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DISTRIBUTION OF RESIDENCE TIME IN REACTORS

Chapter 13

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Introduction

After this chapter you will be able to

- Learn about non-ideal reactors
- Characterize non-ideal reactors

Nothing in life is to be feared. It is only to be understood

Marie Curie

Introduction

Not everything in life is ideal, although we wish it were. Same is with reactor behaviour (size and product distribution).

We like 2 flow patterns: plug and mixed flow

- One or the other is optimum no matter what we are designing for

- These 2 patterns are simple to treat

Introduction

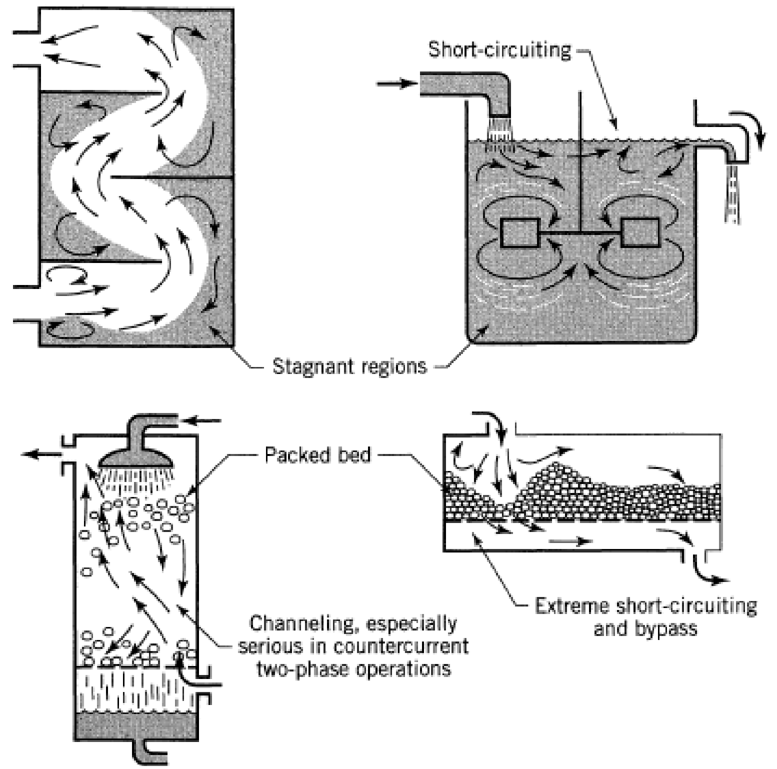
Overall three somewhat interrelated factors make up the contacting or flow pattern:

1. the **RTD** or residence time distribution of material which is flowing through the vessel
2. the **state of aggregation** of the flowing material, its tendency to clump and for a group of molecules to move about together
3. the **earliness and lateness of mixing** of material in the vessel.

Understanding residence time

- Not all molecules are spending the same time in the reactor
- The use of residence time distribution function becomes important as distribution of RT can affect its performance
- Different reactors can exhibit different RTD
- It is characteristic of the mixing that occurs in the chemical reactor

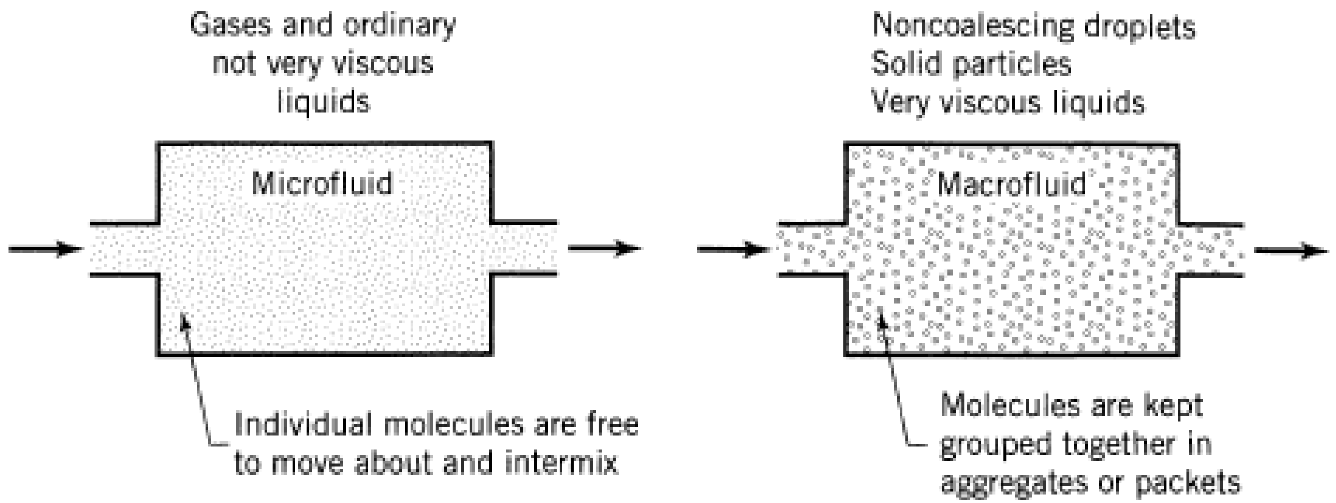
Understanding residence time



[redacted] Nonideal flow patterns which may exist in process equipment.

State of aggregation

Flowing material is in some particular state of aggregation, depending on its nature. In the extremes these states can be called *microfluids* and *macrofluids*,

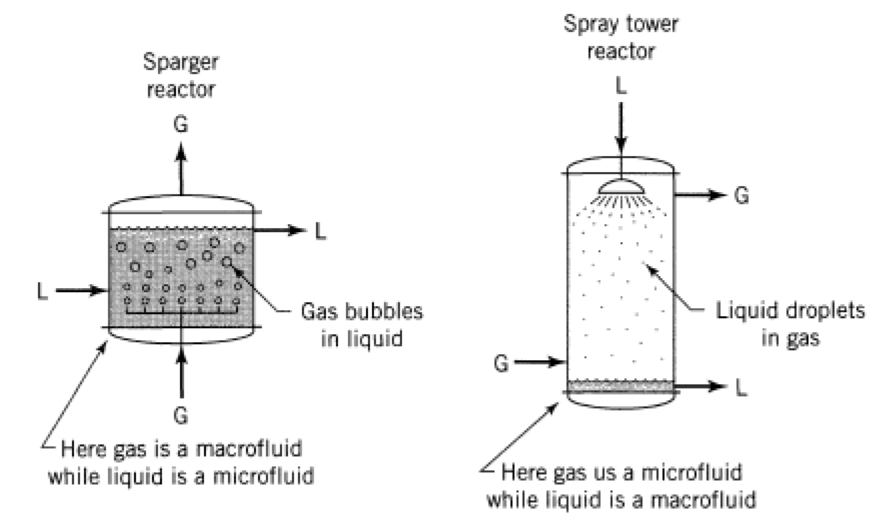


[redacted] Two extremes of aggregation of fluid.

