

# Physics 6A Lab Manual

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# Introduction

## PURPOSE

The laws of physics are based on experimental and observational facts. Laboratory work is therefore an important part of a course in general physics, helping you develop skill in fundamental scientific measurements and increasing your understanding of the physical concepts. It is profitable for you to experience the difficulties of making quantitative measurements in the real world and to learn how to record and process experimental data. For these reasons, successful completion of laboratory work is required of every student.

## PREPARATION

Read the assigned experiment in the manual before coming to the laboratory. Since each experiment must be finished during the lab session, familiarity with the underlying theory and procedure will prove helpful in speeding up your work. Although you may leave when the required work is complete, there are often “additional credit” assignments at the end of each write-up. The most common reason for not finishing the additional credit portion is failure to read the manual before coming to lab. We dislike testing you, but if your TA suspects that you have not read the manual ahead of time, he or she may ask you a few simple questions about the experiment. If you cannot answer satisfactorily, you may lose mills (see below).

## RESPONSIBILITY AND SAFETY

Laboratories are equipped at great expense. You must therefore exercise care in the use of equipment. Each experiment in the lab manual lists the apparatus required. At the beginning of each laboratory period check that you have everything and that it is in good condition. Thereafter, you are responsible for all damaged and missing articles. At the end of each period put your place in order and check the apparatus. By following this procedure you will relieve yourself of any blame for the misdeeds of other students, and you will aid the instructor materially in keeping the laboratory in order.

The laboratory benches are only for material necessary for work. Food, clothing, and other personal belongings not immediately needed should be placed elsewhere. A cluttered, messy laboratory bench invites accidents. Most accidents can be prevented by care and foresight. If an accident does occur, or if someone is injured, the accident should be reported immediately. Clean up any broken glass or spilled fluids.

## FREEDOM

You are allowed some freedom in this laboratory to arrange your work according to your own taste. The only requirement is that you complete each experiment and report the results clearly in your lab manual. We have supplied detailed instructions to help you finish the experiments, especially the first few. However, if you know a better way of performing the lab (and in particular, a different way of arranging your calculations or graphing), feel free to improvise. Ask your TA if you are in doubt.

## LAB GRADE

Each experiment is designed to be completed within the laboratory session. Your TA will check off your lab manual and computer screen at the end of the session. There are no reports to submit. The lab grade accounts for approximately 15% of your course total. Basically, 12 points (12%) are awarded for satisfactorily completing the assignments, filling in your lab manual, and/or displaying the computer screen with the completed work. Thus, we expect every student who attends all labs and follows instructions to receive these 12 points. If the TA finds your work on a particular experiment unsatisfactory or incomplete, he or she will inform you. You will then have the option of redoing the experiment or completing it to your TA's satisfaction. In general, if you work on the lab diligently during the allocated two hours, you will receive full credit even if you do not finish the experiment.

Another two points (2%) will be divided into tenths of a point, called "mills" (1 point = 10 mills). For most labs, you will have an opportunity to earn several mills by answering questions related to the experiment, displaying computer skills, reporting or printing results clearly in your lab manual, or performing some "additional credit" work. When you have earned 20 mills, two more points will be added to your lab grade. Please note that these 20 mills are additional credit, not "extra credit". Not all students may be able to finish the additional credit portion of the experiment.

The one final point (1%), divided into ten mills, will be awarded at the discretion of your TA. He or she may award you 0 to 10 mills at the end of the course for special ingenuity or truly superior work. We expect these "TA mills" to be given to only a few students in any section. (Occasionally, the "TA mills" are used by the course instructor to balance grading differences among TAs.)

If you miss an experiment without excuse, you will lose two of the 15 points. (See below for the policy on missing labs.) Be sure to check with your TA about making up the computer skills; you may be responsible for them in a later lab. Most of the first 12 points of your lab grade is based on work reported in your manual, which you must therefore bring to each session. Your TA may make surprise checks of your manual periodically during the quarter and award mills for complete, easy-to-read results. If you forget to bring your manual, then record the experimental data on separate sheets of paper, and copy them into the manual later. However, if the TA finds that your manual is incomplete, you will lose mills.

In summary:

$$\begin{aligned}\text{Lab grade} &= && (12.0 \text{ points}) \\ &&& - (2.0 \text{ points each for any missing labs}) \\ &&& + (\text{up to } 2.0 \text{ points earned in mills of "additional credit"}) \\ &&& + (\text{up to } 1.0 \text{ point earned in "TA mills"}) \\ \text{Maximum score} &= && 15.0 \text{ points}\end{aligned}$$

Typically, most students receive a lab grade between 13.5 and 14.5 points, with the few poorest students (who attend every lab) getting grades in the 12s and the few best students getting grades in the high 14s or 15.0. There may be a couple of students who miss one or two labs without excuse and receive grades lower than 12.0.

How the lab score is used in determining a student's final course grade is at the discretion of the

individual instructor. However, very roughly, for many instructors a lab score of 12.0 represents approximately B– work, and a score of 15.0 is A+ work, with 14.0 around the B+/A– borderline.

## **POLICY ON MISSING EXPERIMENTS**

1. In the Physics 6 series, each experiment is worth two points (out of 15 maximum points). If you miss an experiment without excuse, you will lose these two points.
2. The equipment for each experiment is set up only during the assigned week; you cannot complete an experiment later in the quarter. You may make up no more than one experiment per quarter by attending another section during the same week and receiving permission from the TA of the substitute section. If the TA agrees to let you complete the experiment in that section, have him or her sign off your lab work at the end of the section and record your score. Show this signature/note to your own TA.
3. (At your option) If you miss a lab but subsequently obtain the data from a partner who performed the experiment, and if you complete your own analysis with that data, then you will receive one of the two points. This option may be used only once per quarter.
4. A written, verifiable medical, athletic, or religious excuse may be used for only one experiment per quarter. Your other lab scores will be averaged without penalty, but you will lose any points that might have been earned for the missed lab.
5. If you miss three or more lab sessions during the quarter for any reason, your course grade will be Incomplete, and you will need to make up these experiments in another quarter. (Note that certain experiments occupy two sessions. If you miss any three sessions, you get an Incomplete.)

# Heart Rate Meter

## APPARATUS

- Computer and Pasco interface
- Heart rate sensor

## INTRODUCTION

This is a short experiment designed to introduce you to computer acquisition of data and the Pasco Science Workshop with its Capstone control program. It is not solely a physics experiment, but also an exercise to acquaint you with the equipment that will be used for “real” labs. If you are already familiar with computers, then this experiment will probably take less than an hour; if not, you should use any remaining time to practice with the computer.

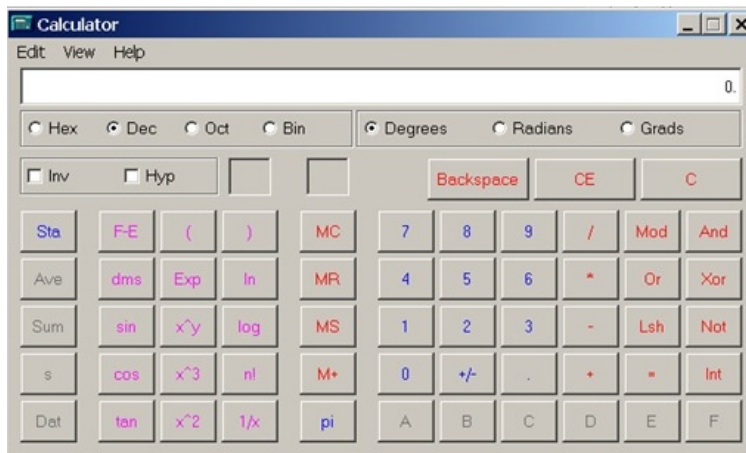
## COMPUTER EXPERIMENTS

Many experiments in the Physics 6A lab series utilize a desktop computer to acquire and analyze data. Most students entering UCLA are already familiar with the basic Windows operations of clicking and dragging with a mouse, pulling down menus, scrolling and resizing windows, and so forth. If you are not familiar with these operations, then consider this experiment a practice session.

It is likely that the computer has already been turned on when you enter the lab. If not, when you turn it on, it will take a minute or two for the system to “boot up”. In the entire Physics 6 lab series, two basic programs are used: Capstone (which controls an interface box to which various experimental sensors can be connected) and the spreadsheet program Microsoft Excel (which allows you to analyze and graph data). After the computer is booted up, you should see shortcut icons for these two programs on the desktop.

## SCIENTIFIC CALCULATOR

You may also have occasion to use an on-screen calculator while working on experiments. Bring up the calculator by clicking on the “Start” menu, go to “Programs”, then to “Accessories”, and finally to “Calculator”. When the calculator is displayed, pull down the “View” menu to “Scientific”, so that the type of calculator on the screen changes to scientific. If you have any difficulty bringing up the scientific calculator, ask your TA for assistance. You should be able to access the scientific calculator quickly at any time during the next three quarters of labs.



## THE PASCO INTERFACE

The Pasco Science Workshop system consists of an interface box controlled by the Capstone computer program, and a variety of different sensors that can measure distances and velocities by echo ranging (via a sonic ranger) or by motion of a smart pulley; as well as by voltage, heart rate, temperature, pressure, light intensity, magnetic fields, and many other physical quantities. Newer interfaces plug into a USB port of the computer, and have inputs for four digital channels and three analog channels. The interface can measure several quantities simultaneously and also has a built-in signal generator which can be controlled to produce 0 – 5 volt signals of DC, AC, and several other different wave forms. The software with the interface permits you to display and analyze the results in a number of different forms: digital meter, analog meter, graph, table, oscilloscope, and so forth.

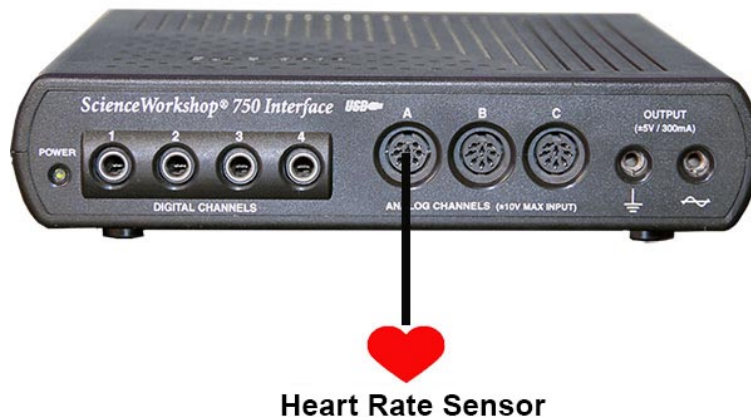


## PROCEDURE

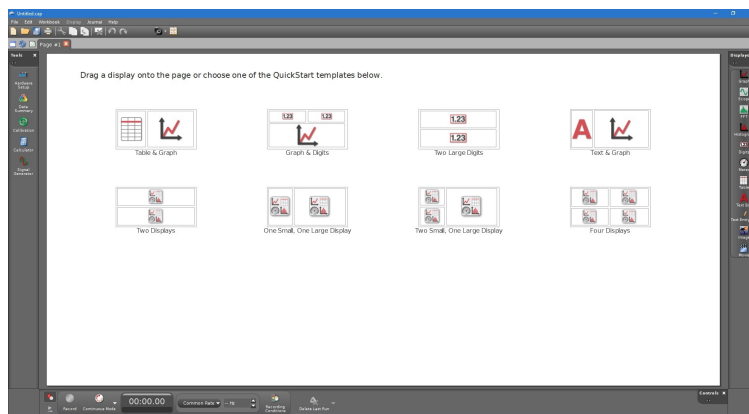
The terms in bold lettering below are basic computer operations with which you will need to be familiar by the end of this first experiment. If you have any difficulty with these operations, ask your TA; perhaps your lab partner can also help you. The partner less familiar with computers should perform most of the operations for this experiment. For the remaining experiments in the Physics 6 lab series, each partner should plan on performing half of the computer operations and half of the experimental setups and adjustments. Your TA has been instructed to intervene if he or she notices one partner doing a disproportionate share of either task.

The heart rate sensor consists of a small box with a multiple pin connector and a clip that attaches onto your ear lobe. The sensor measures the flow of blood through the lobe. As the heart forces blood through the vessels in the ear lobe, the light transmittance of the lobe is changed. The sensor monitors this light with a phototransistor.

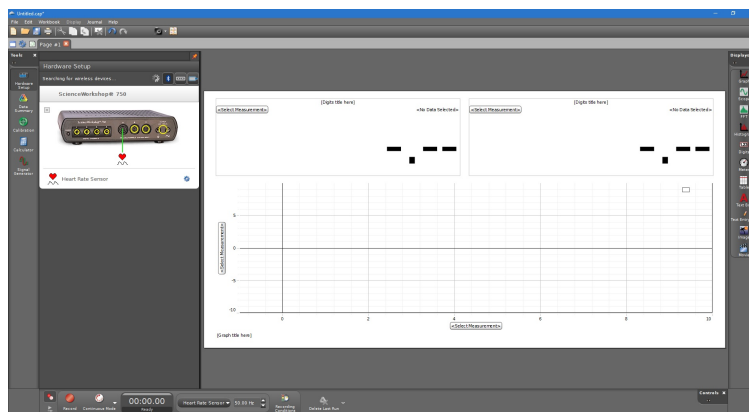
1. Plug the heart rate sensor into analog channel A, and turn on the signal interface.



2. On the computer screen, **double-click** on the PASCO Capstone icon to bring up the program. The illustration below shows the screen when Capstone is first loaded.



3. **Click** on the “Graph & Digits” option. Then **click** the “Hardware setup” button to display the interface. **Click** on Channel A of the interface and choose “Heart Rate Sensor”. Next, **click** the red thumbtack button above the interface to bring both windows into view. Your screen should look like the image below.



4. On the  $y$ -axis of the graph, **click** the “Select Measurement” button and select “Heart Rate

(beats/min)". Above the graph, **click** on the "Select Measurement" button in the digits box and choose "Heart Rate (beats/min)".

5. You are now ready to check your heart rate. Clip the heart sensor to your ear lobe. (The newer sensor boxes also show your heart rate with an LED display on the sensor box.) **Click** "Record" on the bottom tool bar. Observe the data for about one minute, and then **click** "Stop". You should see your resting heart rate (55 – 70 beats per minute for a typical person). The figure changes slightly every few seconds. If you do not see a clear, reasonable rate, try repositioning the ear clip or attaching it to your other ear. As a last resort, ask your TA for help.
6. To see the sensor voltage produced by your heartbeat, **click** on "Heart Rate" on the  $y$ -axis of the graph and select "Voltage (V)". You will see the voltage output data of the heart rate meter as a series of pulses. If there are too many pulses that it looks messy, try zooming in by clicking and dragging on the numbers on the  $x$ -axis of the graph
7. Whenever you **click** "Record" in Capstone, your previous recordings are no longer displayed. However, if you wish to see them again, **click** on "Data Summary" to see all recordings. You can view them again by dragging them to the desired graph or table or you can drag them to the display icons on the right side of the screen.
8. When the graph and digits windows are positioned clearly on the screen, and you have some clear data for your heart rate and sensor voltage, ask your TA to check you off. Congratulations! You have now finished the first experiment.

## ADDITIONAL CREDIT: RAISING AND LOWERING YOUR HEART RATE (2 mills)

Throughout the lab sessions, you will have opportunities to earn "mills", or tenths of a point, which are added to your final lab score. You can earn up to 20 mills, or two points. This additional credit assignment is worth two mills, but first, be sure you and your lab partner are both familiar with the Windows computer operations described above.

The objective of this section is to raise your heart rate to a moderately high value by exercising, and then to produce a graph of heart rate as a function of time as the heart relaxes back to its resting mode. Before proceeding with the instructions below, be sure you have clicked "Stop" if the machine is still monitoring your heartbeat.

The sensor will not record an accurate heart rate when you are moving around. You will need to take off the sensor, exercise, and then hook yourself back up. Raise your heart rate to 140 beats per minute or higher by doing jumping jacks in position, or by going out and running around the building. Do not perform the exercise if you have a health problem. Have your lab partner or another volunteer do it. After exercising, reattach the sensor, and click "Record" to record your heart rate as a function of time. Remain as still as possible while the recording is made — say, for five minutes (or 300 seconds). Click "Stop" when you are finished.

Your graph should show a relatively smooth, decreasing heart rate from 140 beats per minute down toward your resting rate. If you do not obtain a smooth graph, get some more exercise and try again. Try to remain more still while recording, reposition the ear clip, or attach it to your other



ear until you get a nice result. If the appearance of the graph is satisfactory, you can show this graph to your TA to collect two mills.

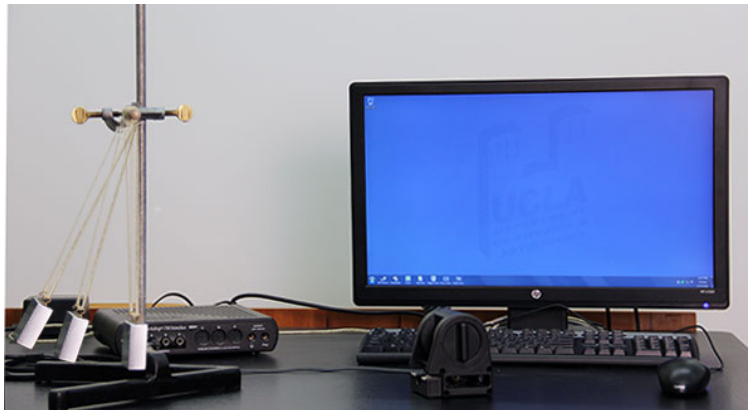
# Kinematics

## APPARATUS

- Computer and Pasco interface
- Motion sensor (sonic ranger)
- Air track, glider with reflector, block to tilt track
- Calipers to measure thickness of block
- Pendulum arrangement

## INTRODUCTION

In this experiment, you will produce position, velocity, and acceleration graphs of your own movements, as well as that of a glider on an air track, using a motion sensor. A motion sensor (sometimes called a sonic ranger) measures the distances to objects by repeated reflection of ultrasonic sound pulses. The software included with this device takes the first and second derivatives of the position measurements to calculate the velocity and acceleration, respectively.



