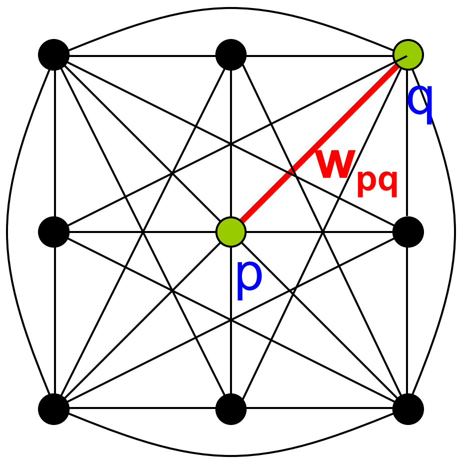
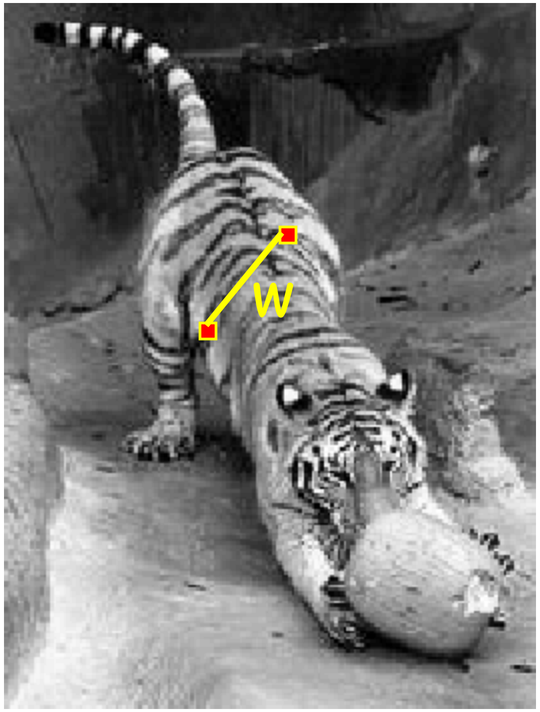
Segmentation, Saliency, Subspace clustering

Images as graphs



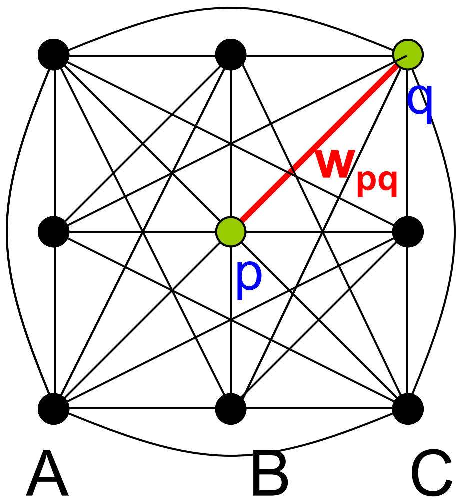


Fully-connected graph

- node (vertex) for every pixel
- link between every pair of pixels, p,q
- affinity weight w_{pq} for each link (edge)
  - w_{pq} measures *similarity*
    - similarity is *inversely proportional* to difference (in color and position…)

Illustration of the graph

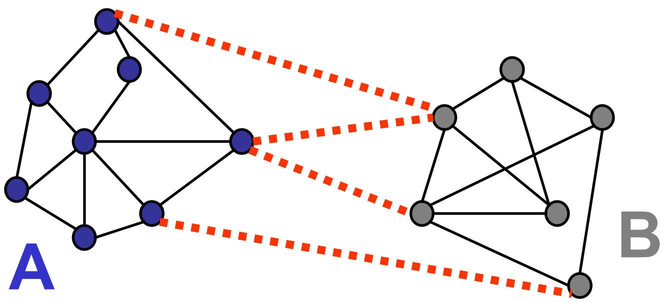
Segmentation by Graph Cuts



Break Graph into Segments

- Want to delete links that cross between segments
- Easiest to break links that have low similarity (low weight)
  - similar pixels should be in the same segments
  - dissimilar pixels should be in different segments

Cuts in a graph: Min cut



Link Cut

- set of links whose removal makes a graph disconnected
- cost of a cut:

$$cut(A,B) = \sum_{p \in A, q \in B} w_{p,q}$$

Find minimum cut

- gives you a segmentation
- fast algorithms exist for doing this

Minimum cut

- Problem with minimum cut:

  Weight of cut proportional to number of edges in the cut; tends to produce small, isolated components.