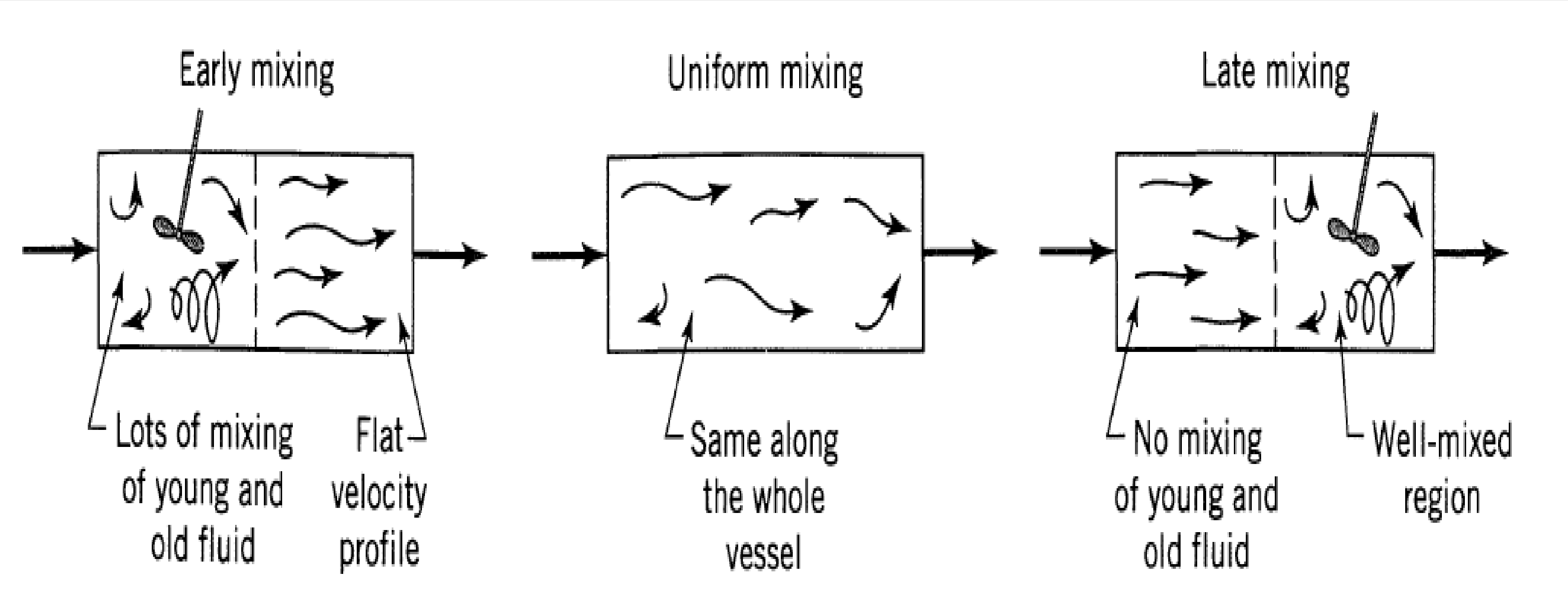
State of aggregation

Single-Phase Systems. These lie somewhere between the extremes of macro- and microfluids.

Two-Phase Systems. A stream of solids always behaves as a macrofluid, but for gas reacting with liquid, either phase can be a macro- or microfluid depending on the contacting scheme being used.



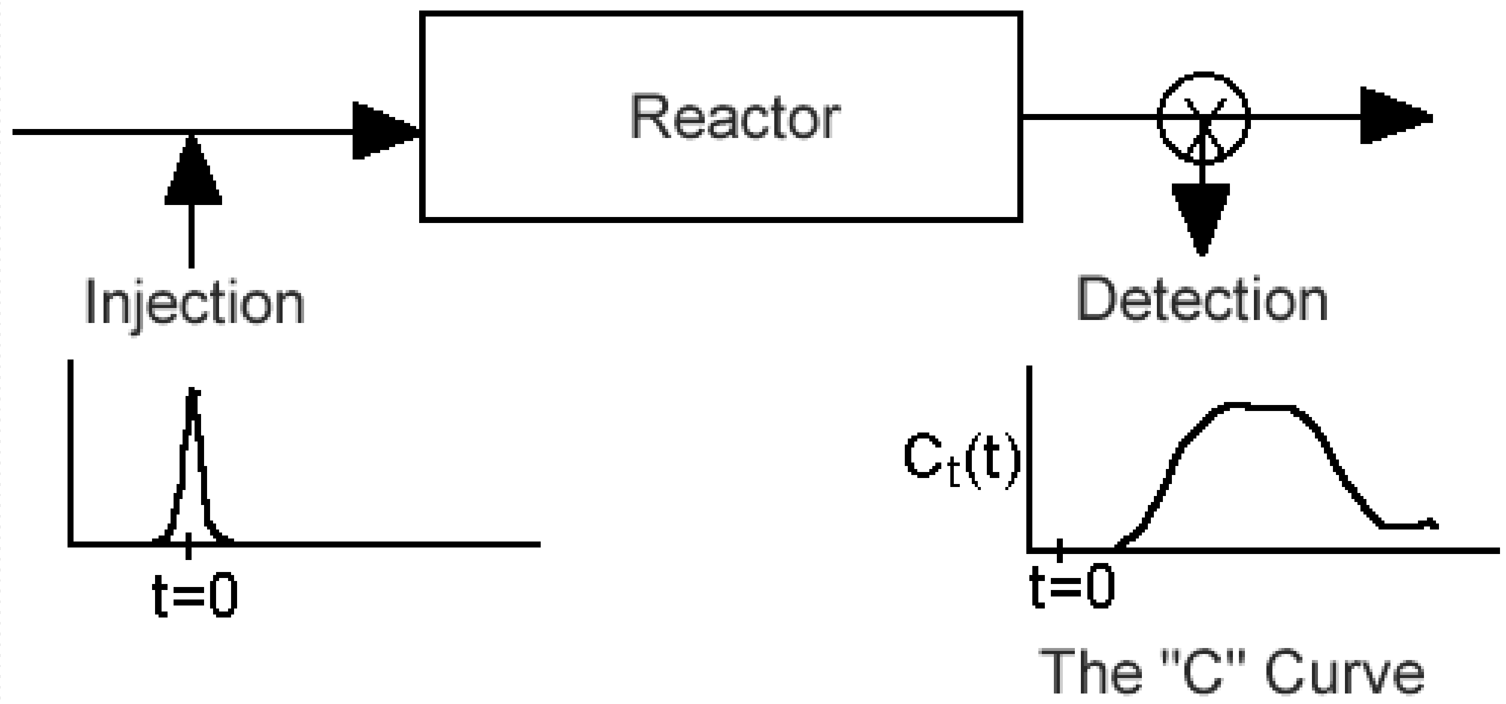
Earliness of Mixing



Measurement of RTD

- Injecting a tracer (inert chemical molecule or atom) into the reactor at time $t = 0$
- Measuring the tracer concentration C in the effluent stream as a function of time
- Non-reactive, detectable, not adsorbed on walls
- Reflect the behaviour of the material flowing through the reactor
- Injection by pulse or step input

Pulse input experiment



Pulse input experiment

- Injection of tracer pulse for single input single output system (no dispersion): one flow carries tracer material across system boundaries
- Choose an increment time Δt sufficiently small so that concentration of tracer $C(t)$ existing between t and $t + \Delta t$ is same
- Amount of tracer material leaving in this time interval $\mathbf{\Delta N = C(t) v \Delta t}$, where v is the effluent volumetric flowrate

Pulse input experiment

- $\Delta N / N_o = (C(t)v / N_o)\, \Delta t$ represents the fraction of material that has residence time between t and t + Δt

For pulse injection

$$\Delta N / N_o = E(t)\, \Delta t$$

E(t) is the RTD function which describes in a quantitative manner how much time different fluid elements have spent in the reactor

Pulse input experiment (The E – curve)

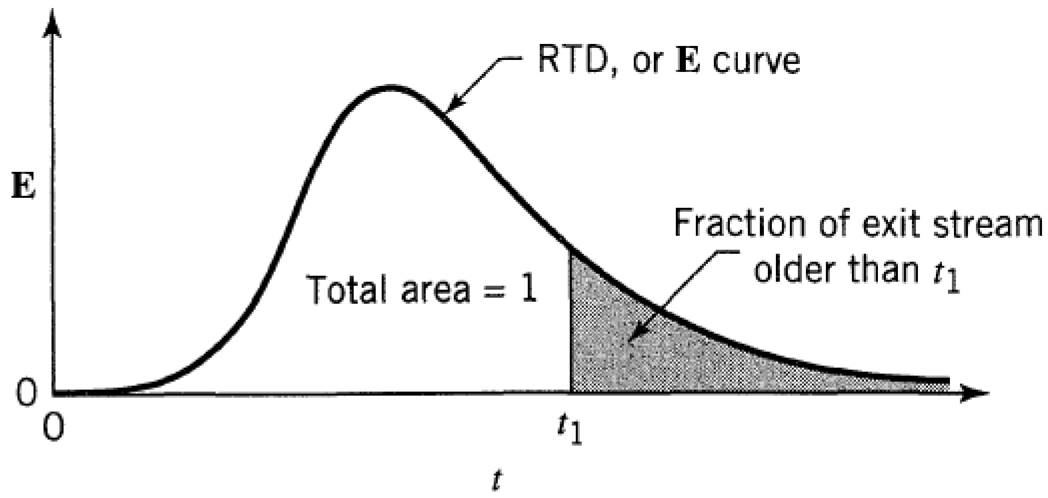


Figure 11.6 The exit age distribution curve **E** for fluid flowing through a vessel; also called the residence time distribution, or RTD.

$$\int_0^\infty \mathbf{E}\,dt = 1$$

The **E** curve is the distribution needed to account for nonideal flow.