To determine distances, the motion sensor emits and receives ultrasound pulses at a frequency of approximately 50 kHz. Since the speed of ultrasound in air at room temperature is known, the software calculates these distances by measuring the time required for the pulse to reflect from an object and return to the sensor. This process is similar to how a bat “sees” using ultrasound, as well as how a Polaroid autofocus camera determines the distance to an object in order to focus properly.

The ultrasonic sound emitted by the motion sensor spreads about  $15^\circ$  off axis. Keep this in mind as you design your experiments. The sensor does not work for objects closer than 0.4 meters. (Some of the newer motion sensors have adjustable width beams and will measure objects as close as 15 centimeters.)

The clicking noise made by the motion sensor is not ultrasound, but a by-product of the mechanism that produces the ultrasound. Most people cannot hear the frequencies emitted by the sensor. If you place your ear near the device, however, you may be able to “feel” the pressure pulse of sound against your eardrum.

## INITIAL SETUP

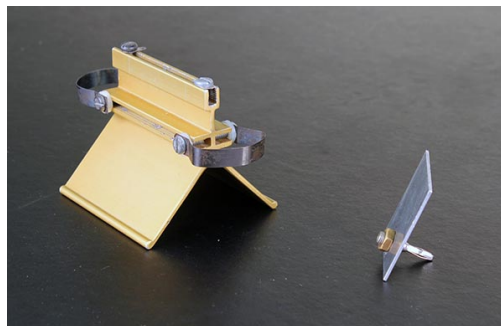
1. Turn on the signal interface and computer, plug the motion sensor into the interface (inserting the yellow-banded plug into digital channel 1 and the other plug into digital channel 2), and call up Capstone.
2. Choose “Table & Graph”, then under Hardware Setup, click on channel 1 of the interface. Choose “Motion Sensor II”.



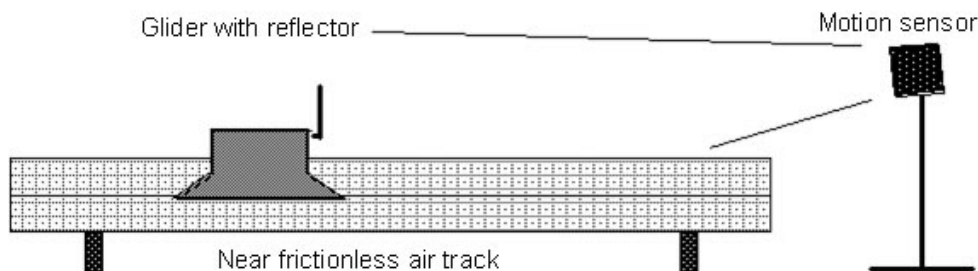
3. Click “Select Measurement” on the  $y$ -axis of the graph and choose “Position (m)”. There is a tool at the top of the graph that says “Add new plot area to the Graph display”. Click on this tool twice to display a total of three graphs.
4. For the second graph, set “Velocity (m/s)” on the  $y$ -axis. For the third graph, set “Acceleration ( $m/s^2$ )” on the  $y$ -axis.
5. Click “Record” to activate the motion sensor. You should hear a series of rapid clicks. Test out the motion sensor by holding a book facing, and approximately 0.5 meter away from, the sensor. (The sensor does not measure distances closer than 0.4 meter.) Move the book to a distance of 1.5 meters from the sensor and then back to 0.5 meter, click “Stop”, and observe the graph. You may want to click the “Scale axes to show all data” button at the left end of the tool bar above the graphs. If you do not obtain a clean trace, try again, being careful to keep the book directly in front of the sensor.

## PROCEDURE

1. Turn on the air track and level it by adjusting the leveling screw such that a glider on the track has no apparent tendency to move in either direction. Since air tracks are often bowed in the middle, place the glider at several different positions on the track to verify that it is as level as possible. Attach a reflector to the glider.



Arrange the sensor so it points slightly downward and can follow the glider along the entire length of track, as shown below. Place your eye at the position you want the sensor to point. Look into its reflective face, and adjust the sensor until you see your reflected image. The sensor is now directed at your eye position.



Tilt the end of the track up by placing the small block under the leveling screw. Give the glider a small hand push up the track. The glider should slow as it moves up, stop momentarily before reaching the top end, and then coast back down the track. After experimenting with several trials, predict the position, velocity, and acceleration of the glider as it moves along the track.

Now set your computer to produce all three graphs aligned vertically. Record the motion of the glider moving up, slowing, stopping, and moving back down.

Practice the attitude that you are going to make the experiment work well and produce a good position graph. If your initial graphs are not clean, adjust the position and direction of the motion sensor, adjust the tilt of the reflector on the glider, etc., until you get a good data set. (Your acceleration graph may still look ragged.) When you have a graph with clean data on the screen, compare this with your predictions. You can print it out to keep for your records.

2. Using the Vernier caliper (see instructions below), measure the height  $h$  of the block which tilts the track and the distance  $D$  between the track supports. The sine of the tilt angle  $\alpha$  is equal to  $h/D$ ; knowledge of  $h$  and  $D$  allows you to determine  $\alpha$ . As the glider moves along the track, Newton's Second Law predicts that its acceleration should be constant and equal in magnitude to  $g \sin \alpha$ . Calculate the value of this acceleration. This is your theoretical prediction for the acceleration of the cart.
3. Acceleration graphs often look ragged, as small errors accumulate when the software takes the second derivative of the position data. We will use the method of linear regression to

determine the the acceleration of the cart. Notice the linearly increasing portion of your velocity graph. Select this region of interest by first clicking on the velocity graph then clicking on the “Highlight range of points in active data” tool. A rectangle will appear. Drag and extend this rectangle over the linearly increasing portion of your data. Click inside the rectangle to highlight the data.

4. Click the drop-down arrow of the “Apply selected curve fits to active...” tool and choose “Linear”. Click this tool again to bring up a display box. You should see a best-fit line appear on top your selected data and a box should pop up that tells you the slope and  $y$ -intercept of this best-fit line. The slope of this line is your experimentally measured acceleration of the cart. Write this value down for your analysis.
5. Another way to measure the acceleration is by looking at the acceleratin graph. In general your graph should be flat (and possibly a a bit ragged). We want to find the average of these data points. To do this, select the data points of interest using the “Highlight range of points in active data” tool. Click the “Display selected statistics for active...” tool. This will display the mean value of the points you selected. Compare this to the value you obtained by making a best-fit line.

Note: By default, Capstone gives only one digit of precision for acceleration measurements and calculations performed using them. You can increase this precision as follows: Open “Data Summary” from the left hand panel and select the name of the measurement in question. In this case, you would select “Acceleration ( $m/s^2$ )” . Next click the blue gear icon (settings), open the “Numerical Format” dropdown menu, and modify the “Number of Decimal Places” as desired.

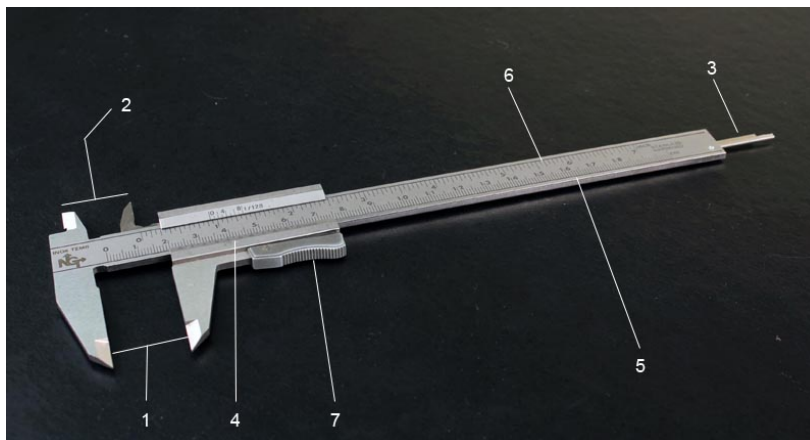
6. Calculate the percentage error between the experimental results  $r_{\text{exp}}$  obtained in steps 4 and 5, and the theoretical value  $r_{\text{th}}$  determined in step 2:

$$\text{percentage error} = (100\%) \times (| r_{\text{exp}} - r_{\text{th}} |) / r_{\text{th}}.$$

The absolute value is taken because we are not interested in the sign of the difference. Typical experimental accuracies in an undergraduate lab range from 3 – 5%, although some quantities can be measured much more accurately (and some much less!).

## VERNIER CALIPER

The **Vernier caliper** is designed to provide a highly precise measurement of length. The numbers in parentheses refer to those found in the figure below.



The **outside jaws** (1) are used to measure around the exterior of an object. The **inside jaws** (2) are used to measure inside the holes of an object. The **depth gauge** (3) is used to measure the depth of holes.

The **Vernier** (4) is used to divide further the **metric scale** (5) or the **English scale** (6) down to 0.01 centimeter or 1/128 inch, respectively. Its operation is described in detail below.

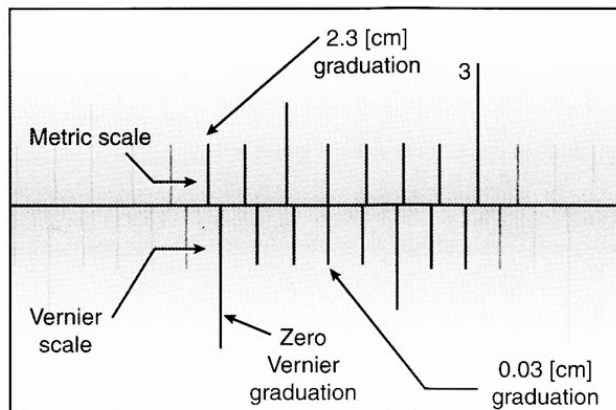
The **thumbwheel** (7) is used to open and close the jaws.

The **locking screw** (8) is used to lock the jaws in position.

The **Vernier** is a device used for estimating fractional parts of distances between two adjacent divisions on a **scale**. The **Vernier scale** subdivides each **main scale** division into as many parts as there are divisions on the scale.

The **Vernier** and **main scales** are placed in contact with each other. The **main scale** is read to the nearest number of whole divisions, while the zero on the **Vernier scale** serves as the index (i.e., the line on the extreme left). One should then estimate the fractional part of the **main scale** reading as a check of the more accurate reading to be made with the aid of the **Vernier scale**.

The **metric scale** is divided into tenths of a centimeter (i.e., millimeters). The **Vernier scale** is divided into 10 divisions, thus representing hundredths of a centimeter. At any given setting, the marks on the **Vernier** and **main scales** coincide at only one point. The mark on the **Vernier scale** which coincides most closely with a corresponding mark on the **main scale** represents the Vernier reading. The figure below should help clarify this.



A reading of 2.33 cm

## DATA

This section is to help you organize your data and guide you through the calculation.

1. Mean value of acceleration = \_\_\_\_\_

Slope of velocity graph (best-fit line) = \_\_\_\_\_

Always record units with your results.

## CALCULATIONS

2.  $g \sin \alpha =$  \_\_\_\_\_

3. Percentage error for mean value of acceleration = \_\_\_\_\_

Percentage error for slope of velocity graph (best-fit line) = \_\_\_\_\_

## QUESTIONS

- What does calculus tell us about the relationship between the position and velocity graphs?
- Suppose you were able to arrange the motion sensor to measure and plot the position, velocity, and acceleration of a ball thrown vertically upward into the air. Neglecting air resistance, how would these graphs compare with those of the glider experiment? Elucidate any differences.

## ADDITIONAL CREDIT: PENDULUM MOTION (3 mills)

Arrange the motion sensor to track a swinging pendulum. For a pendulum bob, use a block with a flat side facing the sensor. Set the pendulum into motion with small-amplitude oscillations, carefully positioning and aligning the sensor so it tracks this motion and produces smooth curves. Display a graph showing position, velocity, and acceleration. Be prepared to discuss which mathematical

function describes each graph, as well as how the graphs are related, and ask the TA to check your work.

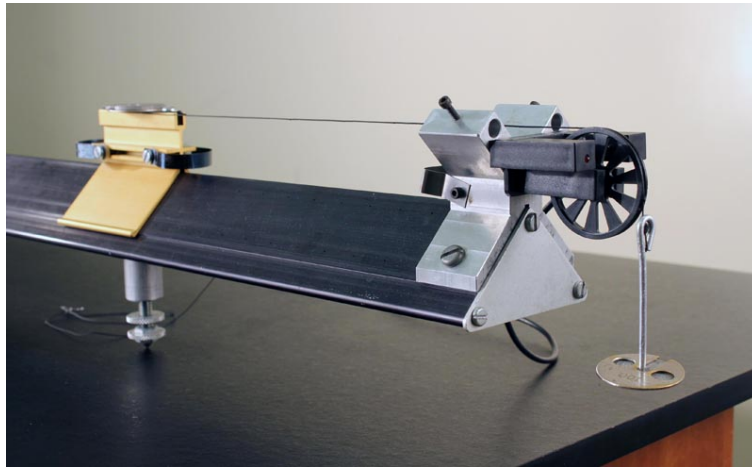


# Newton's Second Law

## APPARATUS

*Shown in the picture below:*

- Air track, smart-pulley mount, and smart pulley
- Small glider
- Mass holder for gliders
- 5-gram mass hanger and three 5-gram disks



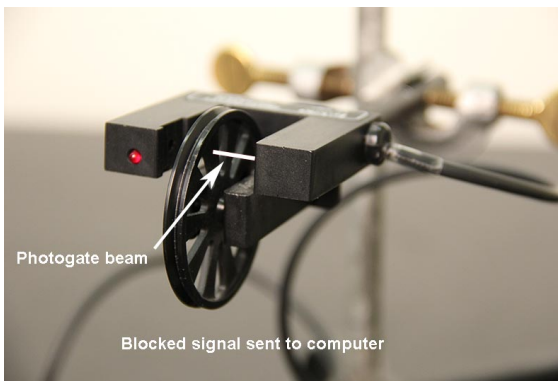
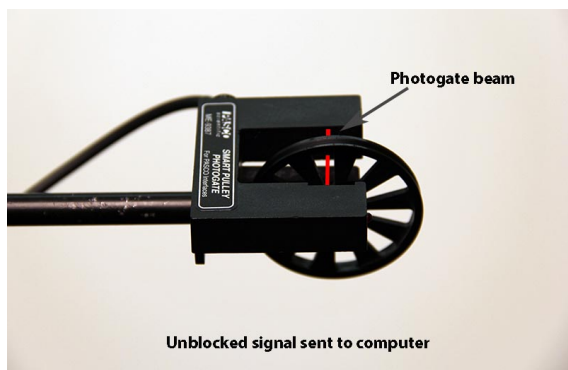
*Not shown in the picture above:*

- Computer and Pasco interface
- Large glider
- Scale and weight set
- Photogate and picket fence
- Rag box for collecting picket fence

## INTRODUCTION

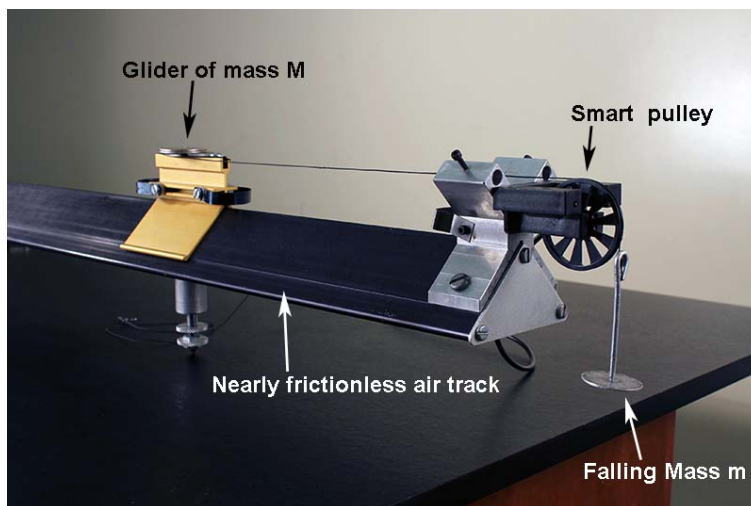
In this experiment, you will test Newton's Second Law by allowing a falling weight (i.e., a known force) to accelerate a glider of known mass along an air track. A string connecting the falling weight to the glider passes over a smart pulley.

The smart pulley has low friction and low inertia, and its rotation is monitored by an attached photogate. One arm of the photogate emits a thin beam of infrared light which is detected by the other arm. The computer discerns whether the beam strikes the detector or is blocked by a spoke in the pulley sheaf. The small LED light in front of one arm illuminates when the beam is blocked. By accurately timing the signals that arrive from the photogate, the computer is able to track the motion of any object linked to the pulley.



