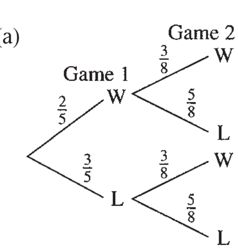
At $(2, -1)$, $\frac{dy}{dx} = 0$

$$0 = 9(2)^2 - 6(2) + C$$
$$= 24 + C$$
$$-24 = C$$
$$\therefore \frac{dy}{dx} = 9x^2 - 6x - 24$$
$$y = \int (9x^2 - 6x - 24)\, dx$$
$$= 3x^3 - 3x^2 - 24x + C$$

Substitute $(2, -1)$:

$$-1 = 3(2)^3 - 3(2)^2 - 24(2) + C$$
$$= -36 + C$$
$$35 = C$$
$$\therefore y = 3x^3 - 3x^2 - 24x + 35$$

4. (a)



(i) $P(\text{WW}) = \frac{2}{5} \times \frac{3}{8}$

$= \frac{3}{20}$

(ii) $P(\text{WL}) + P(\text{LW}) = \frac{2}{5} \times \frac{5}{8} + \frac{3}{5} \times \frac{3}{8}$

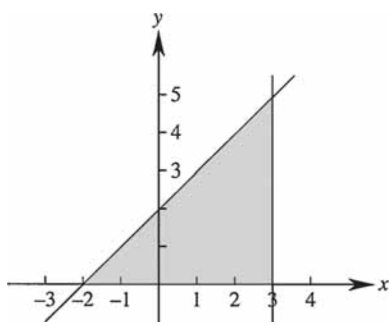
$= \frac{19}{40}$

(iii) $P(\text{at least 1W}) = 1 - P(\text{LL})$

$= 1 - \frac{3}{5} \times \frac{5}{8}$

$= \frac{5}{8}$

(b) (i)



(ii) $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 5$

$= 12.5 \text{ units}^2$

or $A = \int_{-2}^{3} (x + 2)\, dx$

$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{3}$

$= \left[\frac{3^2}{2} + 2(3)\right] - \left[\frac{(-2)^2}{2} + 2(-2)\right]$

$= 12.5 \text{ units}^2$

(iii) $y = x + 2$

$\therefore \quad y^2 = (x + 2)^2$

$V = \pi \int_a^b y^2\, dx$

$= \pi \int_{-2}^{3} (x + 2)^2\, dx$

$= \pi \left[\frac{(x + 2)^3}{1 \times 3}\right]_{-2}^{3}$

$= \pi \left[\frac{(3 + 2)^3}{3} - \frac{(-2 + 2)^3}{3}\right]$

$= \pi \left[\frac{125}{3} - 0\right]$

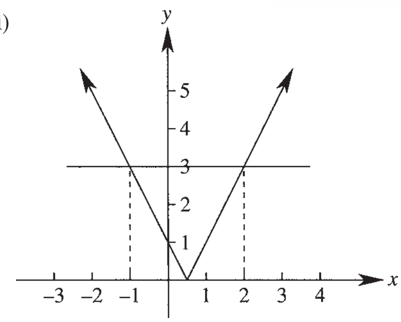
$= \frac{125\pi}{3} \text{ units}^3$

or $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (5)^2 \times 5$

$= \frac{125\pi}{3} \text{ units}^3$

(c) (i)



(ii) $-1 < x < 2$

5. (a) $y = 2x^3 - 9x^2 + 12x - 7$

$\frac{dy}{dx} = 6x^2 - 18x + 12$

$\frac{d^2 y}{dx^2} = 12x - 18$

(i) For stationary points, $\frac{dy}{dx} = 0$

$6x^2 - 18x + 12 = 0$

$x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

$x = 1, 2$

When $x = 1$, $y = 2(1)^3 - 9(1)^2 + 12(1) - 7 = -2$

When $x = 2$, $y = 2(2)^3 - 9(2)^2 + 12(2) - 7 = -3$

So $(1, -2)$ and $(2, -3)$ are stationary points.

At $(1, -2)$ $\frac{d^2 y}{dx^2} = 12(1) - 18 = -6$

$\therefore$ $(1, -2)$ is a maximum turning point

At $(2, -3)$ $\frac{d^2 y}{dx^2} = 12(2) - 18 = 6$

$\therefore$ $(2, -3)$ is a minimum turning point

(ii) For points of inflexion $\frac{d^2 y}{dx^2} = 0$

$12x - 18 = 0$

$12x = 18$

$x = 1.5$

When $x = 1.5$,

$y = 2(1.5)^3 - 9(1.5)^2 + 12(1.5) - 7$

$= -2.5$

Check concavity:

x	1.25	1.5	1.75
$\frac{d^2 y}{dx^2}$	−3	0	3

Concavity changes, so $(1.5, -2.5)$ is a point of inflexion.