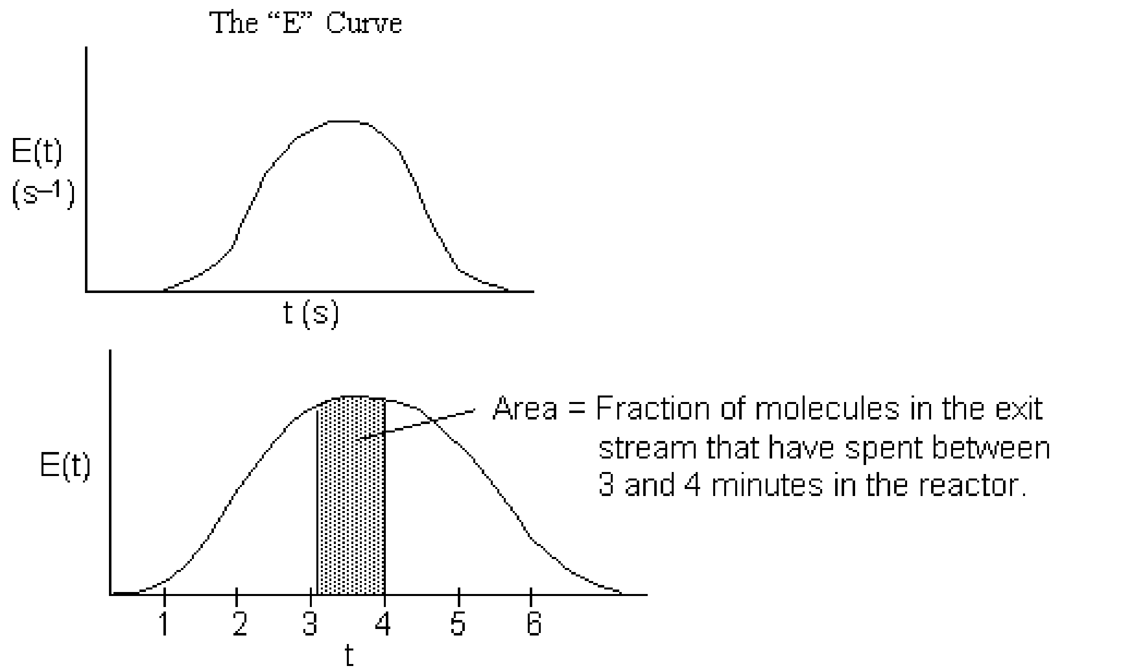
Pulse input experiment (The E – curve)

$$E(t) = \frac{C_T(t)}{\int_0^\infty C_T(t)dt}$$

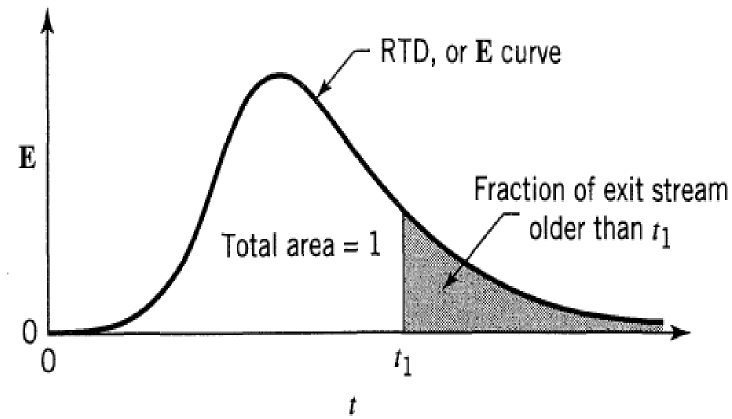
$$E(t)dt$$

↓

Fraction of molecules exiting the reactor that have spent a time between (t) and (t + dt) in the reactor



Pulse input experiment (The E – curve)



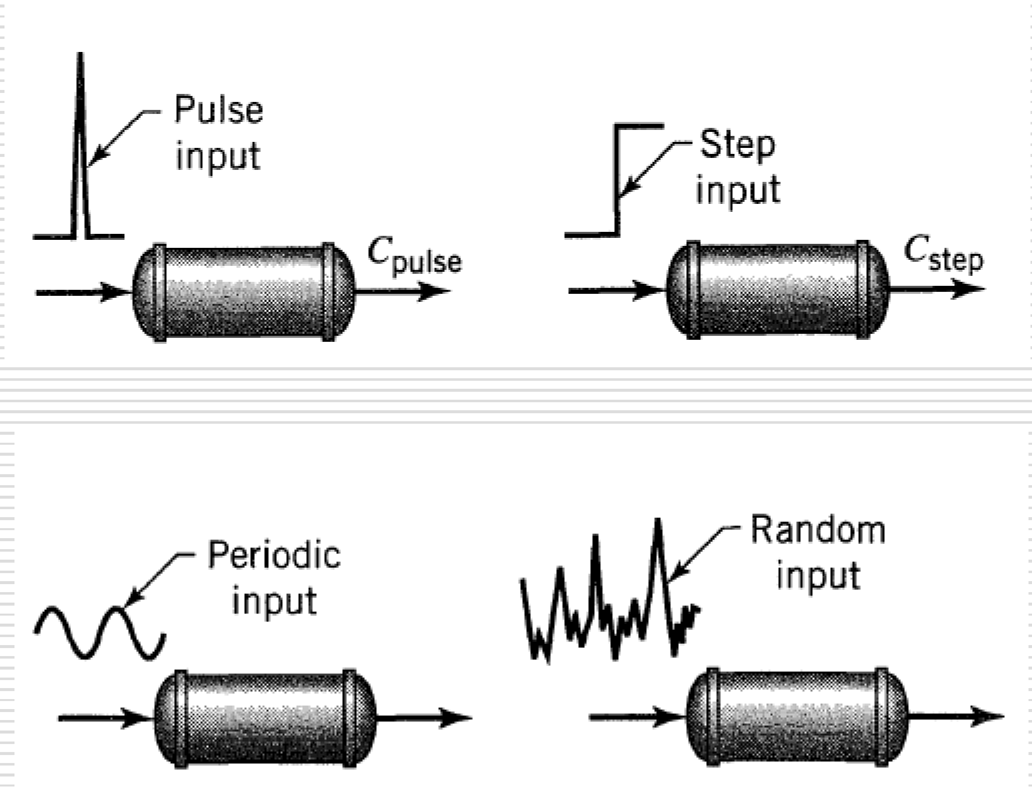
the fraction of material older than t_1

$$\int_{t_1}^{\infty} \mathbf{E}\, dt = 1 - \int_0^{t_1} \mathbf{E}\, dt \qquad [-]$$