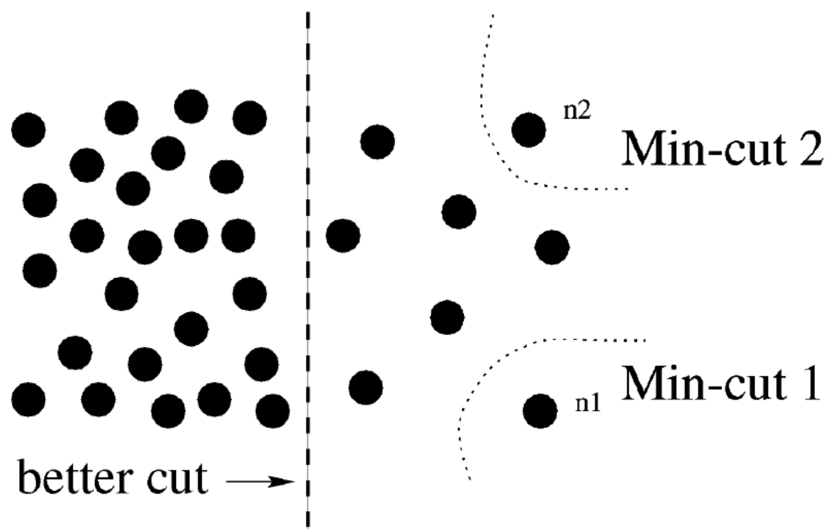
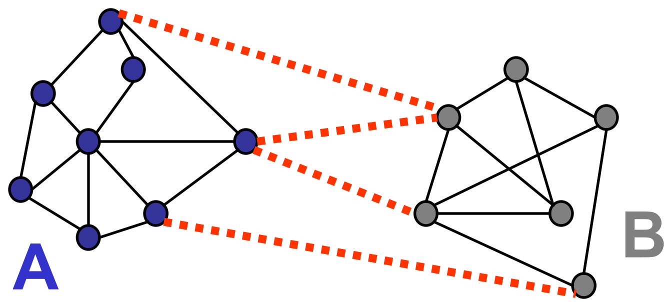


Fig. 1. A case where minimum cut gives a bad partition.

Cuts in a graph: Normalized cut



Normalized Cut

- fix bias of Min Cut by **normalizing** for size of segments:

$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

assoc(A,V) = sum of weights of all edges that touch A

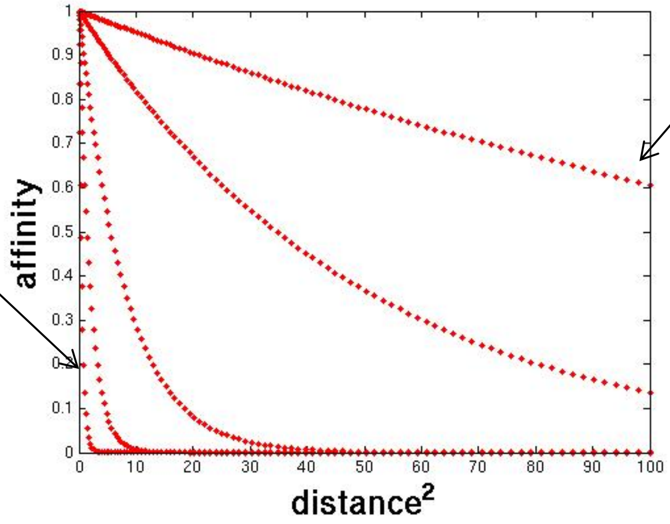
- Ncut value small when we get two clusters with many edges with high weights, and few edges of low weight between them
- Approximate solution for minimizing the Ncut value : generalized eigenvalue problem.

Measuring affinity

- One possibility:

$$aff(x,y) = \exp\left\{-\left(\frac{1}{2\sigma_d^2}\right)\left(\|x-y\|^2\right)\right\}$$

