UNIVERSITEIT VAN AMSTERDAM

FACULTEIT DER NATUURWETENSCHAPPEN, WISKUNDE EN INFORMATICA

1/2

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Datum tentamen 25 - 01 - 2024

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Problem 1

a) Det. wp. for

$\lambda x \rightarrow x^2 - 10x + 37 \quad \{ \lambda x \rightarrow x > 0 \}$

subst. x in post cond. resolves to

$x^2 - 10x + 37 > 0$

$D = b^2 - 4ac$

$= 100 - 4 \cdot 1 \cdot 37 = -48$

Due to the polynomial discrimant being negative we know that it will always be bigger than 0 for all $x \in \mathbb{Z}$

So the precond. $x \geq -6$ is stronger than the wp. and thus the Hoare triple is valid.

e. Det. s.p. for

$\{ \lambda x \rightarrow x \geq -6 \} \ \lambda x \rightarrow x^2 - 10x + 37 \ \{ \ldots \}$

We first det. the inverse of the transform. Which using symbblab resolves to

$5 + \sqrt{x - 12}, \quad 5 - \sqrt{x - 12}$

We can then subst. x with one of these inverse functions

both done using symbolab.

$5 + \sqrt{x - 12} \geq -6$, which holds for $x \geq 12$

$5 - \sqrt{x - 12} \geq -6$, which's solution is $12 \leq x \leq 133$

So the sp is $12 \leq x \leq 133$

b. Det. wp for

$\lambda x \to x + 35 \quad \{\lambda x \to x > -1\}$

Subst. x in post cond. with transform

$x + 35 > -1$

$x > -36$

This is the same as the precond. thus makes a valid Hoare Triple. :-)

e) Det sp. for

$\{\lambda x \to x > -36\} \quad \lambda x \to x + 35 \quad \{\dots\}$

Det inverse of the transformation:

$\lambda x \to x - 35$

Subst x in precond.:

$x + 35 > -36$

$x > -71$

which yields to the sp being $x > -71$.

c. Det. w.p. for

$\lambda x \to x^2 - n^2 \cdot x \quad \{\lambda x \to x \leq n\}, \; n \in \mathbb{N} \quad n > 0$

Subst. x post cond. with transformation

$x^2 - n^2 \cdot x \leq n$

given n is nat number, and should be > 0

for $x = 0$,

$0^2 - n^2 \cdot 0 \leq n$

$0 \leq n$, for $n = 1$ this holds

for $x = 1$

$1^2 - n^2 \cdot 1 \leq n$

$1 - n^2 \leq n$ for $n = 1$ this holds as well

$1 - 1 \leq 1 \Rightarrow 0 \leq 1$

for $x = 2$

$2^2 - n^2 \cdot 2 \leq n$

$4 - 2n^2 \leq n$

$4 - 2 \cdot 1^2 \leq 1$ for $n = 1$

$2 \leq 1$ Which is not true

Thus the precond. $x \geq 0$ does not ensure the post cond. and makes the Hoare triple invalid

d. Det. wp for

$\lambda x \to 2024^x - 25^x$ $\{\lambda x \to x$ is prime $\}$

Subst. x with transformation in postconditon

$2024^x - 25^x$ is prime

~~Of~~ The precond. $x > 0$ does not ensure that $2024^x - 25^x$ is prime

a counter example would be for $x = 2$

$2024^2 - 25^2 = 409591$

Using a prime factors decomposition tool online, we find the following decomposition

$3 \times 683 \times 1999$

Thus the precond. doesn't ensure primality and the Hoare triple is invalid.

2. Det. if $\lambda x \to x > 8$ maces up a valid Hoare triple by det. wp. for

$\lambda x \to x^2 + Ax$ $\{\lambda x \to x > 8\}$, $A \in \mathbb{Z}$

subst. x with transform in post cond.

$x^2 + Ax > 8$

which would hold true for e.g

~~$A = 0$, $x = 4$~~ 3

... ~~$A = -8$ $x = 9$~~

$A = 8$. $x \geq 0$, but not for all A, some counter example

$A = -7$ and $x = 8$ returns 8.

So the post. cond. is not always valid. and the Hoare triple doesn't hold

Det. if $\lambda x \to$ ~~$x > 8$~~ $x \leq A + 8$ maces ~~up~~ up a valid Hoare triple by det wp. for

$\lambda x \to x^2 + Ax$ $\{x \leq A + 8\}$

subst x with transform in post cond.

$x^2 + Ax \leq A + 8$

$x^2 + Ax - A \leq 8$

for $x > 0$ there are counter examples, e.g.

$A = 0$ and $x = 3$ $\Rightarrow$ $9 \leq 8$ which doesn't hold. So this Hoare triple is also invalid for some $A \in \mathbb{Z}$

So ~~both~~ are not always valid, but
$x \to x > 8$ is less strict and thus weaker
compared to $x \leq A + 8$ if A is a low
number.

Problem 2.