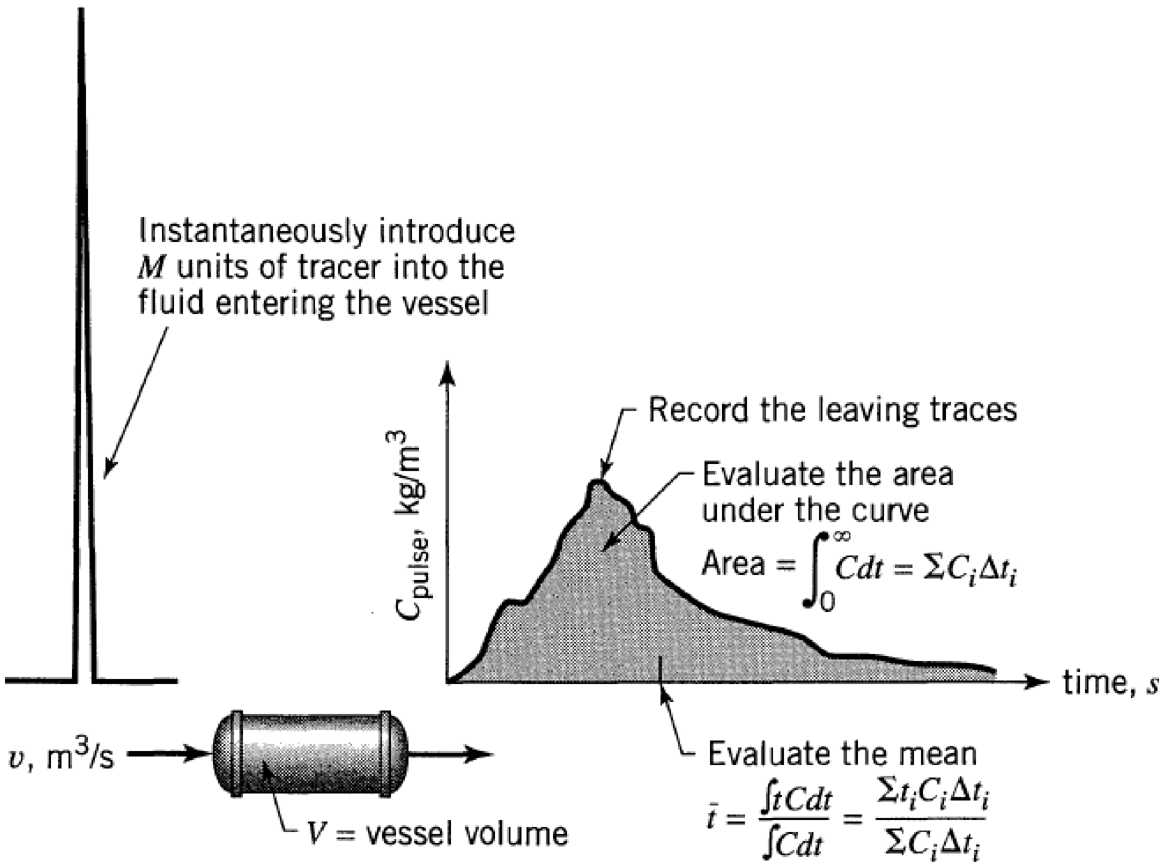
the fraction younger than age t_1 is

$$\int_0^{t_1} \mathbf{E}\, dt \qquad [-]$$

Experimental methods to study RTD



Experimental methods to study RTD



Experimental methods to study RTD

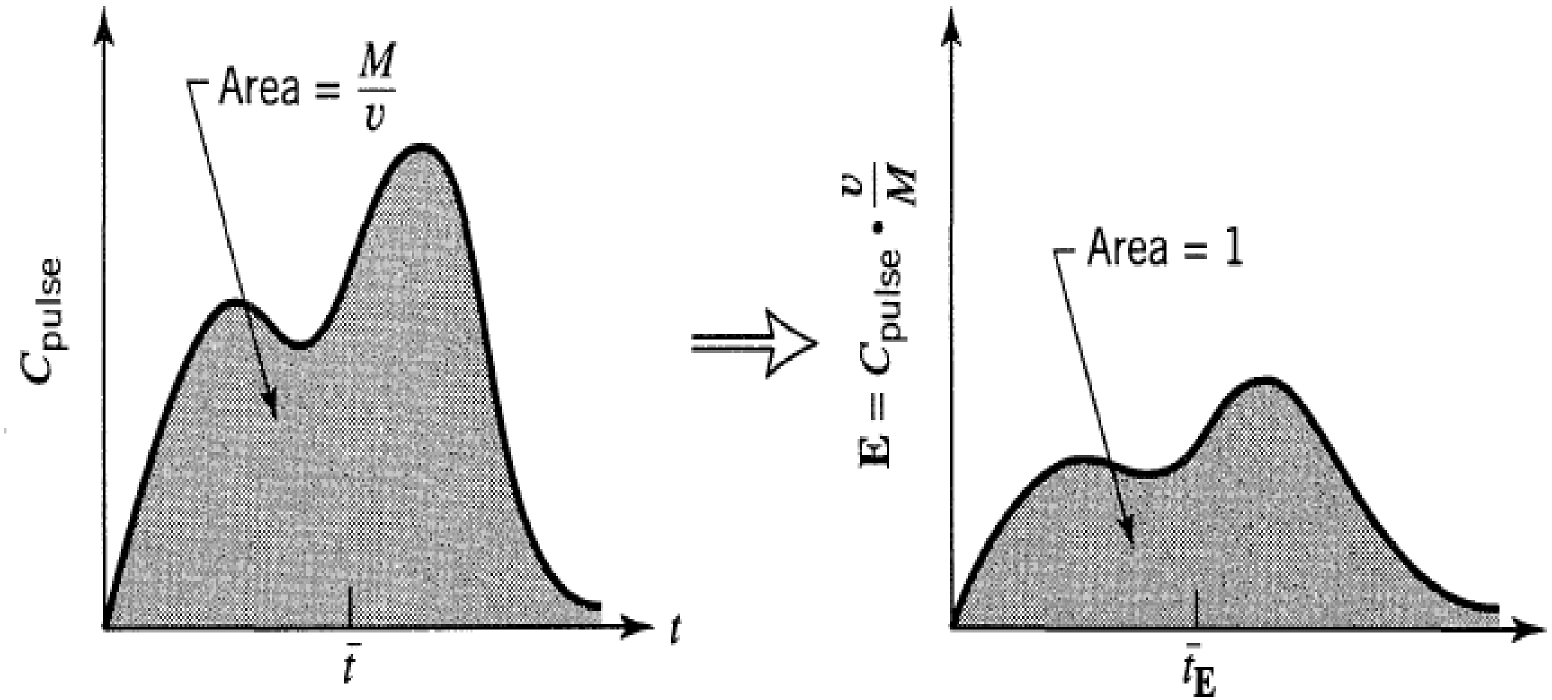
Let us find the **E** curve for a vessel of volume V m^3 through which flows v m^3/s of fluid. For this instantaneously introduce M units of tracer (kg or moles) into the fluid entering the vessel, and record the concentration-time of tracer leaving the vessel. This is the C_{pulse} curve. From the material balance for the vessel we find

$$\left(\begin{array}{c}\text{Area under the}\\ C_{\text{pulse}}\text{ curve}\end{array}\right): \quad A = \int_0^\infty C\,dt \cong \sum_i C_i \Delta t_i = \frac{M}{v} \qquad \left[\frac{\text{kg}\cdot\text{s}}{\text{m}^3}\right] \tag{3}$$

$$\left(\begin{array}{c}\text{Mean of the}\\ C_{\text{pulse}}\text{ curve}\end{array}\right): \quad \bar{t} = \frac{\int_0^\infty tC\,dt}{\int_0^\infty C\,dt} \cong \frac{\sum_i t_i C_i \Delta t_i}{\sum_i C_i \Delta t_i} = \frac{V}{v} \qquad [\text{s}] \tag{4}$$

$$\mathbf{E} = \frac{C_{\text{pulse}}}{M/v}$$

Experimental methods to study RTD



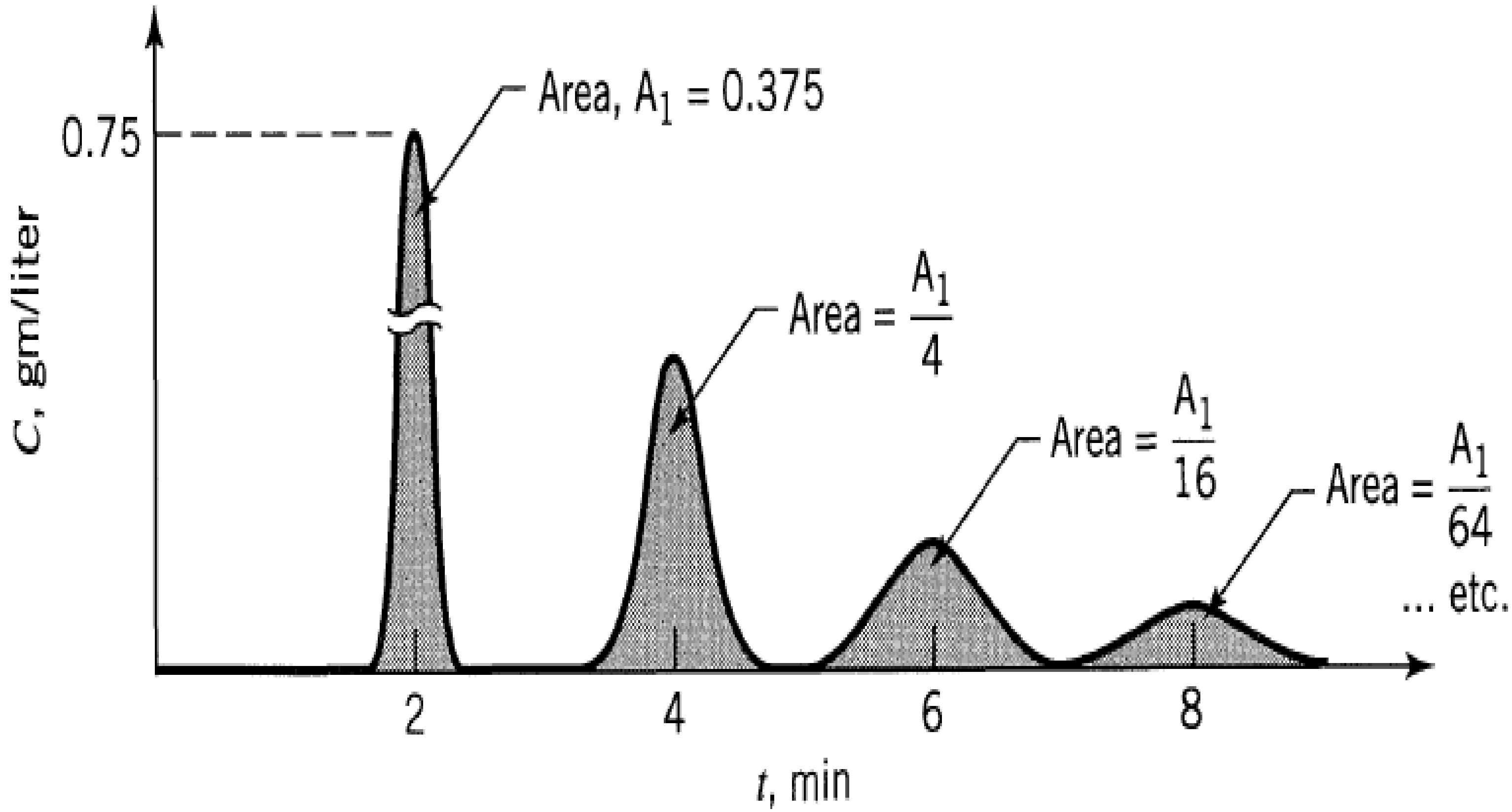
Transforming an experimental C_{pulse} curve into an $\mathbf{E}$ curve.

