Digital Geometry
- Geometry & Its Representation

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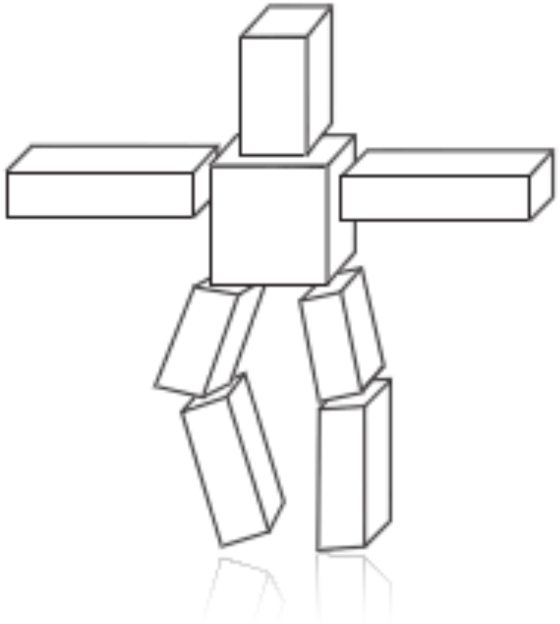
Spring 2019

http://jjcao.github.io/ComputerGraphics/

Music is dynamic, while score is static;
Movement is dynamic, while law is static.

Increasing the complexity of our models

Transformations

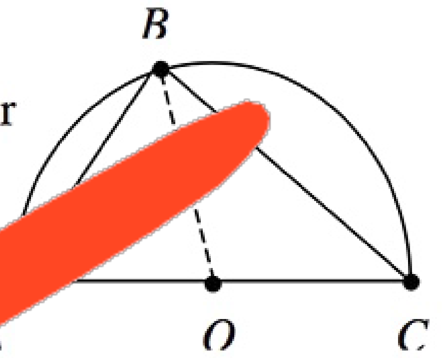


Geometry

Materials, lighting, ...

What is geometry?

THEOREM 9.5. Let ΔABC be inscribed in a semicircle with diameter $\overline{A\ C}$.
Then $\angle ABC$ i... ...t angle.



Proof:

Statement	R...
1. Draw radius OB. Then $OB = OC = OA$	...iven
2. $m\angle OBC = m\angle BCA$ $m\angle OBA = m\angle BAC$	...osceles Triangle Theorem
3. $m\angle ABC = m\angle OBA + m$...	3. Angl... Postulate
4. $m\angle ABC + m\angle BCA$ ... $BAC = 180$	4. The sum o... ...gles of a triangle is 180
5. $m\angle ABC + m$... $+ m\angle OBA = 180$	5. Substitution (line...
6. $2\, m\angle AB$... 180	6. Substitution (line 3)
7. $m\angle A$...$C = 90$	7. Division Property of Equality
8. $\angle ABC$ is a right angle	8. Definition of Right Angle

What is geometry?

"Earth" "measure"

ge•om•et•ry /jēˈämətrē/ *n.*

1. The study of shapes, sizes, patterns, and positions.
2. The study of spaces where some quantity (lengths, angles, etc.) can be *measured*.

Plato: "...the earth is in appearance like one of those balls which have leather coverings in twelve pieces..."

How can we describe geometry?

IMPLICIT

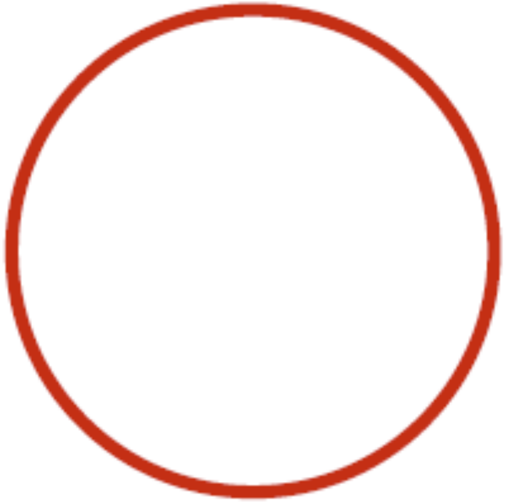
$$x^2 + y^2 = 1$$

LINGUISTIC

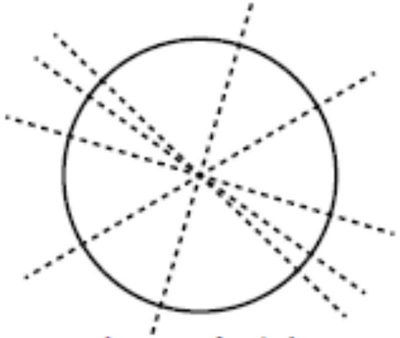
"unit circle"

EXPLICIT

$$(\underbrace{\cos\theta}_{x}, \underbrace{\sin\theta}_{y})$$



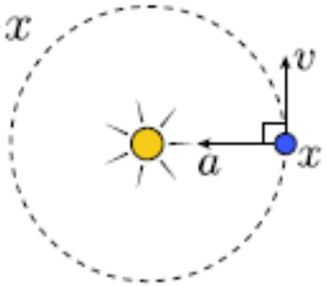
TOMOGRAPHIC



(constant density)

