

# Lesson 4 Presentation

Jacob Jashinsky as Scribe and Caleb Spear as Editor

## Problem 2.3.2

Mark each statement as True or False. Justify each answer.

a. If  $f : A \rightarrow B$  and  $C$  is nonempty subset of  $A$ , then  $f(C)$  is a nonempty subset of  $B$ .

True. For  $f$  to be a function every element in  $A$ , which includes the subset  $C$ , will map to a  $b \in B$ . Therefore,  $f(C)$  will also be a subset of  $B$ .

b. If  $f : A \rightarrow B$  is surjective and  $y \in B$ , then  $f^{-1}(y) \in A$ .

True. Since the function is surjective  $\text{rng } f = B$  and  $f^{-1} : B \rightarrow A$ .

c. If  $f : A \rightarrow B$  and  $D$  is nonempty subset of  $B$  then  $f^{-1}(D)$  is a nonempty subset of  $A$ .

False,  $D$  may include elements that are not in the range of  $f$ , and the inverse of  $f$  will not map elements outside of the range back to  $A$ .

d. The composition of two surjective functions is always surjective.

True, by Theorem 2.3.20 on page 72, we know that composition preserves properties of being injective and surjective.

e. If  $f : A \rightarrow B$  is bijective, then  $f^{-1}B \rightarrow A$  is bijective.

True. Since  $f$  is bijective each  $a \in A$  maps to a unique  $b \in B$ . Therefore the inverse would also map every  $b \in B$  back to every element  $a \in A$ , which would make the inverse bijective.

f. The identity function maps  $\mathbb{R}$  onto  $\{1\}$ .

False. An identity function would map the set  $\mathbb{R}$  back onto  $\mathbb{R}$ .