## THEORY

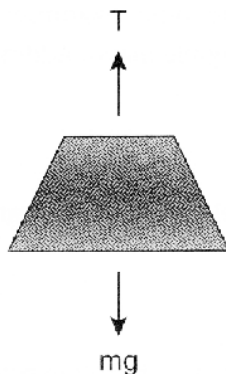
Consider a glider of mass  $M$  on a nearly frictionless air track. This glider is attached to a small mass  $m$  by a string passing over a smart pulley. The Earth exerts a downward force on the small mass which is equal in magnitude to its weight  $mg$ .



Reasoning somewhat intuitively, we can say that this gravitational force causes the entire system of mass  $M + m$  to accelerate. Newton's Second Law can then be written as

$$mg = (M + m)a. \quad (1)$$

More rigorously, let us draw free-body diagrams for each of the two masses. The forces exerted on the small mass  $m$  are its weight  $mg$  (downward) and the tension  $T$  in the string (upward). These two forces cause  $m$  to accelerate downward:



$$\sum F_y = mg - T = ma. \quad (2)$$

Note that we have assigned a positive sign to quantities pointing in the (downward) direction of motion.

The forces exerted on the glider of mass  $M$  are its weight  $Mg$  (downward), the normal force  $N$  from the track (upward), and the tension  $T$  in the string (rightward). Since there is no motion in the vertical direction:



$$\sum F_y = N - Mg = 0 \quad (3)$$

or

$$N = Mg. \quad (4)$$

In the horizontal direction,

$$\sum F_x = T = Ma. \quad (5)$$

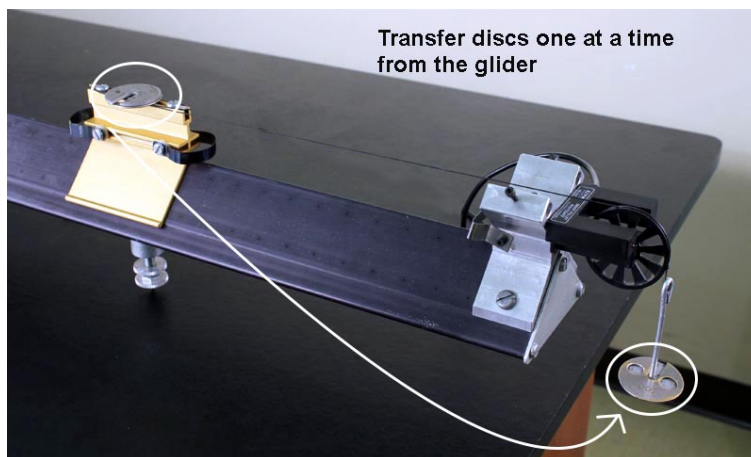
The glider and small mass have the same magnitude of acceleration  $a$  since they are connected by a taut string (of negligible mass). Adding Eqs. 2 and 5, we obtain

$$mg = (M + m)a \quad (6)$$

as postulated above. Thus, the acceleration of the system is

$$a = mg/(M + m). \quad (7)$$

If we wish to test Newton's Second Law, we might think of using different small masses  $m$  and checking whether the acceleration  $a$  is proportional to the gravitational force  $mg$ . Eq. 7, however, shows that  $a$  is not simply proportional to  $m$ , since the denominator also depends on  $m$ . Thus, holding  $M$  constant while increasing  $m$  causes the total mass of the system to increase. On the other hand, if we add, say, three more small masses (each of mass  $m$ ) to the glider and transfer them one at a time to the hanger (also of mass  $m$ ), then the total mass  $M + 4m$  of the system remains constant. Hence, we can test whether the acceleration of the system is proportional to the gravitational force as it increases in magnitude from  $mg$  to  $2mg$ ,  $3mg$ , and  $4mg$ . (Note that when using this method, you need to decide how many measurements to make before doing the experiment, and then add the corresponding masses to the glider.)



## INITIAL SETUP

1. Turn on the air track and level it by adjusting the leveling screw such that a glider on the track has no apparent tendency to move in either direction. Since air tracks are often bowed in the middle, place the glider at several different positions on the track to verify that it is level.
2. Turn on the signal interface and computer, plug the smart pulley into the interface (inserting the plug into digital channel 1), and call up Capstone and choose the "Table & Graph" option.
3. In the "Hardware Setup" tab, click on channel 1 and select "Photogate with Pulley". Click on the "Select Measurement" button on the  $y$ -axis and choose "Position (m)". Click the "Add new plot area..." button above the graph twice to add two additional graphs. Plot "Linear Speed (m/s)" on the  $y$ -axis of one graph and "Linear Acceleration ( $m/s^2$ )" on the  $y$ -axis of the other.
4. Using approximately 1.5 meters of thread, connect the 5-gram mass hanger to a glider. Make sure the thread passes over the smart pulley so the hanger will accelerate a glider down the track.

## PROCEDURE

1. Weigh each glider to obtain its mass, and record the values in the “Data” section.
2. Attach (or tape) three 5-gram masses to the small glider, such that only the 5-gram mass holder accelerates the glider. Turn the air-track blower on, and set one glider on the track as far from the pulley as the thread allows. Click “Record”, and immediately release the glider so that it begins to move. Just before the hanger hits the floor or the glider reaches the end of the track, click “Stop”.
3. Check the graph window and use the “Scale-to-fit” button, if necessary. Check that you are obtaining reasonable plots of the glider’s position, velocity, and acceleration. As usual, the acceleration graph may look ragged.



scale-to-fit button at top left of graph tool bar

4. Read an “eyeball” value of acceleration directly from the graph (This is just an estimate) and record this value in the “Data” section.
5. Click on the velocity graph then click on the “Highlight rang of points...” button. A box will appear in the velocity graph. Drag this box over the data of interest (the linear part). Click inside the box to highlight the data. Click the “Apply selected curve fits...” button and select linear. A box appears telling you the slope of the best-fit line. Record this slope in the “Data” section.
6. We will be recording and graphing data on an Excel spreadsheet. Call up Excel on your computer. When the program has booted up, you will see a menu bar with a set of icons above, and the spreadsheet cells in rows and columns below. Our plan is to record three trials of the acceleration for the accelerating mass, and then to calculate the average acceleration of the each of the three trials. Accordingly, prepare your spreadsheet like the one illustrated below.

	A	B	C	D	E	F
1	Acceleration of an air track glider in m/s <sup>2</sup>					
2						
3	accelerating mass (g)	5	10	15	20	
4	acceleration, trial 1					
5	acceleration, trial 2					
6	acceleration, trial 3					
7	average acceleration					
8						
9						

If you have not used a spreadsheet before, it consists of a sheet of cells labeled by the numbers on the left and the letters on the top. You can enter numbers or text into the cells; the numbers can be manipulated mathematically later. To make an entry into a cell, click on the cell, and begin typing. The text appears on the menu line above, and you can edit it by deleting parts, or by dragging through and typing the corrected material. There are at least four ways of entering typed material into a cell: clicking on the green check, hitting “Enter”,

pressing an arrow key to move to a nearby cell, or clicking on a new cell.

7. Make sure three 5-gram masses are still attached to the small glider, with the only 5-gram mass holder hanging down. Set the system into motion. Obtain the acceleration as described in step 5, and record its value in your spreadsheet. Perform a minimum of three trials. (As data runs accumulate on your velocity graph, you can select run numbers in the box on the graph and delete them to see the current data more clearly.)
8. Transfer one 5-gram mass from the glider to the mass holder, such that the holder and one 5-gram mass accelerate the glider. Set the system into motion, obtain the acceleration, and record its value in your spreadsheet. Perform a minimum of three trials.
9. Transfer another 5-gram mass from the glider to the mass holder, such that the holder and two 5-gram masses accelerate the glider. Set the system into motion, obtain the acceleration as before, and record its value in your spreadsheet. Perform a minimum of three trials.
10. Transfer the final 5-gram mass from the glider to the mass holder, such that the holder and all three 5-gram masses accelerate the glider. Set the system into motion, obtain the acceleration as before, and record its value in your spreadsheet. Perform a minimum of three trials.
11. To calculate the average acceleration of the first three trials, in cell B7 of the Excel illustration above, type the material between the quotes, “=Average(b4..b6)”, and click the green check in the tool bar (or hit “Enter”). The equals sign signals Excel to do a calculation. This operation will average the numbers in cells B4, B5, and B6. It does not matter that the B’s are typed in lower case in the Average function. *Note: If you set up your spreadsheet slightly differently from the one pictured above, the relevant quantities may be in different cells than listed in the preceding lines. In that case, you need to determine which (a) in which cell to place the average function and (b) which cells to average over, in order to obtain your average accelerations in an appropriate place.*
12. Now comes the great virtue of spreadsheets. You do not need to type the Average function into the cells for the other trials. Instead, position the cursor at the lower right corner of cell B7 (the one in which you just did the Average calculation), so that it turns into a lopsided square shape. Now drag over cells C7, D7, and E7. Excel automatically calculates the averages of the other trials with the proper cell references. (By the way, if you were now to change one of the values of the trial accelerations, Excel would automatically and instantly recalculate the average.) Your spreadsheet should now look something like the illustration below:

	A	B	C	D	E	F
1	Acceleration of an air track glider in m/s <sup>2</sup>					
2						
3	accelerating mass (g)	5	10	15	20	
4	acceleration, trial 1	0.1337	0.2855	0.4331	0.5777	
5	acceleration, trial 2	0.1343	0.2846	0.4326	0.5823	
6	acceleration, trial 3	0.1368	0.2850	0.4333	0.5824	
7	average acceleration	0.1349	0.2850	0.4330	0.5808	
8						
9						

(Dragging the calculation across the cells is called the “Fill Right” operation. There is also a “Fill Down” operation for dragging a calculation down a column. Positioning the cursor at

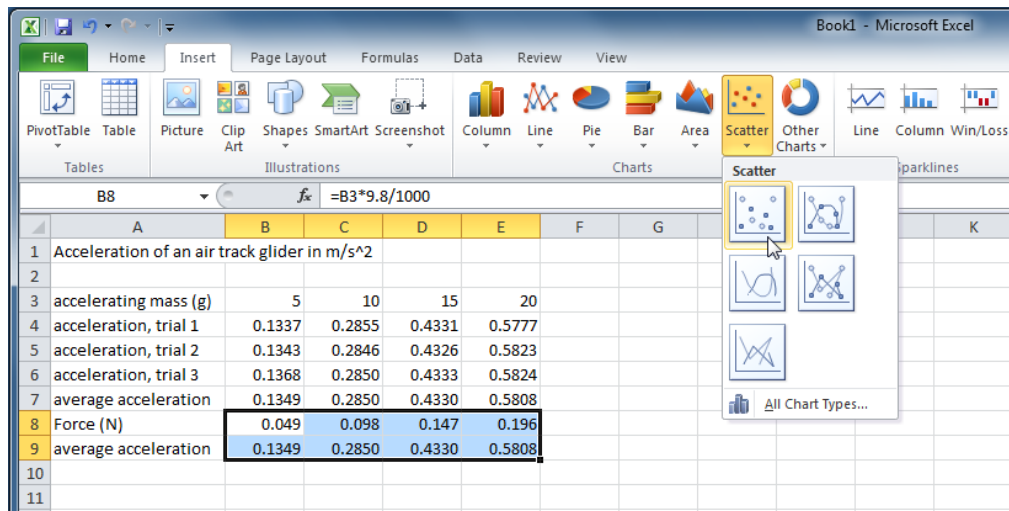
the corner of the cell is a short cut to these operations. They can also be accessed from the “Edit” pull down menu.)

13. In cell A8 of the illustration above, type “Force (N)”. In cell B8, type the expression between the quotes, “=B3\*9.8/1000”. This will take the mass entry in cell B3, divide it by 1000 to convert to kilograms, and multiply by 9.8 to convert to newtons. Then drag this calculation across the other three cells C8 – E8 to calculate the force for each case.
14. We will now have Excel chart this data. First, if we leave the acceleration data above the force data, the acceleration will be plotted on the  $x$ -axis and the force on the  $y$ -axis. Since we consider the force to be the independent variable in this experiment, we would prefer it the other way. Accordingly:
  - Click the label “7” to select row 7,
  - Pull “Edit” down to “Copy” (or simply hit CTRL+C, the control button and the C button together, to copy),
  - Select row 9, and
  - “Paste Special” the “Values” into row 9.

	A	B	C	D	E	F
1	Acceleration of an air track glider in m/s <sup>2</sup>					
2						
3	accelerating mass (g)	5	10	15	20	
4	acceleration, trial 1	0.1337	0.2855	0.4331	0.5777	
5	acceleration, trial 2	0.1343	0.2846	0.4326	0.5823	
6	acceleration, trial 3	0.1368	0.2850	0.4333	0.5824	
7	average acceleration	0.1349	0.2850	0.4330	0.5808	
8	Force (N)	0.049	0.098	0.147	0.196	
9	average acceleration	0.1349	0.2850	0.4330	0.5808	
10						
11						

(If you were to use “Paste” instead of “Paste Special”, Excel would have copied the cell averaging formulas and averaged wrong numbers in row 9.)

15. Now we chart:
  - Select the two rows of cells B8 through E9.
  - In the “Insert” tab menu, select the “Scatter” option without connecting lines:



- After clicking on this line-less scatter option, the chart should appear.
  - From here (and whenever the chart area is active or highlighted) you can work with the “Chart Tools”, which are organized by the “Design”, “Layout”, and “Format” tabs.
  - Within the “Design” tab, in the “Chart Layouts” area, click on the left-most (and upper-most) layout option. This will place titles on the chart that you can edit or delete.
  - Create your own title and labels for the axes.
16. Now we fit a line to the data; Excel calls this a “trendline”. While the chart area is active or highlighted (click on the chart if you need to), select the “Layout” tab of the “Chart Tools” and click on “Trendline” in the “Analysis” area. Select “Linear Trendline”. We want to find to find a numerical expression for this linear fit to the data: Again, click on “Trendline” but now select “More Trendline Options...”. In the pop-up window, make sure “Linear” is selected, and check the box beside “Display Equation on chart”. Click on “Close”. Since we are plotting  $a = F/m_{\text{total}}$ , the slope of the equation (the factor multiplying  $x$ ) is  $1/m_{\text{total}}$ . Calculate this experimental  $m_{\text{total}}$ , enter it in the “Data” section, compare it to the value of  $m_{\text{total}}$  obtained by weighing the glider plus the 20 grams of small masses used to accelerate the glider ( $M + 4m$ ), and calculate the experimental error. Show your results to your TA.

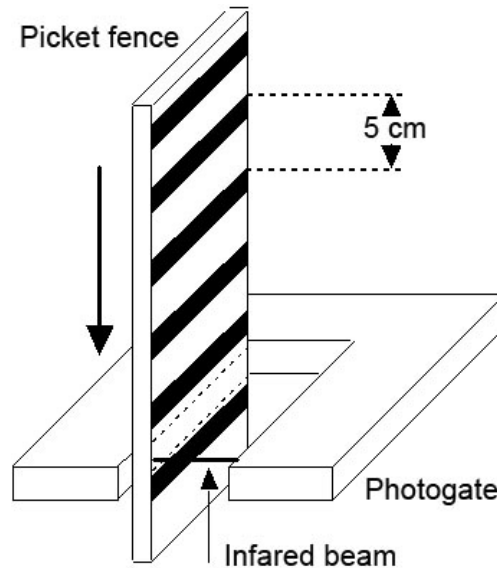
## DATA

- Mass of small glider = \_\_\_\_\_  
Mass of large glider = \_\_\_\_\_
- “Eyeball” value of acceleration = \_\_\_\_\_
- Slope of velocity graph = \_\_\_\_\_
- Experimental mass = \_\_\_\_\_  
Experimental error = \_\_\_\_\_



**ADDITIONAL CREDIT: FREE-FALLING PICKET FENCE (3 mills)**

Here you will need a photogate, a picket fence, and a rag box to catch the falling fence. The picket fence is a strip of clear plastic with evenly spaced black bars. When the fence is dropped through a photogate, the light beam is interrupted by the bars; since the fence accelerates while falling, the bars interrupt the beam with increasing frequency. The software calculates the distance fallen, as well as the corresponding velocity and acceleration. This acceleration should be constant and equal in magnitude to  $g$ .



Under “Hardware Setup”, select “Picket Fence” to add the sensor. Insert the physical plug of the photogate into the appropriate digital channel of the interface. Drag a new graph over from the graph icon on the right side of the screen. Click the “Select Measurement” button on the  $y$ -axis and select “Acceleration ( $m/s^2$ )”. Arrange the photogate in such a way that the falling picket fence will be caught by the rag box on the floor.

Click “Record”, drop the picket fence through the photogate, and click “Stop”. Since the entire motion occurs within a fraction of a second, you may not see much on the graph. Expand the time scale using the “Scale-to-fit” button. Again, the acceleration graph may look ragged. Use the “Display selected statistics...” button to find the mean value of the acceleration. Record the magnitude of the acceleration due to gravity with experimental error below.

