CS 2230
CS II: Data structures

Meeting 11: Asymptotic analysis

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Today’s big ideas

- Algorithms can be evaluated based on efficiency in time and space
- Time for an algorithm to finish can be expressed as a function f of input size n
- We usually focus on the big picture by looking at the **growth rate** of $f(n)$
  - so...we don’t always need to run timing experiments!

Let’s start with an experiment

Question: how long does it take to insert an integer into a sorted array?

Hypothesis: the average time to insert an integer will increase with the size of the array

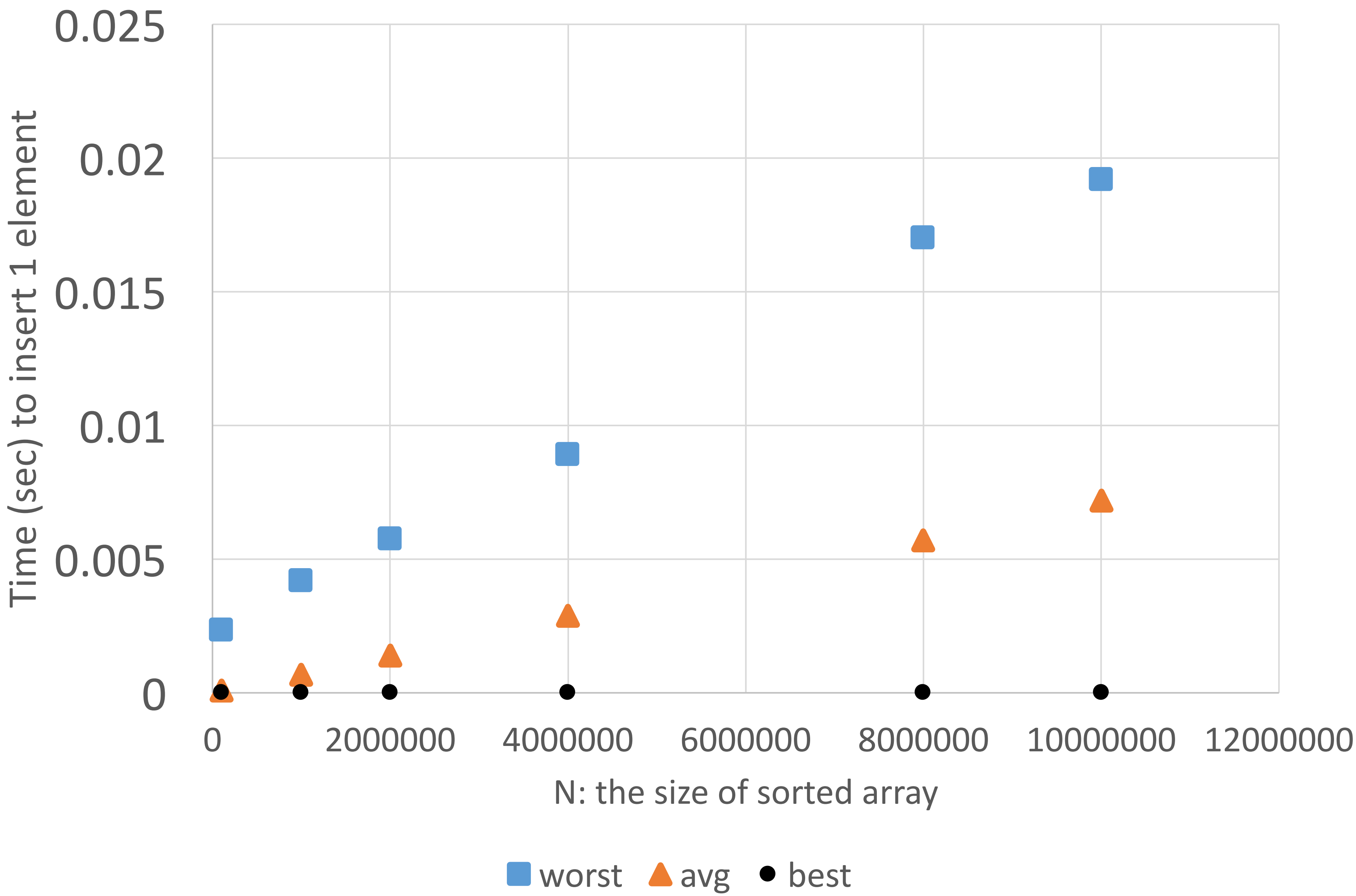
Methodology:

1. create random sorted array of size N
2. insert T integers into the sorted array, timing each insert
3. repeat experiment for more values of N

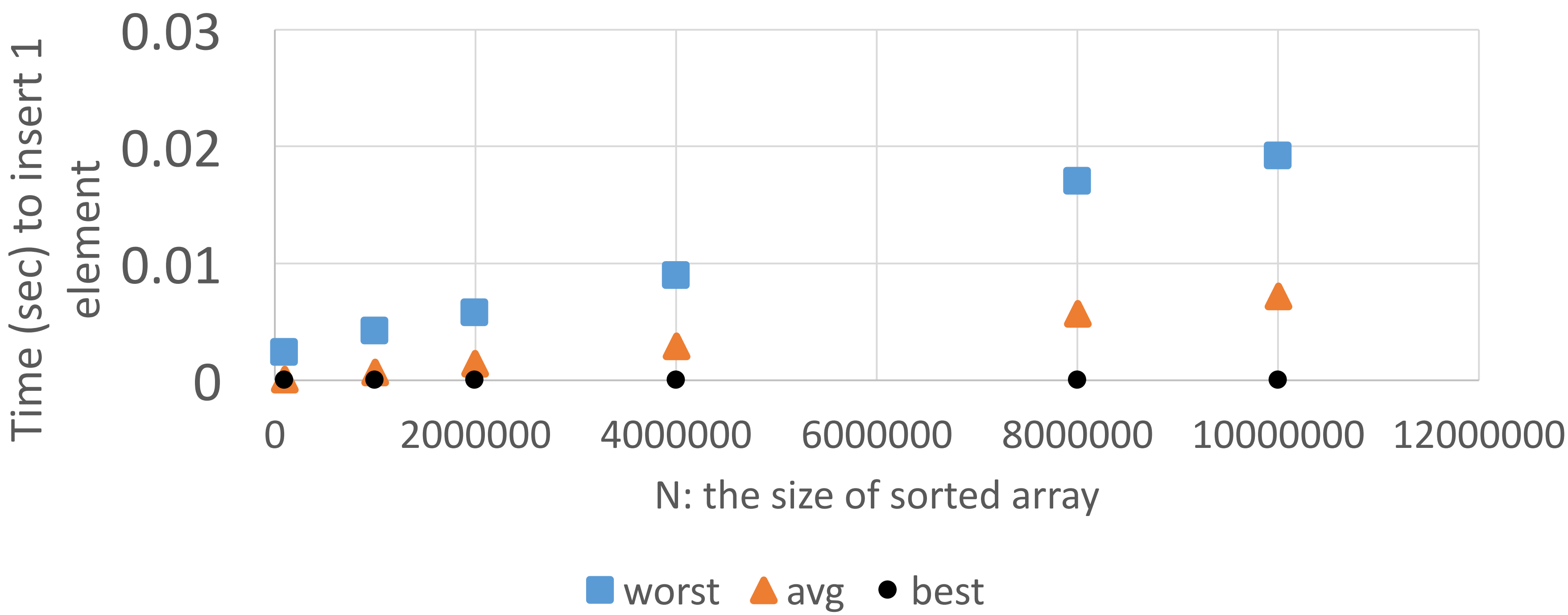
Code to run the experiment

```
// create an sorted array of N random integers; N given by command line argument
System.out.println("creating and sorting array");
final int N = Integer.parseInt(args[0]);
final Random r = new Random();
final int[] array = r.ints(N).toArray();
Arrays.sort(array);

// generate 1000 random integers to insert into the sorted array
System.out.println("creating test inputs");
final int T = 1000;
final int[] trialElements = r.ints(T).toArray();
final long[] results = new long[T];

// run 1000 experiments
System.out.println("running experiments");
for (int t=0; t<T; t++) {
    long trial_start = System.nanoTime();
    // always just insert at the end
    insertSorted(array, N-1, trialElements[t]);
    long trial_end = System.nanoTime();
    results[t] = trial_end - trial_start;
}
```

