Math 301 Vocabulary

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Sections 1.1 - 1.2

Statement: This is a declarative statement that is classified as either true or false. It cannot be true or false in some cases but always. No ambiguity is allowed.

Negation: It is the logical opposite of a statement. If p represents a true statement then $\sim p$ is a false statement.

Conjunction: combining of two statements. p and q is the same as $p \wedge p$. The conjunction is true only when both statements are true.

Disjunction: Combining two statements by using the term or, denoted $p \vee q$. Using only the inclusive definition, meaning that both p and q could happen. It's not just one or the other.

Implication or Conditional: If p, then q, denoted $p \Rightarrow q$. It's an if-statement. An implication is false if the antecedent is true and the consequent is false.

Antecedent: In the statement, If p, then q, the p is the antecedent.

Consequent: In the statement, If p, then q, the q is the consequent.

Biconditional: The statement of the form p if and only if q, denoted $p \iff q$. Which is implying two conditional statements, $p \Rightarrow q$ and $q \Rightarrow p$, hence the prefix bi.

Tautology: When a compound statement is true in all cases it is called a tautology. "the negation of a conjunction is logically equivalent to the dis-junction of the negations." See page 25 for a list.

Universal Quantifier: is the symbol, $\forall$, which mean, for all. So for example, $\forall\ x, p(x)$, for all of x, the function is true.

Existential Quantifier: There exists, denoted $\exists$ so for example. $\exists\ x \ni p$ which reads, there exists and x such that the statement p is true.

Such that: is the symbol, $\ni$.

Sections 1.3 - 1.4

Inductive Reasoning : Drawing a conclusion from observations. Inductive reasoning leads to a conjucture.

Counterexample: Using an example to prove the original statement false, this can be used when $\forall$ statements are used.

Deductive reasoning: Applying a general principle to a particular case. (the general case could mean using n).

Hypothesis: When an implication is a theorem. The antecedent is the hypothesis.

Conclusion: When an implication is a theorem, the consequent is the conclusion.

Contrapositive: The contrapostive of the statement $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$. They are logically equivalent.

Converse: The converse of the statement $p \Rightarrow q$ is $q \Rightarrow p$. They are not always logically equivalent.

Inverse: The inverse of the statement $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$. They are not always logically equivalent.

Contradiction: A contradiction can be shown by negating a statement and showing that there is a contradiction.

Building blocks of proof:

Proof by Contradiction: To prove by contradiction we assume p and $\sim q$ and deduce the contradiction.

Proof by cases: Some proofs are large or encompasses a general region. The proof could be broken up into cases that each could have their own proof.

Sections 2.1 - 2.2

Subset: $A \subseteq B$, A is a subset of B. A is contained in B.

Proper subset: $A \subset B$, A is a subset of B. A is contained in B, but there exists an element in B that is not in A.

Equal Sets: $A = B$, if $A \subseteq B$ and $B \subseteq A$.

Interval: Is a set of real numbers, (a,b), [a,b], [a,b), (a,b].

Empty set: A set with no elements. $\varnothing$. An empty set is a subset of every set.

Union: $A \cup B$, read "A union B", or read "A or B".

Intersection: $A \cap B$, read "A intersect B", or read "A and B".

Complement: $A \setminus B = \{x :\in A \cap x \notin B\}$. This is the complement of B.

Disjoint sets: Two set that do not intersect. Also called mutually exclusive.

Indexed family: $\bigcap_{j \in J} A_j = \{x : x \in A_j \text{ for some} j \in J\}$ and also $\bigcup_{j \in J} A_j = \{x : x \in A_j \text{ for some } j \in J\}$

Ordered pairs: $(a, b) = \{\{a\}, \{a, b\}\}$

Cartesian product: $A \times B$ is the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$. $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$.

Relation between A and B: Is a subset of the ordered pairs of a cartesian product.

Equivalence Relation: Is a relation that has the properties of being reflexive, symmetric, and transitive for all $x, y, z \in S$.

Reflexive: $x\mathrm{R}x$

Symmetric: If $x\mathrm{R}y$, then $y\mathrm{R}x$.

Transitive: If $x\mathrm{R}y$ and $y\mathrm{R}z$, then $x\mathrm{R}z$ exists.

Partition: Is a subset of some set S. Every element in S is in a subset and there is no overlap between the subsets.

Sections 2.2 - 2.3

Domain of f: denoted dom$f$. Set A is the domain of the function $f : A \to B$.

Range of f: denoted dom$f$, is the set of all second elements of members of f, where b is the second element of the ordered pairs $(a, b) \in f$.

Codomain: Set B is the codomain of the function $f : A \to B$.