Experimental  $g$  = \_\_\_\_\_

Experimental error = \_\_\_\_\_

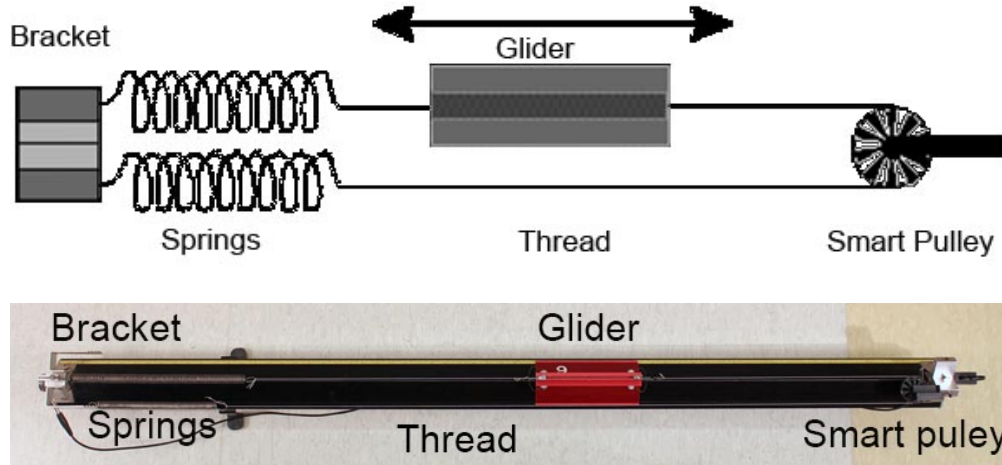
This short experiment illustrates the power of computer measurement. It appears easy, but be sure you understand what is happening in the measurement and can explain the results to your TA.

# Conservation of Energy

## APPARATUS

*Shown in the diagram and picture below (both with a top-down view):*

- Air track, springs and bracket, thread, glider
- Smart pulley and mount
- Dumb pulley (on the same mount)



*Not shown in the images above:*

- Computer and Pasco interface
- Scale and weight set
- Mass hanger

## THEORY

In this experiment, you will test the Law of Energy Conservation by monitoring an oscillating air track glider connected to springs by a thread which passes over a smart pulley.

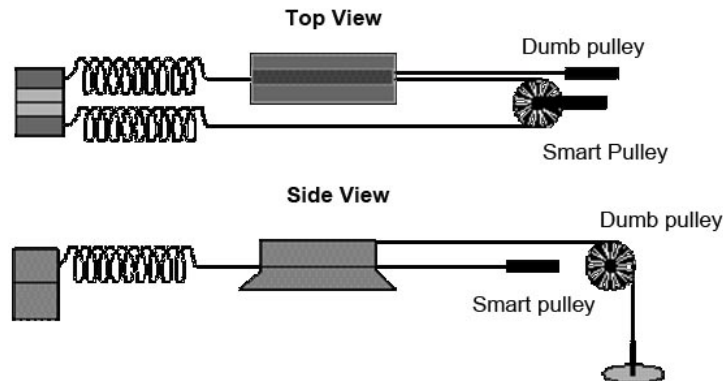
Consider a glider of mass  $M$  on a nearly frictionless air track. As the glider oscillates back and forth, there is a continuous exchange of mechanical energy between two forms: kinetic energy contained in the moving glider, and potential energy stored in the stretched or compressed springs. The Law of Energy Conservation tells us that the total mechanical energy of the system (i.e., the sum of the kinetic and potential terms) remains constant in time. In reality, however, a small amount of mechanical energy may be lost to friction.

## INITIAL SETUP

1. Turn on the air track and level it by adjusting the leveling screw such that a glider on the track has no apparent tendency to move in either direction. Since air tracks are often bowed in the middle, place the glider at several different positions on the track to verify that it is

level.

2. Weigh the glider to obtain its mass, and record this value in the “Data” section.
3. Assemble the springs, glider, smart pulley, and thread as shown in the figure below. One spring is attached to a bracket at the end of air track and to one end of the glider. The other end of the glider is attached to a second spring by a thread which passes over the smart pulley. The second spring is then attached to the bracket. The springs should be tensioned so that you can get an end-to-end glider motion over a distance of at least 40 cm without either spring being completely compressed.
4. Mount a dumb pulley (which has low friction, but is not connected to the computer) on the same fixture that holds the smart pulley, albeit in a vertical plane. Using a long piece of thread, connect the mass hanger to the glider. Make sure the thread passes over the dumb pulley so that weights added to the hanger will displace the glider.



## PROCEDURE PART 1: MEASURING THE SPRING CONSTANT $k$

1. Our first task is to measure  $k$ , the force constant in  $F = -kx$  of a Hooke's Law spring. Call up a blank Excel worksheet and prepare three columns to record the total mass in the hanger (including the mass of the hanger itself) in grams, its weight in newtons, and the position of the glider in meters. In the mass column, type “0” and “10” in the first two cells. Select these two cells, position the cursor at the lower left corner of the bottom cell until it turns into a lopsided square, and drag down the column to fill it with a series up to 60 grams. The running yellow box shows how far the series continues.
2. Fill down the next column with the force values. Remember how to do this? Type “=A3\*9.8/1000” in the cell next to the mass value of zero, being sure to use the correct cell designation corresponding to your spreadsheet (where we typed “A3” above). Then fill down the forces next to the masses.
3. Now make the measurements. Turn on the air blower, and help the glider come to equilibrium. Add masses to the mass hanger, one at a time, and read the distance (in meters!) from the scale on the air track aligned with one corner of the glider. On your spreadsheet, record the distance corresponding to the addition of each mass in the mass hanger. Be sure that you use values corresponding to the entries in the mass column (including the mass of the hanger itself). If you need to use different masses, change the mass entries; the forces will be

recalculated instantly.

4. When you have filled in the distance column next to the force column, chart these variables against each other in Excel, and find the slope. Here is a reminder of how this is done:
  - Select the cells with numbers in the force and distance columns.
  - In the “Insert” tab menu, select the “Scatter” option without connecting lines. After clicking on this line-less scatter option, the chart should appear.
  - Within the “Design” tab of the “Chart Tools”, in the “Chart Layouts” area, click on the left-most (and upper-most) layout option. This will place titles on the chart that you can edit or delete.
  - Create your own title and labels for the axes.
  - In the “Layout” tab of the “Chart Tools”, click on “Trendline” in the “Analysis” area. Select “More Trendline Options...”. In the pop-up window, make sure “Linear” is selected, and check the box beside “Display Equation on chart”. Click on “Close”.

Convince yourself that the slope is  $1/k$  and not  $k$ . Record the value of  $k$  in the “Data” section.

## PROCEDURE PART 2: PLOTTING ENERGIES

1. Unhook the mass hanger string. Hook up the smart pulley physically and virtually, in Data Studio. Double-click on the smart pulley icon, and check “Position”, “Velocity”, and “Acceleration”. Arrange a graph to plot the position  $x$ , the velocity  $v$ , and the acceleration  $a$  as a stack of three plots on a single graph window. With the air blower on, pull the glider out, click “Start”, let the glider oscillate several times, and click “Stop”. The velocity and acceleration graphs resemble the sinusoidal oscillations of a simple harmonic oscillator, but the position graph consists of a series of S-shaped curves increasing in  $y$  value. This shape results because the smart pulley does not distinguish between the forward and reverse directions of motion; it merely counts the number of times the spokes block the photosensor and records the result as positive distance. Thus, each S-shaped curve on the position graph is produced as the oscillator moves from one endpoint of its motion to the other.
2. Now prepare Data Studio to calculate the kinetic energy in real time. Click the calculator button in the graph tool bar (or in the top tool bar). In the equation area of the pop-up window, type “ $y = 0.5*m*v^2$ ” (for  $(1/2)mv^2$ ). You will be asked to define the variables  $m$  and  $v$  below. Here  $m$  is a constant (the measured mass of the glider in kilograms), but  $v$  is a data measurement variable (the velocity, Ch. 1 (m/s)). When you have finished defining  $m$  and  $v$ , click “Accept” again.
3. The kinetic energy calculation now appears in the “Data” column. Drag it to a graph, and perform a run with the glider oscillating to check that you are obtaining reasonable-looking kinetic energy curves. (The kinetic energy never quite goes to zero at the end points.)
4. Calculating the potential energy with the smart pulley is trickier. First, as demonstrated above, the pulley does not distinguish between forward and backward motion, so we can look at only the first half-oscillation. Second, when we pull the glider out and “Start” the distance measurement, the software assigns zero to the first distance measurement. However, we want to assign zero to the equilibrium position. In other words, we want to calculate

$PE = (1/2)k(x - x_0)^2$ , with  $x_0$  the equilibrium position.

To accomplish this, record one full S-shaped curve of position: Move the glider away from its equilibrium position to a point where the smart-pulley LED just turns OFF. Wiggle the glider back and forth to locate the ON/OFF transition point as precisely as you can. If the LED is ON, then move away from equilibrium until the light turns OFF. The photogate is now unblocked. Within the first millimeter of motion, it will be blocked by a spoke, and the timing will begin.

Have your partner click “Start”; then release the glider. Click “Stop” just after the glider has reached its maximum position on the other side of equilibrium. Check your position graph, and repeat the experiment until you have a nice S-shaped curve containing 25 – 30 data points with a few points from the next S-curve.

If you are accumulating too many data runs, cluttering up your data column, set the top of the data column to “By Run”, instead of “By Measurement”. Then delete any unneeded runs.

5. Layer a graph with position and velocity from your best run. The velocity first swings through a minimum (near zero when the glider reaches the endpoint opposite where it was released. Select the position values from  $t = 0$  through this endpoint, and have Data Studio calculate the mean value of the position. This is your  $x_0$  in  $PE = (1/2)k(x - x_0)^2$ ; record it in the “Data” section.
6. Now click “New” in the calculator window again, and type in the equation “ $y2 = 0.5*k*(x-x0)^2$ ”, and define its variables  $k$ ,  $x$ , and  $x0$  appropriately. (You can change the name of the variable  $y2$  if you wish.) Click “Accept”.
7. Finally, click “New” in the calculator window again, and type in the equation “ $y3 = P + K$ ”. This is to be the total energy, so define  $P$  as the data measurement variable  $y2$  (the potential energy calculation) and  $K$  as the data measurement variable  $y$  (the kinetic energy calculation).
8. These data calculations all appear in the “Data” column. Get a new layered graph with your variables  $y$ ,  $y2$ , and  $y3$  plotted on the  $y$ -axis, and time on the  $x$ -axis. Title your graph and show it to your TA before printing it out. Be prepared to comment to your TA on whether your graphs show reasonable data for the kinetic, potential, and total energies.

## DATA

- (Initial Setup, step 2) Mass of glider = \_\_\_\_\_
- (Procedure Part 1, step 4) Spring constant  $k$  = \_\_\_\_\_
- (Procedure Part 2, step 5) Equilibrium position  $x_0$  = \_\_\_\_\_

## ADDITIONAL CREDIT PART 1 (5 mills)

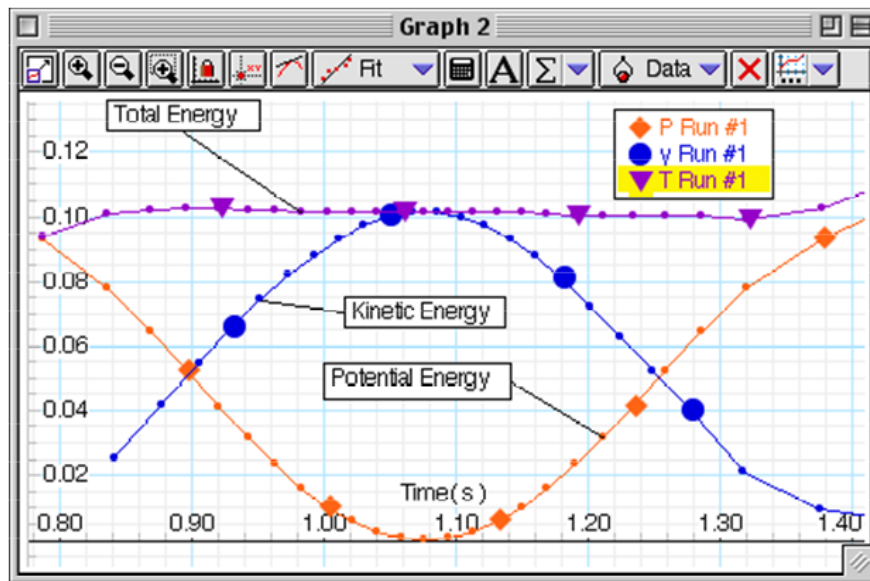
This is a harder experiment to do well, requiring more careful experimental technique. After you started, you may have found that you had not tensioned the springs enough to get 25 points on the S-shaped curve without compressing one of the springs. Changing the spring tension may require

remeasuring  $k$ . (Should  $k$  change significantly?)

Repeat the experiment more carefully. If necessary, retension the springs so you can get at least 25 points on the S-shaped curve for the half-oscillation, and remeasure  $k$  carefully.

Averaging the distance values to obtain  $x_0$  may not be the best method of determining the true  $x_0$ . From studying your various measurements, what would be another method to obtain a value for  $x_0$ ? Does this agree with the averaging method? Include this method in you repeat of the experiment, if you think it would improve the measurement of  $x_0$ . Print out the final results.

Full additional credit for this assignment requires obtaining good  $PE$ ,  $KE$ , and total  $E$  curves without significant assistance from your TA. The total energy curve should be a nearly-level line, decreasing slightly through the motion owing to frictional losses. Label your curves as in the illustration below. Include an explanation of any improvements you made in the measuring techniques.



### ADDITIONAL CREDIT PART 2 (2 mills)

The glider-spring system is a simple harmonic oscillator. You will study the physics of such systems later in the course. The frequency of oscillation is given by

$$f = (1/2\pi)\sqrt{k/m}. \quad (1)$$

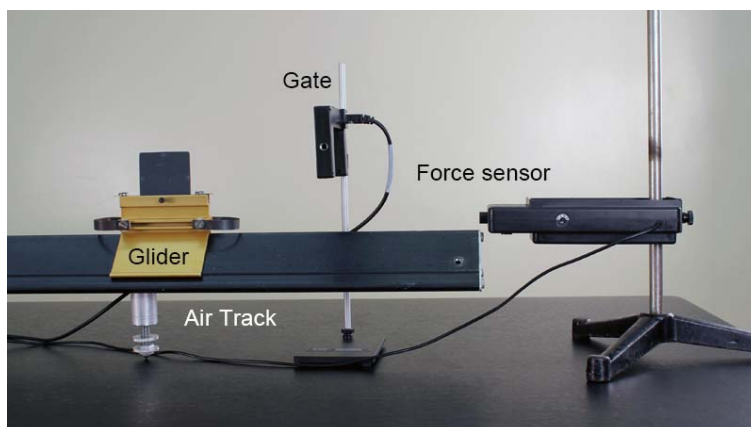
Devise a way to obtain the frequency of oscillation from your Data Studio measurements (or make new measurements), and use your values of  $k$  and  $m$  to check this formula. Report the results with experimental error.

# Momentum and Impulse

## APPARATUS

*Shown in the picture below:*

- Air track
- Glider with bumper and flag
- Photogate
- Force sensor



*Not shown in the picture above:*

- Computer and Pasco interface
- Set of masses in 50-g increments to 600 g
- Vernier calipers or meter stick to measure glider flag
- Scale and weight set

## THEORY

Newton's Second Law tells us that the net force acting on an object is equal to the object's mass multiplied by its acceleration:  $\mathbf{F}_{\text{net}} = m\mathbf{a}$ . Using  $\mathbf{a} = d\mathbf{v}/dt$ , where  $\mathbf{v}$  is the object's velocity, we can rewrite this law as

$$\mathbf{F}_{\text{net}} = m d\mathbf{v}/dt. \quad (1)$$

Newton himself believed that this relation should also account for the possibility that the mass is varying:

$$\mathbf{F}_{\text{net}} = d(m\mathbf{v})/dt. \quad (2)$$

Examples of varying masses include rain falling into a rolling open box car and a rocket expelling gases. The above equation can be rewritten as

$$\mathbf{F}_{\text{net}} = d\mathbf{p}/dt, \quad (3)$$

where  $\mathbf{p} = m\mathbf{v}$  is the *momentum* of the object. Eq. 3 is the most general definition of force: the change of momentum with time. If we write it as a differential equation,

$$d\mathbf{p} = \mathbf{F}_{\text{net}} dt, \quad (4)$$

and integrate with respect to time, then Eq. 3 becomes

$$\Delta\mathbf{p} = \mathbf{p}_2 - \mathbf{p}_1 = \int \mathbf{F}_{\text{net}} dt. \quad (5)$$

The right side of Eq. 5 is known as the *impulse*, and the left side is the change in momentum. The notion of impulse is often associated with a force that acts for a short period of time. Examples of such forces include a bat hitting a ball and the impact between two objects moving at relatively high speeds.

In this experiment, you will verify Eq. 5 by allowing a glider on an air track to pass through a photogate and strike a force sensor. The sensor allows you to measure the force on the glider as a function of time. This time interval is relatively short, so the impulse approximation is valid. The velocity of the glider is measured when it crosses the photogate, just before and just after the collision. These two velocity measurements, along with knowledge of the glider's mass, allow you to calculate the change in momentum (i.e., the left side of Eq. 5). The sensor generates a force-versus-time curve on the computer, which can be integrated to obtain the impulse (i.e., the right side of Eq. 5). The glider has a foam bumper, so its collision with the force sensor is *inelastic*. In other words, kinetic energy is not conserved during the collision, but the change in momentum is still equal to the impulse.