Results



0.025
0.02
0.015
0.01
0.005
0
Time (sec) to insert 1 element
0
2000000
4000000
6000000
8000000
10000000
12000000
N: the size of sorted array
worst
avg
best

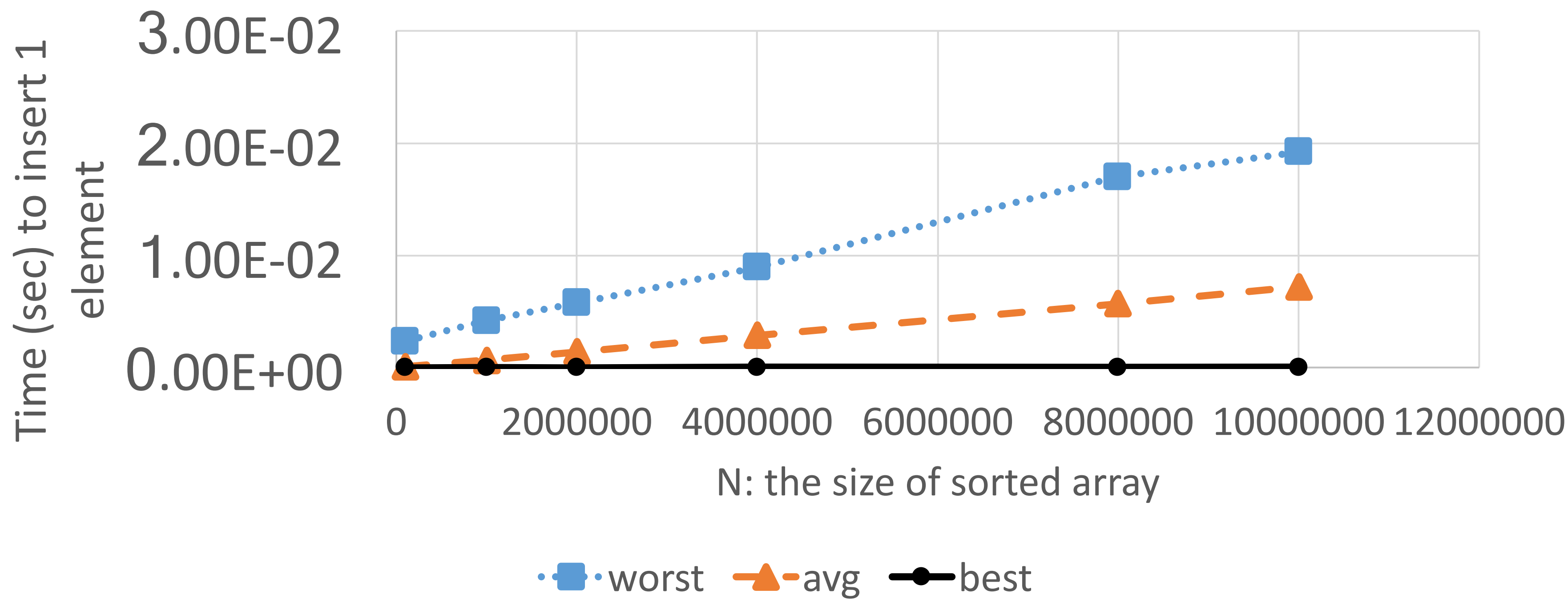


What is the **main cause** of the difference between worst, average, and best case points for a given value of N?

a) the integer inserted in a trial of the experiment may take longer or shorter depending on its value
b) larger integers take up more storage space in the computer so they take longer to swap through the array
c) variability in the computer while it executes each trial causes some trials to take different amounts of time
d) longer arrays take longer to insert elements into

https://b.socrative.com/login/student/
room CS2230X ids 1000-2999
room CS2230Y ids 3000+

Can we find an analytical model for worst/avg/best case time as a function of N?