DYNAMIC

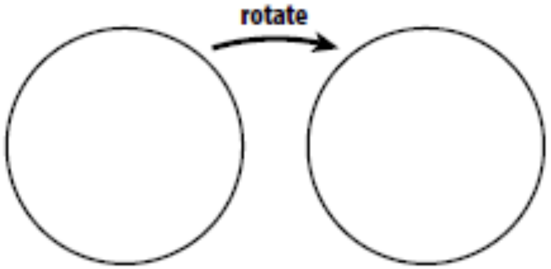
$$\frac{d^2}{dt^2}x = -x$$



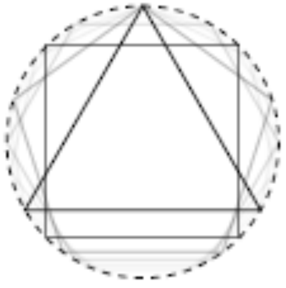
CURVATURE

$$\kappa = 1$$

SYMMETRIC



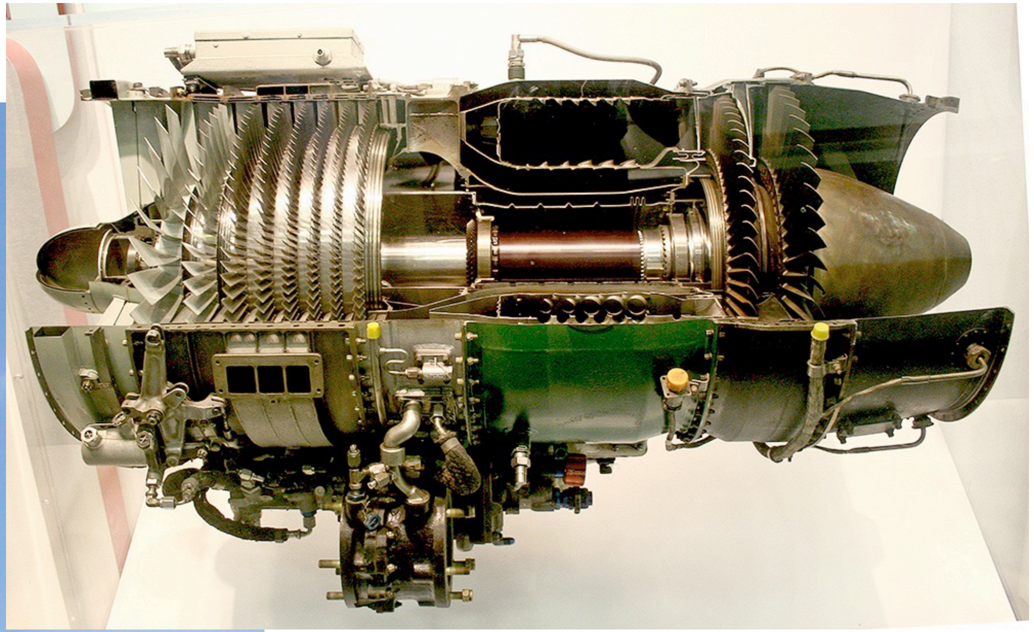
DISCRETE



$n \to \infty$

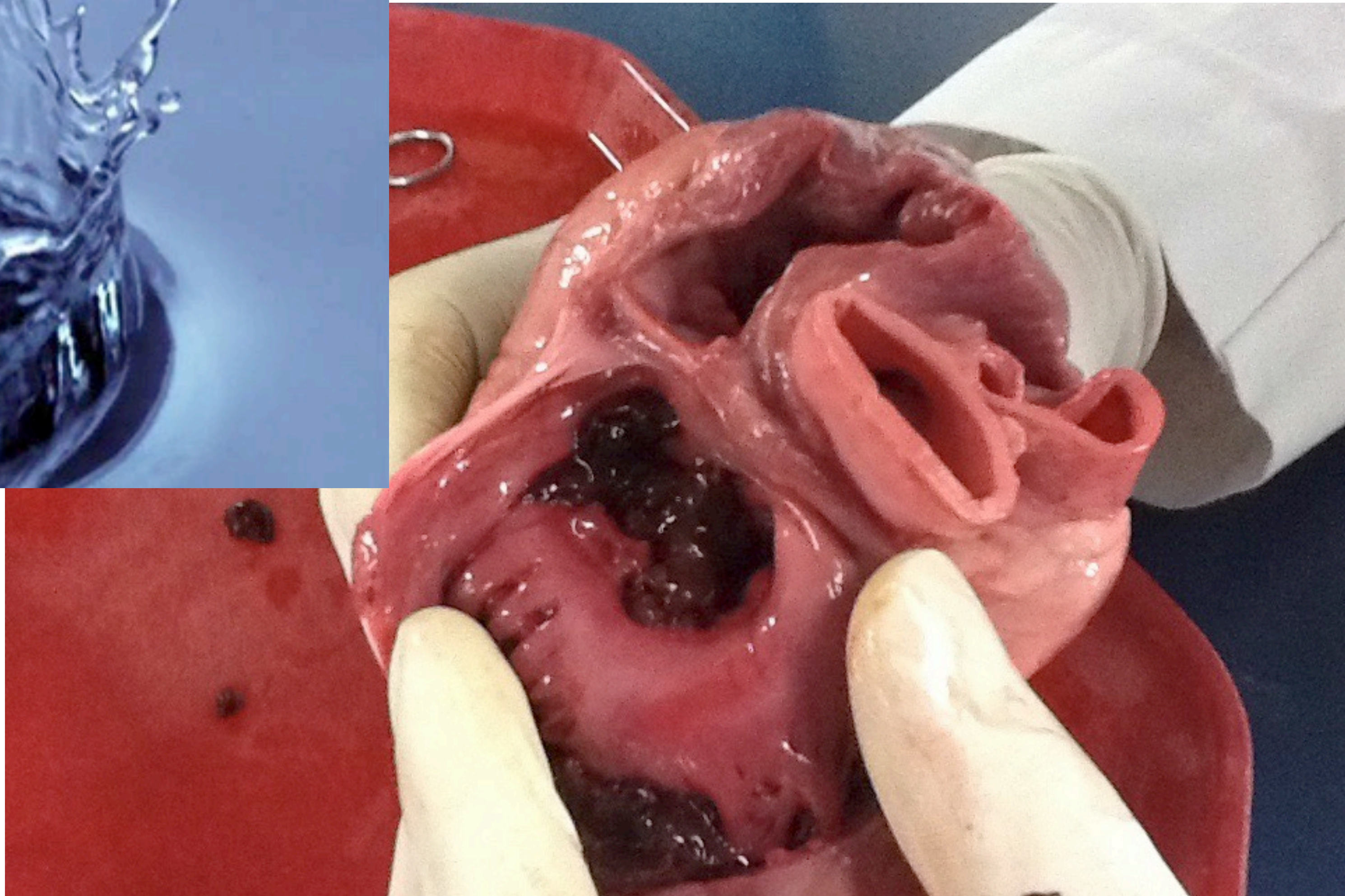
Given all these options, what’s the best way to encode geometry on a computer?

