Problem 2.1.6, parts d-f

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1 Problem

6. Let A and B be subsets of a universal set U. Simplify each of the following expressions.
(d) $A \cup [B \cap (U \setminus A)]$
(e) $(A \cup B) \cap [A \cup (U \setminus B)]$
(f) $(A \cap B) \cup [A \cap (U \setminus B)]$

References on page 44 of the textbook
Theorem a: $A \cup (U \setminus A) = U$
Theorem d: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Theorem e: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

2 Solution

(d)

$$A \cup [B \cap (U \setminus A)] \tag{1}$$

Using theorem d we get

$$(A \cup B) \cap [A \cup (U \setminus A)] \tag{2}$$

Using theorem a we get

$$(A \cup B) \cap U \tag{3}$$

The union of A and B is a subset of U, the intersection of the two sets is just the union of A and B.

$$(A \cup B) \tag{4}$$

(e)

$$(A \cup B) \cap [A \cup (U \setminus B)] \tag{5}$$

Using theorem e we get

$$[(A \cup B) \cap A] \cup [(A \cup B) \cap (U \setminus B)] \tag{6}$$

Using theorem e again we get

$$[(A \cap A) \cup (B \cap A)] \cup [((U \setminus B) \cap A) \cup ((U \setminus B) \cap B)] \tag{7}$$

We simplify to get

$$(A \cup (B \cap A)) \cup ((U \setminus B) \cap A) \cup \emptyset \tag{8}$$

$B \cap A$ is a subset of A so the union is just A.

$$A \cup ((U \setminus B) \cap A) \tag{9}$$

$(U \setminus B) \cap A$ is a subset of A so it reduces to just A.

$$A \tag{10}$$

(f)

$$(A \cap B) \cup [A \cap (U \setminus B)] \tag{11}$$

Using theorem e we get

$$[(A \cap B) \cup A] \cap [(A \cup B) \cup (U \setminus B)] \tag{12}$$

Left of intersection reduces to just A. Right of the intersection reduces to U.

$$A \cap U \tag{13}$$

This simplifies to

$$A \tag{14}$$