Axiom of Completeness

Every nonempty set of real numbers that is bounded above had a least upper bound.

Convergence of a sequence

(a_n) converges to a real number a if for every $\varepsilon > 0$ there exists an $n \in \mathbb{N}$ such that whenever $n \geq N$ it follows that $|a_n - a| < \varepsilon$.

Monotone Convergence Theorem

If a_n is monotone and bounded then the sequence converges

Convergence of a series

Let b_n be a sequence and the infinite series be defined as $\sum b_n$. Also let the sequence s_n be defined as the $s_n = b_0 + b_1 + b_2 + ...b_n$. The series converges to B if $(s_n) \to B$.

Balzono Weierstrass Theorem

Every bounded sequence contains a convergent subsequence.

Cauchy sequence

The sequence a_n is Cauchy if for every $\varepsilon > 0$ there exists an $N \in \mathbb{N}$ such that whenever $n, m > N$ implies $|a_n - a_m| < \varepsilon$

Heine Borel Theroem

Let K be a subset of R. If one is satisfied then it implies the others i. K is compact ii. K is closed an bounded iii. Every open cover of K has a finite subcover.

Def of funciton limit

Let $f : A \to \mathbb{R}$, and c be a limit point of A. $\lim_{x \to c} f(x) = L$ if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that whenever $0 < |x - c| < \delta$ and $x \in A$ implies that $|f(x) - L| < \varepsilon$.

Continuity at a point

A function $f : A \to \mathbb{R}$ is continuous at a point c if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that whenever $|x - c| < \varepsilon$ and $x \in A$ implies that $|f(x) - f(c)| < \varepsilon$.

Extreme value theorem

If $f : K \to \mathbb{R}$ is continuous on K then f attains a maximum and minimum somewhere on the domain.

Uniform Continuity

$f : A \to \mathbb{R}$ is uniformly continuous on A if for every $\varepsilon > 0$ there exists a δ such that for all $x, y \in A$, $|x - y| < \delta$ implies that $|f(x) - f(y)| < \varepsilon$.

Intermediate Value Theorem

Let $f : [a,b] \to \mathbb{R}$ be continuous. If L is a real number such that $f(a) < L < f(b)$ or $f(b) < L < f(a)$, then there exists a point $c \in [a,b]$ where $f(c) = L$.

Derivative at a point

Let $f : A \to \mathbb{R}$ be defined on A. Given a point $c \in A$, the derivative of f at c is defined by $g'(c) = \lim_{x \to c} \frac{g(x)-g(c)}{x-c}$, provided that the limit exists.

Mean Value Theorem

If $f : [a,b] \to \mathbb{R}$ is continuous on $[a,b]$, and differentiable on (a,b), then there exists a point $c \in (a,b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Uniform Convergence of a sequence of functions

Let (f_n) be defined on the set $A \subseteq \mathbb{R}$. Then f_n converges uniformly on A to a limit function f defined on A if for every $\varepsilon > 0$ there exists an $N \in \mathbb{N}$ such that whenever $n \geq N$ and $x \in A$ implies that $|f_n(x) - f(x)| < \varepsilon$.

Weierstrass M-Test

For each $n \in \mathbb{N}$, let f_n be a function defined on the set $A \subseteq \mathbb{R}$ and let $M_n > 0$ be a real number satisfying $|f_n(x)| \leq M_n$ for all $x \in A$. If $\sum M_n$ converges then $\sum f_n$ converges uniformly on A.