Lab 6

Discrete Fourier Series & Discrete-Time Fourier Transform

Problem 1 (3 points)

Matlab filename must be `exer01.m`.

A discrete signal received by LIGO (Laser Interferometer Gravitational-Wave Observatory) is provided in file `din.dat`. The underlying analog signal is sampled at 4096 Hz. The signal can be loaded into Matlab using the following command:

```
>> load din.dat;
>> stem(din);
```

The variable `din` in Matlab is a column vector with the samples of the discrete signal. First let us assume that the `din` signal is periodic and its period is $N =$ `length(din)`.

a) Calculate the Discrete Fourier Series coefficients (c_k) of signal `din`. Plot the coefficients $|c_k|$ against a frequency axis in Hz showing positive and negative frequencies.

b) Frequency resolution of the signal's spectrum in Hz.

c) Make $c_k = 0$ for frequencies beyond 500 Hz, compute the inverse Discrete Fourier Series and plot it with the original signal. Please note that you must take into account the frequency in Hertz in order to discard the corresponding c_k.

d) Plot the error signal and calculate the Signal-to-Error Ratio (dB) of the approximated signal.

e) The approximation in our case is good, what is the minimum required sampling rate and what would be the length of the data obtained using that sampling rate. Compare with the original data and discuss the advantages of using a lower sampling rate.

Problem 2 (4 points)

Matlab filename must be `exer02.m`.

Now let us assume that signal `din` is non-periodic. In this case the Fourier Transform to employ must the Discrete-Time Fourier Transform.

a) Sample the whole Discrete-Time Fourier Transform of `din` with $N = 2048$ points and plot the resulting spectrum along with the $|c_k|$ coefficients computed above. Use a frequency axis in Hertz to display the results.

b) Reconstruct the time-domain signal by computing the Inverse Discrete-Time Fourier Transform by using numerical integration with the samples obtained in a). Plot the reconstructed signal along with the original signal.

c) Discuss the result obtained in b)

Exercise 3 (3 points)

Matlab filename must be `exer03.m`.

A radar system transmits a pulsed signal with the following characteristics:

- The sampling frequency used to generate the pulse is 1.023 MHz.
- The pulse is a pseudo-random noise sequence that you can generate using the Matlab function `cacode()`. The sequence length of the pulse is 1023 samples, which means that the duration of the pulse is 1ms. The output of `cacode()` are '1s' and '0s'. Map the '1s' to -1 and the '0s' as '1';
- The duty cycle of the pulsed signal is 2%. (This means that the chirp is active only 2% of the time, being the signal equal to zero when not active)
- The number of chirp pulses sent by the transmitter are $N = 20$.

The final result is a sequence of duration 1 second with 20 pseudo-random pulses of duration 1 ms each uniformly separated between them. After hitting the target, the signal is detected by the radar. A extremely simplified model of the incoming signal can be written as $y[n] = \alpha \cdot s[n - k] + w[n]$, where $s[n]$ is the transmitted signal, $\alpha < 1$ models the channel losses, k (a positive integer) is the time it takes the signal to travel from the transmitter to the receiver (in number of samples), and $w[n]$ is additive white gaussian noise (AWGN).

Let us assume that in our case $\alpha = 0.1$ and $k = 20$.

The radar system will correlate the incoming signal with the pseudo-random pulse of duration 1 ms. That will hopefully produce a peak in the cross-correlation operation. The position of the peak indicates an estimation of the delay k.

You must provide one delay estimation per second using the $N = 20$ pseudo-random pulses.