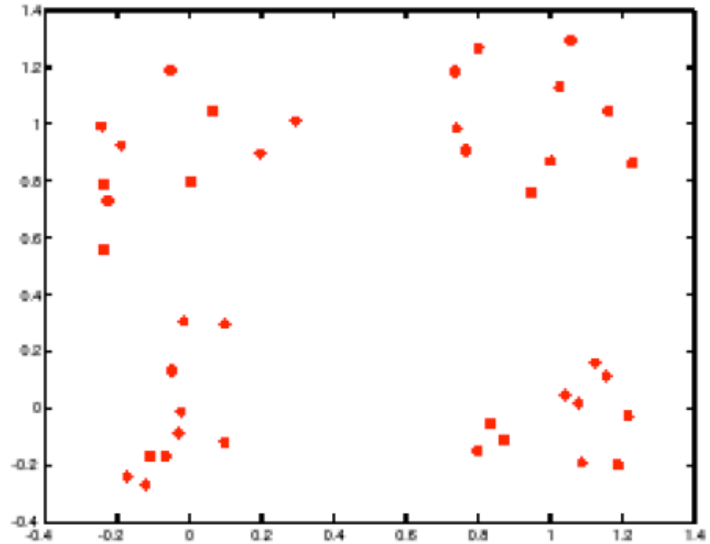
Small sigma: group only nearby points



Large sigma: group distant points

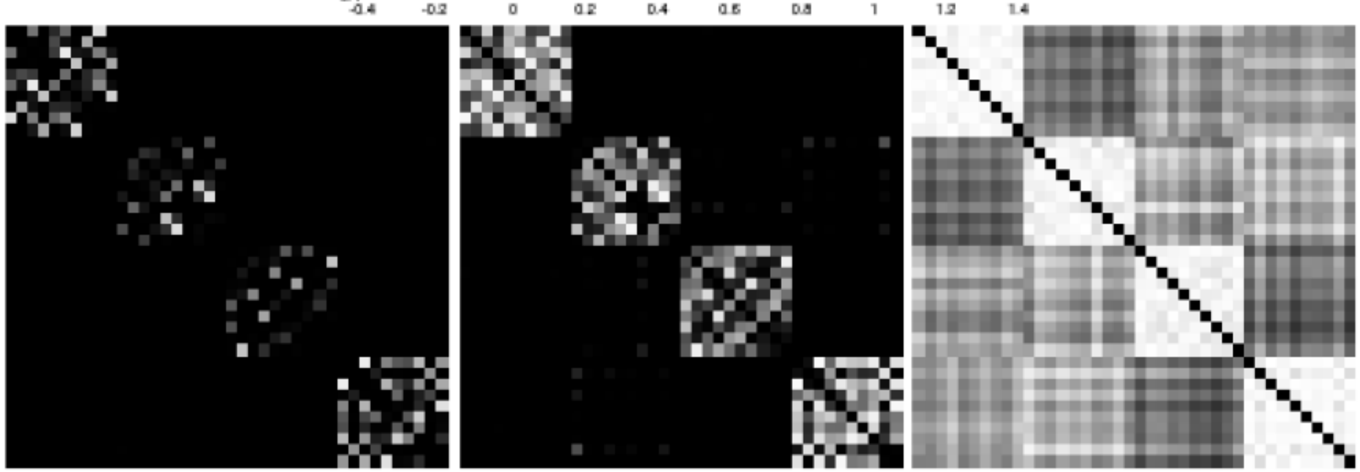
Measuring affinity

Data points



σ=.2

Affinity matrices



σ=.1 σ=.2 σ=1

Example results

Results: Berkeley

http://www.cs.berkeley.edu/~fowlkes/BSE/

Normalized cuts: pros and cons

Pros:

- Generic framework, flexible to choice of function that computes weights (“affinities”) between nodes
- Does not require model of the data distribution

Cons:

- Time complexity can be high
  - Dense, highly connected graphs → many affinity computations
  - Solving eigenvalue problem
- Preference for balanced partitions

Evaluation metric

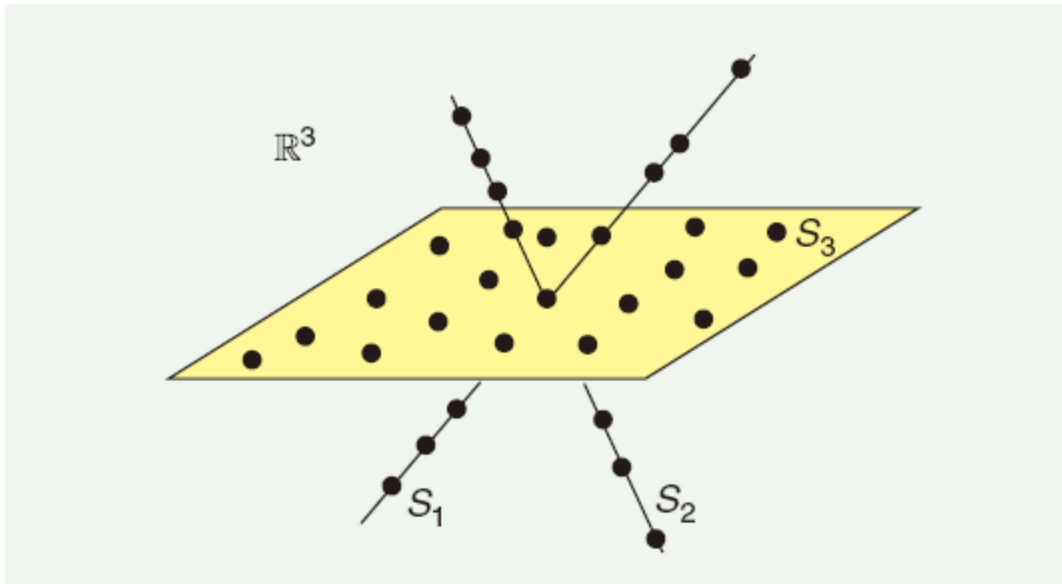
- ACC: accuracy
- NMI: normalized mutual information

