HW #10 | 0 Hours | Jacob Jashinsky
5.2.3, 5.2.5, 5.2.11 | Math 461-01
5.3.1, 5.3.3, 5.3.7, 5.3.11 | November 19, 2018

Exercise 5.2.3

a) Use definition 5.2.1 to produce the proper formula for the derivative of $h(x) = 1/x$.

Solution:

x<1

b) Combine the result in part (a) with the Chain Rule to supply a proof for part (iv) of thm. 5.2.4

Solution:

x<1

c) Supply a direct proof of thm. 5.2.4 (iv) by algebraically manipulating the difference quotient for (f/g) in a style similar to the proof of thm. 5.2.4 (iii).

Solution:

x<1

Exercise 5.2.5

Let

$$f(X=x)=\begin{cases} x^a\ , & \text{if } x>0 \\ 0\ , & \text{if } x\leq 0 \end{cases}$$

a) For which values of a is f continuous at zero?

Solution:

x<1

b) For which values of a is f differentiable at zero? In this case, is the derivative function continuous?

Solution:

x<1

c) For which values of a is f twice-differentiable?

Solution:

x<1

Exercise 5.2.11

Assume that g is differentiable on $[a,b]$ and satisfies $g'(a) < 0 < g'(b)$.

a) Show that there exists a point $x \in (a,b)$ where $g(a) > g(x)$, and a point $y \in (a,b)$ where $g(y) < g(b)$.

Solution:

x<1

b) Now complete the proof of Darboux's Theorem started earlier.

Solution:

x<1

Exercise 5.3.1

Recall from Exercise 4.4.9 that a function $f : A \to \mathbb{R}$ is Lipschitz on A if there exists an $m > 0$ such that

$$\left| \frac{f(x) - f(y)}{x - y} \right| < M$$

for all $x \neq y$ in A.

a) Show that if f is differentiable on a closed interval $[a,b]$ and if f' is continuous on $[a,b]$, then f is Lipschitz on $[a,b]$.

Solution:

x<1

b) Review the definition of a contractive function in Exercise 4.3.11. If we add the assumption that $|f'(x) < 1|$ on $[a,b]$, does if follow that f is contractive on this set?

Solution:

x<1

Exercise 5.3.3

Let h be a differentiable function defined on the interval $[0,3]$, and assume that $h(0) = 1$, $h(1) = 2$, and $h(3) = 2$.

a) Argue that there exists a point $d \in [0,3]$ where $h(d) = d$

Solution:

x<1

b) Argue that at some point c we have $h'(c) = 1/3$.

Solution:

x<1

c) Argue that $h'(x) = 1/4$ at some point in the domain.

Solution:

x<1

Exercise 5.3.7

A fixed point of a function f is a value x where $f(x) = x$. show that if f is differentiable on an interval with $f'(x) \neq 1$, then f can have at most one fixed point.

Solution:

x<1

Exercise 5.3.11

a) Use the Generalized Mean Value theorem to furnish a proof of the $0/0$ case if L'Hospital's Rule.

Solution:

x<1

b) If we keep the first part of the hypothesis of Theorem 5.3.6 the same but we assume that

$$\lim_{x \to a} \frac{f'(x)}{g'(x)} = \infty$$

does it necessarily follow that

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \infty \text{ ?}$$

Solution:

x<1