

# Lecture2

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## 1 Monte Carlo: Lecture 2 Excercises

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Implement a Normal random generator, i.e., generate values of the random variable  $x$  from uniformly distributed random numbers such that  $x$  follows the Normal distribution

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right],$$

where  $\mu$  is the mean and  $\sigma^2$  the variance, both treated as input parameters.

**Goal: Box–Muller method**

1. Generate two independent uniformly distributed random numbers,  
 $0 < u_1 < 1$  and  $0 < u_2 < 1$ .
2. Using  $u_1$  and  $u_2$  define

$$r = \sqrt{-2 \ln u_1}, \quad \text{radius}$$

$$\varphi = 2\pi u_2, \quad \text{angle / phase}$$

3. Compute two independent standard normal random variables,

$$z_0 = r \cdot \cos \varphi,$$

$$z_1 = r \cdot \sin \varphi.$$

so that variables  $z_0$  and  $z_1$  follow a normal distribution with zero mean  $\mu = 0$  and unit standard deviation  $\sigma = 1$ .

4. Obtain normally distributed variables with arbitrary mean  $\mu$  and standard deviation  $\sigma > 0$  by adjusting the average value and the standard deviation  $\sigma$ :

$$x_0 = \mu + \sigma \cdot z_0,$$

$$x_1 = \mu + \sigma \cdot z_1.$$

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[1]: # Libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
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from IPython.display import display, clear_output

[2]: # Function to create an array that follows a normal distribution
def generate_normal_dist(mean=0., std=1., L:int=1):
    # We need to create one u that stores the uniformly distributed values
    # with size L+1 when L is odd in order to take it by half when constructing
    # values
    total_size = L + L%2
    u = np.random.uniform(size=total_size)

    r = np.sqrt(-2*np.log(u[:int(total_size/2)]))
    phi = 2*np.pi*u[int(total_size/2):]

    z_0 = r * np.cos(phi)
    z_1 = r * np.sin(phi)

    x_0 = mean + std * z_0
    x_1 = mean + std * z_1
    return np.append(x_0,x_1)[:L]

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[3]: mean=0.
std=1.
num_sigma=4.
num_bins=20
num_iter=1000

# Define Boundaries of the distribution
lower_boundary = mean - std * num_sigma
higher_boundary = mean + std * num_sigma

# Create Bin Limits within the defined limits
bin_limits = np.linspace(
    start = lower_boundary,
    stop = higher_boundary,
    num = num_bins+1
)
frequency_data = np.zeros(num_bins)

# Precompute bin centers/width for plotting
bin_widths = np.diff(bin_limits)
bin_centers = (bin_limits[:-1] + bin_limits[1:]) / 2

# --- Live plot setup ---
plt.ion()
fig, ax = plt.subplots()
bars = ax.bar(bin_centers, frequency_data, width=bin_widths, align="center")
ax.set_xlim(lower_boundary, higher_boundary)

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ax.set_xlabel("x")
ax.set_ylabel("Frequency Count")
ax.set_title("Live histogram")

# Generate Random Value, Register in Frequency Data and Graph in Real Time
for i in range(num_iter):

    # Get Value from the Normal Distribution Generator
    current_val = generate_normal_dist(L=1)[0]

    # Classify the value in between the bin limits using numpy's searchsorted
    idx = np.searchsorted(bin_limits, current_val, side="right") - 1

