PH4505/PAP723 Homework 1

AY17/18 Semester 2

Instructions for PH4505 students: You are to choose 2 of the 3 questions to submit as your graded assignment. Every plot produced by your programs should have clear x and y axis labels. If more than one curve is shown on a single plot, the two curves should be labeled clearly. For full marks, code must follow good programming style. Your code should be commented; there should be no cryptic variable and function names (like abc); and the program structure should be modular (e.g. numerical constants should be defined in one place instead of being scattered throughout the code).

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1. **Quantum uncertainty in the harmonic oscillator**

In units where all the constants are 1, the wave function of the nth energy level of the one-dimensional quantum harmonic oscillator - i.e., a spinless point particle in a quadratic potential well - is given by

$$\psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} \mathrm{e}^{-x^2/2} H_n(x), \tag{1}$$

for $n = 0 \ldots \infty$, where $H_n(x)$ is the nth Hermite polynomial. Hermite polynomials satisfy a relation somewhat similar to that for the Fibonacci numbers, although more complex:

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x). \tag{2}$$

The first two Hermite polynomials are $H_0(x) = 1$ and $H_1(x) = 2x$.

(a) Write a user-defined function $H(n, x)$ that calculates $H_n(x)$ for given x and any integer $n \geq 0$. Use your function to make a plot that shows the harmonic oscillator wavefunctions for $n = 0, 1, 2, 3$, all on the same graph, in the range $x = -4$ to $x = 4$. Hint: There is a function `factorial` in the `math` package that calculates the factorial of an integer.

(b) Make a seperate plot of the wavefunction for $n = 30$ from $x = -10$ to $x = 10$. Hint: If your program takes too long to run in this case, then you are doing the calculation wrong - the program should take only a second or so to run.

(c) The quantum uncertainty in the position of a particle in the nth level of a harmonic oscillator can be quantified by its root-mean-square position $\sqrt{\langle x^2 \rangle}$, where

$$\langle x^2 \rangle = \int_{\infty}^{-\infty} x^2 \left|\psi_n(x)\right|^2 \mathrm{d}x\,. \tag{3}$$

Write a program that evaluates this integral using Gaussian quadrature on 100 points, then calculate the uncertainty (i.e., the root-mean-square position of the particle) for a given value of n. Use your program to calculate the uncertainty for $n = 5$. You should get an answer in the vicinity of $\sqrt{\langle x^2 \rangle} = 2.3$.

2. **Gravitational pull of a uniform sheet**

A uniform square sheet of metal is floating motionless in space:

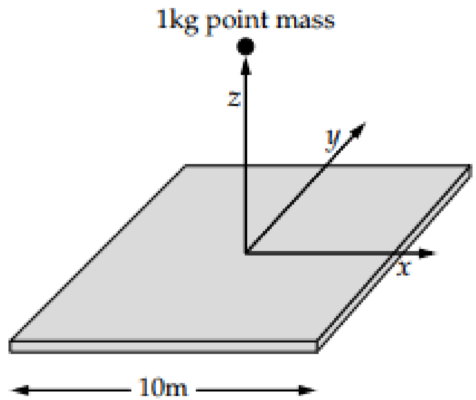

Figure 1: Floating plate

The sheet is 10m on a side and of negligible thickness, and it has a mass of 10 metric tonnes. Consider the gravitational force due to the plate felt by a point mass of 1kg a distance z from the center of the square, in the direction perpendicular to the sheet, as shown in Fig. 1. The component of the force along the z-axis is

$$F_z = G\sigma z \int\int_{-L/2}^{L/2} \frac{\mathrm{d}x\mathrm{d}y}{(x^2+y^2+z^2)^{3/2}}, \tag{4}$$

where $G = 6.674 \times 10^{-11}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$ is Newton's gravitational constant and σ is the mass per unit area of the sheet.

(a) Write a program to calculate and plot the force as a function of z from $z = 0$ to $z = 10$m. For the double integral, use (double) Gaussian quadrature with 100 sample points along each axis.

3. **Volume of a torus**

A ring torus is the product of 2 circles, generated by revolving a circle around an axis which forms another circle.

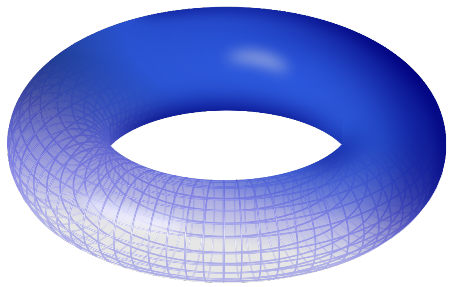

Figure 2: Torus

(a) Write a program to calculate the volume of a torus where the major radius $R = 1$ and minor radius $r = 0.5$. You are to use the Monte Carlo integration method to determine the volume.

(b) Using the data that you obtain from the Monte Carlo integration, make a 3-dimensional scatter plot of the results to visualize the torus.