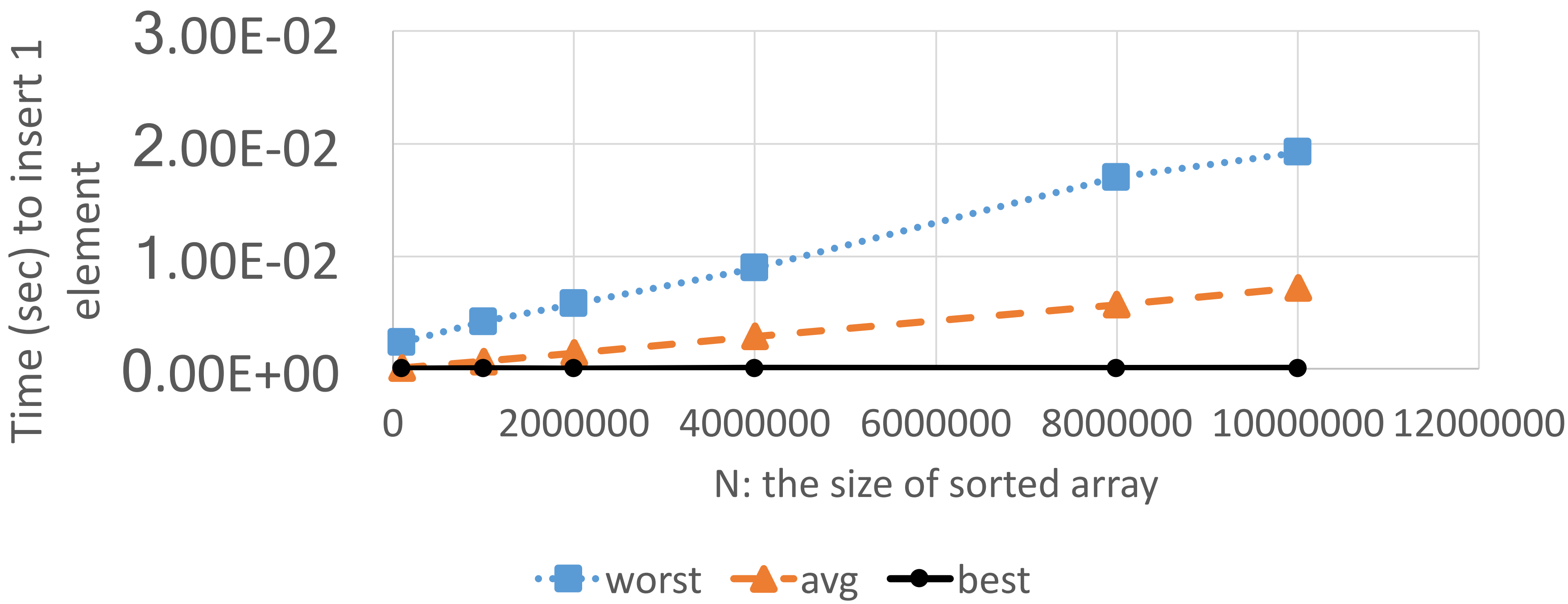
$worstTime(N) = ?$

c = time it takes to to do 1 swap

$avgTime(N) = ?$

$bestTime(N) = ?$

Can we find an analytical model for worst/avg/best case time as a function of N?



