Exercise 4.4.9

(Lipschitz Function) A function $f : A \to \mathbb{R}$ is called Lipschitz if there exists a bound $M > 0$ such that

$$|\frac{f(x) - f(y)}{x - y}| \leq M$$

for all $x \neq y \in A$. Geometrically speaking, a function f is Lipschitz if there is a uniform bound on the magnitude of the slopes of lines drawn through any two points in the graph of f.

a) Show that if $f : A \to \mathbb{R}$ is lipschitz, then it is uniformly continuous on A.

Solution:

x<1

b) Is the converse statement true? Are all uniformly coninuous functions necesarily Lipschitz?

Solution:

x<1

Exercise 4.5.2ab

Provide an example of each of the following, or explain why the request is impossible.

a) A contuinuous function defined on an open interval with range equal to a closed interval.

Solution:

x<1

b) A continuous function defined on a closed inteval with a range equal to an open interval.

Solution:

x<1

Exercise 4.5.3

A function f is increasing on A if $f(x) \leq f(y)$ for all $x < y$ in A. Show that if f is increasing on $[a,b]$ and statisfies the intermediate value property (Def 4.5.3), then f is continuous on $[a,b]$.

Solution:

x<1

Exercise 4.5.7

Let f be a continuous function on the closed interval $[0,1]$ with range also contained in $[0,1]$. Prove that f must have a fixed point; that is, show $f(x) = x$ for at least one value of $x \in [0,1]$

Solution:

x<1