

Praktikum 3 08.03

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Aufgabe 1

$$D_f(x_1, x_2) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(x_1, x_2) & \frac{\partial f_2}{\partial x_1}(x_1, x_2) \\ \frac{\partial f_1}{\partial x_2}(x_1, x_2) & \frac{\partial f_2}{\partial x_2}(x_1, x_2) \end{pmatrix} = \begin{pmatrix} -18 & -4x_2 \\ -4x_2 & 12x_2^2 - 4x_1 \end{pmatrix}$$

$$\frac{\partial f_1}{\partial x_1} = -18$$

$$\frac{\partial f_2}{\partial x_1} = -4x_2$$

$$\frac{\partial f_1}{\partial x_2} = -4x_2$$

$$\frac{\partial f_2}{\partial x_2} = -4x_1 + 24x_2$$

$$x^{(0)} = (1.1, 0.9)^T$$

$$D_f(1.1, 0.9) \delta^{(0)} = -f(1.1, 0.9) \Rightarrow \begin{pmatrix} -18 & -3.6 \\ -3.6 & 5.32 \end{pmatrix} \cdot \delta^{(0)} = -\begin{pmatrix} -1.92 \\ -1.044 \end{pmatrix}$$

$$\Rightarrow \delta^{(0)} = \begin{pmatrix} -0.10405445 \\ 0.12582781 \end{pmatrix}$$

$$x^{(1)} = x^{(0)} + \delta^{(0)} = \underline{\underline{\begin{pmatrix} 0.9959455481972038 \\ 1.0258278145695365 \end{pmatrix}}}$$

$$D_f(0.99595, 1.02583) \cdot \delta^{(1)} = -f(0.99595, 1.02583)$$

$$\begin{pmatrix} -18 & -4.10331 \\ 4.10331 & 8.64409 \end{pmatrix} \cdot \delta^{(1)} = \begin{pmatrix} -0.03167 \\ 0.23133 \end{pmatrix}$$

$$\delta^{(1)} = \begin{pmatrix} 0.00392 \\ -0.0249 \end{pmatrix}$$

$$x^{(2)} = x^{(1)} + \delta^{(1)}$$

$$= \begin{pmatrix} 0.99595 \\ 1.02583 \end{pmatrix} + \begin{pmatrix} 0.00392 \\ -0.0249 \end{pmatrix}$$

$$x^{(2)} = \underline{\underline{\begin{pmatrix} 0.99987 \\ 1.00093 \end{pmatrix}}}$$

$$\Rightarrow \|f^{(0)}\|_2 = \left\| \begin{pmatrix} -1.42 \\ 1.044 \end{pmatrix} \right\|_2 = \sqrt{(-1.42)^2 + (1.044)^2} = \underline{\underline{1.76248}}$$

$$\Rightarrow \|x^{(1)} - x^{(0)}\|_2 = \left\| \begin{pmatrix} 0.99595 \\ 1.02583 \end{pmatrix} - \begin{pmatrix} 1.1 \\ 0.9 \end{pmatrix} \right\|_2 = \left\| \begin{pmatrix} -0.10405 \\ 0.12583 \end{pmatrix} \right\|_2 = \sqrt{(-0.10405)^2 + 0.12583^2} = \underline{\underline{0.16328}}$$

$$\Rightarrow \|f^{(1)}\|_2 = \left\| \begin{pmatrix} -0.03167 \\ 0.23133 \end{pmatrix} \right\|_2 = \sqrt{(-0.03167)^2 + 0.23133^2} = \underline{\underline{0.23349}}$$

$$\Rightarrow \|x^{(2)} - x^{(1)}\|_2 = \left\| \begin{pmatrix} 0.99987 \\ 1.00093 \end{pmatrix} - \begin{pmatrix} 0.99595 \\ 1.02583 \end{pmatrix} \right\|_2 = \left\| \begin{pmatrix} 0.00392 \\ -0.0249 \end{pmatrix} \right\|_2 = \sqrt{0.00392^2 + (-0.0249)^2} = \underline{\underline{0.02521}}$$