VBUDSII: The second "individual" homogenization approach.

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This document presents the formulation for the second homogenization approach for VBUDSII. This method has been developed because the original formulation described in the MS thesis by Chris Dembia does not yield sufficiently accurate results. The first homogenization formulation is described in Dembia's thesis.

The method is presented for a four-cell implementation. The four cells are:

1. AA: cell pair A, antipin (e.g. UOX fuel).
2. AP: cell pair A, pin (e.g. UOX's coolant).
3. BA: cell pair B, anitpin (e.g. IMF fuel).
4. BP: cell pair B, pin (e.g. IMF's coolant).

We obtain a mean free path in each cell as a function of energy group:

$$\lambda^{AA}(E) = 1/\Sigma_t^{AA}(E) \tag{1}$$
$$\lambda^{AP}(E) = 1/\Sigma_t^{AP}(E) \tag{2}$$
$$\lambda^{BA}(E) = 1/\Sigma_t^{BA}(E) \tag{3}$$
$$\lambda^{BP}(E) = 1/\Sigma_t^{BP}(E) \tag{4}$$

We use these mean free paths to obtain the homogenization parameter in each cell for each energy group.

$$h^{AA} = h(\lambda^{AA}(E)) \tag{5}$$
$$h^{AP} = h(\lambda^{AP}(E)) \tag{6}$$
$$h^{BA} = h(\lambda^{BA}(E)) \tag{7}$$
$$h^{BP} = h(\lambda^{BP}(E)) \tag{8}$$

where $h(\lambda)$ is given by:

$$h(\lambda) = \frac{1}{2}\left(1 + \mathrm{erf}[k(\lambda - \lambda^h)]\right) \tag{9}$$

but could be given by a similar expression. The quantities k and λ^h are parameters that the user can change; λ^h should be set to some characteristic dimension of the cell and there is little to be said as of yet about how k should be chosen. The value of h is always less than or equal to 1.

In each column of the $\mathbf{\Pi}$ matrix, two terms represent probabilities of remaining in the cell pair, and two represent probabilities of moving to the other cell pair. The latter of these two, in each column, are scaled down by the factor h.

The $\mathbf{\Pi}$ matrix has the following form:

$$\mathbf{\Pi} = \begin{bmatrix} \Pi^{AA\leftarrow AA} & \mathbf{\Pi^{AA\leftarrow BA}} & \Pi^{AA\leftarrow AP} & \mathbf{\Pi^{AA\leftarrow BP}} \\ \mathbf{\Pi^{BA\leftarrow AA}} & \Pi^{BA\leftarrow BA} & \mathbf{\Pi^{BA\leftarrow AP}} & \Pi^{BA\leftarrow BP} \\ \Pi^{AP\leftarrow AA} & \mathbf{\Pi^{AP\leftarrow BA}} & \Pi^{AP\leftarrow AP} & \mathbf{\Pi^{AP\leftarrow BP}} \\ \mathbf{\Pi^{BP\leftarrow AA}} & \Pi^{BP\leftarrow BA} & \mathbf{\Pi^{BP\leftarrow AP}} & \Pi^{BP\leftarrow BP} \end{bmatrix} \tag{10}$$

The entries representing out-of-pair transport are emboldened, and it is these quantities that are reduced by the appropriate h. This introduces the issue that the columns of the matrix no longer sum to 1. This is managed by scaling the remaining two terms in each column by a single factor that brings the sum back t 1. For the first column, we have:

$$\gamma(\Pi^{AA\leftarrow AA} + \Pi^{AP\leftarrow AA}) + h(\Pi^{BA\leftarrow AA} + \Pi^{BP\leftarrow AA}) = 1 \tag{11}$$

so that γ is given by:

$$\gamma = \frac{1 - h(\Pi^{BA\leftarrow AA} + \Pi^{BP\leftarrow AA})}{\Pi^{AA\leftarrow AA} + \Pi^{AP\leftarrow AA}} \tag{12}$$

The final form of the $\mathbf{\Pi}$ matrix is then:

$$\mathbf{\Pi} = \begin{bmatrix} \gamma^{AA}\Pi^{AA\leftarrow AA} & h^{BA}\mathbf{\Pi^{AA\leftarrow BA}} & \gamma^{AP}\Pi^{AA\leftarrow AP} & h^{BP}\mathbf{\Pi^{AA\leftarrow BP}} \\ h^{AA}\mathbf{\Pi^{BA\leftarrow AA}} & \gamma^{BA}\Pi^{BA\leftarrow BA} & h^{AP}\mathbf{\Pi^{BA\leftarrow AP}} & \gamma^{BP}\Pi^{BA\leftarrow BP} \\ \gamma^{AA}\Pi^{AP\leftarrow AA} & h^{BA}\mathbf{\Pi^{AP\leftarrow BA}} & \gamma^{AP}\Pi^{AP\leftarrow AP} & h^{BP}\mathbf{\Pi^{AP\leftarrow BP}} \\ h^{AA}\mathbf{\Pi^{BP\leftarrow AA}} & \gamma^{BA}\Pi^{BP\leftarrow BA} & h^{AP}\mathbf{\Pi^{BP\leftarrow AP}} & \gamma^{BP}\Pi^{BP\leftarrow BP} \end{bmatrix} \tag{13}$$