Examples of geometry

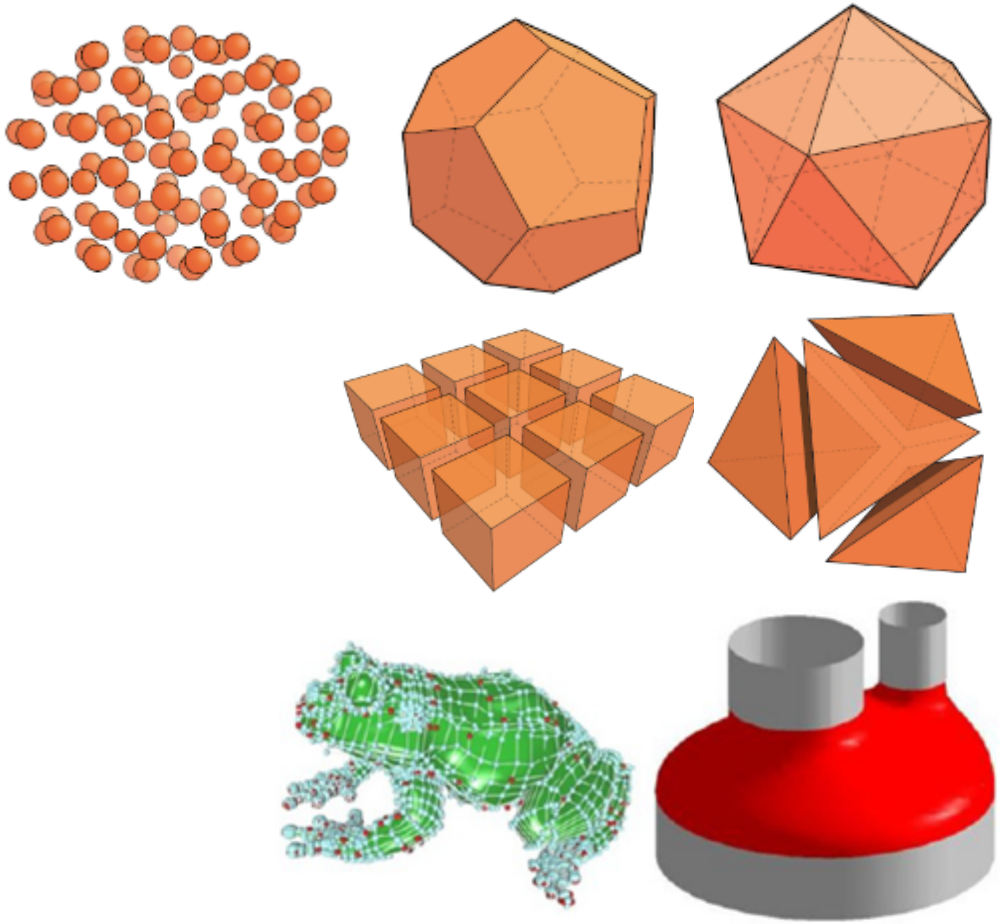


Examples of geometry

Examples of geometry



Many ways to digitally encode geometry



- EXPLICIT
  - point cloud
  - polygon mesh
  - subdivision, NURBS
  - L-systems
  - ...
- IMPLICIT
  - level set
  - algebraic surface
  - ...
- **Each choice best suited to a different task/type of geometry**

What do we need from shapes in CG?