- Purity
- ARI: adjusted rand index
- F-score
- Precision
- Recall

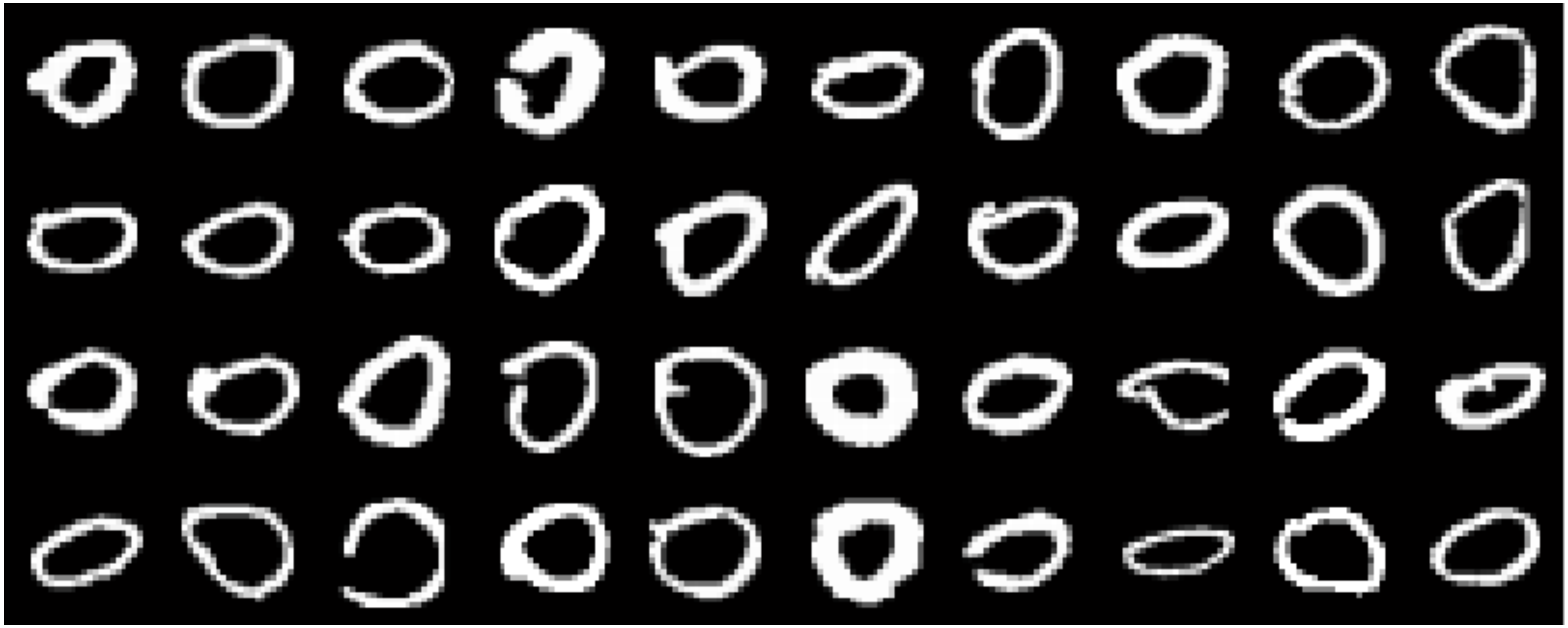
Evaluation metric

- result = ClusteringMeasure(Y, predY)
- ACC=result(1)
- NMI=result(2)
- Purity=result(3)
- [AR,RI,MI,HI]=RANDINDEX(c1,c2) returns the adjusted Rand index, the unadjusted Rand index, "Mirkin's" index and "Hubert's" index.
- [f,p,r] = compute_f(T,H)

Subspace segmentation



Many vision data have the structure of multiple subspaces.



Different transformations of a single digit are well approximated by a 7-dimensional manifold [1]. As illustrated by Hastie et. al. [2] these 7-dimensional manifolds are in turn well approximated by 12-dimensional subspaces.

[1]. P. Y. Simard, Y. LeCun and J. Denker. Eiffcient pattern recognition using a new transformation distance. In Advances in Neural Information Processing Systems, Morgan Kaufman, San Mateo, CA, pages 50-58, 1993.
[2] T. Hastie and P. Y. Simard. Metrics and Models for Handwritten Character Recognition. Statistical Science, 13, 1998

For a Lambertian object, it has been shown that the set of all images taken under all lighting conditions forms a cone in the image space, which can be well approximated by a low-dimensional subspace. [1]

[1] J. Ho, M. H. Yang, J. Lim, K.C. Lee, and D. Kriegman, "Clustering appearances of objects under varying illumination conditions," in Proc. IEEE Conf. Computer Vision and Pattern Recognition, 2003, pp. 11–18.

Motion segmentation

Data Analysis

Face recognition

Find single low dimensional subspace or manifold

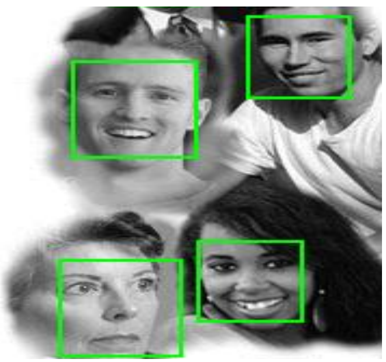
Dimensionality Reduction: PCA, LDA, LPP, MFA, etc

Manifold Learning: ISOMAP, LLE, LE, HE, LTSA, etc

Sparsity based methods: RPCA, Sparse Representation.