## INITIAL SETUP: CALIBRATING THE FORCE SENSOR

1. Arrange the force sensor so you can hang masses from it. Set up the sensor in Data Studio, and plug it in. Double-click on the force sensor icon in the setup window, click the “Measurement” tab, and check the box for “Voltage”. The sensor will produce force readings, but we are going to ignore these and do our own calibration of the sensor, so we can convert its voltage readings to forces. We will hang masses from the sensor, type in the force, and let the computer read the corresponding voltage. Note that the sensor has two interchangeable end parts: a screw hook for hanging masses in the calibration phase of the experiment, and a rubber bumper for the impulse measurements in the actual experimental runs. When calibrating the force sensor, use the screw hook, and mount the sensor vertically downward on the horizontal bar of the ring stand.
2. Click “Sampling Options” in the setup window tool bar, check the box for “Keep data values only when commanded”, and type in “mass” for a name and “grams” for units. Drag a table to the force sensor, and drag the voltage data and the Keyboard 1 data (the mass values) over to the table, so that you get two columns showing these data. We will convert the masses to forces later in Excel.
3. Click “Start”. The “Start” button changes to a keep-and-stop button as shown below:



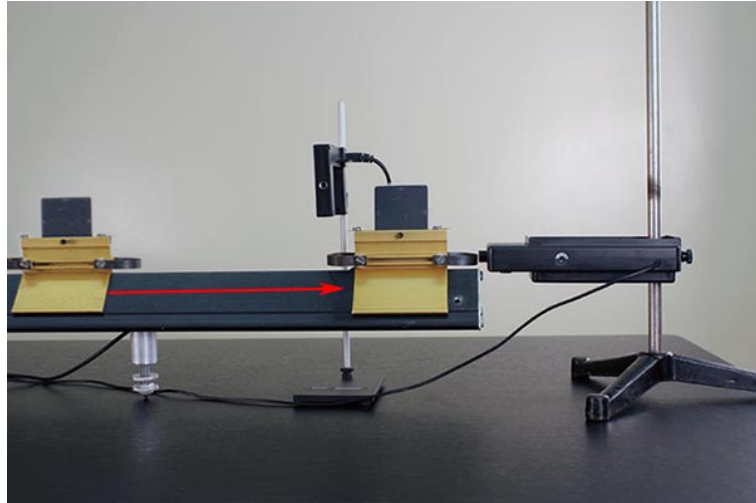
Also, you will see in your table the running voltage reading. With no mass hanging on the force sensor, push the “Tare” button on the side of the sensor to zero the reading, and click



- “Keep”. You will be prompted to enter the mass value (zero in this case). Repeat this five times, so that you obtain five values of the voltage (nearly the same) for the zero-mass case.
- Now add the 50-gram mass holder. Repeat five times the procedure of pressing “Keep” and entering a mass of 50. (We are using a minus sign for all the masses, since the glider will push the force sensor in the opposite direction during the actual experimental runs.) Perform another 50-gram step, and then perform 100-gram steps up to a total of 500 grams, taking five voltage readings for each mass value. Remember to type in minus signs in front of the mass values. Click the red “Stop” button when you are finished.
  - Your table now contains a column of five voltage readings for each mass. Call up an Excel worksheet, and copy the voltage column of this table into column A of Excel so you can work on it. (That is, select the data in the Data Studio column, and paste in the data.) In column B, copy your mass values from the Data Studio column. In column C, type in the mass series, just once for each mass value. In column D, convert these mass values to forces in newtons. In column E, take the average of the appropriate voltage readings for each mass. Be sure you average only the voltages that correspond to a given mass. For example, you can use “AVERAGE(A4..A8)” to average the values in cells A4 through A8. It is easy to make a mistake here, so have your lab partner check your entries in the AVERAGE function.
  - Copy the force values from column D and “Paste Special” (the values) into column F, select the data in columns E and F, and graph the force as a function of voltage. Remember that you need to choose “XY scatter”. (If you had not copied column D over to F, you would have obtained a graph of voltage as a function of force.) Insert a trendline with linear regression, and include its equation on the chart. This equation gives the required conversion from voltage to force. Write the equation in the “Data” section.

## PROCEDURE

- Level the air track carefully. Mount the force sensor horizontally on the vertical rod of the ring stand at the end of the track so the glider bumper will strike the sensor as the glider moves down the track. Replace the screw hook of the force sensor with the rubber bumper. Set up the photogate so the glider flag clears the gate by a few centimeters before the bumper strikes the sensor. Refer to the picture below.



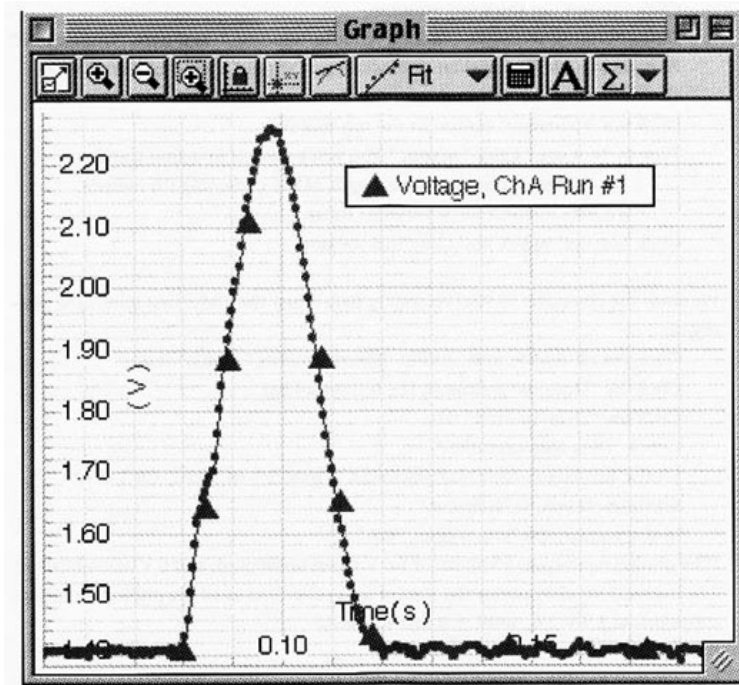
The glider then bounces off the sensor and passes through the photogate again. In an experimental run, you should record the two photogate velocity readings and the force-versus-time curve from the sensor.

2. Weigh the glider, and record its mass (in kilograms) in the “Data” section.
3. Measure the length of the glider flag, and record its length (in meters) in the “Data” section.
4. We want to set up the photogate to measure the velocity of the glider. We will measure the time the photogate is blocked by the glider flag, and then do a calculation to find the velocity.
  - a. Disengage “Keep data only when commanded” in the “Sampling Options” button.
  - b. Set up the photogate in Data Studio, and plug it in.
  - c. Click “Timers” in the setup window.
  - d. Under “Timing Sequence Choices”, choose “Blocked”, and then “Unblocked”.
  - e. Pictures of a blocked and an unblocked gate should appear in the window.
  - f. Close the window.
  - g. Click the “Calculate” button in the top tool bar.
  - h. Type in “ $v = d/t$ ”. Click “Accept”. You will be asked to define  $d$  and  $t$ .
    - $d$  is a constant: the measured length of the glider flag (in meters).
    - $t$  is a data measurement variable: timer 1.
  - i. Click “Accept” again, and close this window.
5. We want the computer to start recording data when the glider first enters the photogate.
  - a. Click the photogate icon, and in “Measurements”, check “State”.
  - b. Click the “Sampling Options” button in the setup tool bar.

- c. Click the “Delayed Start” tab.
- d. Check “Data Measurement”.
- e. Set the first box (which may start with “Timer 1”) to “State, Ch 1 (V)”.
- f. Set the next box to “Is Below”.
- g. In the voltage text box, type in “4.9”.

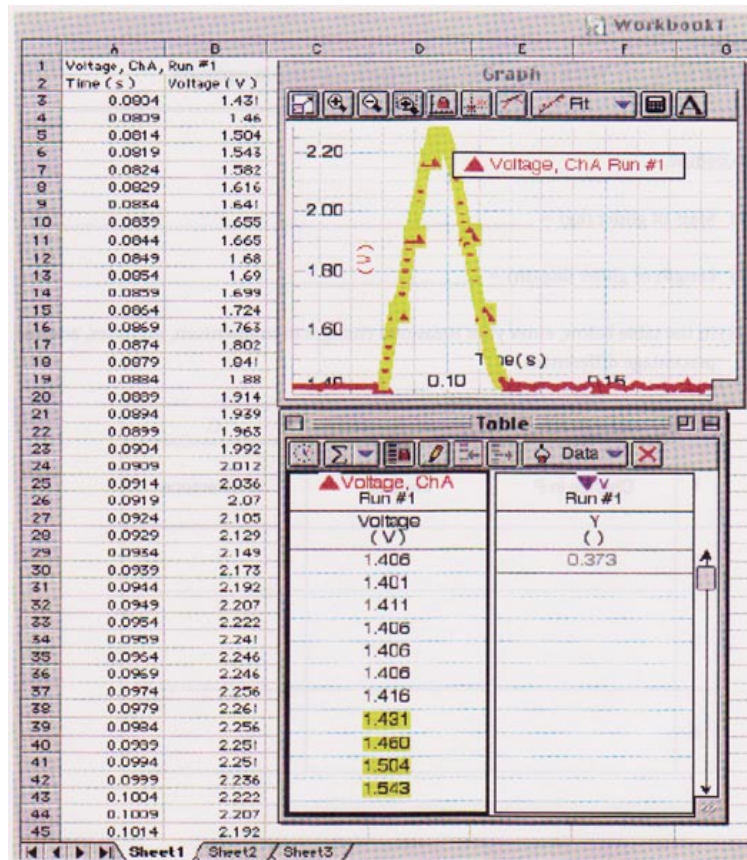
(The photogate outputs a voltage of 5.0 V when unblocked, and 0 V when blocked. The instructions above delay the start of data collection until the photogate voltage drops below 4.9 V, i.e., until it is first blocked.)

- 6. Double-click on the force sensor icon. Set the sampling rate to 2000 Hz. Note that the computer will then take a force reading every  $1/2000$ , or 0.0005, second. Drag a table over to the force sensor, and set it to read a column of voltages and a column of velocities (by dragging over the appropriate data). You do not need the time measurement column and can get rid of it by clicking the clock symbol on the table tool bar.
- 7. Turn on the air track, and click “Start”. Push the “Tare” button on the side of the force sensor to zero its readings. Send the glider down the track so it passes through the photogate, strikes and bounces off the force sensor, and crosses the gate again. Click “Stop”. Your table should show a long list of force and voltage readings, as well as the two velocity readings at the top. You may need to use the scroll bar on the table to see all the readings, particularly the second velocity reading. The second velocity value is smaller since the collision is inelastic.
- 8. Drag a graph to the force sensor on the computer, set it to read the force sensor voltages. You should see a nice graph of the impulsive force. Use the “Scale-to-Fit” button, if necessary, to locate the impulse. Use the “Zoom Select” button so you can see the impulse clearly.



If all is well, you are ready to take data. If not, check over your steps.

9. Have each lab partner make three measurements with different glider speeds to check the relation  $\Delta \mathbf{p} = \int \mathbf{F}_{\text{net}} dt$ . To calculate the impulse, click on the column of the table with the voltage readings, and select an area of the graph that just covers the impulsive force (i.e., when the graph leaves the base line and returns to the base line, before any oscillations are encountered). These voltage readings are automatically highlighted in the table and correspond to the area chosen in the graph. Pull down the “Edit” menu to “Copy”, and paste the selected voltage data on an Excel sheet. (The time readings may copy over also, but we do not need them.) The figure below shows the Data Studio graph and table overlying an Excel sheet. The appropriate area of the graph has been selected, and you can see part of the highlighted table below. Only one velocity value is shown in the Data Studio table; you would need to scroll it to see the other.



10. In the next Excel column, use the force calibration equation determined earlier to convert the voltages to forces. The impulse is the area under the force-versus-time curve:

$$\int \mathbf{F}_{\text{net}} dt = \sum \mathbf{F}_i \Delta t_i = \Delta t \sum F_i, \quad (6)$$

since the time intervals  $\Delta t_i = 0.0005$  second are all equal. You can obtain the force sum with an Excel function such as “=Sum(b3..b147)”. When you calculate the change in momentum from  $mv_1$  to  $mv_2$ , should you add or subtract these two numbers?

11. In the “Data” section, record the three values of change in momentum and impulse, as well as the percentage difference between each set of values.

## DATA

Initial Setup, step 6: Write in your voltage-to-force conversion equation:

### Procedure

- Mass of glider (kg) = \_\_\_\_\_
- Length of glider flag (m) = \_\_\_\_\_

12. In the table below, enter your measured changes in momentum, impulses, and the percentage differences.

	A	B	C
1	Change in P	Impulse	% difference
2			
3			
4			

### ADDITIONAL CREDIT (3 mills)

Data Studio itself has an integral function on the “Special” button of the calculator window. This additional credit is for figuring out how to obtain the integral of just the area under the impulse without significant TA assistance. The procedure is not trivial, and may require some thinking on how to formulate the integral function, use of the smart cursor and its delta function, and reference to the help section of Data Studio. Don’t forget conversion of voltage to force. When you get all this worked out, display a table comparing the Data Studio integrals with the Excel integrals.

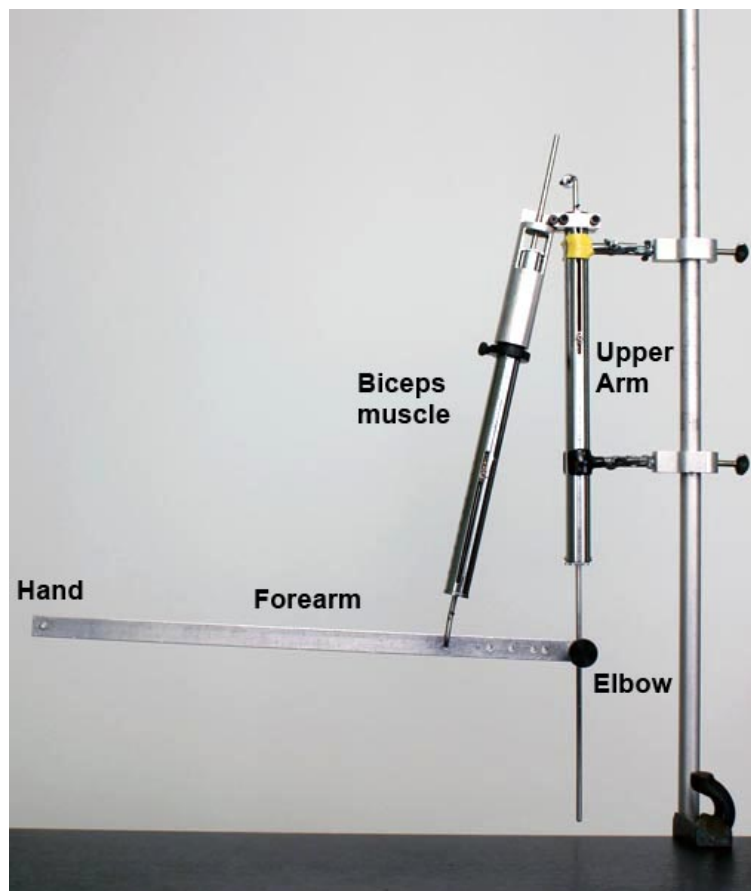
# Biceps Muscle Model

## APPARATUS

- Biceps model
- Large mass hanger with four 1-kg masses
- Small mass hanger for “hand” end of forearm bar with five 100-g masses
- Meter stick
- Centimeter ruler
- Weighing scales

## INTRODUCTION

In this experiment, you will use a mechanical model to measure the force exerted by the biceps muscle on a human forearm, as well as the force of compression on the upper-arm (humerus) bone when the arm lifts a weight.



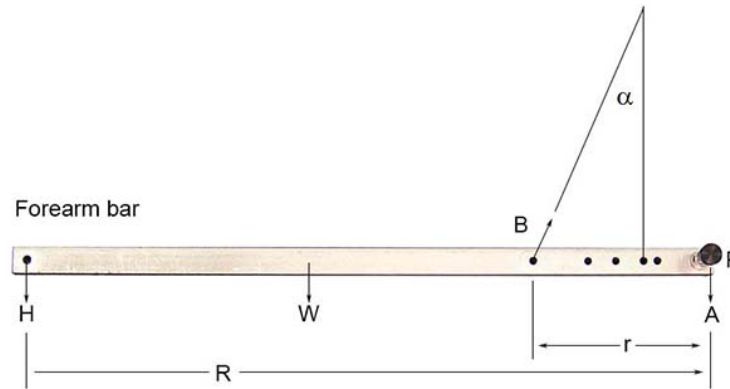
Two tension-compression gauges are employed to determine the compression of the humerus bone and the tension in the biceps muscle as various weights are placed in the “hand” (i.e., hung from the end of the forearm). The point at which the biceps muscle connects to the forearm can be changed,

and the effects arising from different attachment points can be studied.

Note that the gauge readings are in kilograms or pounds. In the metric system, a kilogram is a unit of mass, not of force. Nevertheless, many force scales (such as the gauges in this experiment and the metric scales for weighing people) read in kilograms. Near the Earth's surface, force in newtons is equal to mass in kilograms multiplied by the gravitational acceleration  $g = 9.8 \text{ m/s}^2$ . In this experiment, we ignore the distinction between force and mass, and record “force” in kilograms.

