1) $I = \int_1^2 \ln(x^2)\,dx$ fehler $= 10^{-5}$

$$\left|\int_a^b f(x)\,dx - Rf(h)\right| \le \frac{h^2}{24}\cdot(b-a)\cdot \max_{x\in[a,b]} |f''(x)|$$

$$\left|\int_a^b f(x)\,dx - Tf(h)\right| \le \frac{h^2}{12}\cdot(b-a)\cdot \max_{x\in[a,b]} |f''(x)|$$

$$\left|\int_a^b f(x)\,dx - Sf(h)\right| \le \frac{h^4}{2880}\cdot(b-a)\cdot \max_{x\in[a,b]} |f^{(4)}(x)|$$

$f(x) = \ln(x^2)$

$f'(x) = \frac{2}{x}$

$f''(x) = -\frac{2}{x^2}$

$f'''(x) = \frac{4}{x^3}$

$f^{(4)}(x) = -\frac{12}{x^4}$

Rechteckregel

$\left|\int_a^b f(x) - Rf(h)\right| \le 10^{-5}$

$10^{-5} \le \frac{h^2}{24}(2-1)\cdot \max_{x\in[1,2]}\left(\frac{2}{x}\right)$

$10^{-5} \le \frac{h^2}{24}\cdot 1 \cdot 2$

$10^{-5} \le \frac{h^2}{12}$

$10^{-5}\cdot 12 \le h^2$

$\sqrt{10^{-5}\cdot 12} \le h$

$h \approx 0.0109545$

$n = \frac{b-a}{h} = \frac{2-1}{0.0109545} \approx \underline{\underline{92}}$

Trapezregel

$\left|\int_a^b f(x) - Tf(h)\right| \le 10^{-5}$

$10^{-5} \le \frac{h^2}{12}\cdot 2$

$10^{-5}\cdot 6 < h^2$

$h \ge \sqrt{10^{-5}\cdot 6}$

$h \approx 0.007745$

$n = \frac{b-a}{h} = \frac{2-1}{0.007745} \approx \underline{\underline{130}}$

Simpsonregel

$\left|\int_a^b f(x) - Sf(h)\right| \le 10^{-5}$

$10^{-5} \le \frac{h^4}{2880}\cdot 1\cdot \max_{x\in[1,2]} |f^{(4)}(x)|$

$10^{-5} \le \frac{h^4}{2880}\cdot \max_{x\in[1,2]}\left(-\frac{12}{x^4}\right)$

$10^{-5} \le \frac{h^4}{2880}\cdot 12$

$h \ge \sqrt[4]{\frac{10^{-5}\cdot 2880}{12}}$

$h \approx 0.2213$

$n = \frac{b-a}{h} = \frac{2-1}{0.2213} \approx \underline{\underline{5}}$