Data Analysis

Find single low dimensional subspace or manifold



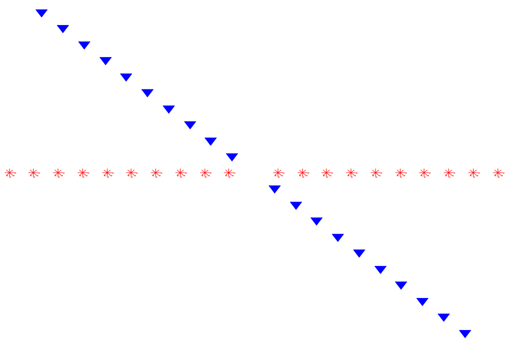
Find multiple low dimensional subspaces or manifolds

Subspace Segmentation: SSC, LRR, GPCA, RANSAC, etc

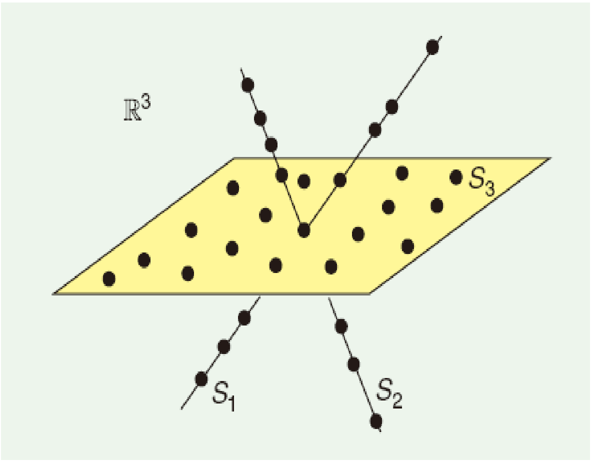
Manifold Clustering: SMMC, KSCC, KSSC, etc

Subspace segmentation

independent



disjoint



Input

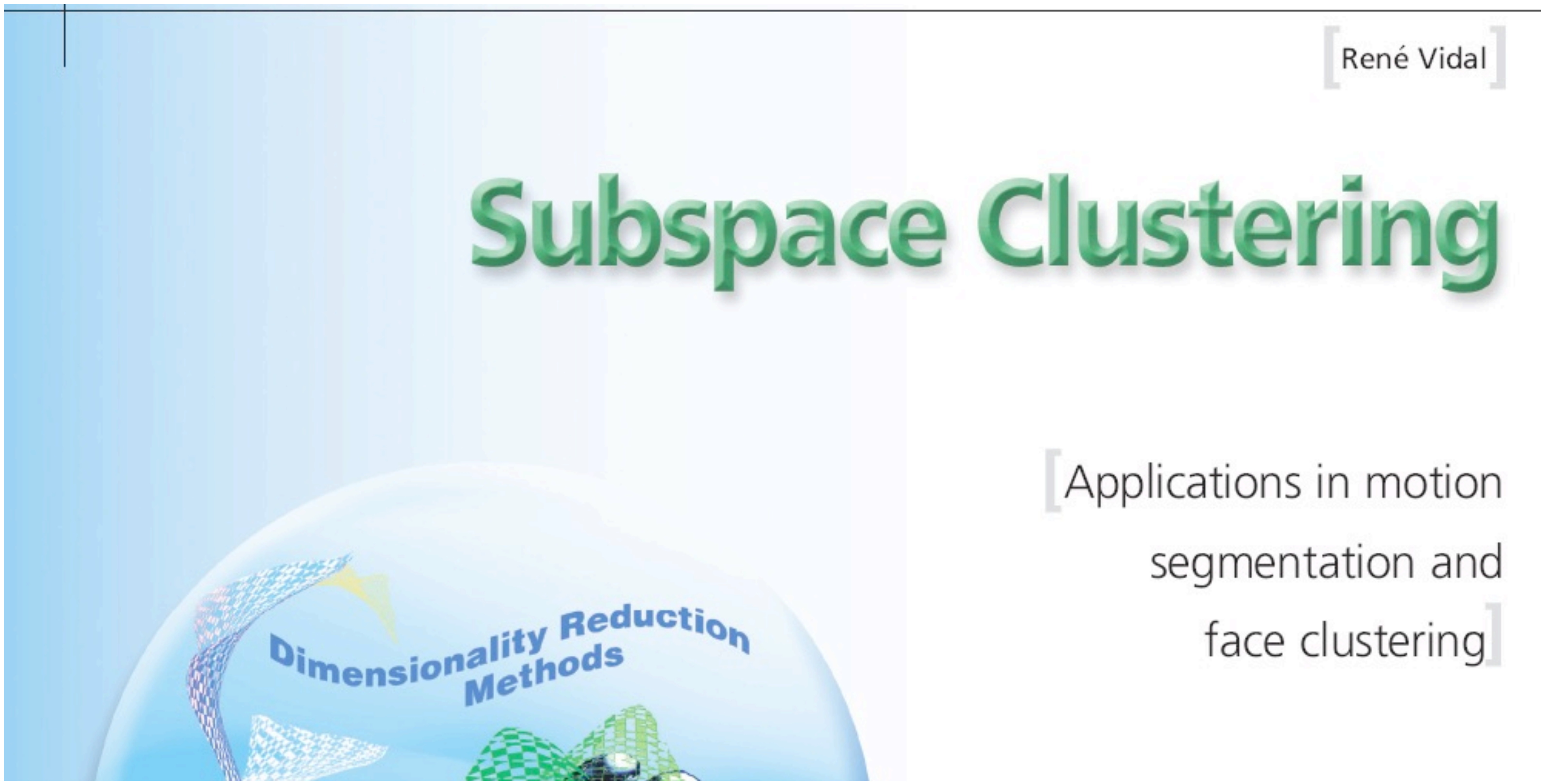
- Data
- The number of the subspaces/manifolds
- The dimension of each subspace/manifold