## THEORY

Below is a free-body diagram showing all the forces exerted on the forearm bar. According to the Laws of Statics (i.e., Newton's First Law and the lever rule), the net force on the stationary bar must be zero, and the net torque on the bar must also be zero. The “hand” end of the bar can be moved up and down, compressing and extending the gauges. Before taking measurements, always adjust the elbow attachment point on the upper-arm gauge so the bar is horizontal. This makes the ensuing analysis simple.



The forces acting on the forearm are its weight  $W$  (which points downward and may be assumed to act on the forearm's center-of-gravity), the weight of the “hand”  $H$  (which points downward), the force from the biceps muscle  $B$  (which pulls upward on the forearm at a small angle  $\alpha$  with respect to the vertical), and the force from the humerus bone  $A$  (which pushes downward on the elbow). Since the biceps force has a small horizontal component of magnitude  $B \sin \alpha$  directed toward the elbow, the upper-arm gauge must push back in the opposite direction with a horizontal force of magnitude  $P$ , so that the net force in the horizontal direction is zero:

$$\sum F_x = B \sin \alpha - P = 0. \quad (1)$$

Furthermore, since the net force in the vertical direction is zero, Newton's First Law can be written as

$$\sum F_y = B \cos \alpha - H - W - A = 0 \quad (2)$$

or

$$B \cos \alpha = H + W + A. \quad (3)$$



If we choose the elbow joint as the pivot about which torques are calculated, then the forces  $A$  and  $P$  do not contribute to the torque about this pivot because their moment arms are zero. (In other words, the lines of action for  $A$  and  $P$  pass through the elbow joint.) The force  $H$  acts at a perpendicular distance  $R$  from the pivot, so its moment arm is  $R$ . The force  $W$  acts at a perpendicular distance  $R/2$  from the pivot, so its moment arm is  $R/2$ . The component of  $B$  perpendicular to the forearm —  $B \cos \alpha$  — acts at a perpendicular distance  $r$  from the pivot, so its moment arm is  $r$ . Since the net torque about the elbow joint is zero, the lever rule can be written as

$$\sum \tau = (H)(R) + (W)(R/2) - (B \cos \alpha)(r) = 0 \quad (4)$$

or

$$B \cos \alpha = (H + W/2)R/r. \quad (5)$$

Thus, the magnitude of the biceps force is

$$B = (H + W/2)R/(r \cos \alpha). \quad (6)$$

Note that if we plot  $B$  as a function of  $H$  while keeping  $R$ ,  $r$ , and  $\alpha$  constant, then we will obtain a linear graph. On the other hand, if we keep  $H$  constant while varying  $r$ , then we must plot  $B$  as a function of  $R/(r \cos \alpha)$  to obtain a linear graph.

The magnitude of the humerus force can now be obtained from Eqs. 3 and 5:

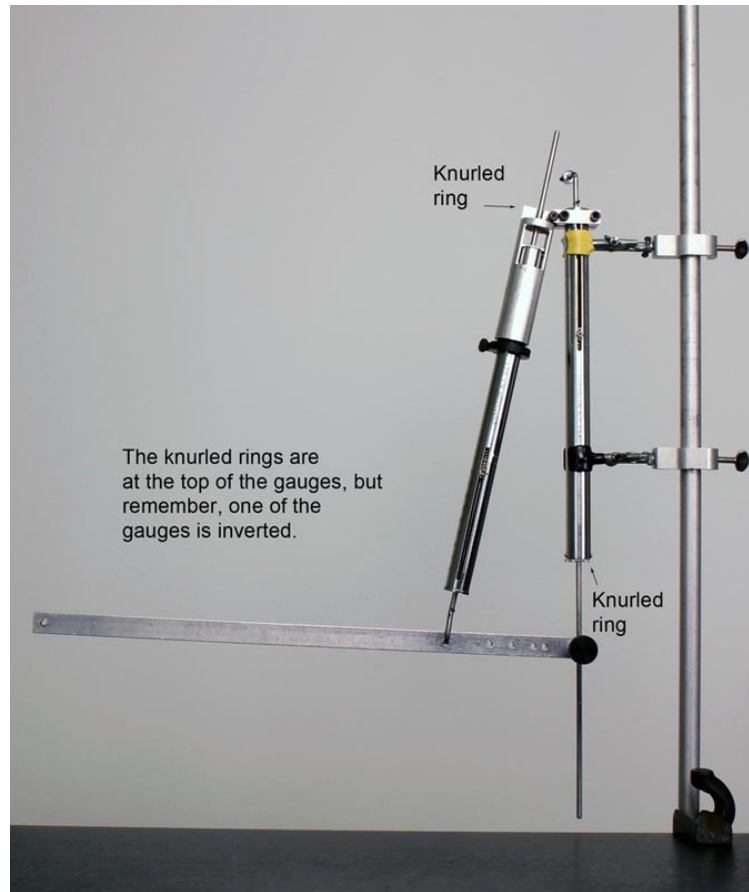
$$A = B \cos \alpha - H - W \quad (7)$$

$$= (H + W/2)R/r - H - W \quad (8)$$

$$= (R/r - 1)H + (R/2r - 1)W. \quad (9)$$

## PROCEDURE

1. Remove the forearm bar completely. The biceps tension-compression gauge is now suspended vertically so you can hang masses from it. Use the small keeper ring with the thumbscrew on the short right-angle section to prevent the masses from falling off. Notice that you can zero the scale by rotating the knurled ring at the top of the gauge.

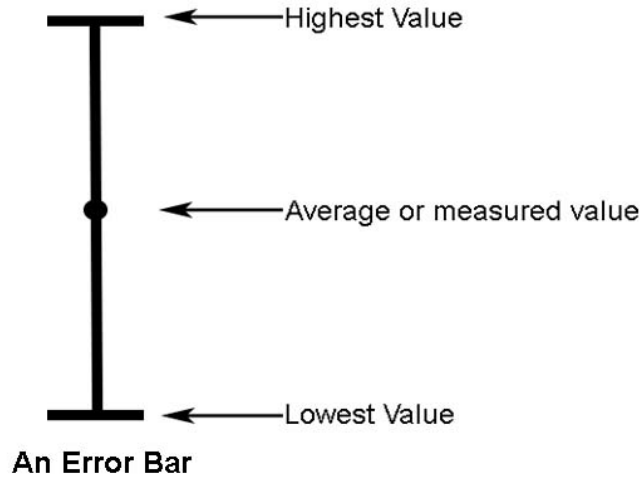


2. Hang one to four 1-kg masses from the gauge in increments of 1 kg. How accurately could the scale be read if there were no friction in the gauge? Record this estimate in the “Data” section.
3. With the heavier masses, there may be some friction in the gauge which brings it to rest over a small interval of readings. Move the mass hanger up and down near its equilibrium position, and record the maximum and minimum readings at which the pointer sticks. The average of these two values is a good estimate of the “true” mass, and their difference is a good estimate of the experimental error. Record the actual mass, maximum mass, minimum mass, average (“true”) mass, and difference in mass (“error”) in the “Data” section. State whether you observe any systematic error in the scale (i.e., whether the scale is off by a constant ratio).
4. Repeat parts 2 and 3 with the upper-arm tension-compression gauge. Notice that the upper-arm gauge has a stiffer spring and a different scale than the biceps gauge. To hang masses from this gauge, you will need to invert the apparatus in its holding clamps. You may also need to rotate the gauge in the clamps so you can see its readings. Have your lab partner hold the biceps gauge out of the way, or tie it back with a piece of string. Adjust the knurled ring on the upper-arm gauge so the scale reads zero. Perform the same set of measurements as in parts 2 and 3, and record your results in the “Data” section.
5. Measure the distance between the elbow hole and each of the five biceps attachment holes, as well as the distance between the elbow hole and the mass hanger hole at the “hand” end of the forearm bar. Record these values in the “Data” section.

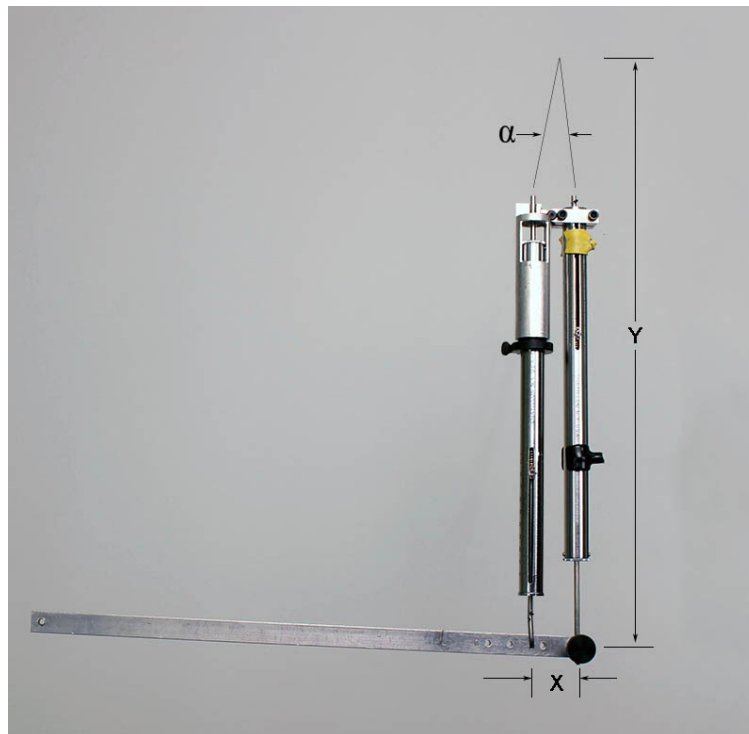
6. Measure the mass of the forearm bar without the upper-arm attachment piece, and record this value in the “Data” section.
7. Adjust the knurled rings of the gauges so they read zero with no weight attached. Arrange the apparatus as shown in Figure 1, with the biceps gauge attached to the farthest hole from the elbow, which is approximately 12 cm from the hole through which the upper-arm gauge passes. When clamping the forearm gauge, make sure the pointer can move freely through its range and is not obstructed by the clamp jaws. Use the small keeper ring with the thumbscrew on the short right-angle section to retain the biceps gauge in position. Attaching the 50-g mass hanger to the end of the forearm bar, add masses in increments of 100 g up to 550 g, and record the readings of the two gauges in the “Data” section. As you change the masses in the “hand,” adjust the position of the upper-arm attachment point at the “elbow” so the bar comes to rest in a horizontal position. You may need to push up forcefully on the bar when using the heavier masses, compressing the upper-arm gauge before tightening the attachment.

If you find that there seems to be an unusual amount of friction in your scale readings, check that the scales are not twisted in their clamps. All persons doing experiments in the real world soon realize that nature can be difficult, and not everything works as it is supposed to or according to the simple instructions. This principle has been canonized in variations of Murphy’s Law: “If anything can go wrong, it will,” “Nature sides with the hidden flaw,” etc. Throughout this lab series, you will often need to use common sense and resort to your own ingenuity to get through the parts that don’t seem to work quite right. Ask your TA for assistance when necessary, but first try to solve the problem yourself. Gradually, you will learn to proceed with confidence that you are doing to make the experiments work and yield good data.

8. With a total mass of 150 g hanging from the end of the forearm bar, take readings with the biceps gauge attached to the various holes, which are approximately 3, 4, 6, 8, and 12 cm from the elbow. Each time you adjust to a new position, make sure the bar is horizontal. For certain biceps attachment positions (e.g., 3 and 4 cm from the elbow), you may find it necessary to move the elbow attachment point a considerable distance along the upper-arm gauge rod, forcefully compressing the gauge to keep the bar horizontal.
9. Make a neat graph of the biceps force  $B$  as a function of the hand weight  $H$  in the “Data” section. Label the axes with units. Add error bars (which show the uncertainty associated with any measurement, as depicted below), and construct the straight line predicted by Eq. 6. Also, make a neat graph of the humerus force  $A$  as a function of the hand weight  $H$ . You may plot this data on the same graph. Add error bars, and construct the straight line predicted by Eq. 9. Label the two curves, and title the graph.



10. Make a neat graph of the biceps force  $B$  as a function of  $R/(r \cos \alpha)$  for the various hole positions  $r$  in the "Data" section. Obtain the angle  $\alpha$  from  $\tan \alpha = x/y$  (you already know the value of  $x$ , but will need to extend the lines of the gauge rods to determine the value of  $y$ ). Note that the variables  $B$  and  $R/(r \cos \alpha)$  are chosen so the plot will be linear. Label the vertical axis with units, and note that the horizontal axis is dimensionless. Add error bars, and construct the straight line predicted by Eq. 6.



## DATA

1. Estimated error in scale reading = \_\_\_\_\_

2. Biceps tension-compression gauge with one 1-kg mass

Actual mass = \_\_\_\_\_

Maximum mass = \_\_\_\_\_

Minimum mass = \_\_\_\_\_

Average mass = \_\_\_\_\_

Difference in mass = \_\_\_\_\_

Biceps tension-compression gauge with two 1-kg masses

Actual mass = \_\_\_\_\_

Maximum mass = \_\_\_\_\_

Minimum mass = \_\_\_\_\_

Average mass = \_\_\_\_\_

Difference in mass = \_\_\_\_\_

Biceps tension-compression gauge with three 1-kg masses

Actual mass = \_\_\_\_\_

Maximum mass = \_\_\_\_\_

Minimum mass = \_\_\_\_\_

Average mass = \_\_\_\_\_

Difference in mass = \_\_\_\_\_

Biceps tension-compression gauge with four 1-kg masses

Actual mass = \_\_\_\_\_

Maximum mass = \_\_\_\_\_

Minimum mass = \_\_\_\_\_

Average mass = \_\_\_\_\_

Difference in mass = \_\_\_\_\_

3. Upper-arm tension-compression gauge with one 1-kg mass

Estimated error in scale reading = \_\_\_\_\_

Actual mass = \_\_\_\_\_

Maximum mass = \_\_\_\_\_

Minimum mass = \_\_\_\_\_

Average mass = \_\_\_\_\_

Difference in mass = \_\_\_\_\_

Upper-arm tension-compression gauge with two 1-kg masses

Estimated error in scale reading = \_\_\_\_\_

Actual mass = \_\_\_\_\_

Maximum mass = \_\_\_\_\_

Minimum mass = \_\_\_\_\_

Average mass = \_\_\_\_\_

Difference in mass = \_\_\_\_\_

Upper-arm tension-compression gauge with three 1-kg masses

Estimated error in scale reading = \_\_\_\_\_

Actual mass = \_\_\_\_\_

Maximum mass = \_\_\_\_\_

Minimum mass = \_\_\_\_\_

Average mass = \_\_\_\_\_

Difference in mass = \_\_\_\_\_

Upper-arm tension-compression gauge with four 1-kg masses

Estimated error in scale reading = \_\_\_\_\_

Actual mass = \_\_\_\_\_

Maximum mass = \_\_\_\_\_

Minimum mass = \_\_\_\_\_

Average mass = \_\_\_\_\_

Difference in mass = \_\_\_\_\_

4. Distance between elbow hole and:

First biceps attachment hole = \_\_\_\_\_

Second biceps attachment hole = \_\_\_\_\_

Third biceps attachment hole = \_\_\_\_\_

Fourth biceps attachment hole = \_\_\_\_\_

Fifth biceps attachment hole = \_\_\_\_\_

Mass hanger hole at “hand” end = \_\_\_\_\_

5. Mass of forearm bar = \_\_\_\_\_

6. Biceps attached 12 cm from elbow, with total mass of 150 g

Biceps tension-compression gauge:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Upper-arm tension-compression gauge:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Biceps attached 12 cm from elbow, with total mass of 250 g

Biceps tension-compression gauge:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Upper-arm tension-compression gauge:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Biceps attached 12 cm from elbow, with total mass of 350 g

Biceps tension-compression gauge:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Upper-arm tension-compression gauge:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Biceps attached 12 cm from elbow, with total mass of 450 g

Biceps tension-compression gauge:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Upper-arm tension-compression gauge:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Biceps attached 12 cm from elbow, with total mass of 550 g

Biceps tension-compression gauge:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_



Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Upper-arm tension-compression gauge:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

7. Total mass of 150 g in “hand”

Biceps attached 3 cm from elbow:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Biceps attached 4 cm from elbow:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Biceps attached 6 cm from elbow:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Biceps attached 8 cm from elbow:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

Biceps attached 12 cm from elbow:

Maximum reading = \_\_\_\_\_

Minimum reading = \_\_\_\_\_

Average reading = \_\_\_\_\_

Difference in readings = \_\_\_\_\_

8. Plot the graph of the biceps force  $B$  as a function of the hand weight  $H$ , as well as the graph of the humerus force  $A$  as a function of the hand weight  $H$ , using one page of graph paper at the end of this workbook. Remember to label the curves, title the graphs, and add error bars to your data points.
9. Plot the graph of the biceps force  $B$  as a function of  $R/(r \cos \alpha)$  using one page of graph paper at the end of this workbook. Remember to label the curve, title the graph, and add error bars to your data points.