Problem

A large tank (860 liters) is used as a gas-liquid contactor. Gas bubbles up through the vessel and out the top, liquid flows in at one part and out the other at 5 liters/s. To get an idea of the flow pattern of liquid in this tank a pulse of tracer (M = 150 gm) is injected at the liquid inlet and measured at the outlet, as shown in Fig. E11.2*a*.

(a) Is this a properly done experiment?
(b) If so, find the liquid fraction in the vessel.
(c) Determine the **E** curve for the liquid.
(d) Qualitatively what do you think is happening in the vessel?

Solution



Area, A1 = 0.375
0.75
C, gm/liter
Area = A1/4
Area = A1/16
Area = A1/64
... etc.
2
4
6
8
t, min

Solution

Check the material balance against the tracer curve

$$\text{Area} = \frac{M}{v} = \frac{150\ \text{gm}}{5\ \text{liters/s}} = 30\,\frac{\text{gm}\cdot\text{s}}{\text{liter}} = 0.5\,\frac{\text{gm}\cdot\text{min}}{\text{liter}}$$

From the tracer curve

$$\text{Area} = A_1\left(1 + \frac{1}{4} + \frac{1}{16} + \cdot\ \cdot\ \cdot\right) = 0.375\left(\frac{4}{3}\right) = 0.5\,\frac{\text{gm}\cdot\text{min}}{\text{liter}}$$

These values agree. The results are consistent.

Solution

(b) For the liquid, ~~Eq. 4~~ gives

$$\bar{t}_l = \frac{\int tC\,dt}{\int C\,dt} = \frac{1}{0.5}\left[2A_1 + 4 \times \frac{A_1}{4} + 6 \times \frac{A_1}{16} + 8 \times \frac{A_1}{64} + \cdot \cdot \cdot\right] = 2.67 \text{ min}$$

Thus the liquid volume in the vessel is

$$V_l = \bar{t}_l v_l = 2.67(5 \times 60) = 800 \text{ liters}$$

and the volume fraction of phases is

$$\left.\begin{aligned} \text{Fraction of liquid} &= \frac{800}{860} = 93\% \\ \text{Fraction of gas} &= 7\% \end{aligned}\right\} \qquad \textbf{(b)}$$

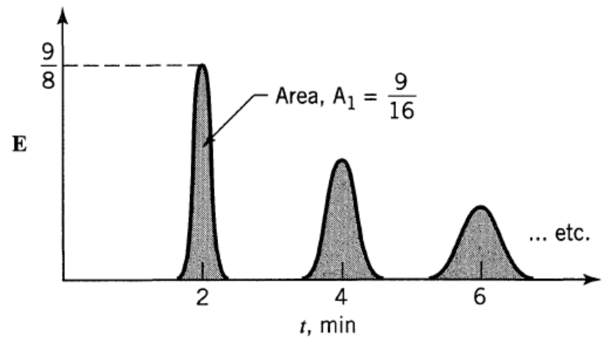
Solution

(c) Finally, from ~~Eq. 5~~ we find the **E** curve, or

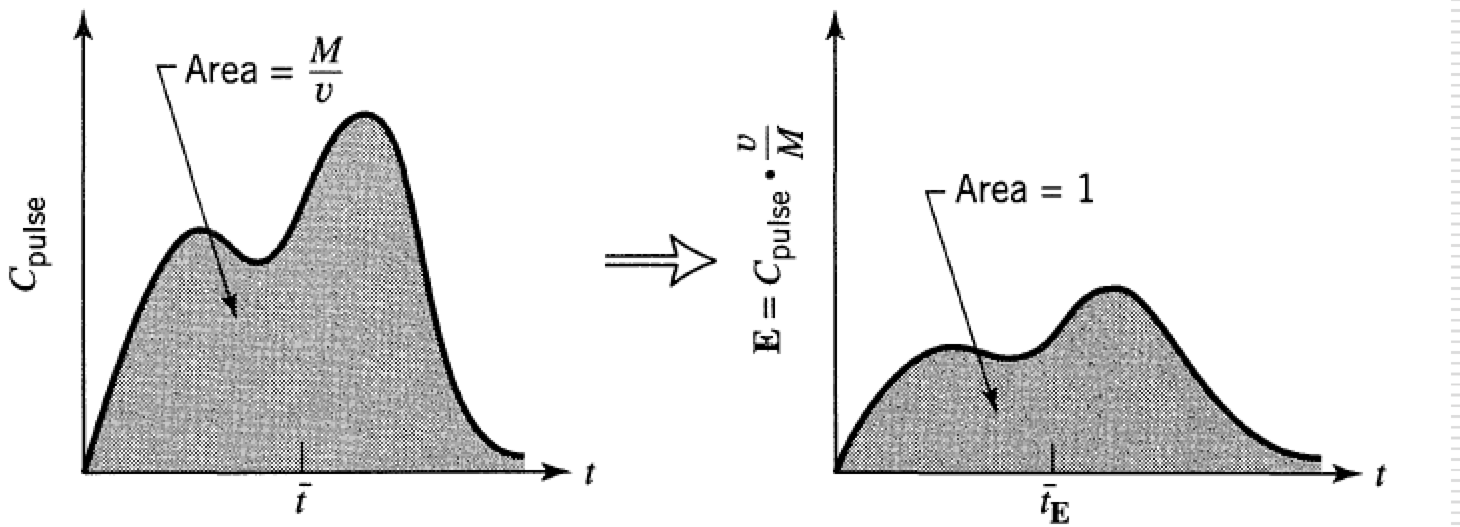
$$\mathbf{E} = \frac{C_{\text{pulse}}}{M/v} = \frac{0.75}{0.5}\,\mathrm{C} = 1.5\,\mathrm{C}$$

Thus the **E** curve for the liquid is as shown in ~~Fig. E11.2*b*~~. **(c)**

(d) The vessel has a strong recirculation of liquid, probably induced by the rising bubbles. **(d)**