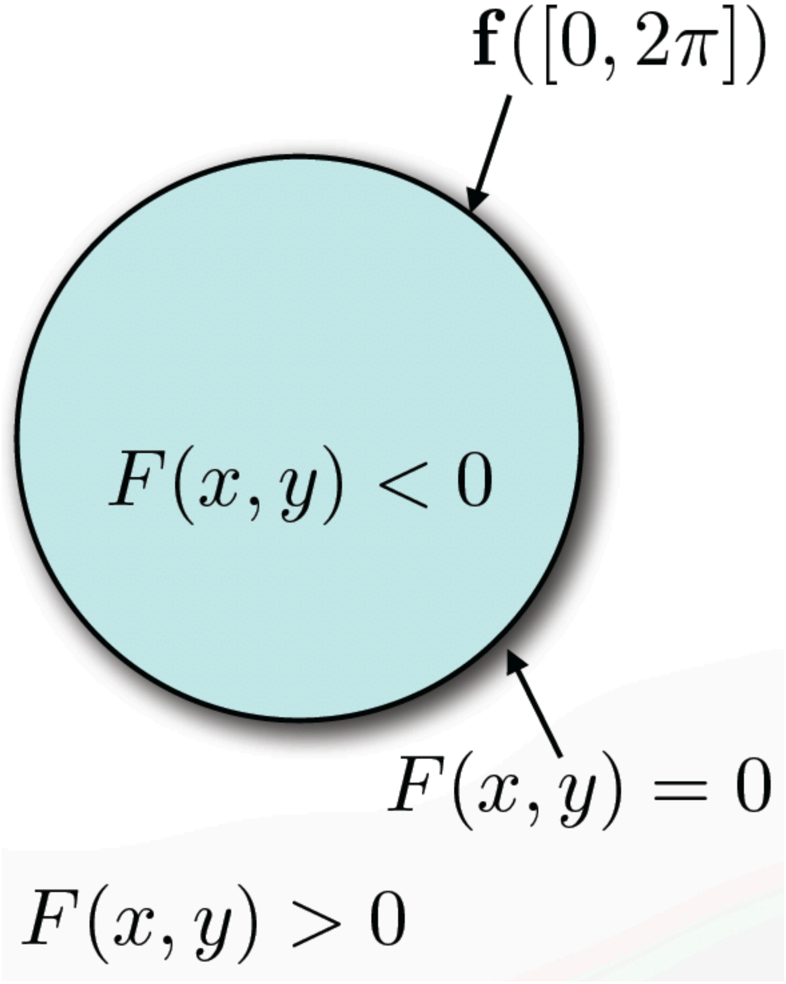
- Modeling
  - Model anything with arbitrary precision (in principle)
  - Easy to build and modify (local control)
  - Smoothness and continuity
  - Ability to evaluate derivatives
- Efficient computations (for rendering, collisions, etc.)
- Easy to implement (a minor consideration...)

- **No single technique solves all problems!**

“Implicit” Representations of Geometry

- Points aren’t known directly, but satisfy some relationship
  - E.g., unit sphere is all points x such that **x^2+y^2+z^2=1**
  - More generally, **f(x,y,z) = 0**
- Represent a surface as the zero set of a (regular) function defined in R^3.

$$K = g^{-1}(0) = \{\mathbf{p} \in \mathrm{R}^3 : g(\mathbf{p}) = 0\}$$



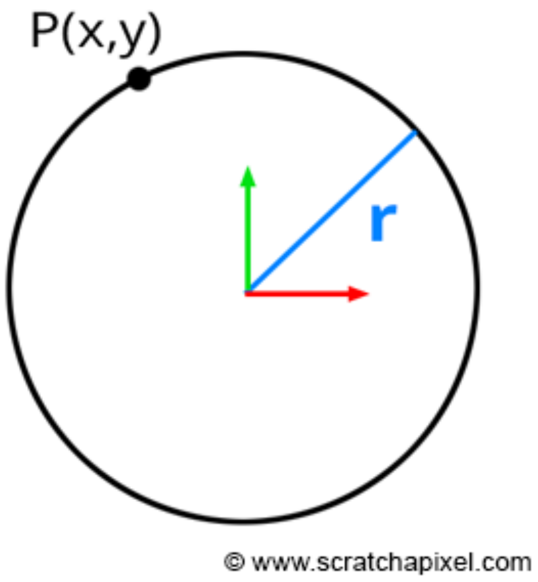
Let’s play a game:

I’m thinking of an implicit surface $f(x,y,z)=0$.

Find *any* point on it.

Give up?

- My function was $g(x,y,z) = x^2 + y^2 + z^2 - r^2$



- Implicit surfaces make some tasks hard (like sampling).

Let's play another game.

I have a surface $f(x,y,z) = x^2 + y^2 + z^2 - 1$

I want to see if a point is inside it.