Function from A to B: A function from A to B, $f : A \to B$ is nonempty relation $f \subseteq A \times B$ that satisfies the following assumptions. 1. Existence: For all a in A there exists a b in B such that $(a, b) \in f$. 2. Uniqueness: If $(a, b) \in f$ and $(a, c) \in f$, then $b = c$. (vertical line test).

Surjective: A function is surjective if $B = \text{rng} f$.

Injective: A function is injective (one to one) if $f(a) = f(a')$ and $a = a'$.

Bijective: A function is bijective if it is both injective and surjective.

Image of a set: For the function $f : A \to B$, let $C \subseteq A$. $f(C)$ is called the image of C in B.

Pre-image of a set: For the function $f : A \to B$, let $D \subseteq B$. $f^{-1}(D)$ is called the Pre-image of D in A.

Composition of functions: $f \circ g : A \to C$, where $f : A \to B$ and $g : B \to C$.

Inverse function: $f^{-1} = \{(y, x) \in B \times A : (x, y) \in f\}$

Identity function: A function defined on set A that maps element in A onto itself.

Equinumerous: $S \sim T$, the two sets are set equivalent if there exists a bijective function that maps S on to T, $f : S \to T$.

Finite: A set S is finite if $S = \varnothing$, or if there exists $n \in \mathbb{N}$ and bijective function $f : \{1, 2, 3, ..., n\} \to S$.

Infinite: If a set is not finite it is infinite.

Cardinal number: It is the number of elements in a set or the n in the set I_n.

Transfinite: If a cardinal number is not finite then it is transfinite.

Denumerable: A set is denumerable if there exists a bijection $f : \mathbb{N} \to S$.

Countable: It is a finite or denumerable set.

Uncountable: It is a set that is not countable.

Power set: $\wp(S)$ is the collection of all the subsets of S.

Section 3.1

Well-ordering property of N: If S is a nonempty subset of $\mathbb{N}$, then there exists an element $m \in S$ such that $m \leq k$ for all $k \in S$.

Principle of mathematical induction: Let $P(n)$ be a statement that is either true or false for each $n \in \mathbb{N}$. then $P(n)$ is true for all $n \in \mathbb{N}$, provided that,

a. $P(1)$ is true, and

b. for each $k \in \mathbb{N}$, if $P(k)$ is true, then $P(k+1)$ is true.

Basis for induction: Part (a) of the principle of mathematical induction.

Induction step: Part (b) of the principle of mathematical induction.

Induction hypothesis: Assuming that $P(k)$ is true in part (b) is known as the induction hypothesis.

Section 3.3

Upper bound: Let S be s subset of $\mathbb{R}$. If there exists a real number m such that $m \geq s$ for all $s \in S$, then m is called an upper bound, and S is bounded above.

Lower bound: Let S be s subset of $\mathbb{R}$. If there exists a real number m such that $m \leq s$ for all $s \in S$, then m is called an lower bound, and S is bounded below.

Bounded set: The set S is bounded if it is bounded above and below.

Maximum: If m is an upper bound of S and $m \in S$, then m is called the maximum of S. $m = \max S$.

Minimum: If m is a lower bound of S and $m \in S$, then m is called the minimum of S. $m = \min S$.

Supremum: If S is bounded above the least upper bound of S is called its supremum, denoted, $\sup S$, iff

a. $m \geq s$ for all $s \in S$ and,

b. if $m' < m$, then there exists $s' \in S$ such that $s' > m'$.

Infimum: If S is bounded below the greatest upper bound of S is called its Infimum, denoted, $\inf S$, iff

a. $m \leq s$ for all $s \in S$ and,

b. if $m' > m$, then there exists $s' \in S$ such that $s' < m'$.

Completeness axiom: Every nonempty subset S of $\mathbb{R}$ that is bounded above has a least upper bound. That is, $\sup S$ exists and is a real number.

Archimedean property: The set $\mathbb{N}$ of natural numbers is unbounded above in $\mathbb{R}$.

Density of the rationals: $\mathbb{Q}$ is done in $\mathbb{R}$, or that between any two real numbers there is a rational number.

Sections 3.4 and 3.5

Neighborhood: Let $\epsilon > 0$ and $x \in \mathbb{R}$. A neighborhood of x is a set of the form $N(x;\epsilon) = \{y \in \mathbb{R} : |x-y| < \epsilon\}$

Deleted neighborhood: Let $\epsilon > 0$ and $x \in \mathbb{R}$. A deleted neighborhood of x is a set of the form $N^*(x;\epsilon) = \{y \in \mathbb{R} : 0 < |x-y| < \epsilon\}$

Interior Point: Let $S \subseteq \mathbb{R}$. A point x in $\mathbb{R}$ is an interior point of S if there exists a neighborhood N of x such that $N \subseteq S$

Boundary Point: If for every neighborhood N of x, $N \cap S \neq \varnothing$ and $N \cap (\mathbb{R} \setminus S) \neq \varnothing$, then x is called a boundary point of S.