Learning

- Unsupervised： data is the only input.
- Supervised： both data and label are input.
- Semi-supervised： make use of both labeled data and unlabeled data.

Before LRR, SSC

- Rene Vidal



After LRR, SSC

- http://sunju.org/research/subspace-segment/

Subspace Segmentation

Geometric modeling of structured data with low-dimensional subspaces/manifolds, with applications in signal processing, robust control, and computational vision (segmentation). (**Update: June 23 2014**)

2014

1. Subspace clustering of dimensionality-reduced data
2. Learning Subspaces of Different Dimension
3. Robust Subspace Segmentation with Block-diagonal Prior (CVPR)
4. Smooth Representation Clustering (CVPR)

2013

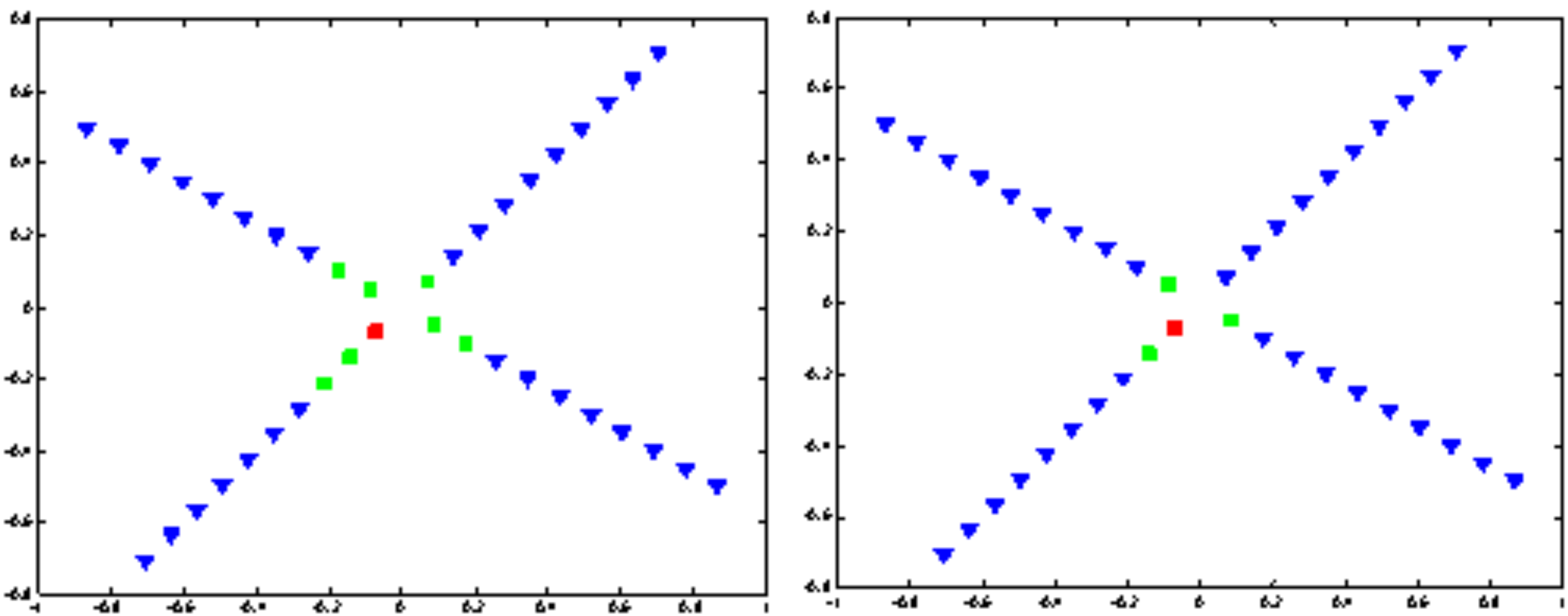
1. Greedy feature selection for subspace clustering (Manuscript)
2. Robust subspace clustering (Manuscript)
3. Noisy sparse subspace clustering (ICML)
4. Subspace clustering via thresholding and spectral clustering (ICCASP)

