HW # | 0 Hours | Jacob Jashinsky
3.4.9a, 4.2.2abc, 4.2.5abc | | Math 461-01
4.2.6abcd, 4.3.1ab | | October 29, 2018

Exercise 3.4.9a

Let $\{r_1, r_2, r_3, \ldots\}$ be an enumeration of the rational numbers, and for each $n \in \mathbb{N}$ set $\varepsilon_n = 1/2^n$. Define $O = \bigcup_{n=1}^{\infty} V_{\varepsilon_n}(R_n)$, and let $F = O^c$.

a. Argue that F is a closed, nonempty set consisting only of irrational numbers.

Solution:

x<1

Exercise 4.2.2abc

For each stated limit, find the largest possible δ-neighborhood that is a proper response to the given ε challenge.

a. $\lim_{x\to 3}(5x-6)=9$, where $\varepsilon=1$.

Solution:

x<1

b. $\lim_{x\to 4}\sqrt{x}=2$, where $\varepsilon=1$.

Solution:

x<1

c. $\lim_{x\to \pi}\lfloor x\rfloor=3$, where $\varepsilon=1$.

Solution:

x<1

Exercise 4.2.5abc

Use Definition 4.2.1 to supply a proper prof for the following limit statements.

a. $\lim_{x\to 2}(3x+4) = 10$

Solution:

x<1

b. $\lim_{x\to 0} x^3 = 0$

Solution:

x<1

c. $\lim_{x\to 2}(x^2+x-1) = 5$

Solution:

x<1

Exercise 4.2.6

Decide if the following are true or false, and give short justifications for each conclusion.

a. If a particular δ has been constructed as a suitable response to a particular ε challange, then any smaller positive δ will also suffice.

Solution:

x<1

b. If $\lim_{x\to a} f(x) = L$ and a happens to be in the domain of f, then $L = f(a)$.

Solution:

x<1

c. If $\lim_{x\to a} f(x) = L$, then $\lim_{x\to a} 3[f(x)-2]^2 = 3(L-2)^2$.

Solution:

x<1

d. If If $\lim_{x\to a} f(x) = 0$, then $\lim f(x)g(X) = 0$ for any function g (with domain equal to the domain of f.)

Solution:

x<1

Exercise 4.3.1

Let $g(x) = \sqrt[3]{x}$

a. Prove that g is continuous at $c = 0$.

Solution:

x<1

b. Prove that g is continuous at a point $c \neq 0$. (The Identity $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ will be helpful.)

Solution:

x<1