    # Sum 1 to the count array
    if 0 <= idx < num_bins:
        frequency_data[idx] += 1

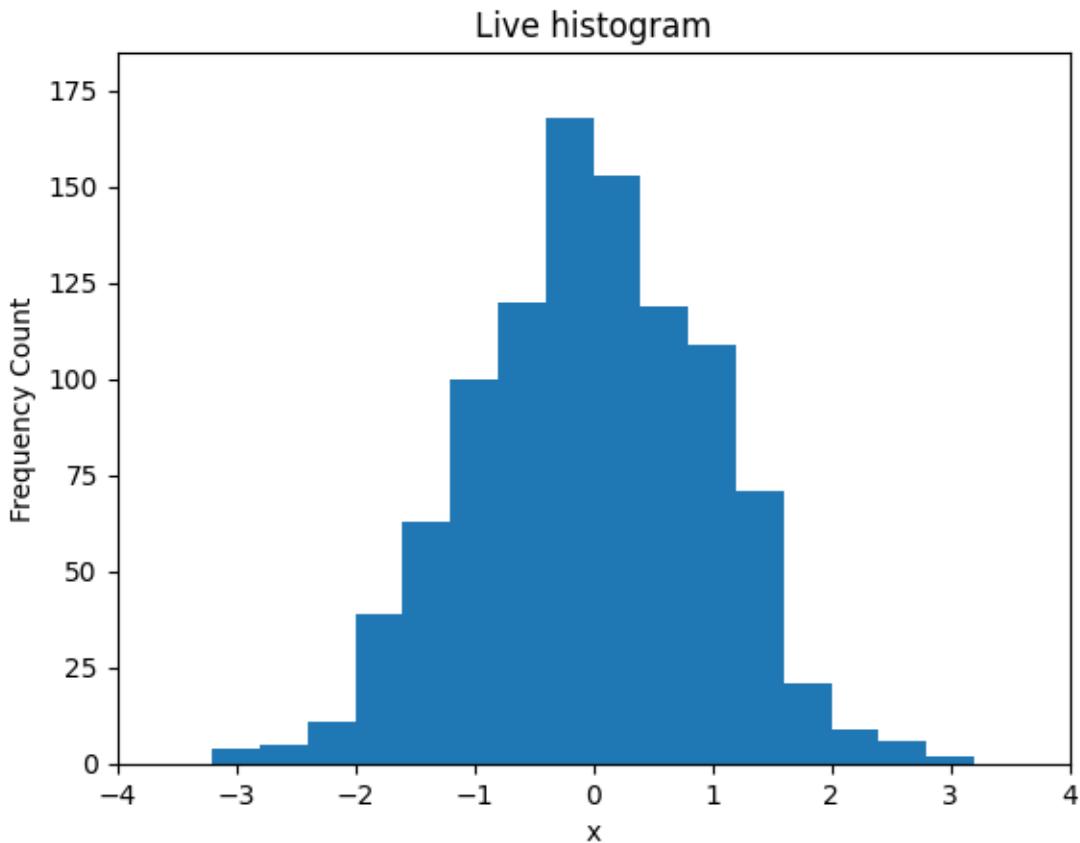
    # update graph
    for b, h in zip(bars, frequency_data):
        b.set_height(h)

    ax.set_ylim(0, max(1, frequency_data.max() * 1.1))

    clear_output(wait=True)
    display(fig)
    # plt.pause(0.01)

plt.close(fig)

```



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[4]: pd.DataFrame({  
    "Lower Bin Limit": bin_limits[:-1],  
    "Upper Bin Limit": bin_limits[1:],  
    "Frequency Count": frequency_data  
})
```

	Lower Bin Limit	Upper Bin Limit	Frequency Count
0	-4.0	-3.6	0.0
1	-3.6	-3.2	0.0
2	-3.2	-2.8	4.0
3	-2.8	-2.4	5.0
4	-2.4	-2.0	11.0
5	-2.0	-1.6	39.0
6	-1.6	-1.2	63.0
7	-1.2	-0.8	100.0
8	-0.8	-0.4	120.0
9	-0.4	0.0	168.0
10	0.0	0.4	153.0
11	0.4	0.8	119.0
12	0.8	1.2	109.0

13	1.2	1.6	71.0
14	1.6	2.0	21.0
15	2.0	2.4	9.0
16	2.4	2.8	6.0
17	2.8	3.2	2.0
18	3.2	3.6	0.0
19	3.6	4.0	0.0

[5]: *# Testing number generator*

```
plt.hist(generate_normal_dist(mean=10, std=8, L=100000 ), bins=100)  
plt.show()
```