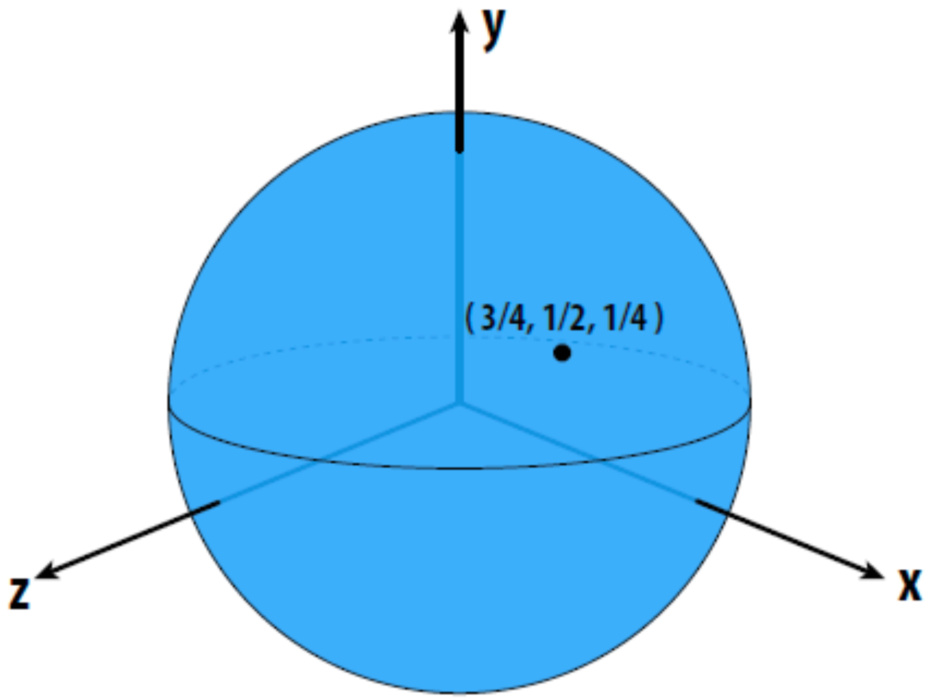
Check if this point is inside the unit sphere

How about the point (3/4, 1/2, 1/4)?

9/16 + 4/16 + 1/16 = 7/8

7/8 < 1

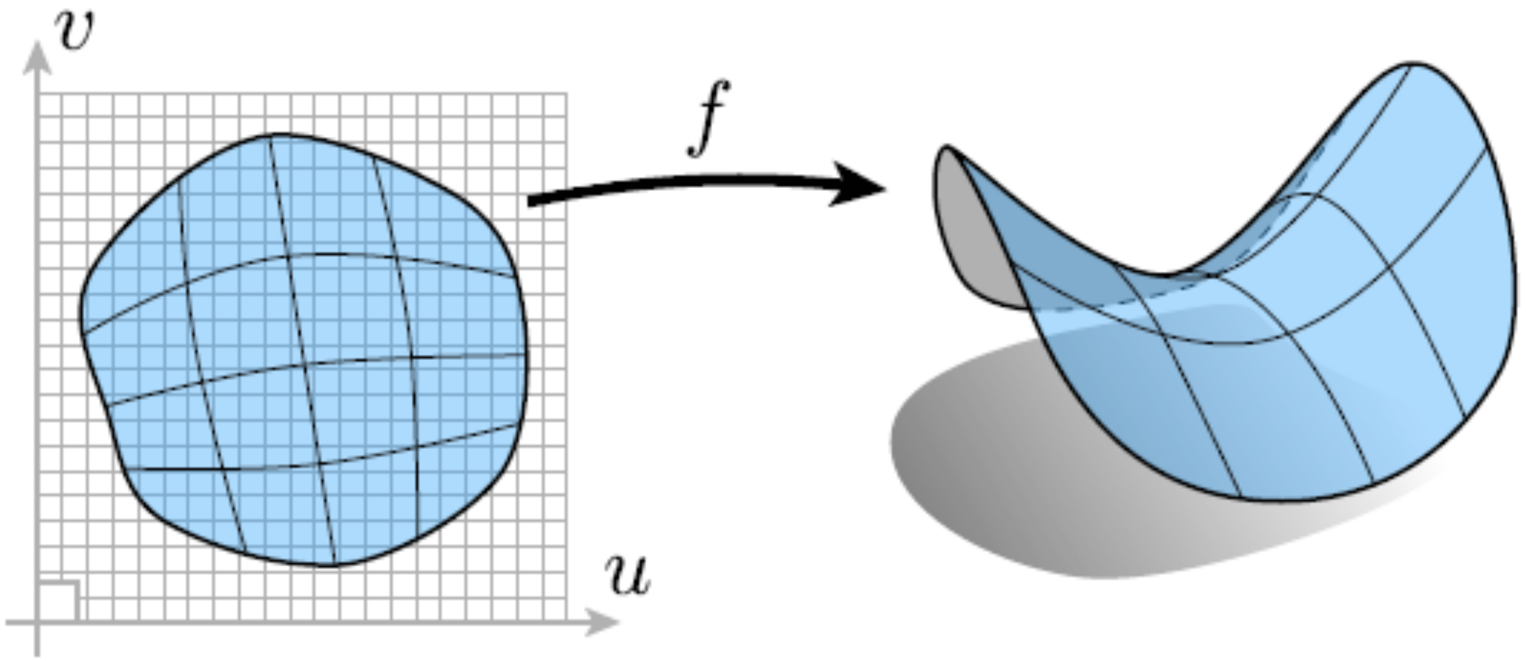
YES.



