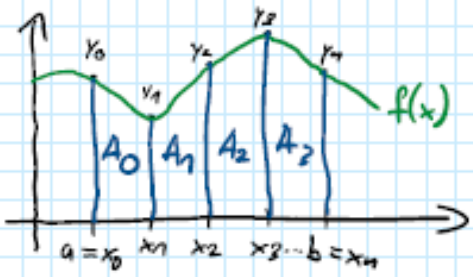
1) a)



$$Tf = \frac{f(a)+f(b)}{2} \cdot (b-a)$$

$$Tf = \sum_{i=0}^{n-1} A_i = \sum_{i=0}^{n-1} \frac{y_i + y_{i+1}}{2} \cdot (x_{i+1} - x_i)$$

b)

$h = \frac{b-a}{n}$, $x_i = a + i \cdot h$

$\Rightarrow b = n \cdot h - a$, $\Rightarrow a = x_i - i \cdot h$

$$Tf = \sum_{i=0}^{n-1} A_i = \frac{f(a)+f(b)}{2} \cdot (b-a) \text{ , für } n = 1$$

$$n = 2: \quad \frac{f(x_0)+f(x_1)}{2} \cdot h + \frac{f(x_1)+f(x_2)}{2} \cdot h$$

$$n = 3: \quad \frac{f(x_0)+f(x_1)}{2} \cdot h + \frac{f(x_1)+f(x_2)}{2} \cdot h + \frac{f(x_2)+f(x_3)}{2} \cdot h$$

$$= h \cdot \left(\frac{f(x_0)+f(x_1)+f(x_2)+f(x_3)}{2} \right)$$

$\Rightarrow$ für n (beliebig):

$$h \cdot \left(\frac{f(x_0)+f(x_n)}{2} + \sum_{i=0}^{n-1} f(x_i) \right)$$

$$= h \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=0}^{n-1} f(x_i) \right)$$

2a) $RS(h) = h \cdot \sum_{i=0}^{n-1} S\left(x_i + \frac{h}{2}\right)$

$h = -3$

$\sum_{i=0}^{n-1} S\left(x_i + \frac{h}{2}\right) = S(20-1{,}5) + S(17-1{,}5) + S(14-1{,}5) + S(11-1{,}5) + S(8-1{,}5) = -1{,}46077$

$RS(h) = 4{,}3823$

Fehlerabschätzung max. = 0,755

Abs. Fehler = 0,0898

2b) $TS(h) = h \cdot \left(\frac{S(a)+S(b)}{2} + \sum_{i=1}^{n-1} S(x_i)\right)$

$h = -3$

$\frac{S(a)+S(b)}{2} = \frac{\left(\frac{\sqrt{5}}{20} + \frac{2\sqrt{5}}{5}\right)}{2}$

$\sum_{i=1}^{n-1} S(x_i) = S(20) + S(17) + S(14) + S(11) + S(8) = -1{,}16142$

$TS(h) = 3{,}4842 + 0{,}675 \cdot \sqrt{5}$

Fehlerabschätzung max. = 1,509

Abs. Fehler $= -0{,}9879 + 0{,}675 \cdot \sqrt{5}$

2c) $SS(h) = \frac{h}{3}\left(\frac{1}{2} \cdot S(a) + \sum_{i=1}^{n-1} S(x_i) + 2\sum_{i=1}^{n} S\left(\frac{x_{i-1}+x_i}{2}\right) + \frac{S(b)}{2}\right)$

$SS(h) = 0{,}225 \cdot \sqrt{5} + 3{,}2579$

Fehlerabschätzung max. = 0,03566

Abs. Fehler $= 1{,}21424 - 0{,}225\sqrt{5}$