## QUESTIONS

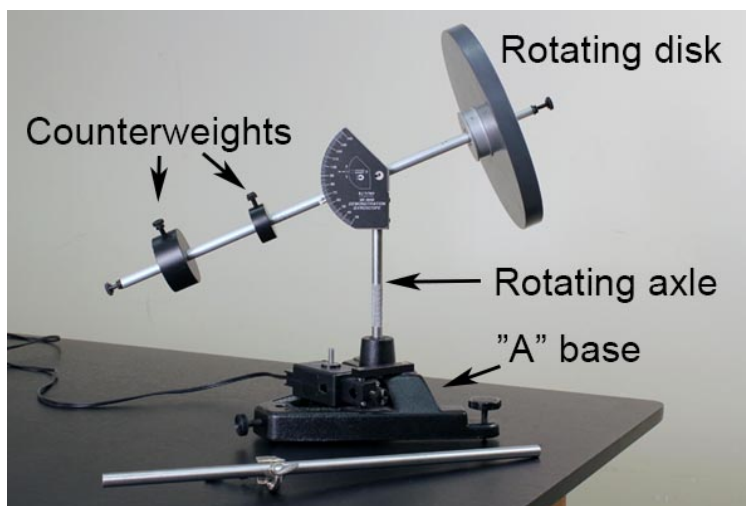
- a. What is the largest value of  $P$  (the horizontal force exerted by the upper-arm tension-compression gauge) that would have arisen in your measurements?
- b. People who “work out” regularly with weights can “curl” 25 to 50 or more pounds with one hand (i.e., they can take the weight in one hand held horizontally and raise it to their shoulder). The actual biceps muscle attachment point is approximately 5 cm from the elbow. How large a force would the biceps muscle be exerting when one “curls” 50 pounds?

# Rotation and Gyroscopic Precession

## APPARATUS

*Shown in the picture below:*

- Pasco gyro assembly with rotation sensor on base
- Support rod and clamp for gyro



*Not shown in the picture above:*

- Rotator with movable weights on stand with rotation sensor
- Vernier calipers
- Pan balance to weigh add-on weight
- Meter stick
- Weight hanger and weights
- Digistrobe
- String to spin wheel
- Computer and Science Workshop Interface

## NOTE TO INSTRUCTORS

This experiment consists of two parts. Part 1 is a relatively straightforward measurement of the angular acceleration produced by different torques on an apparatus with variable rotational inertia. Part 2 involves the measurement of a gyroscope's precession as a function of its rotational inertia and rotational angular velocity, and requires somewhat more experimental ingenuity.

Most students cannot complete these two parts in one lab session, so you should choose which part you would like them to perform. The default option (in which you do not express a preference) is part 1. Another option, which you would need to choose at the beginning of the quarter in conjunction with the other instructors, is to reduce the “Biceps” experiment to one week by omitting parts 1 – 4 and to allow two weeks for this “Rotation” experiment.

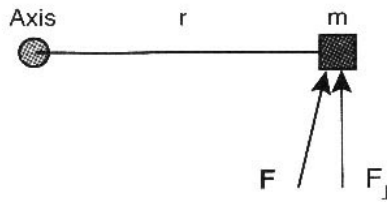
## TORQUE AND ROTATIONAL INERTIA

We are all aware that a massive wheel has rotational inertia. In other words, it is hard to start the wheel rotating; and, once moving, the wheel tends to continue rotating and is hard to stop. These effects are independent of friction; it is hard to start a wheel rotating even if its bearings are nearly frictionless. *Rotational inertia* is a measure of this resistance to rotational acceleration, just as inertia is a measure of resistance to linear acceleration.

Automobile piston engines use a *flywheel* for this very purpose. The gasoline explosions in the piston chambers deliver jerk-like forces to the rotating crankshaft, but the large rotational inertia of the flywheel on the crankshaft smooths out the otherwise jerky rotational motion.

We also have an intuitive idea of *torque*, the tendency of a force to rotate a body. To produce the maximum rotational acceleration, we want to push perpendicular to the rotation axis, and at as large a distance  $r$  from the rotation axis as possible.

Consider a small mass  $m$  at a distance  $r$  from the rotation axis. If a force  $\mathbf{F}$  acts on it, the linear acceleration of the mass around the circle will be  $a = F_{\perp}/m$ , where  $F_{\perp}$  is the component of  $F$  perpendicular to the radius arm. The angular velocity  $\omega = v/r$  of the mass is increasing, but the angular acceleration  $\alpha = a/r$  is constant.



Multiply each side of the equation  $F_{\perp} = ma$  by  $r$ , and manipulate the  $r$ 's:

$$rF_{\perp} = mr^2a/r = mr^2\alpha, \quad (1)$$

or

$$\text{torque} = \tau = I\alpha. \quad (2)$$

We recognize the torque  $\tau = rF_{\perp}$  in vector form  $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ . The rotational inertia  $I$  is equal to  $mr^2$  for a small particle of mass  $m$ . For an assembly of small particles, each of mass  $m_i$ , we sum to get the rotational inertia:

$$I = \sum m_i r_i^2. \quad (3)$$

And if the mass distribution is continuous, we integrate:

$$I = \int r^2 dm. \quad (4)$$

We can continue to define the rotational analogs to linear motion. For example, the *angular momentum*  $L$  is

$$L = I\omega, \quad (5)$$

in analogy to

$$p = mv, \quad (6)$$

and the *rotational kinetic energy* is

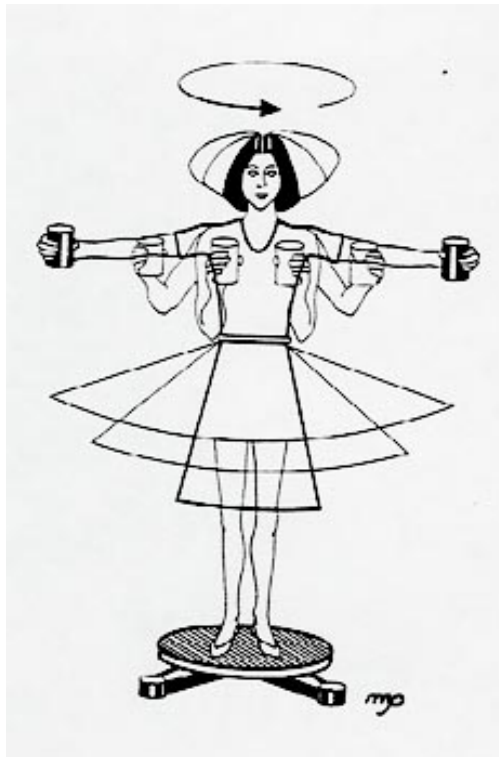
$$\text{rotational } KE = (1/2)I\omega^2, \quad (7)$$

in analogy to

$$\text{translational } KE = (1/2)mv^2. \quad (8)$$

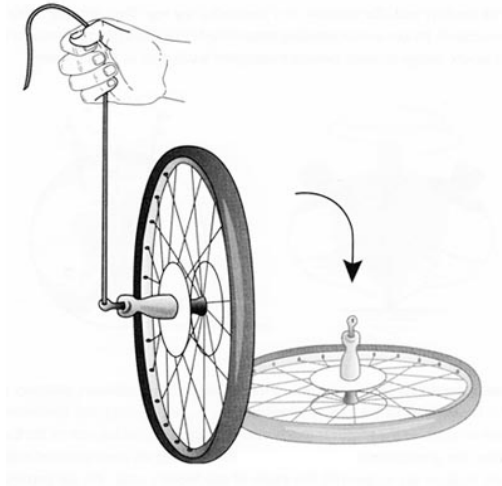
Linear momentum  $p = mv$  is conserved in the absence of external forces. Likewise, angular momentum  $L = I\omega$  is conserved in the absence of external torques. One interesting difference between rotational motion and linear motion is that; since rotational inertia depends on the positions of the masses, it is easy to change the rotational inertia “on the fly”, so to speak. A spinning ballerina, ice skater, or star with a large rotational inertia  $I$  and small angular velocity  $\omega$  can increase the angular velocity of spin by pulling mass in to reduce the rotational inertia: the ballerina and ice skater by pulling in their arms and legs, and the star by collapsing smaller by gravity.

$$I\omega = I\omega.$$

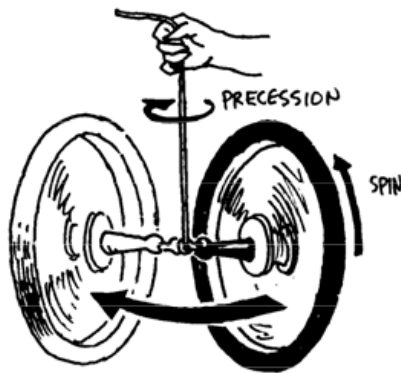


## PRECESSION

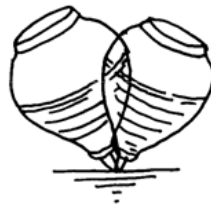
A common lecture demonstration of gyroscopic precession is to hang a bicycle wheel by one end of its axle. If the bicycle wheel is not spinning, it flops down.



But if the wheel is spinning, it doesn't fall. Instead it *precesses* around: its axle rotates in a horizontal plane.



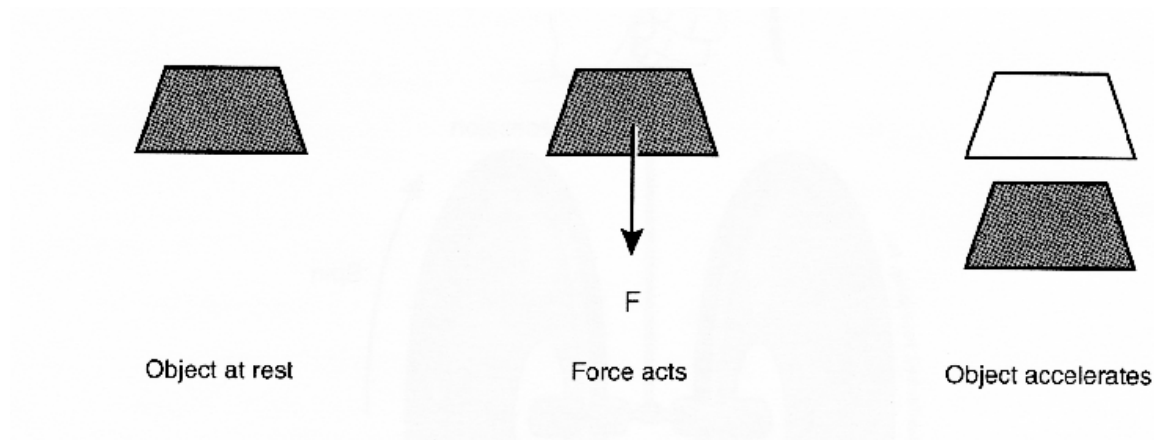
We are all familiar with the example of a precessing toy top. The spinning Earth also precesses around. Its axis is now pointing toward the North Star in the sky, but over time the axis slowly swings around, making a complete revolution in 26,000 years.



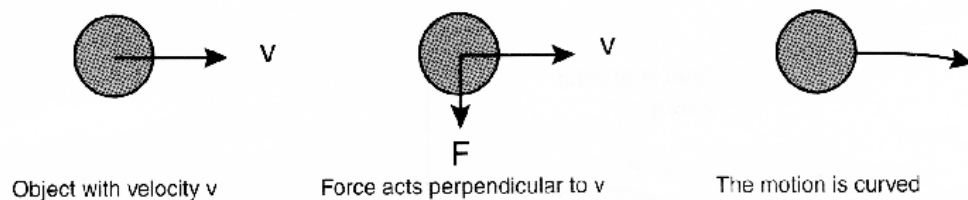


A necessary condition for precession is a torque aligned in a different direction from the spin. In the case of the bicycle wheel and the toy top, gravity acts downward on the center of mass so the torque is in a horizontal direction. In the case of the Earth's precession, the gravitational force from the Moon is acting on the equatorial bulge of the Earth to align the equatorial bulge with the plane of the Moon's orbit. We are particularly interested in the case when the torque is perpendicular to the spin axis.

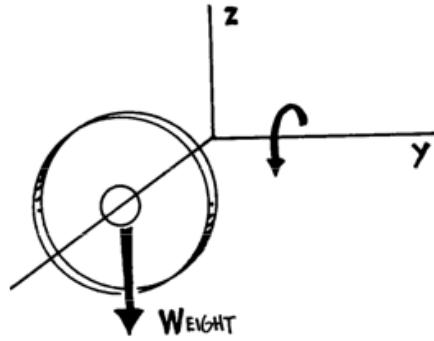
Let's try to understand precession in general. Consider linear motion first. If an object is at rest, and a force acts on it, the force will increase the speed of the object in the direction of the force.



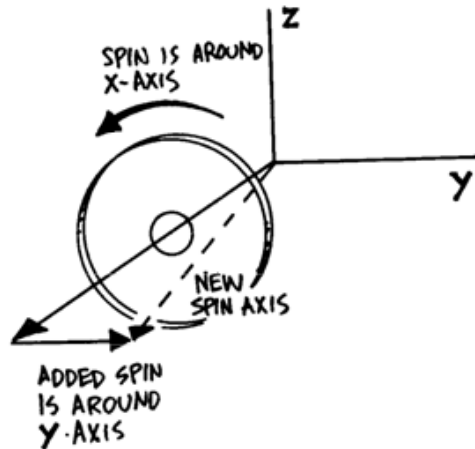
But if the object is already moving and the force acts perpendicular to the motion, the speed will not be changed. Instead the force will curve the velocity around, producing uniform circular motion if the force is always perpendicular to the velocity.



Something similar happens with rotational motion. When the wheel is not spinning, the torque from the weight produces an angular velocity about the torque axis, in this case the  $y$ -axis.



But if the wheel is already spinning, it has spin (angular velocity) about the  $x$ -axis. The torque of the weight adds some spin about the  $y$ -axis, perpendicular to the original spin. The resulting spin axis is turned a little in the  $xy$ -plane. The torque doesn't change the value of the spin; instead it "curves" the spin. (Again, note that here the torque axis is perpendicular to the spin axis. This need not be true in general, but we are considering this simplified case.)



Mathematically,

$$\tau = d\mathbf{L}/dt, \quad (9)$$

which is the rotational analog of

$$\mathbf{F} = d\mathbf{p}/dt. \quad (10)$$

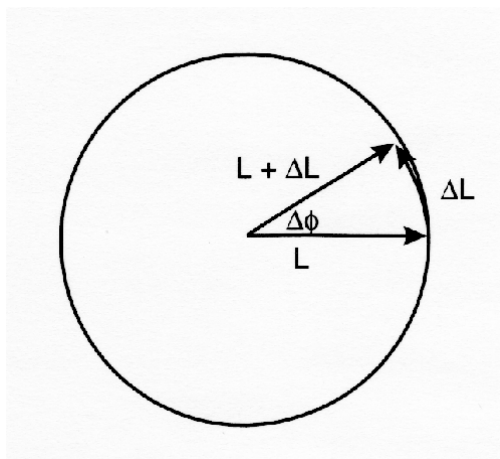
To keep the ideas clear, we will call the angular velocity of the spinning object itself its *spin*  $\omega$ , and the turning around of the spin axis the *precession* angular velocity  $\Omega$ . The angular momentum  $\mathbf{L}$  of the spin is  $L = I\omega$ , where  $\omega$  is the angular velocity of the spin, and  $I$  is the rotational inertia of the wheel.

Thus, in a time  $\Delta t$ , the torque produces a change in the angular momentum of the spin given by

$$\Delta L = \tau \Delta t. \quad (11)$$

But for small changes in the angle  $\phi$  of  $L$ ,  $\Delta L = L\Delta\phi$ .





Thus,

$$\Delta L = L \Delta \phi = \tau \Delta t. \quad (12)$$

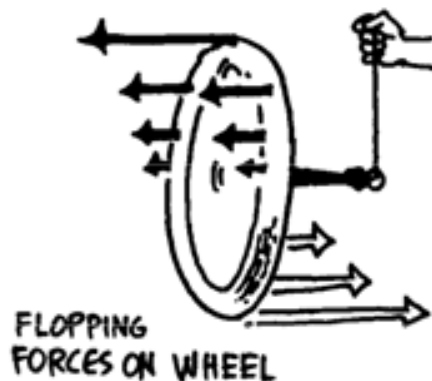
The angular velocity of precession  $\Omega = \Delta \phi / \Delta t = \tau / L$ , the last equality following from the equation above. Since  $L = I\omega$ , we have finally

$$\Omega = \tau / I\omega. \quad (13)$$

This then is our basic equation relating the precession angular velocity to the rotational inertia, spin, and torque.

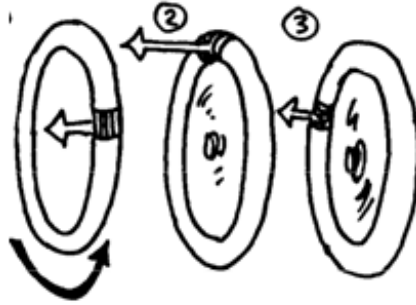
We have shown that precession can be understood from the principles of rotational motion, torque, rotational inertia, angular momentum, etc., and we have even derived the equation for the magnitude of the precession above. But these concepts must be based on the simpler principles of force and acceleration. Let's see if we can understand precession from just the concepts of force and acceleration.

When the gyro bicycle wheel is in the hanging position, the torque exerted by gravity exerts an outward force on the top half of the wheel, and an inward force on the bottom half of the wheel — forces that would make the wheel flop over if it were not spinning.



Now look at a small piece of the wheel as it spins. As the piece comes over the top half of the wheel, the outward force on it grows to a maximum at the top, and decreases to zero at the far

side position. Then, the force becomes negative, and grows to a maximum pointing inward at the bottom position, and decreases to zero again at the near side position.