(iii) When $x = -3$,

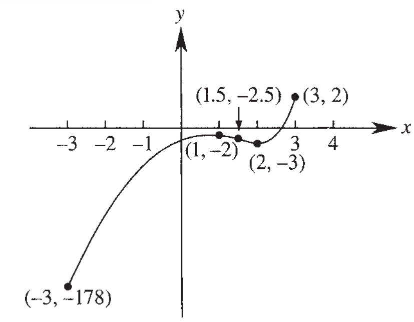
$y = 2(-3)^3 - 9(3)^2 + 12(-3) - 7$

$= -178$

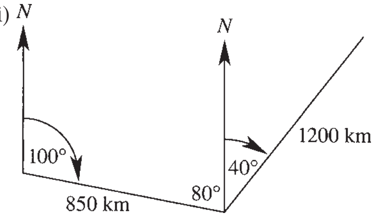
When $x = 3$,

$y = 2(3)^3 - 9(3)^2 + 12(3) - 7$

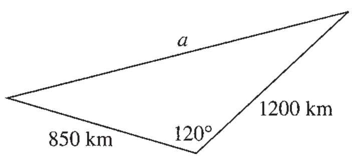
$= 2$



(b) (i)



(ii)



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 850^2 + 1200^2 - 2 \times 850 \times 1200 \cos 120°$$
$$= 3\,182\,500$$
$$c = \sqrt{3182500}$$
$$= 1784$$

So the plane is 1784 km from the airport.

(c) $(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)$

$= \operatorname{cosec}^2\theta - \cot^2\theta$

$= 1 + \cot^2\theta - \cot^2\theta$

$= 1$

6. (a) (i) $\dfrac{dy}{dx} = 2x$

At $(-2, 4)$ $\dfrac{dy}{dx} = 2(-2)$

$\therefore \quad m_1 = -4$

Normal is perpendicular to tangent

$\therefore \quad m_1 m_2 = -1$

$-4m_2 = -1$

$m_2 = \dfrac{1}{4}$

$y - y_1 = m(x - x_1)$

$y - 4 = \dfrac{1}{4}(x - -2)$

$4y - 16 = x + 2$

$0 = x - 4y + 18 \qquad (1)$

(ii) $y = x^2 \qquad (2)$

Substitute (2) in (1):

$0 = x - 4x^2 + 18$

$4x^2 - x - 18 = 0$

$(x + 2)(4x - 9) = 0$

$x + 2 = 0, \; 4x - 9 = 0$

$x = -2, \; 4x = 9$

$x = 2\dfrac{1}{4}$

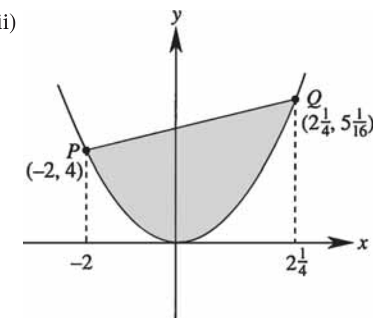
Substitute $x = 2\dfrac{1}{4}$ in (2):

$y = \left(2\dfrac{1}{4}\right)^2$

$= 5\dfrac{1}{16}$

$\therefore \quad Q = \left(2\dfrac{1}{4}, 5\dfrac{1}{16}\right)$

(iii)



PQ: $x - 4y + 18 = 0$

$x + 18 = 4y$

$\dfrac{x}{4} + \dfrac{18}{4} = y$

$$\text{Area} = \int_{-2}^{2\frac{1}{4}} \left(\frac{x}{4} + \frac{18}{4} - x^2\right) dx$$
$$= \left[\frac{x^2}{8} + \frac{18x}{4} - \frac{x^3}{3}\right]_{-2}^{2\frac{1}{4}}$$
$$= \left[\frac{\left(2\frac{1}{4}\right)^2}{8} + \frac{18\left(2\frac{1}{4}\right)}{4} - \frac{\left(2\frac{1}{4}\right)^3}{3}\right] - \left[\frac{(-2)^2}{8} + \frac{18(-2)}{4} - \frac{(-2)^3}{3}\right]$$
$$= 12.8 \text{ units}^2$$

(b) (i) The particle is at the origin when $x = 0$, i.e. at t_1, t_3 and t_5

(ii) At rest, $\dfrac{dx}{dt} = 0$ (at the stationary points, i.e. t_2 and t_4)