LSA

- Select neighbors
- Fit subspace
- Definite distance

Limitation of LSA

- The problem about the intersection

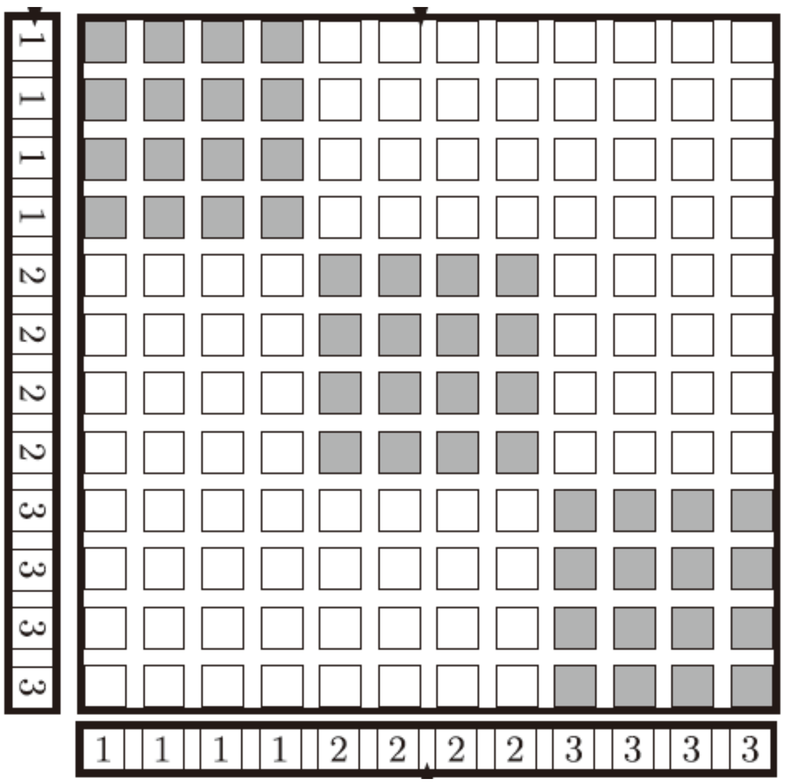


Spectral clustering-based methods

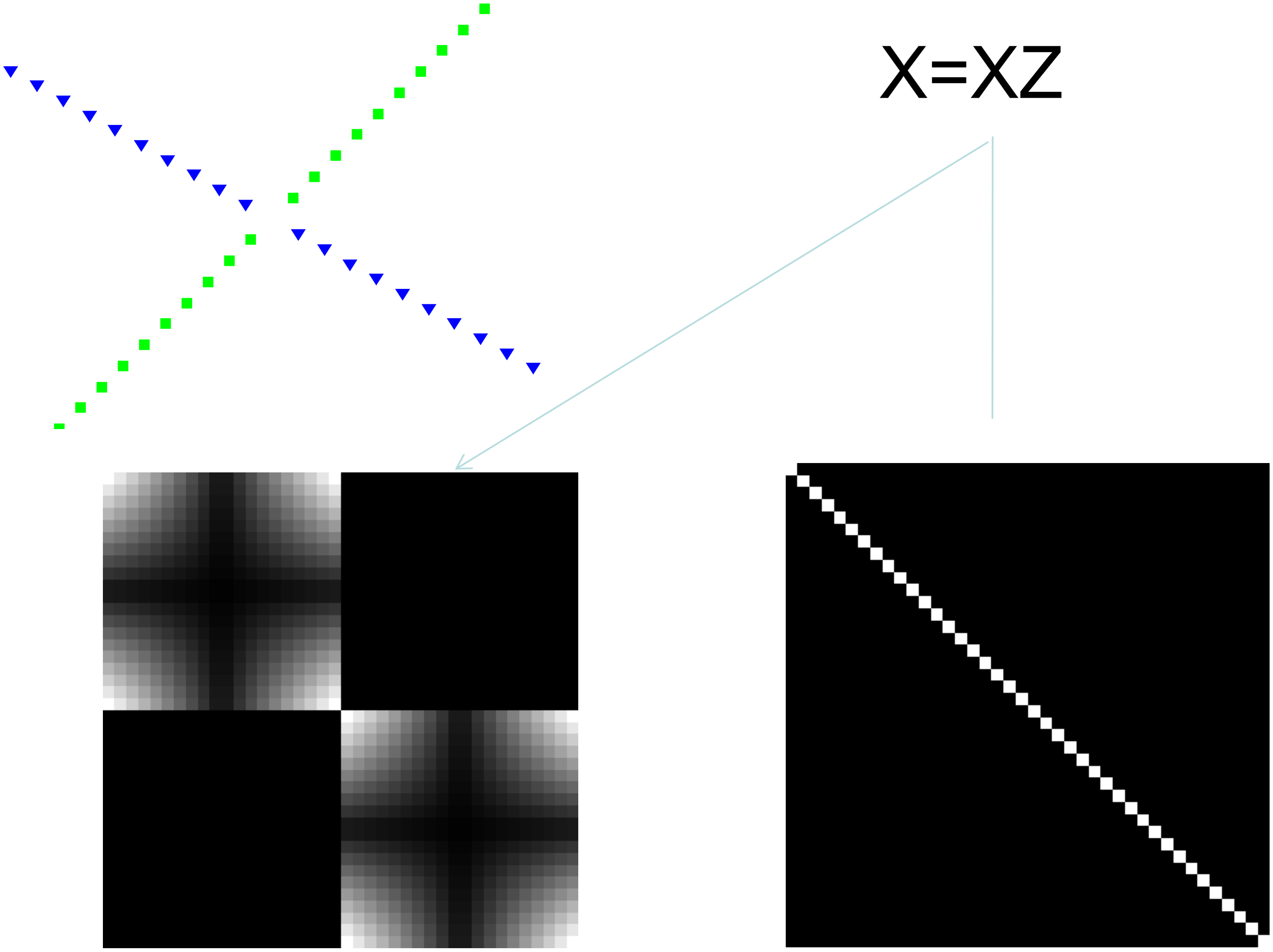
- Input: data, number of subspace
- ***Compute the affinity matrix***
- Spectral clustering