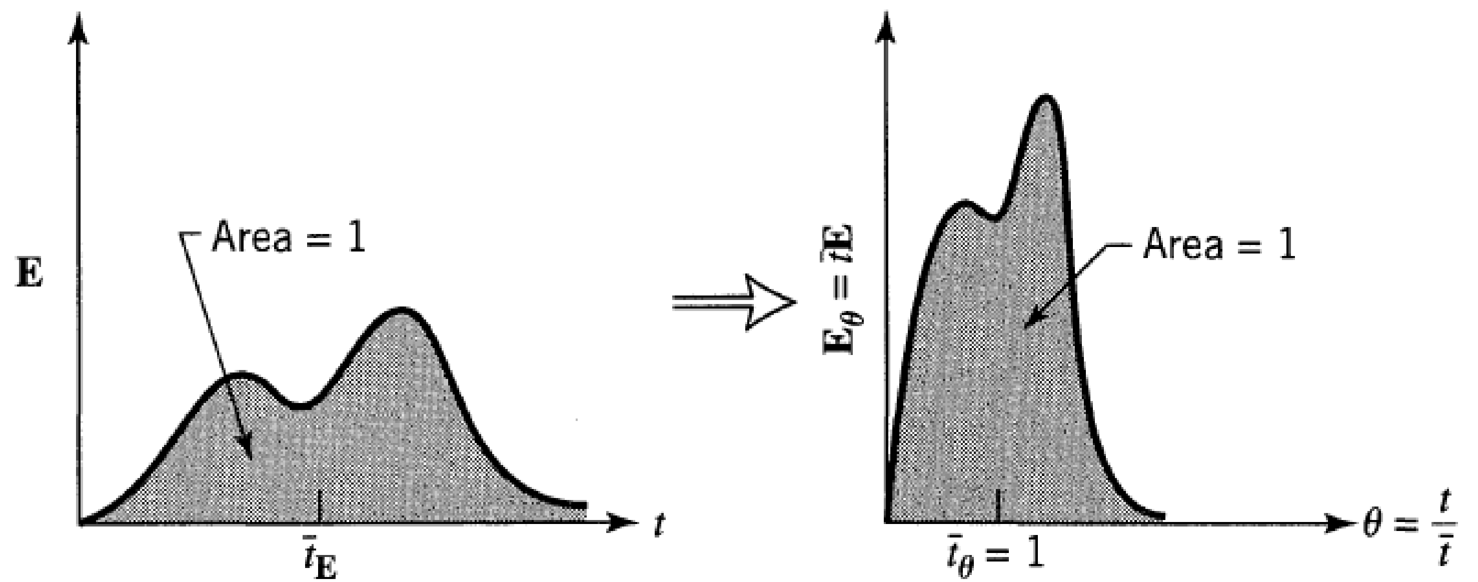
Experimental methods to study RTD



We have another RTD function $\mathbf{E}_\theta$. Here time is measured in terms of mean residence time $\theta = t/\bar{t}$. Thus

$$\mathbf{E}_\theta = \bar{t}\mathbf{E} = \frac{V}{v} \cdot \frac{C_{pulse}}{M/v} = \frac{V}{M} C_{pulse}$$

Experimental methods to study RTD

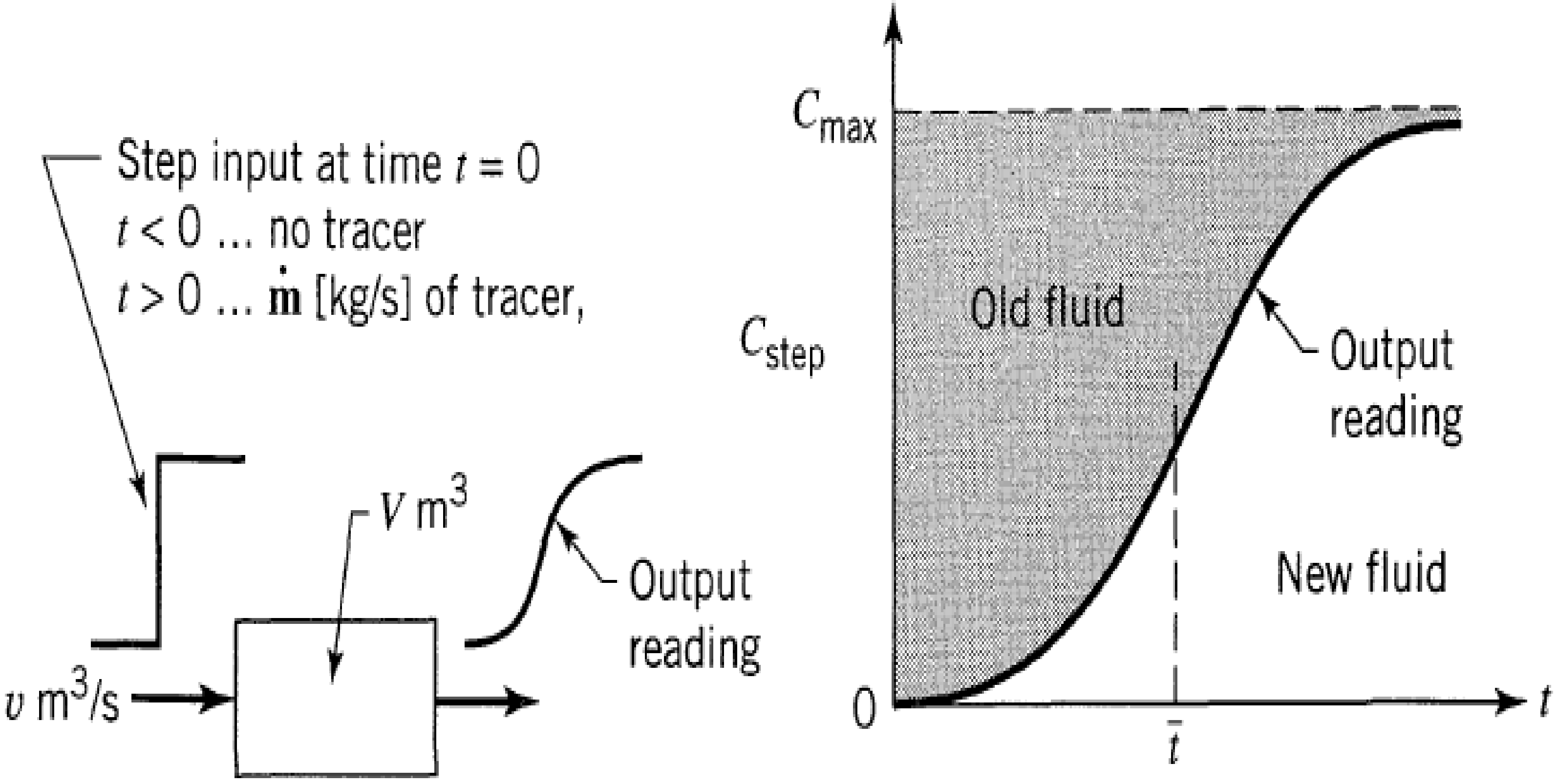


Transforming an **E** curve into an $\mathbf{E}_\theta$ curve.

Step Experiment

Consider v m^3/s of fluid flowing through a vessel of volume V. Now at time t = 0 switch from ordinary fluid to fluid with tracer of concentration $C_{max} = \left[\dfrac{\text{kg or mol}}{\text{m}^3}\right]$, and measure the outlet tracer concentration C_{step} versus t, as shown in Fig. 11.11.

Step Experiment



Step Experiment

$$C_{\max} = \frac{\dot{m}}{v}\left[\frac{\text{kg}\cdot\text{s}}{\text{m}^3}\right]$$

$$\begin{pmatrix}\text{shaded area}\\ \text{of Fig. 11.11}\end{pmatrix} = C_{\max}\bar{t} = \frac{\dot{m}V}{v^2}\left[\frac{\text{kg}\cdot\text{s}^2}{\text{m}^3}\right]$$

$$\bar{t} = \frac{\int_0^{C_{\max}} t\,dC_{\text{step}}}{\int_0^{C_{\max}} dC_{\text{step}}} = \frac{1}{C_{\max}}\int_0^{C_{\max}} t\,dC_{\text{step}}$$

$\dot{m}$ [kg/s] is the flow rate of tracer in the entering fluid.

The dimensionless form of the C_{step} curve is called the **F** curve.