(c) $T = T_0 e^{-kt}$

When $t = 0$, $T = 97$

$\therefore \quad T_0 = 97$

$T = 97e^{-kt}$

When $t = 5$, $T = 84$

$84 = 97e^{-k \times 5}$

$\dfrac{84}{97} = e^{-5k}$

$\ln\dfrac{84}{97} = \ln e^{-5k}$

$= -5k \ln e$

$= -5k$

$\dfrac{\ln\frac{84}{97}}{-5} = k$

$0.029 = k$

So $\quad T = 97e^{-0.029t}$

(i) When $t = 15$

$T = 97e^{-0.029 \times 15}$

$= 63$

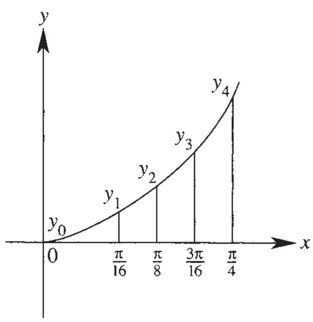
So the temperature is 63°C after 15 minutes.

(ii) When $T = 20$

$$20 = 97\, e^{-0.029t}$$
$$\frac{20}{97} = e^{-0.029t}$$
$$\ln \frac{20}{97} = \ln e^{-0.029t}$$
$$= -0.029t \ln e$$
$$= -0.029t$$
$$\frac{\ln \frac{20}{97}}{-0.029} = t$$
$$54.9 = t$$

So the temperature is 20°C after 54.9 minutes.

7. (a) (i)



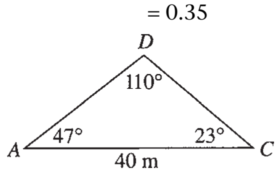
$$\int_a^b f(x)\,dx \doteqdot \frac{h}{3}[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2]$$

$$\int_0^{\frac{\pi}{4}} \tan x\,dx \doteqdot \frac{\frac{\pi}{16}}{3}\left[\left(\tan 0 + \tan\frac{\pi}{4}\right) + 4\left(\tan\frac{\pi}{16} + \tan\frac{3\pi}{16}\right) + 2\tan\frac{\pi}{8}\right]$$
$$\doteqdot 0.35 \text{ units}^2$$

(ii) $$\frac{dy}{dx} = \frac{-\sin x}{\cos x}$$
$$= -\tan x$$

(iii) $$\int_0^{\frac{\pi}{4}} \tan x\,dx = \left[-\ln(\cos x)\right]_0^{\frac{\pi}{4}}$$
$$= \left[-\ln\left(\cos\frac{\pi}{4}\right)\right] - [-\ln(\cos 0)]$$
$$= 0.35$$

(b) (i)



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{AD}{\sin 23°} = \frac{40}{\sin 110°}$$
$$AD = \frac{40\sin 23°}{\sin 110°}$$
$$= 16.6 \text{ m}$$

(ii) $$\sin 47° = \frac{BD}{16.6}$$
$$16.6 \sin 47° = BD$$
$$12.2 = BD$$

So the height is 12.2 m

(c) $\angle CBE = 50°$ (base $\angle$s of isosceles Δ)

$\angle DCB = 50° + 50°$ (exterior $\angle$ of ΔCBE)

$= 100°$

$\angle ABC = 130° - 50°$

$= 80°$

$\angle DAB = 360° - (100° + 80° + 80°)$

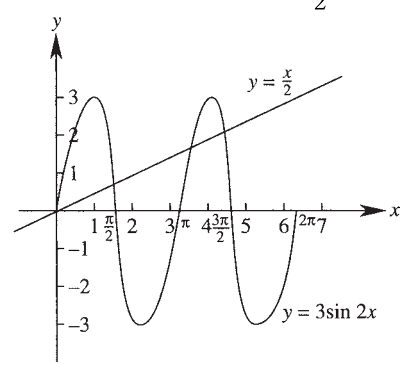
($\angle$ sum of quadrilateral)

$= 100°$

$\therefore$ $\angle DAB = \angle DCB$ and $\angle ABC = \angle ADC$

$\therefore$ ABCD is a parallelogram (opposite $\angle$s equal)

8. (a) (i) & (ii) Amplitude = 3, period $= \frac{2\pi}{2} = \pi$



(iii) 4 points of intersection, so 4 roots

(b) $2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$ (1st, 2nd quadrants)

$x = 30°, 180° - 30°$

$= 30°, 150°$

(c) (i) $\log_x 12 = \log_x (2^2 \times 3)$

$= \log_x 2^2 + \log_x 3$

$= 2\log_x 2 + \log_x 3$

$= 2q + p$

(ii) $\log_x 2x = \log_x 2 + \log_x x$

$= q + 1$

(d) (i) $1 + 3 + 5 + \ldots$ is an arithmetic series with $a = 1$, $d = 2$

When $n = 12$

$T_n = a + (n-1)d$

$T_{12} = 1 + (12 - 1) \times 2$

$= 23$

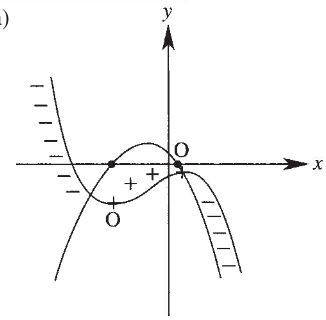
So there are 23 oranges in the 12th row.