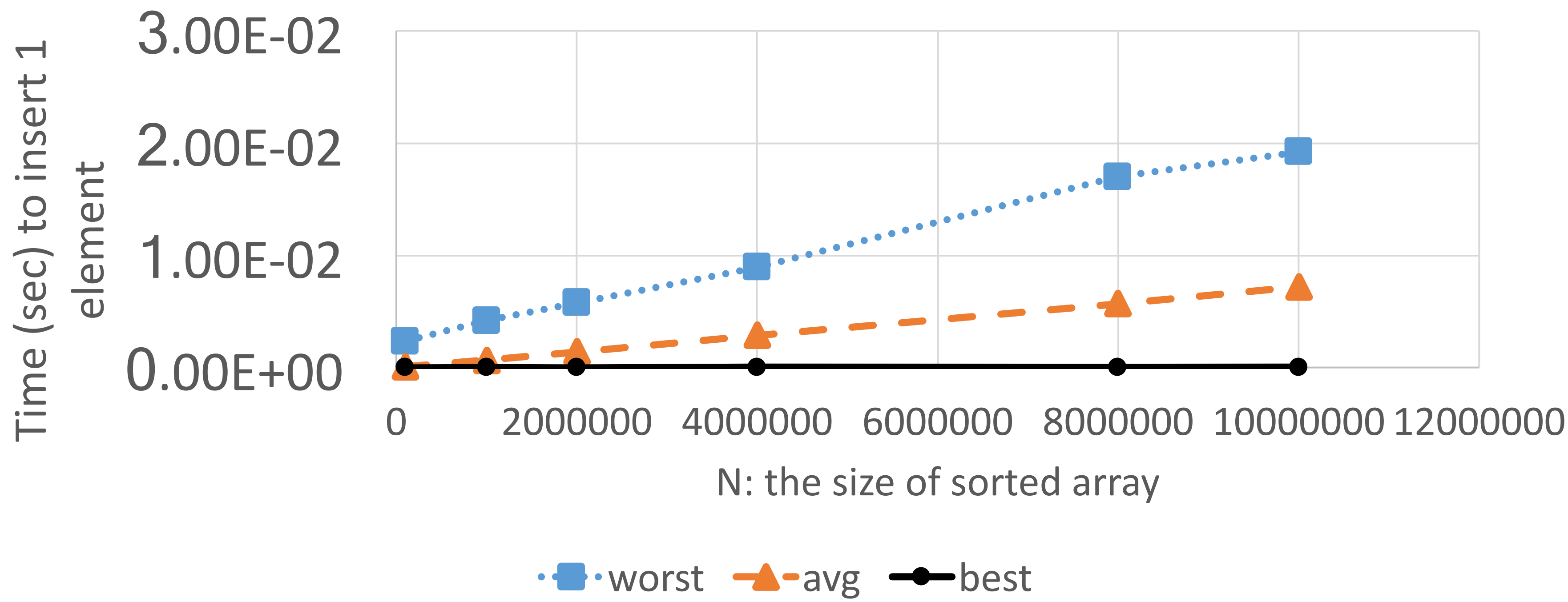
$worstTime(N) = c * N$

c = time it takes to to do 1 swap

$avgTime(N) = ?$

$bestTime = ?$

Can we find an analytical model for worst/avg/best case time as a function of N?