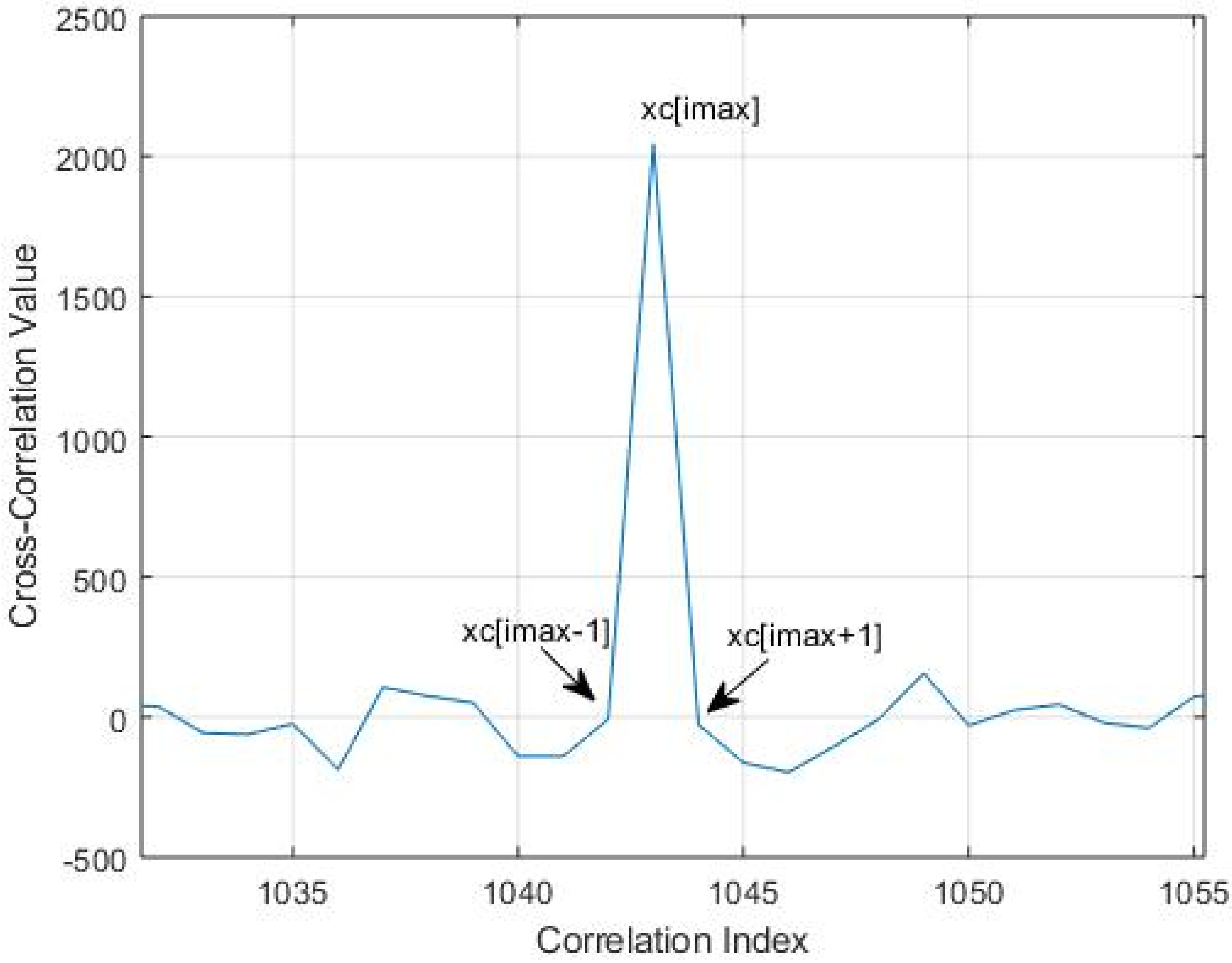

Figure 6.1: Correlation peak and delay estimation

1. Use an appropriate cross-correlation function to correlate signals $s[n]$ and $y[n]$ and detect the signal delay in $y[n]$ in the absence of noise ($w[n] = 0$).

2. Now, add AWGN noise to the received signal so that you have a SNR=-25dB and determine the mean (μ) and standard deviation (σ) of the estimated delay.

3. Generate a plot $SNR - \sigma$ for different values of SNR, for example from SNR=-43dB to SNR=-33dB.

```
% Use of cacode() function generates 1023 length C/A Codes for GPS ...
   PRNs 1-37
% function g=cacode(sv,fs)
% sv is the satellite code (integer between 1-37). You can ...
   generate several codes if
% you use a vector instead of an scalar
```

```
% fs is the number of samples per chip (fs ≥ 1, can be a ...
    fractional number)
% To generate the CA code for PRN 6 with one sample per chip
g=cacode(6,1);
% You can check that g is a vector of length 1023
```

```
% The operation of cross-correlation can be done using different ...
    approaches:
% Remember the definition of correlation
%       r[l] = sum(n=-inf..inf) x[n]*y[n-l]
% First we are going to use the xcorr() function
r1 = xcorr(x,y); % length(r1) = 2*max(length(x),length(y))-1
% We can also use the conv() function
r2 = conv(x,fliplr(y)); % length(r2) = length(x)+length(y)-1
% Finally we can also use the filter() function
r3 = filter(fliplr(y),1,x); % length(r3) = length(x)
```