2'a.) $w(x,t) = \sin(x+ct)$

$$\frac{\partial w}{\partial t}(x,t) = \cos(x+ct)\cdot(0+c\cdot 1) = \cos(x+ct)\cdot c$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\cdot(\cos(x+ct)\cdot c) = -\sin(x+ct)\cdot(0+c\cdot 1)\cdot c = -\sin(x+ct)\cdot c^2$$

$$\frac{\partial w}{\partial x}(x,t) = \cos(x+ct)\cdot(1+c\cdot 0) = \cos(x+ct)\cdot 1 = \cos(x+ct)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\cdot(\cos(x+ct)) = -\sin(x+ct)\cdot(1+c\cdot 0) = -\sin(x+ct)\cdot 1 = -\sin(x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = c^2\cdot\frac{\partial^2 w}{\partial x^2} \iff c^2\cdot -\sin(x+ct) = c^2\cdot -\sin(x+ct)$$

2a.) $v(x,t) = \sin(x+ct) + \cos(2x+2ct)$

$$\frac{\partial v}{\partial t} = v'(x,t) = \cos(x+ct)\cdot(0+c\cdot 1) + -\sin(2x+2ct)\cdot(0+2\cdot c\cdot 1)$$
$$= c\cdot\cos(x+ct) + 2c\cdot -\sin(2x+2ct)$$

$$\frac{\partial^2 v}{\partial t^2} = \frac{\partial}{\partial t}\cdot(c\cdot\cos(x+ct) + 2c\cdot -\sin(2x+2ct)) = c\cdot -\sin(x+ct)\cdot(0+c\cdot 1) + 2c\cdot -\cos(2x+2ct)\cdot(0+2\cdot c\cdot 1)$$
$$= c\cdot c\cdot -\sin(x+ct) + 2c\cdot 2c\cdot -\cos(2x+2ct) = c^2\cdot -\sin(x+ct) + 4c^2\cdot -\cos(2x+2ct)$$

$$\frac{\partial v}{\partial x} = \cos(x+ct)\cdot(1+c\cdot 0) + -\sin(2x+2ct)\cdot(2+2\cdot c\cdot 0) = 1\cdot\cos(x+ct) + 2\cdot -\sin(2x+2ct)$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial}{\partial x}\cdot(\cos(x+ct) + 2\cdot -\sin(2x+2ct)) = -\sin(x+ct)\cdot(1+c\cdot 0) + 2\cdot -\cos(2x+2ct)\cdot(2+2\cdot c\cdot 0)$$
$$= -\sin(x+ct) + 4\cdot -\cos(2x+2ct)$$

$$\frac{\partial^2 v}{\partial t^2} = c^2\frac{\partial^2 v}{\partial x^2} \Leftrightarrow \underbrace{c^2\cdot(-\sin(x+ct) + 4\cdot -\cos(2x+2ct))}_{\frac{\partial^2 v}{\partial t^2}} = c^2\cdot\underbrace{(-\sin(x+ct) + 4\cdot -\cos(2x+2ct))}_{\frac{\partial^2 v}{\partial x^2}}$$