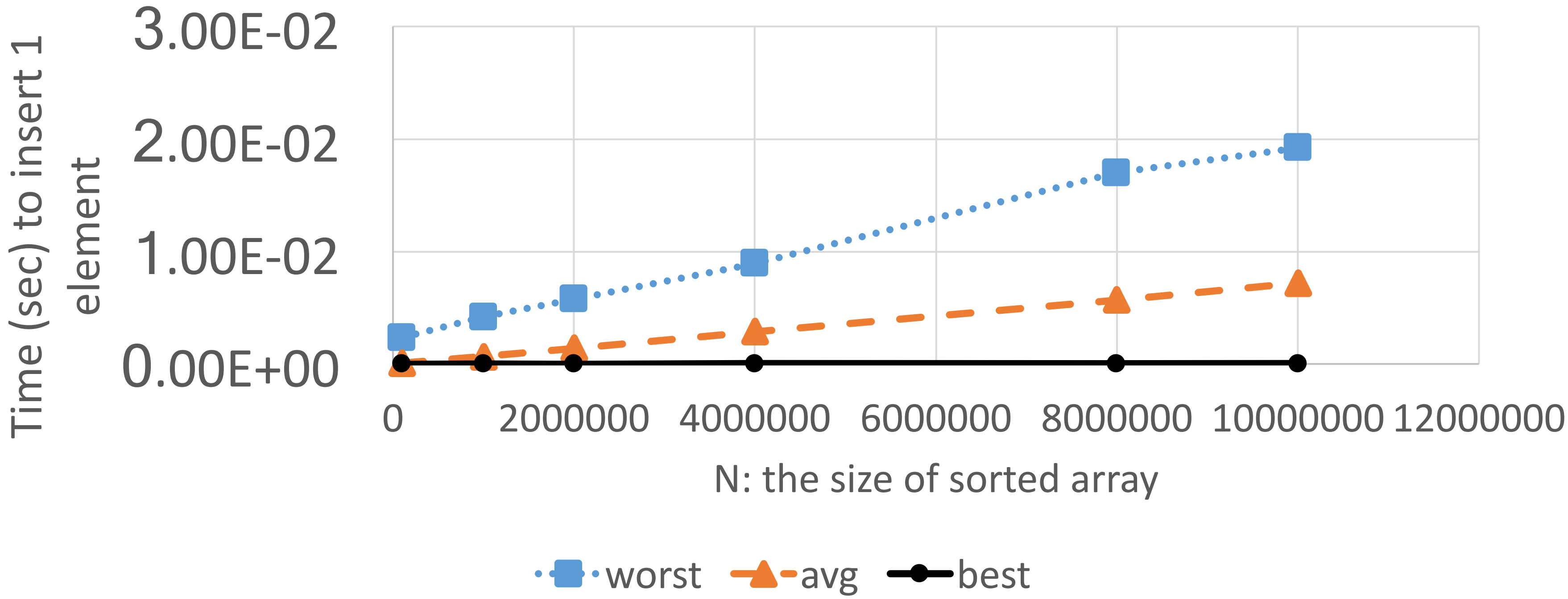
$$worstTime(N) = c * N$$

c = time it takes to to do 1 swap

$$avgTime(N) = \frac{c}{2} * (N + 1)$$

$$bestTime(N) = ?$$

Can we find an analytical model for worst/avg/best case time as a function of N?



$$worstTime(N) = c * N$$

c = time it takes to to do 1 swap

$$avgTime(N) = \frac{c}{2} * (N + 1)$$

$$bestTime(N) = c * 1$$

Common functions in algorithm analysis

constant	logarithm	linear	n log n	quadratic	cubic	polynomial	exponential
1	$\log n$	n	$n \log n$	n^2	n^3	n^d	a^n
							$a > 1$

from Goodrich, Tamassia, Goldwasser

Common functions in algorithm analysis

constant	logarithm	linear	n log n	quadratic	cubic	polynomial	exponential
1	$\log n$	n	$n \log n$	n^2	n^3	n^d	a^n

