Unit 3 Presentation 4.3.11

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April 2^{nd} 2018

1 4.3.11

Prove theorem 4.3.3 for a bounded decreasing sequence.

1.1 Theorem 4.3.3

A monotone sequence is convergent if and only if it is bounded.

1.2 Definition 4.3.1

A sequence s_n is increasing if $s_n \leq s_{n+1}$ for all $n \in \mathbb{N}$. A sequence is decreasing if $s_n \geq s_{n+1}$ for all $n \in \mathbb{N}$. A sequence is monotone if it is increasing or decreasing.

1.3 Completeness axiom

Every nonempty subset S of $\mathbb{R}$ that is bounded above has a least upper bound. It follows that every subset S of $\mathbb{R}$ that is bounded below has a greatest lower bound., that is to say $\inf S$ exists and is a real number.

1.4 Proof

Let s_n be a bounded decreasing sequence and let S denote the nonempty bounded set $\{s_n : n \in \mathbb{N}\}$ By the completeness axiom, S has a greatest lower bound, and we let $s = \inf S$ and claim that $\lim s_n = s$. Given any $\epsilon > 0, s + \epsilon$ is not a lower bound of S, thus there exists a natural number N such that $s_N < s + \epsilon$. Furthermore since s_n is decreasing and s is a lower bound of S we have

$$s + \epsilon > s_N \geq s_n \geq s$$

for all $n \geq N$. Hence s_n converges to s.