Measure the angular position of this small piece of the wheel with zero angle  $\theta = \omega t$  at the top position, increasing to  $\pi/2$  radians at the far side position, etc. We can represent the force on the small piece then as

$$F = F_0 \cos \omega t. \quad (14)$$

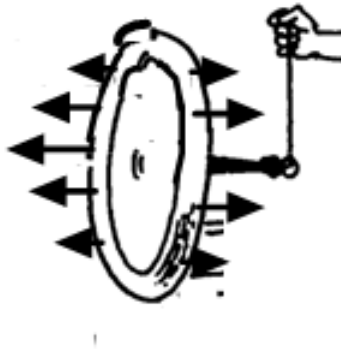
According to Newton's Second Law, the acceleration is proportional to the force:

$$a = a_0 \cos \omega t. \quad (15)$$

But, how does the small piece actually move? Its velocity is the integral of  $a$ ,  $v = \int a \, dt$ , and the integral of  $\cos \omega t$  is proportional to  $\sin \omega t$ .

$$v = v_0 \sin \omega t. \quad (16)$$

The velocity reaches a maximum pointing outward at the far side position, is zero at the top and bottom, and is maximum pointing inward (negative) at the near side position.



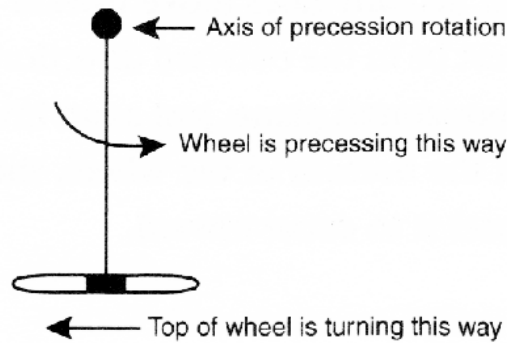
Thus, the forces from the torque don't push the wheel down; they push it around!

Finally, if we integrate the velocity, we should get the distance the wheel element moves:

$$x = \int v \, dt = \int v_0 \sin(\omega t) \, dt = -(v_0/\omega) \cos \omega t = -x_0 \cos \omega t. \quad (17)$$

This equation suggests that the wheel element is moving opposite the force on it:  $F = F_0 \cos \omega t$ . In other words, the wheel element should be moving inward at the top of the turn, while the force is outward. Can this possibly be true?

Indeed it can! Study the top views of the wheel as it is precessing around. The wheel element is just coming over the top of the wheel:



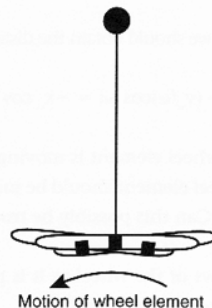
A few moments earlier, the wheel and wheel element were in this position:



A few moments later, the wheel and wheel element will be in this position:



Combine the pictures of the wheel:



So you see the wheel element actually does move inward at the top while the force is outward. In fact, the force must be in the outward direction to produce the curved path of the wheel element in the horizontal plane, just as an inward force produces uniform circular motion. Similarly, at the bottom of the wheel, the inward force on the wheel element from the torque causes it to curve inward.

## PROCEDURE FOR ROTATION

1. You have a rotator device connected to a rotary motion sensor. The rotator device has movable masses on a rod. On the other end of the rotational motion sensor is a pulley with two wheels of different diameter. Plug the yellow plug of the rotational sensor into #1 digital channel of the Science Workshop interface, and the black plug into #2 channel. In the setup window of Data Studio, double-click on the rotary motion sensor. Now double-click on the rotary motion sensor connected to the Science Workshop in the picture, click the measurements tab, and check the box for angular acceleration. Drag a graph over to the rotary motion sensor, and set it to read angular acceleration. Ready a falling weight with string wound around one of the drums, click “Start”, let the weight fall (spinning up the rotator), and check that you are getting readings of angular acceleration. To record a value of the acceleration below, you can select an area of the chart, and use the statistics button to get a mean value of the acceleration.



2. Remove the rod and mass assembly from the sensor, and separately weigh the masses and the rod (without its screw), and the pulley wheel. Measure the diameters of the two pulley wheels

with the Vernier calipers, and convert to radii in meters. Record the information below.

Mass of movable weights in kilograms = \_\_\_\_\_

Mass of rod without screw = \_\_\_\_\_

Mass of pulley wheel = \_\_\_\_\_

Radius of small pulley in meters = \_\_\_\_\_

Radius of large pulley in meters = \_\_\_\_\_

**Warning:** In rotational experiments it is especially important to keep mass units (as in rotational inertia) and force units (as in torque) clearly distinguished.

3. According to the textbook, the rotational inertia of a rod of mass  $m$  and length  $l$  about an axis perpendicular to the rod and through its center is  $(1/12)ml^2$ . Calculate the rotational inertia of the rod. Does it make any difference that the rod is hollow?

$I$  of rod in  $\text{kg} \cdot \text{m}^2$  = \_\_\_\_\_

4. From the mass of the pulley wheel and its volume (which you can calculate from the radii of the two parts), you can calculate its density. From this you can calculate the rotational inertia of the pulley wheel. You may have to do some estimating.

$I$  of pulley wheel in  $\text{kg} \cdot \text{m}^2$  = \_\_\_\_\_

5. Do a trial experimental run in which the masses on the rotator are at the ends of the rod so that the rotational inertia is large. Use 100 – 200 g on the weight hanger and wind the string around the smaller pulley wheel. (This should be the smallest angular acceleration case.) Notice that the angular acceleration on the graph quickly jumps up and reaches an approximately constant value. You can select an area of this nearly constant value, and use the statistics button to get its mean value for the “measured angular acceleration” in the table below. Now do a trial experimental run in which the masses on the rotator are close to the center so that the rotational inertia is small. Use 100 – 200 g on the weight hanger and wind the string around the larger pulley wheel so that the torque will spin the rotator up to high speed. (This should be the largest angular acceleration case.) On the graph, notice that the acceleration quickly reaches a peak, and then falls off with time. Why isn’t the of acceleration constant in this case? Where on the acceleration curve should you average the values of acceleration to get a good result to compare with the predicted acceleration? How can you adjust the apparatus or experimental parameters to get a better result for the small rotational inertia case?

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6. Using masses of 50 – 200 grams, hook the string on the pulley pin, wind the string around the pulley, and let the mass fall while you measure the rotational acceleration on the computer. Do

three experimental trials for each case, and average them to find the rotational acceleration.

Do four cases: two different positions of the masses on the rod (as close to the center as possible, and as far out as possible), and two different pulley sizes. Compute the rotational inertia of the rod with masses (add the effects of the rods, the pulley wheel, and the masses), the torque, the predicted rotational acceleration, and the experimental error, and fill in the table below.

	A	B	C	D	E
1	torque	rotational inertia	predicted rotational	measured angular	experimental
2			acceleration	acceleration	error
3					
4					
5					
6					

When you add the rotational inertias of the rod, masses, and pulley wheel to get the rotational inertia of the entire rotating assembly, are you leaving out anything?

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## PROCEDURE FOR GYROSCOPE

1. We want to check the precession, equation 13,  $\Omega = \tau/I\omega$ . Study your gyroscope for a moment. As you rotate the arm for precession, the rotary motion sensor at the base should move freely. (If not, adjust the screws that hold the sensor.) The rotary sensor from the first part of the experiment may still be set up on Data Studio. Unplug this rotary sensor and delete all the sensors and data. (Or quit Data Studio, and open it up new.) Plug the rotary sensor of the gyro precession into the Science Workshop interface, and drag a digits window to the rotary motion sensor symbol, and set it to measure the rotational angular velocity  $\Omega$ . (You may have to double-click on the icon of the rotary motion sensor connected to the Science Workshop, and check the “Angular Velocity Ch 1 & 2 (ras/s)” box.)



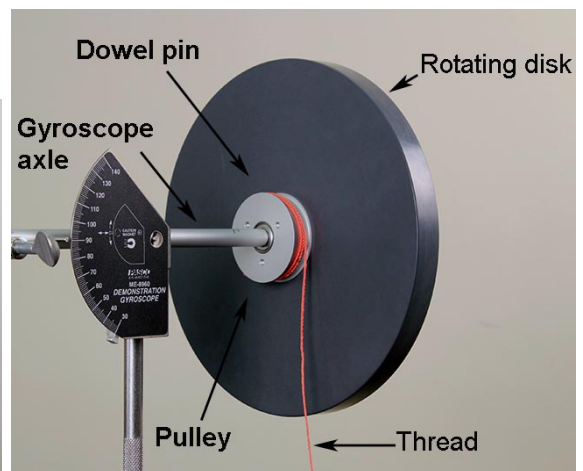
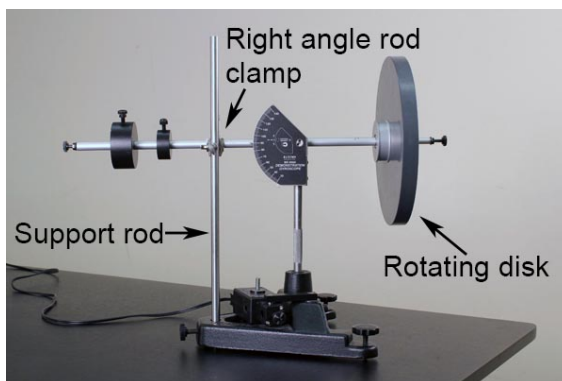
Check that you can get a reading of  $\Omega$  from the computer. Notice that the angular velocity reading in the digits window jumps around, making it difficult to get a definite reading.

Drag a graph window to the rotation sensor, and set to it read angular velocity. Note that the reading on the graph also jumps around a little also as the gyro arm turns; but after you record the data, you can select an area of the graph, and use the statistics button to get a mean value for the angular velocity.

2. We will be measuring the angular velocity  $\omega$  of the gyro wheel with the Digistrobe. The Digistrobe meter reads in rpm, revolutions per minute. You will need to convert this to radians per second. Determine the conversion factor  $f$  in  $\omega = f \times (\text{rpm reading})$ .

$$f = \underline{\hspace{2cm}}$$

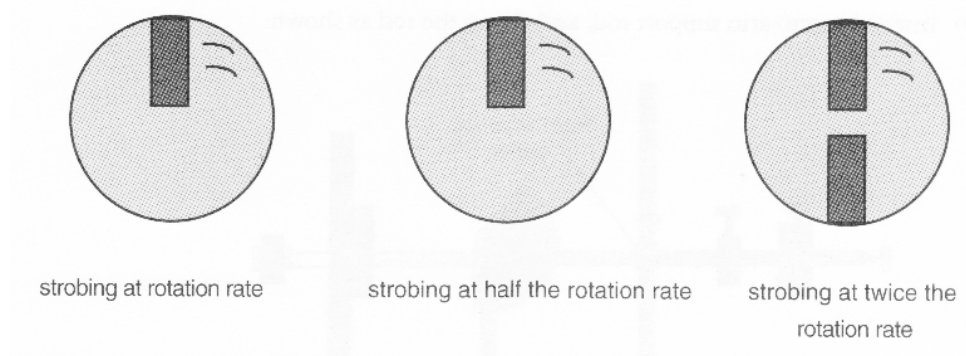
3. Insert the gyro arm support rod and clamp the rod as shown.



Wind a string around the gyro wheel pulley, and pull it off to spin up the wheel.

Turn on the Digistrobe, face it toward the spinning wheel, and adjust the flash rate until the mark on the wheel is stopped.

Now think for a few moments. If the strobe is flashing at the rate the wheel is rotating, the mark will, of course, appear stationary. However, if the strobe were flashing at half the rotation rate (or a third, quarter, etc.), the mark would also appear stationary. On the other hand, if the strobe were flashing at twice the rotation rate, you would see two marks, half a revolution apart.



Given these pieces of information, how will you determine the actual rotation rate from the strobe rate? (Write a brief answer below.)

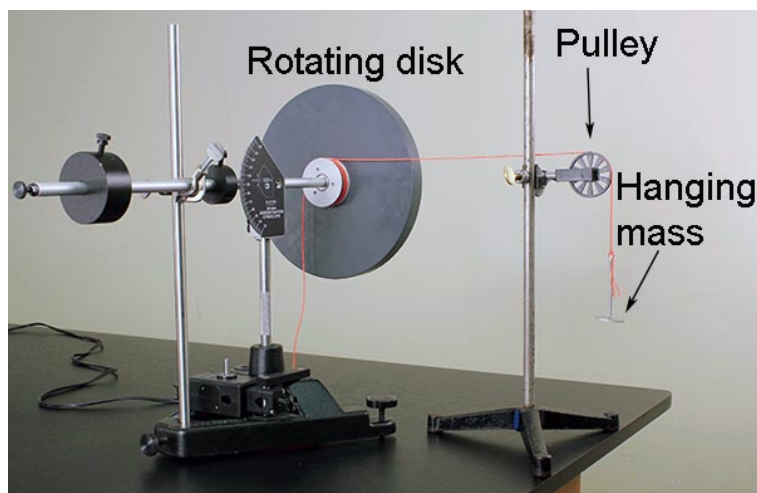
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4. We need the rotational inertia  $I$  of the wheel. With the apparatus still clamped as above, arrange a known weight to fall through a measured distance  $h$  to accelerate the wheel. We leave the exact arrangement up to you. Measure the final velocity of the wheel with the strobe, and calculate the rotational inertia of the wheel from

$$(1/2)I\omega^2 = mgh. \quad (18)$$



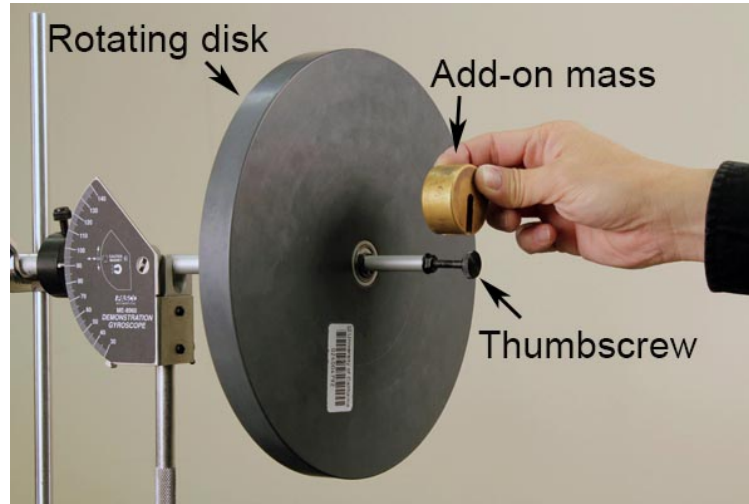
Be sure to keep your force and mass units straight. Make several trials until you have the strobe rate approximately correct, and need only to adjust it slightly just after the weights fall. Record  $I$  below.



$I =$  \_\_\_\_\_

- We need the tipping torque  $\tau$ . Unclamp the gyro arm. The counterweight arm has a large and small weight. Adjust the positions of these weights so that the gyro arm is exactly balanced in the horizontal position. The small weight can be used to fine tune the balance. To produce the turning torque, you will attach the small add-on weight to the front of the gyro wheel. Weigh this add-on weight (in newtons!), and measure the distance of its center from the axis of rotation. From these measurements you can calculate the torque.

$\tau =$  \_\_\_\_\_



- Now we are ready to precess! Spin up the wheel with a piece of string. Help the gyro start precessing with the arm horizontal by moving it along in the correct direction at the correct speed, and then release gently. Measure  $\Omega$  on the computer, and  $\omega$  with the strobe.

$\omega =$  \_\_\_\_\_

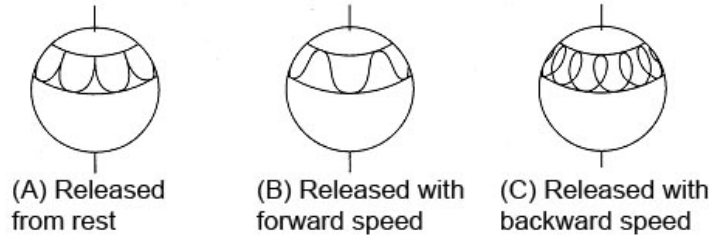
measured  $\Omega =$  \_\_\_\_\_

calculated  $\Omega = \tau / I\omega =$  \_\_\_\_\_

percentage error = \_\_\_\_\_

## NUTATION

- Spin up the wheel to high speed using the string. With the add-on weight producing a tipping torque, hold the shaft horizontally, and release suddenly. The gyroscope undergoes nutation, as in pattern A below.



2. Start again with a smaller torque (just shift the counterweights a little), and with the wheel spinning and arm starting absolutely horizontally (supported by your finger), and then release suddenly. (Use the angle scale on the gyro support to determine the  $90^\circ$  horizontal position). When the gyro is precessing and the nutation motion is damping out, the wheel end of the arm is somewhat below the horizontal position (read the angle scale). (After several precession revolutions, the nutation may be almost completely damped out with the gyro still precessing at a tilted angle. It may take a little experimentation with the torque and wheel speed to produce this situation.) Can you think of a general physics principle explaining why the gyro arm must be tilted down while the gyro is precessing when started from a horizontal position with sudden release? Answer below with brief explanation.

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In fact, there is yet another very general physics principle explaining why the gyro arm must be tilted down during precession if it is released from the horizontal position. Answer below with brief explanation.

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### ADDITIONAL CREDIT (3 mills)

Perform the precession measurement three times with three significantly different  $\omega$ 's, and fill in the table below. Each lab partner must do this separately, doing the computer work him/herself, while directing the other lab partner to assist with the measurements.

	A	B	C	D
1	w(rad/s)	W (exp)	W(th)	% error
2				
3				
4				