1. a) if R is coreflexive it is automatically
transitive.
The two properties are not comparable:
$R = \{(1,1)\}$ with $A = \{1,2\}$ is
coreflexive transitive, but not assymetric
transitive (since not irreflexive)
And $R = \{(1,2)\}$ is assymmetric transitive,
but not coreflexive transitive.
So one is not a subset of the other and
thus not comparable.

b) See code :)

2. a) $A = \{0,1,2,3\}$, ~~R~~
$R = \{(0,2), (3,1)\}$
$R^{-1} = \{(2,0), (1,3)\}$
$\Delta_A = \{(0,0), (1,1), (2,2), (3,3)\}$
$R \cup \Delta_A = \{(0,0), (0,2), (1,1), (2,2), (3,1), (3,3)\}$
$(R \cup \Delta_A) \circ R^{-1} = \{(0,0) (1,3), (2,0), (3,3)\}$
This doesn't equal to R and is
~~doe~~ thus not true.

we start with $R = R \cup \Delta_A$

b) we build the smallest equiv. E on A
such that $(R \cup \Delta_A) \circ R^{-1} \subseteq E$
1. by first ensuring reflexivity
add $(1,1)$ $(2,2)$
2. by ensuring symmetry
add $(3,1)$ $(0,2)$
3. by ensuring transitivity
already transitive thus
$E = \{(0,0), (0,2) (1,1) (1,3) (2,0) (2,2) (3,1) (3,3)\}$

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Problem 3

1. numNRI n = num n for simplification

num 0 = 0

num 1 = 0

num $n = 2^{n^2-1} + 2^{2n-2} \cdot$ num $(n-1)$

Proof by induction

Base case, for domain size n

n = 0, no rels possible, thus $\emptyset$ => 0
which is correct num 0 = 0

n = 1, only one element ~~and~~, so
R(1) => {{}, {(a,a)}},
none are valid rel. which are
neither reflex or irreflexive thus also 0
which is also valid num 1 = 0

Induction step.

Found the following formula on geeksforgeeks
Which calculates the cardinality:

$R(n) = (2^n - 2)(2^{n^2-n})$

We assume this to be true for all $n > 1$ $n \in \mathbb{N}$
and thus say that num n = R(n)

So our IH is num n = R(n)

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~~PROOF~~

need to prove that

$$num\ (n+1) = (2^{n+1}-2)\ (2^{(n+1)^2-(n+1)}$$
$$= (2^{n+1}-2)\ (2^{n^2+n})$$

Proof:

$$num\ (n+1) = 2^{(n+1)^2-1} + 2^{2(n+1)-2} \cdot num\ ((n+1)-1)$$
$$= 2^{n^2+2n} + 2^{2n} \cdot num\ n$$
$$\overset{ih}{=} 2^{n^2+2n} + 2^{2n}\ (2^n-2)\ (2^{n^2-n})$$

used wolfram alpha to get to alternate form

$$= 2^{n^2+n+1}\ (2^n-1)$$

~~=~~ get 1 out of exponent

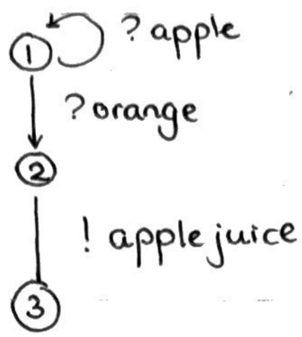
$$= (2^{n^2+n}) \cdot 2\ (2^n-1)$$
$$= 2^{n^2+n}\ (2^{n+1}-2)$$
$$= (2^{n+1}-2)\ (2^{n^2+n})$$

Thus ~~two ofas~~ by induction we have proved the function num NRI' to be correct.

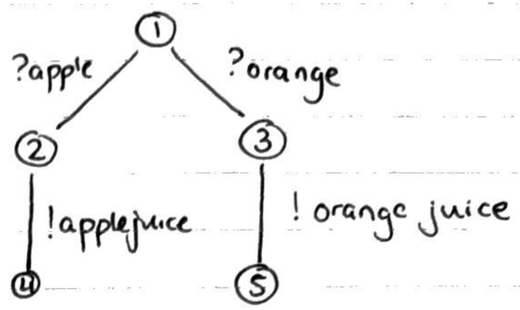
Problem 4.

1.

implementation



model



input output conformance $\forall \sigma \in straces(m)$:

$i \text{ ioco } m \Rightarrow out(i \text{ after } \sigma) \subseteq out(m \text{ after } \sigma)$

$out(i \text{ after } ?orange) = \{ !apple juice \}$

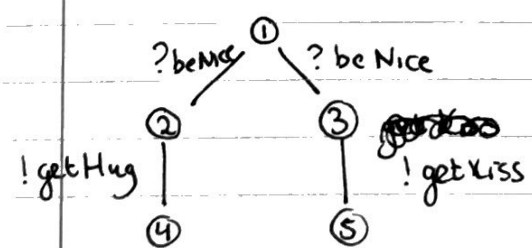
$\not\subseteq$

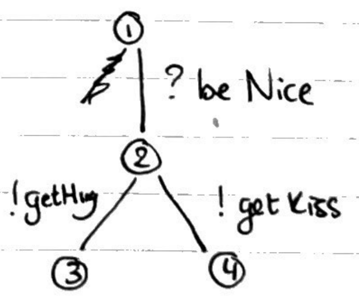
$out(m \text{ after } ?orange) = \{ !orange juice \}$

any output of i has to be foreseen by m, which in this case does not hold. Therefore the ~~sys~~ model and imp are not in ioco

3. a [0]
   b [2,4]
   c. [3]
   d. [5]
   e. [] not possible if output label present.

4. The result from question 2 would ~~differ~~ not differ, but the input would. The function would have to take into account for the quiesence action δ. Two models with similar straces would not necessarily have to be the same.





both have the same suspension traces:
e.g. { ~~ε~~ δ } { δ, ? be Nice } , etc.
but their implementation is not the same.

Problems

1\. I ~~rewrote~~ used the recurrence equation
P3: ~~to~~ and substituted x_{t+1} and x_t
with corona to form a property.
~~The function unf~~
P1: Besides that I ~~ma~~ added an initial condition
for ~~xt~~ t=0 which should be the same
as x_0.
P2: Furthermore, ~~the~~ if ~~s~~ r or s is
bigger than 0 this ~~would~~ should increase
the amount of infected.

Unfortunately the function will fail because
it tries to divide by 0.
This already makes the first property fail.