Implicit surfaces make other tasks easy (like inside/outside tests).

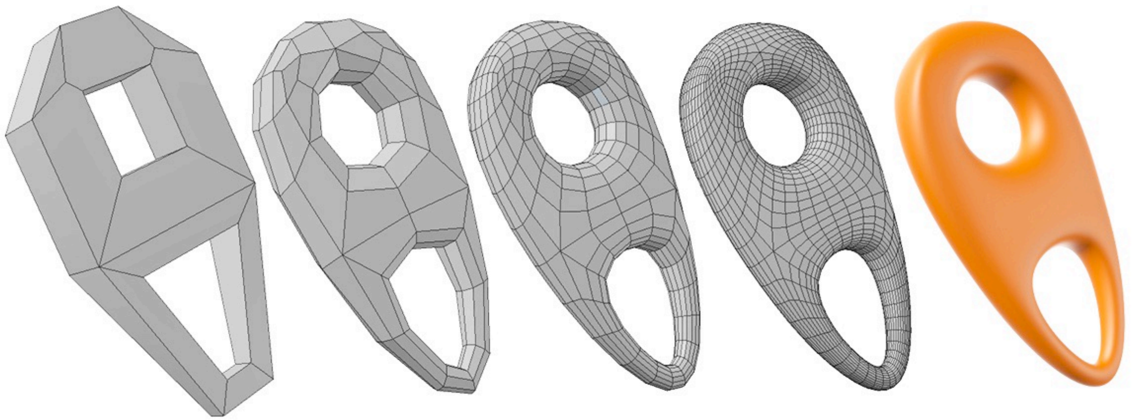
"Explicit" Representations of Geometry

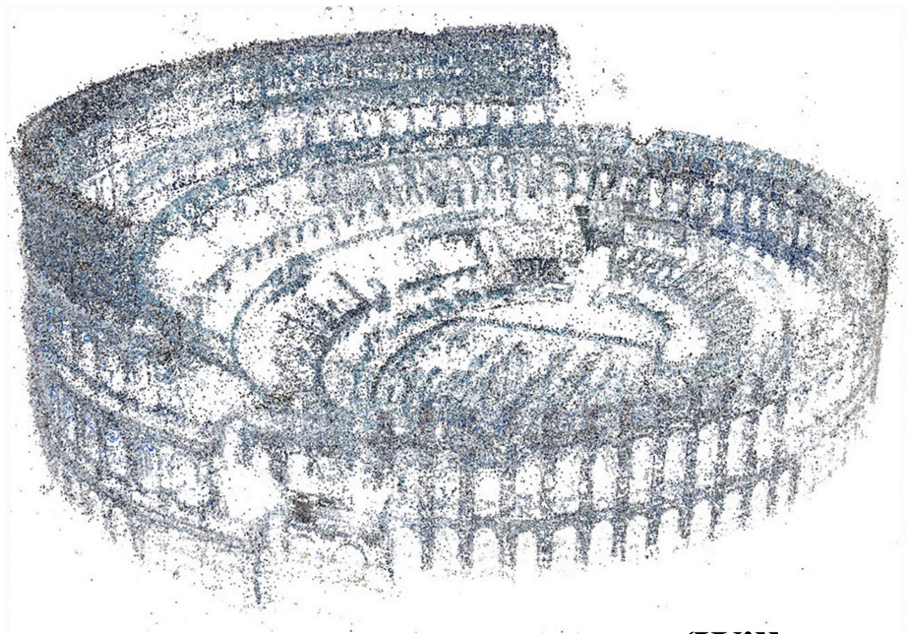
- **All points are given directly**
- **E.g., points on sphere are** $(\cos(u)\sin(v), \sin(u)\sin(v), \cos(v)),$
  $\text{for } 0 \leq u < 2\pi \text{ and } 0 \leq v \leq \pi$
- **More generally:** $f : \mathbb{R}^2 \to \mathbb{R}^3; (u, v) \mapsto (x, y, z)$

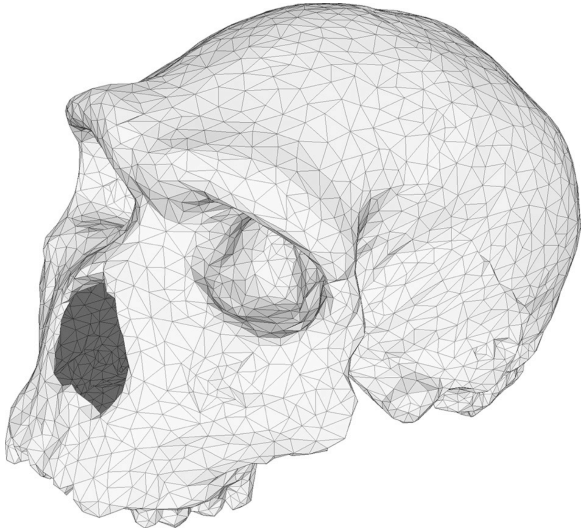


Many explicit representations in graphics

- triangle meshes
- polygon meshes
- subdivision surfaces
- NURBS
- point clouds
- ...







(Will see some of these a bit later.)

