Homework 6
3.3.2abcd, 3.3.3, 3.3.5abc
3.3.11, 3.4.1, 3.4.4ab

0.5 Hours

Jacob Jashinsky
Math 461-01
October 22, 2018

Exercise 3.3.2 abcd

Decide which of the following sets are compact. For those that are not compact, show how Definition 3.3.1 breaks down. In other words, give an example of a sequence contained in the given set that does not possess a subsequence converging to a limit in the set.

a) $\mathbb{N}$

> **Solution:**
>
> The set is not compact. The subsequence $\{1, 3, 5, ...\}$ is contained in $\mathbb{N}$ but it does not converge to a limit.

b) $\mathbb{Q} \cap [0, 1]$

> **Solution:**
>
> The set is not compact

c) The Cantor set

> **Solution:**
>
> The set is compact

d) $\{1 + 1/2^2 + 1/3^2 + ... + 1/n^2 : n \in \mathbb{N}\}$

> **Solution:**
>
> The set is not compact

Exercise 3.3.3

Prove the converse of theorem 3.3.4 by showing that if a set $K \subseteq \mathbb{R}$ is closed and bounded then it is compact.

Solution:

Since K is bounded every sequence in K will also be bounded. By the Bolzano Weierstass theorem, every bounded sequence will contain a convergent subsequence. Theorem 3.2.5 states that these subsequences in A converge to the limit points of A.

Those subsequence will converge to a limit within k because the set is closed and contains its limit points, thus the assumptions of compactness is met.

Exercise 3.3.5 abc

Decide whether the following propositions are true or false. If the claim is valid, supply a short proof, and if the claim is false, provide a counterexample.

a) The arbitrary intersection of compact sets is compact.

Solution:

x<1

b) The arbitrary union of compact sets is compact.

Solution:

x<1

c) Let A be arbitrary, and let K be compact. Then, the intersection $A \cap K$ is compact.

Solution:

x<1

Exercise 3.3.11

Consider each of the sets listed in Exercise 3.3.2. For each one that is not compact, find an open cover for which there is no finite subcover.

Solution:

x<1

Exercise 3.4.1

If P is a perfect set and K is compact, is the intersection $P \cap K$ always compact? Always perfect?

Solution:

x<1

Exercise 3.4.4

Repeat the Cantor construction from Section 3.1 starting with the interval $[0, 1]$. This time, however, remove the open middle fourth from each of the component.

    a) Is the resulting set compact? Perfect?

Solution:

   x<1

    b) Using the algorithms from section 3.1, compute the length and dimension of this cantor-like set.

Solution:

   x<1