$a > 1$

$bestTime(N) = c * 1$

$worstTime(N) = c * N$

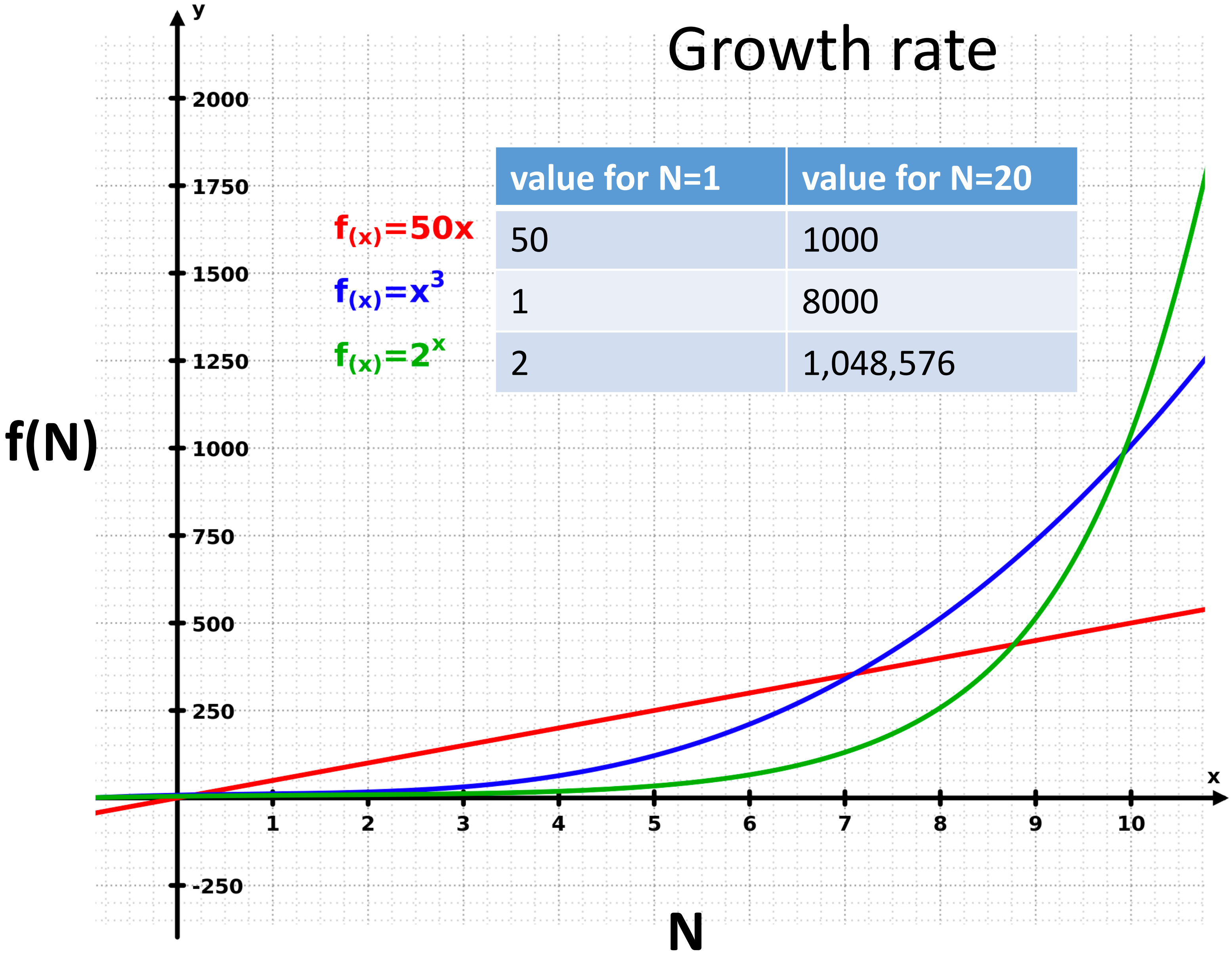
$avgTime(N) = \frac{c}{2} * (N + 1)$

classifying the
running time models
from our experiment

from Goodrich, Tamassia, Goldwasser

Growth rate

	value for N=1	value for N=20
$f_{(x)}=50x$	50	1000
$f_{(x)}=x^3$	1	8000
$f_{(x)}=2^x$	2	1,048,576



https://upload.wikimedia.org/wikipedia/commons/thumb/6/64/Exponential.svg/2000px-Exponential.svg.png

Peer instruction

Order these functions from slowest to fastest growing

A. $4n^2 + nlog(n^2)$
B. $2n^3 + n^2 + nlog(n)$
C. $3n^2 + 500n$
D. $n! + n^2$
E. $2^n + 4n^3$
F. $n^2\log(n)$
G. $nlog^2(n)$

Examples

- Problems with running time **linear** in N
  - search an unsorted array of size N for a value
- Problems with running time **quadratic** in N
  - sort N students using our `insertSorted` algorithm (put new element at end of array and swap into sorted position)
- Problems with running time **exponential** in N
  - find the quickest route for a UPS truck with N packages to deliver to the destinations

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