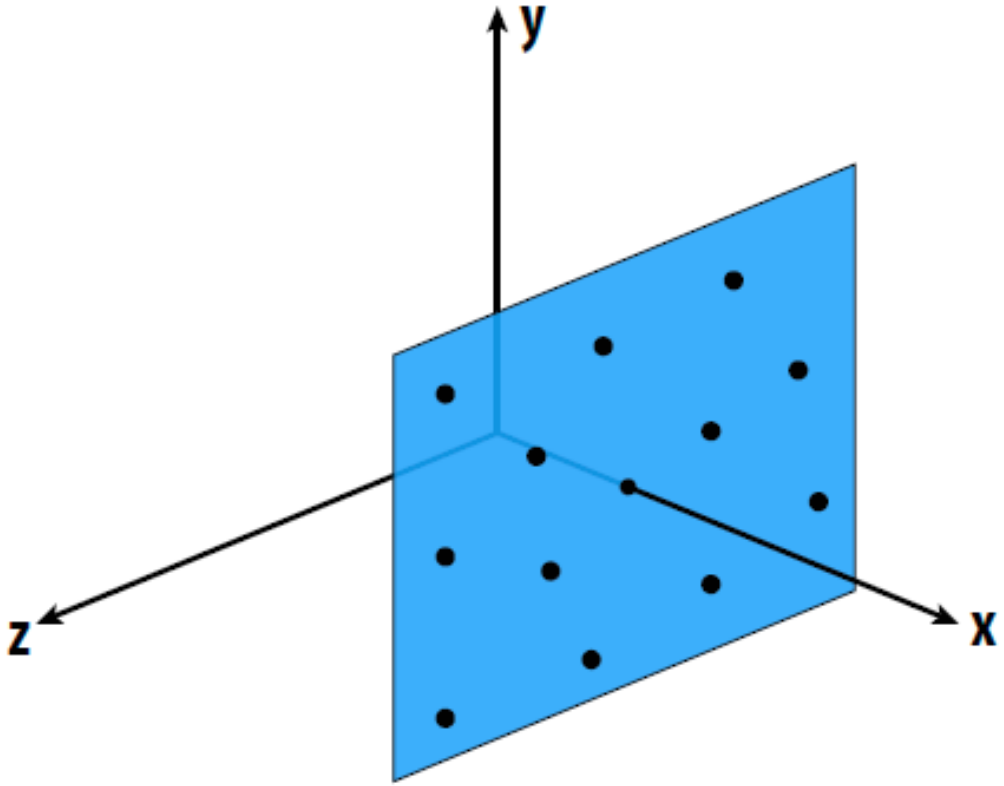
But first, let’s play a game:

I’ll give you an explicit surface.

You give me some points on it.

Sampling an explicit surface

- My surface is f(u, v) = (1.23, u, v).
- Just plug in any values (u,v)!



- Explicit surfaces make some tasks easy (like sampling).

Let’s play another game.

I have a new surface $f(u,v)$.

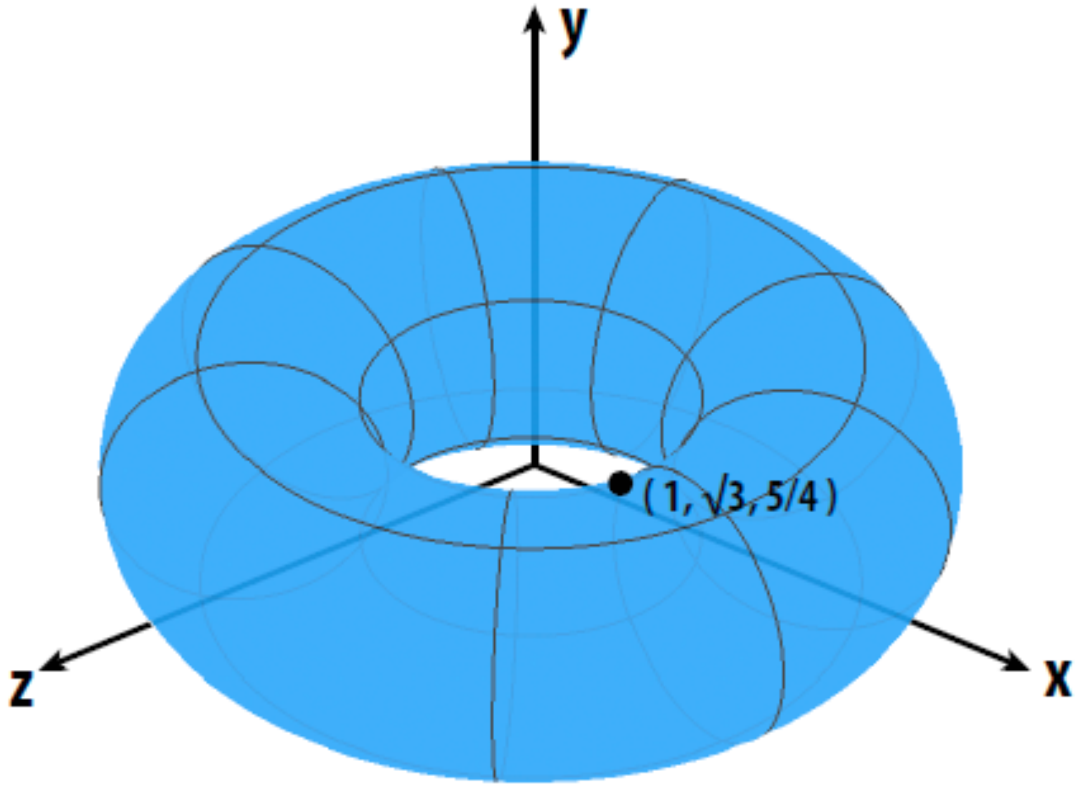
I want to see if a point is inside it.