Closed set: A set S is closed iff its complement $\mathbb{R} \setminus S$ is open.

Open set: A set S is open iff $S = \text{int } S$.

Accumulation point: Let S be a subset of $\mathbb{R}$. A point x in $\mathbb{R}$ is an accumulation points of S if every deleted neighborhood of x contains a point of S. The set is denoted S'

Isolated point: If $x \in S$ and $x \notin S'$, then x is called an isolated point of S.

Closure of a set: Denoted cl S. cl $S = S \cup S'$, or that cl $S = S \cup \text{bd}S$.

Compact Set: A set S is said to be compact if whenever it is contained in the union of a family F of open sets, it is contained in the union of some finite number of the sets in F.

Open Cover: If F is a family of open sets whos union contains S, then F is called an open cover of S.

Subcover: If $Z \subseteq F$ and Z is also an open cover of S, then Z is called a subcover of S.

Heine-Borel theorem: A subset of $\mathbb{R}$ is compact iff S is closed and bounded.

Bolzano-Weierstrass theorem: If a bounded subset S of $\mathbb{R}$ contains infinitely many points, then there exists at least one points in $\mathbb{R}$ that is an accumulation point of S.

Section 4.1

Sequence: A sequence is a function whose domain is the set $\mathbb{N}$ of natural numbers, denoted (s_n).

Convergent Sequence: (s_n) is said to converge to the real numbers s, provided that

For every $\epsilon > 0$, there exists a $N \in \mathbb{N}$ such that $\forall\ n \in \mathbb{N}$, $n \geq N$ implies that $|s_n - s| < \epsilon$.

Limit of a Sequence: If (s_n) converges to s, then s is called the limit of the sequence (s_n). $\lim s_n = s$

Divergent Sequence: A series that does not converge to a real number is divergent.

Bounded Sequence: (s_n) is said to be bounded if the range $\{s_n : n \in \mathbb{N}\}$ is a bounded set, that is, if there exists an $M \geq 0$ such that $|s_N| \leq M$ for all $n \in \mathbb{N}$.

Sections 4.2 - 4.3

4.2.1 Limit Theorems: Suppose $\lim s_n = s$ and $\lim t_n = t$, then

a. $\lim(s_n + t_n) = s + t$

b. $\lim(ks_n) = ks$ and $\lim(k + s_n) = k + s$.

c. $\lim(s_n t_n) = st$

d. $\lim(s_n/t_n) = s/t$

Diverge to $+\infty$: A sequence (s_n) diverges to $+\infty$, and we write $\lim s_n = +\infty$ provided that for every $m \in \mathbb{R}$ there exists an $N \in \mathbb{N}$ such that $n \geq N$ implies that $s_n > M$.

Diverge to $-\infty$: A sequence (s_n) diverges to $-\infty$, and we write $\lim s_n = -\infty$ provided that for every $m \in \mathbb{R}$ there exists an $N \in \mathbb{N}$ such that $n \geq N$ implies that $s_n < M$.

Theorem 4.2.4: Suppose $\lim s_n = s$ and $\lim t_n = t$. If $s_n \leq t_n$ for $n \in \mathbb{N}$ then $s < t$.

In other words. so long as you can show that s_n is always less than t_n than the limit of s_n is less than the limit of t_n.

Theorem 4.2.12: Let $s_n \leq t_n$ for all $n \in \mathbb{N}$.

a. If $\lim s_n = +\infty$, then $\lim t_n = +\infty$.

b. If $\lim t_n = -\infty$, then $\lim s_n = -\infty$.

Because the we know the limit of one we can show the limit of the other because we know the relationship $s_n \leq t_n$.

Theorem 4.2.13: Let s_n be a sequence of positive numbers, then $\lim(s_n) = +\infty$ iff $\lim(1/s_n) = 0$.

Increasing Sequence: The sequence s_n is increasing if $s_n \leq s_{n+1}$ for all $n \in \mathbb{N}$.

Decreasing Sequence: The sequence s_n is decreasing if $s_n \geq s_{n+1}$ for all $n \in \mathbb{N}$.

Monotone Sequence: A sequence is monotone if it is increasing or decreasing.

Cauchy Sequence: A sequence (s_n) of real numbers is said to be a Cauchy sequence if for each $\epsilon > 0$ there exists an $N \in \mathbb{N}$ such that $m, n \geq N$ implies that $|s_n - s_m| < \epsilon$.

Cauchy Convergence Criterion: A sequence of real numbers is convergent iff it is a Cauchy sequence.