(ii) Total number of oranges is 289, so $S_n = 289$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$289 = \frac{n}{2}[2 \times 1 + (n-1)2]$$
$$578 = n[2 + 2n - 2]$$
$$= n \times 2n$$
$$= 2n^2$$
$$289 = n^2$$
$$\sqrt{289} = n$$
$$17 = n$$

So there are 17 rows of oranges altogether.

9. (a)



(b) The statement would only be true if there were equal numbers of each colour. It is probably false.

(c)
$$\ln x^2 = \ln(2x+3)$$
$$\therefore \quad x^2 = 2x+3$$
$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x = 3, -1$$
But $x \neq -1$ (ln −1 does not exist)

so the solution is $x = 3$

(d) (i) $\dfrac{dy}{dx} = e^x$

When $x = k$

$$\frac{dy}{dx} = e^k$$

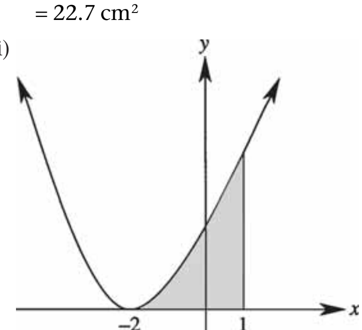
So gradient $m = e^k$

(ii) When $x = k$, $y = e^k$
$$y - y_1 = m(x - x_1)$$
$$y - e^k = e^k(x-k)$$
$$= e^k x - ke^k$$
$$y = e^k x - ke^k + e^k$$
$$= e^k(x - k + 1)$$

(iii) Substitute (2, 0) into the equation
$$0 = e^k(2 - k + 1)$$
$$= e^k(3-k)$$
$$3 - k = 0$$
$$3 = k$$

(e) $180° = \pi$ radians
$$1° = \frac{\pi}{180°}$$
$$\therefore \quad 53° = \frac{\pi}{180°} \times 53°$$
$$= \frac{53\pi}{180}$$
$$A = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 7^2 \times \frac{53\pi}{180}$$
$$= 22.7 \text{ cm}^2$$

10. (a) (i)



(ii) $y = (x+2)^2$

$\therefore \quad y^2 = (x+2)^4$
$$V = \pi\int_a^b y^2\,dx$$
$$= \pi\int_{-2}^{1} (x+2)^4\,dx$$
$$= \pi\left[\frac{(x+2)^5}{1 \times 5}\right]_{-2}^{1}$$
$$= \pi\left[\frac{(1+2)^5}{5} - \frac{(-2+2)^5}{5}\right]$$
$$= \pi\left[\frac{243}{5} - 0\right]$$
$$= \frac{243\pi}{5} \text{ units}^3$$

(b) (i) $s = \dfrac{d}{t}$

So $t = \dfrac{d}{s}$
$$= \frac{3000}{s}$$

Cost of trip taking t hours:

$C = (s^2 + 7500)t$
$$= (s^2 + 7500)\frac{3000}{s}$$
$$= 3000s + \frac{7500 \times 3000}{s}$$
$$= 3000\left(s + \frac{7500}{s}\right)$$

(ii) $C = 3000\left(s + \dfrac{7500}{s}\right)$
$$= 3000(s + 7500s^{-1})$$
$$\frac{dC}{ds} = 3000(1 - 7500s^{-2})$$
$$= 3000\left(1 - \frac{7500}{s^2}\right)$$

For minimum cost, $\dfrac{dC}{ds} = 0$
$$3000\left(1 - \frac{7500}{s^2}\right) = 0$$
$$1 - \frac{7500}{s^2} = 0$$
$$1 = \frac{7500}{s^2}$$
$$s^2 = 7500$$
$s = \sqrt{7500}$ (speed is positive)
$$= 86.6 \text{ km/h}$$

Check:
$$\frac{d^2C}{ds^2} = 3000(15\,000s^{-3})$$
$$= 3000\left(\frac{15\,000}{s^3}\right)$$

When $s = 86.6$
$$\frac{d^2C}{ds^2} = 3000\left(\frac{15\,000}{86.6^3}\right)$$
$$> 0$$

Concave upwards

So minimum when $s = 86.6$

(iii) $C = 3000\left(86.6 + \dfrac{7500}{86.6}\right)$
$$= 519\,615 \text{ cents}$$
$$= \$5196.15$$