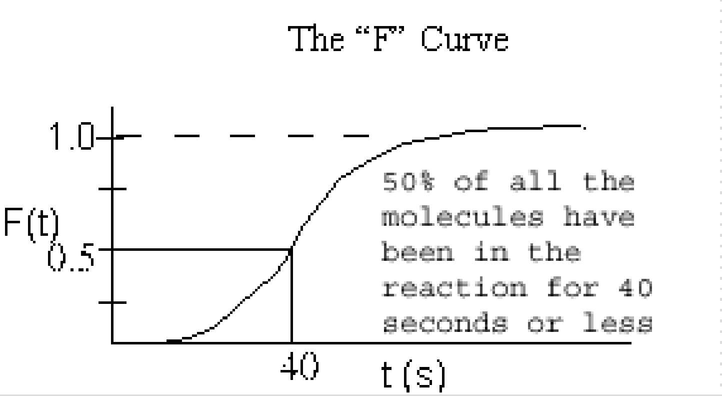
The cumulative distribution function

$$F(t) = \int_0^t E(t)dt$$

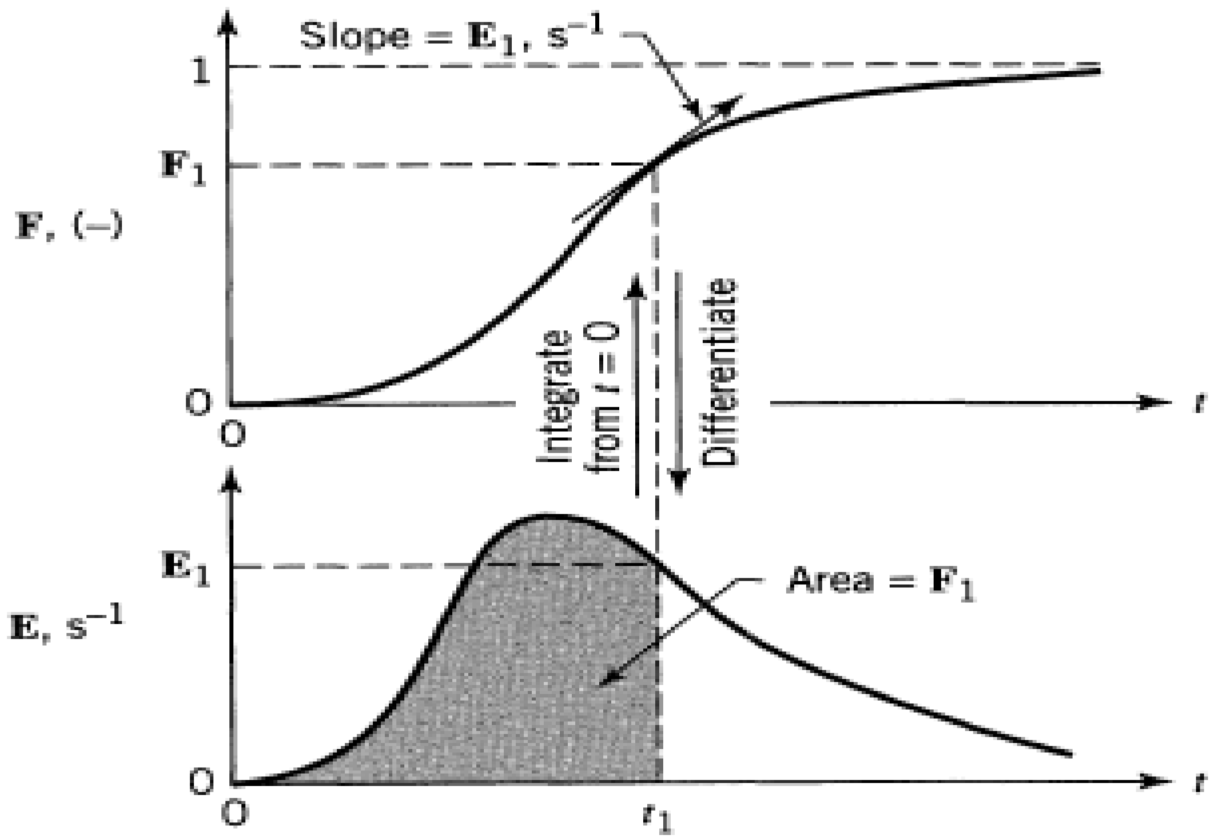
Fraction of molecules exiting the reactor that have spent a time t or less in the reactor

$$1 - F(t)$$

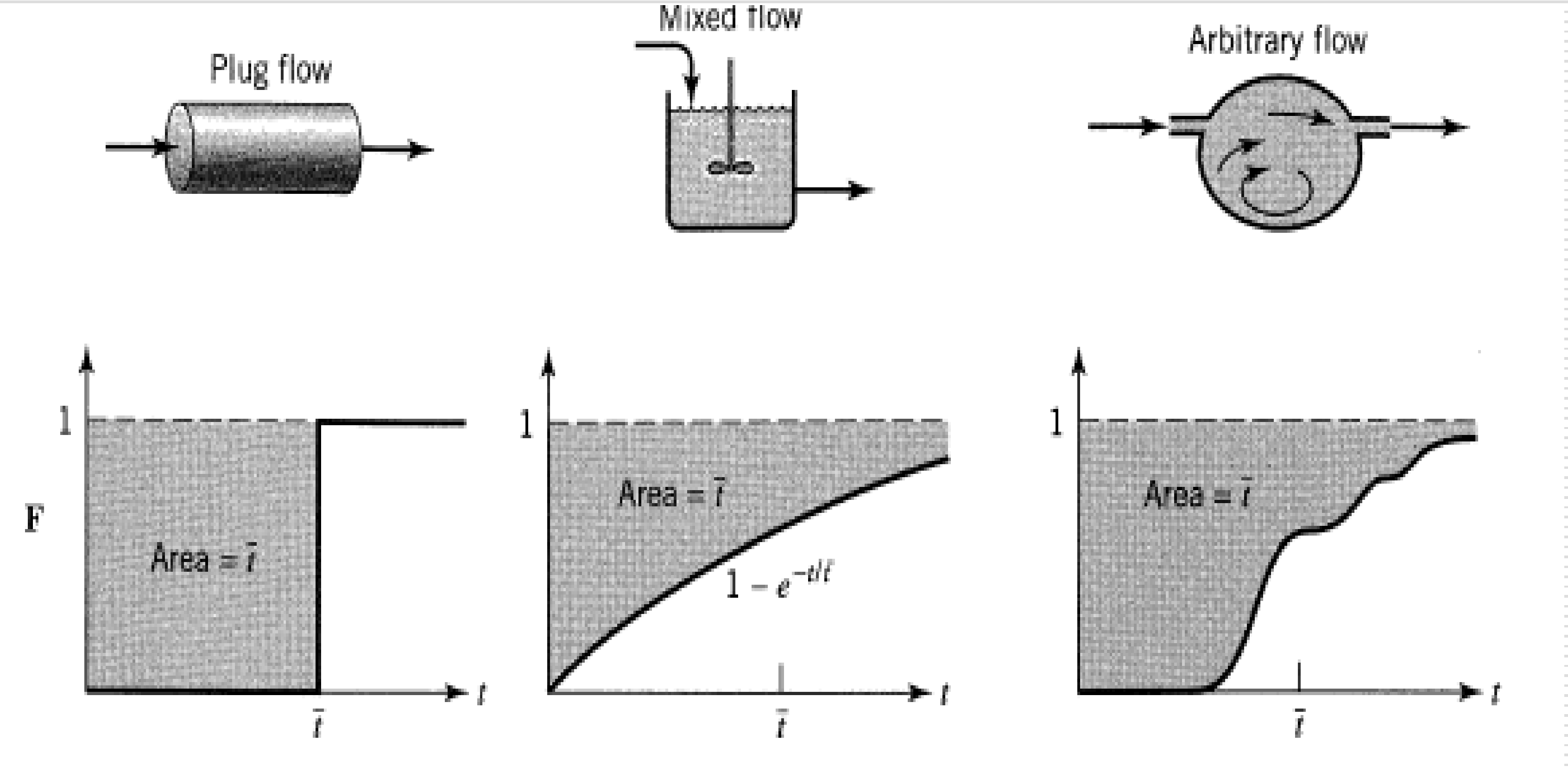
Fraction of molecules that have spent a time t or greater in the reactor. Integral from t to ∞