Check if this point is inside the torus

My surface is f(u,v) = (2+cos(u))cos(v), 2+cos(u))sin(v), sin(u))

How about the point (1,√3,5/4)?

...NO!

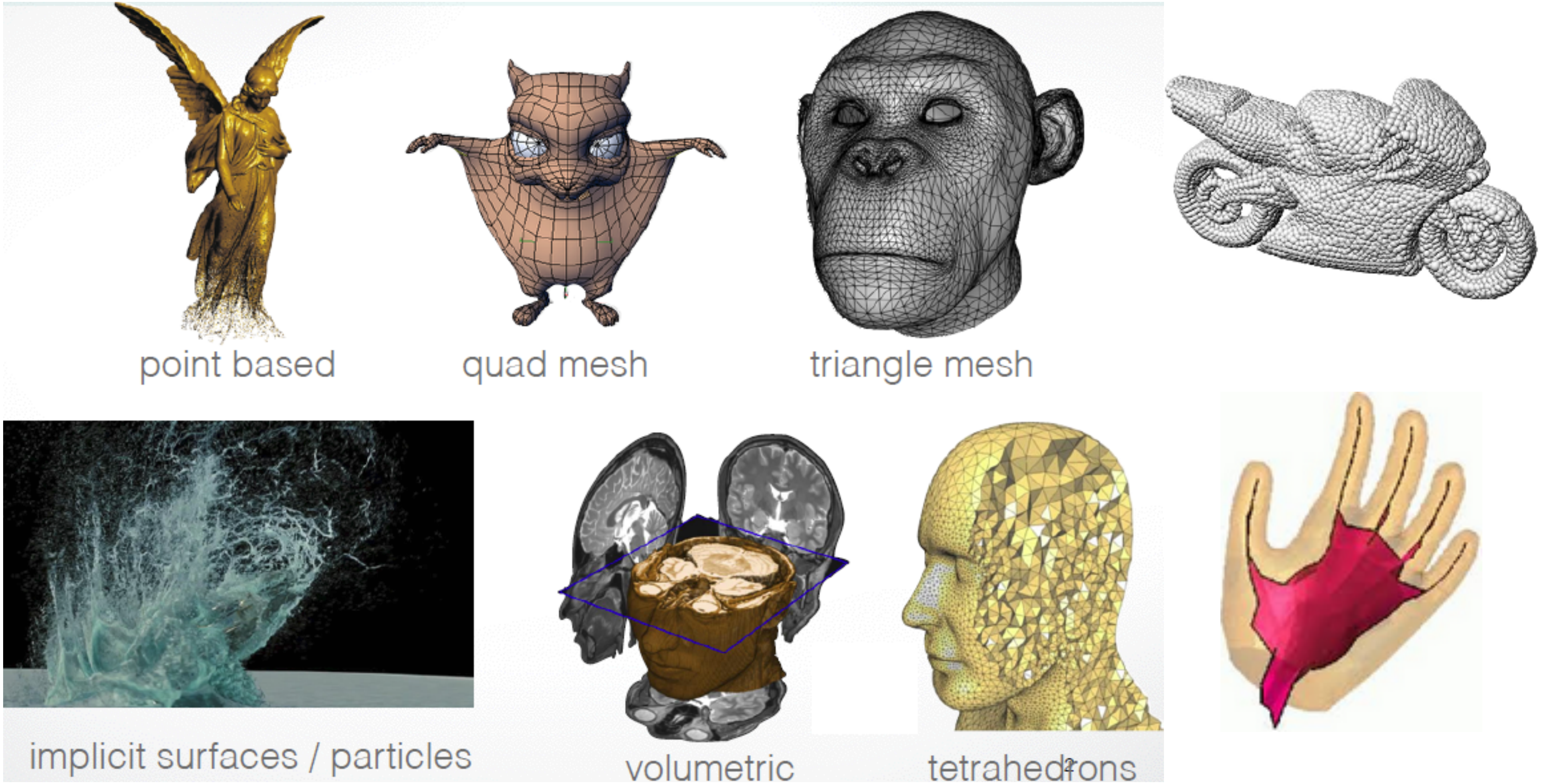


Explicit surfaces make other tasks hard (like inside/outside tests).

CONCLUSION:

Some representations work better than others—depends on the task!

Summary


point based
quad mesh
triangle mesh
implicit surfaces / particles
volumetric
tetrahedrons

What do we need from shapes in Computer Graphics?

- Local control of shape for modeling
- Ability to model what we need
- Smoothness and continuity
- Ability to evaluate derivatives
- Ability to do collision detection
- Ease of rendering

No single technique solves all problems!

Resources

- read & display a mesh: jjcao_plot/eg_trisurf.m
- Read & display a huge point set (100k to 1 million points)
  - PC_processing_1.0
  - jjcao_code/tools/pcd_viewer