Lab 5

Fourier Series and Fourier Transform

Exercise 1

Matlab filename must be `exer01.m`.

One period of a periodic signal $x_p(t)$ is given by the expression below:

$$x_p(t) = \begin{cases} \cos^2\left(2\pi \frac{t}{t_0}\right) & |t| \le \frac{t_0}{4} \\ 0 & \frac{t_0}{4} \le |t| \le \frac{T}{2} \end{cases}$$

The period of this signal is T, where

$$T = 6 \cdot 10^{-3} s$$
$$t_0 = 2.7 \cdot 10^{-3} s$$

Part 1: Fourier Series (5 points)

a) Obtain manually the mathematical expression of the Fourier Series coefficients c_k of $x_p(t)$. Be careful with the expression of c_k at $k = 0$. You can use Matlab Symbolic Math Toolbox to check the result.

b) Define two anonymous functions in Matlab to describe $x_p(t)$ and c_k using T, t_0 as input parameters.

c) Reconstruct the time domain signal using $2N + 1$ coefficients ($k = -N \dots N$). Determine N such as the error in the reconstruction is 96dB below the original signal power. Determine the maximum frequency present in the $x_p(t)$ under that criteria.

d) Numerically compute the c_k coefficients in Matlab (let us call the new coefficients $\hat{c}_k$). You must sample $x_p(t)$ during a period T at a frequency that takes into account the previous result and obtain M samples. Plot $|\hat{c}_k|$ and $|c_k|$ for $k = -2N, \dots, 2N$ in the same figure.

e) Discuss the above result

Part 2: Fourier Transform (5 Points)

Now consider a signal $x(t)$ such that:

$$x(t) = \begin{cases} x_p(t) & |t| \leq \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$$

a) Determine the mathematical expression of the Fourier Transform of $x(t)$, $X(f)$. Again, you can use Maple to check the result.

b) Define an anonymous function to describe $X(f)$.

c) Compute the energy of signal $x(t)$, $E_x = \int_{-\infty}^{\infty} x(t)^2 dt$.

d) Using the Parseval Relation and the analytical expression of $X(f)$ in b), find the bandwidth of the signal whose energy E_s is such that $10 \log_{10} \left(\frac{E_x}{E_x - E_s} \right) > 96$ dB.

e) Numerically compute the Inverse Fourier Transform of $X(f)$, $\hat{x}(t)$ (use result in b)). To compute the integration, use the bandwidth calculated in d) and a frequency step of $\Delta f = 50Hz$. Evaluate $\hat{x}(t)$ in the interval $t \in [-5T, 5T]$ and plot the result in the same figure as $x(t)$.

f) Discuss the above result.