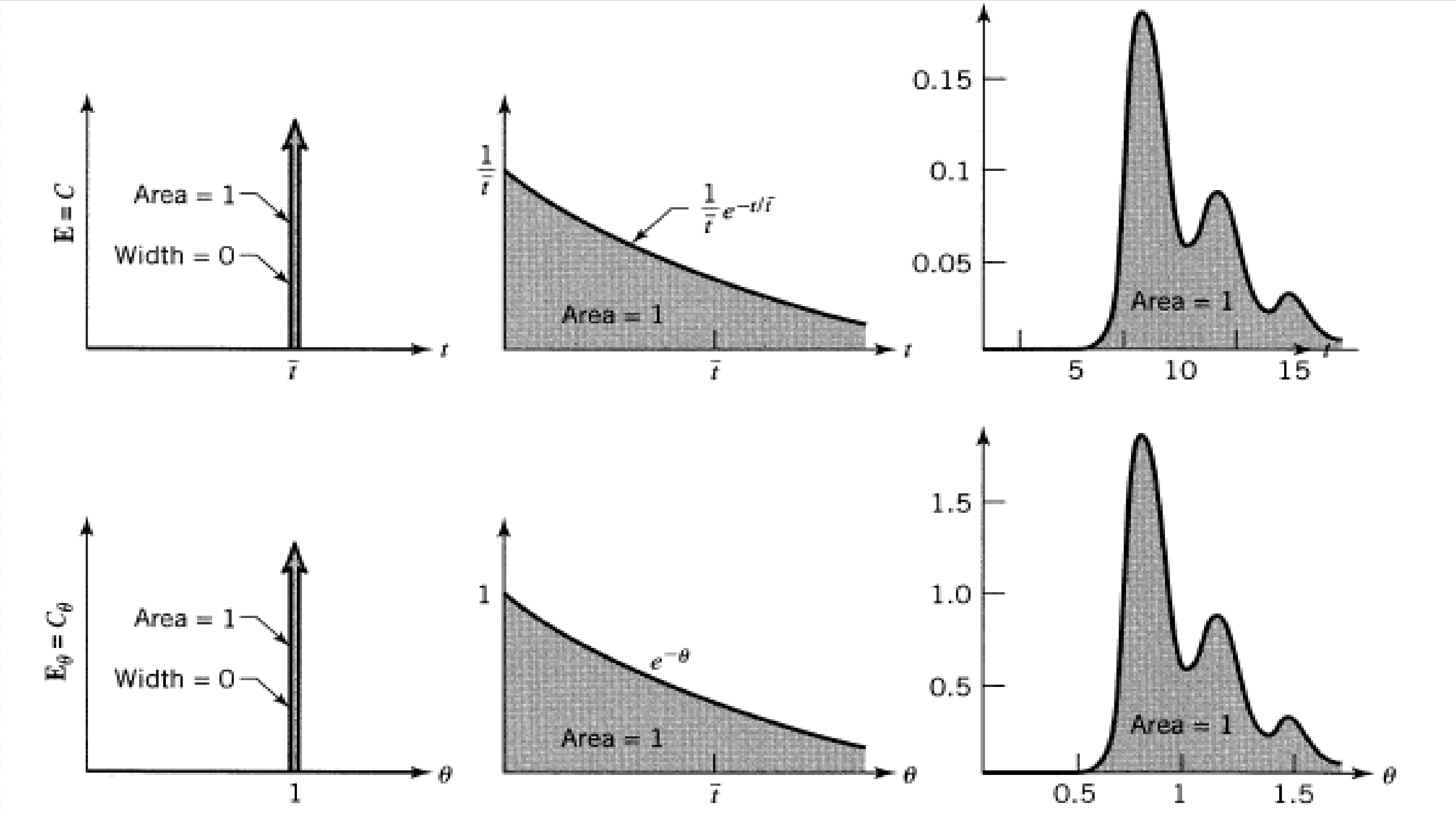
The cumulative distribution function



The cumulative distribution function



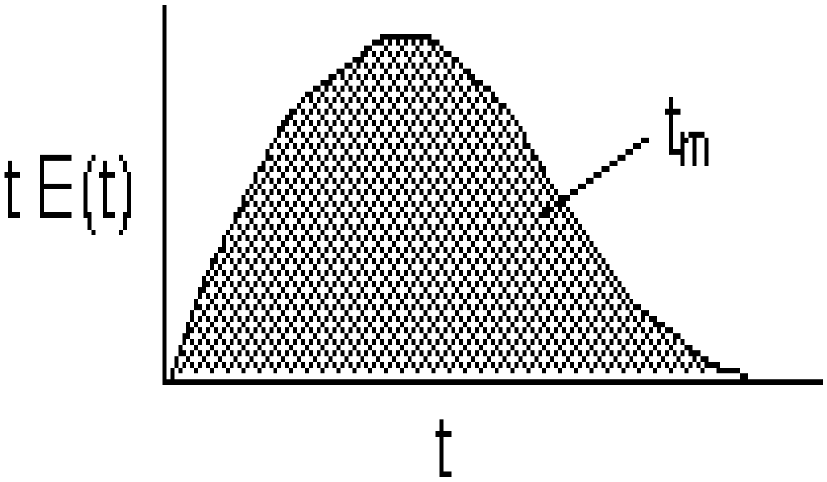
The cumulative distribution function



Mean residence Time

- For $v = v_o$, it is equal to space time and gives the average time the effluent molecules spent in the reactor (dispersion is absent)

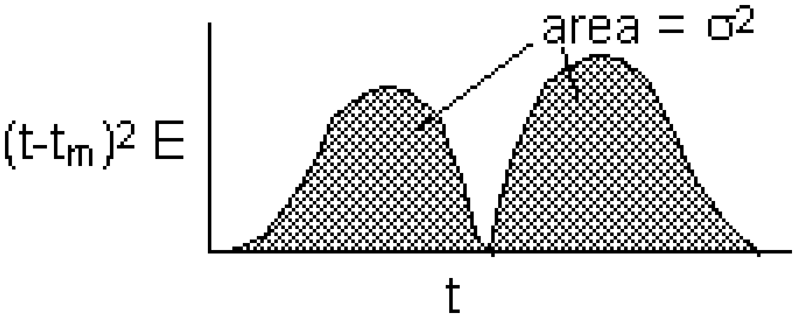
$$t_m = \int_0^\infty t\,E(t)\,dt$$



Other moments of RTD

- First moment is the mean residence time
- Second moment is the variance or square of standard variation. Its magnitude gives the spread of the distribution

$$\sigma^2 = \int_0^\infty (t - t_m)^2 E(t)dt$$



Other moments of RTD

At any time these curves are related as follows:

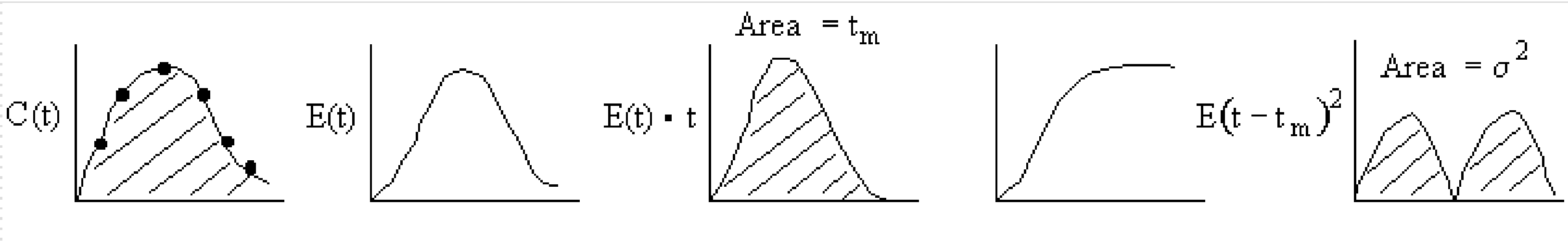
$$\mathbf{E} = \frac{v}{\dot{\mathbf{m}}} \cdot C_{\text{pulse}}, \quad \mathbf{F} = \frac{v}{\dot{\mathbf{m}}} \cdot C_{\text{step}}, \quad \mathbf{E} = \frac{d\mathbf{F}}{dt},$$

$$\bar{t} = \frac{V}{v}, \quad \theta = \frac{t}{\bar{t}}, \quad \bar{\theta}_{\mathbf{E}} = 1, \quad \mathbf{E}_{\theta} = \bar{t}\mathbf{E}$$

θ, $\mathbf{E}_{\theta}$, $\mathbf{F}$ . . . all dimensionless, $\mathbf{E} = [\text{time}^{-1}]$

To summarize

t (min) :	0	1	2	3	4	5	6
C(mg/m^3) :	0	0	0.1	0.2	0.3	0.1	0



$$\text{Area} = \int_0^\infty C(t)dt \qquad E(t) = \frac{C(t)}{\int_0^\infty C(t)dt} \qquad t_m = \int_0^\infty tE(t)dt \qquad F(t)\int_0^t E(t)dt \qquad \sigma^2 = \int_0^\infty (t - t_m)^2 E(t)dt$$

Problem

The following data were obtained from a tracer test to a reactor.

1) Plot Ct(t).
2) Find E(t).
3) Find the fraction of material that spends between 15 and 20 seconds in the reactor.
4) Find F(t) and, the fraction of material that spends 25 seconds or less in the reactor.
5) Evaluate mean residence time.
6) Evaluate the variance.

t(s):	0	5	10	15	20	25	30	35
C_t(mg/dm^3):	0	0	0	5	10	5	0	0

Problem

The concentration readings in Table E11.1 represent a continuous response to a pulse input into a closed vessel which is to be used as a chemical reactor. Calculate the mean residence time of fluid in the vessel t, and tabulate and plot the exit age distribution **E**.

Table E11.1

Time t, min	Tracer Output Concentration, C_{pulse} gm/liter fluid
0	0
5	3
10	5
15	5
20	4
25	2
30	1
35	0