Affinity matrix

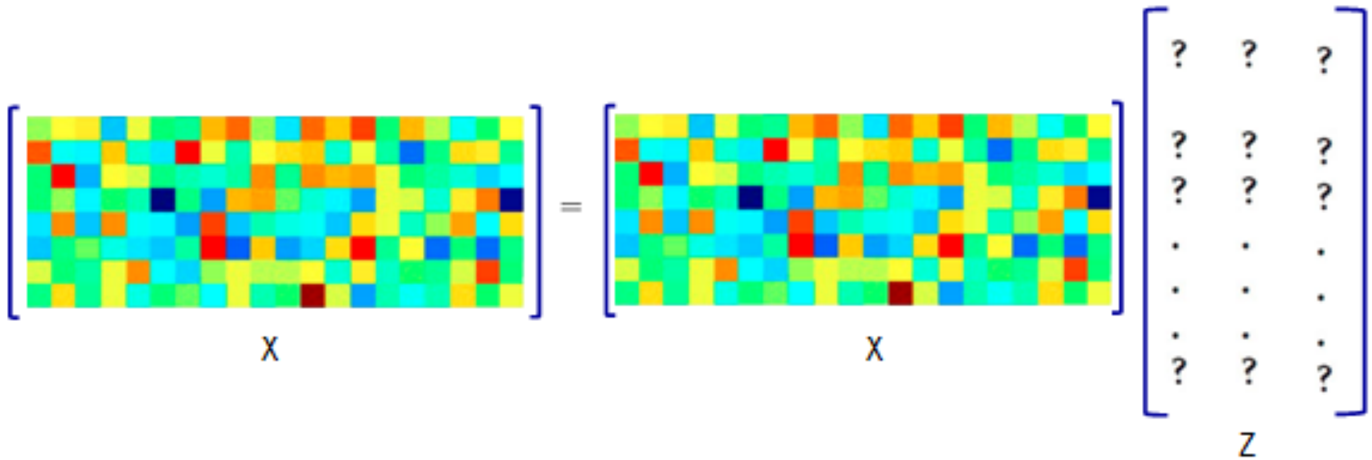
suppose that the data points are arranged to satisfy the true segmentation result



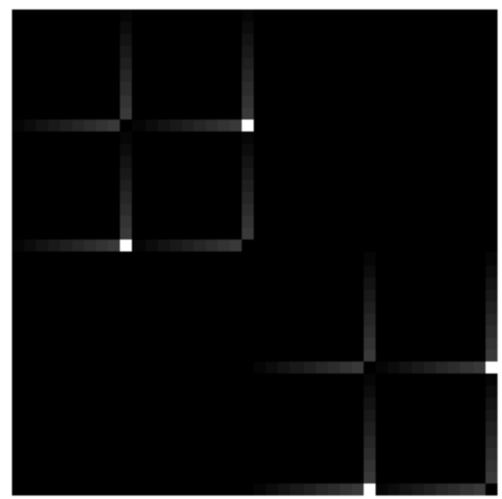
Affinity matrix



Sparse Subspace Clustering (SSC)

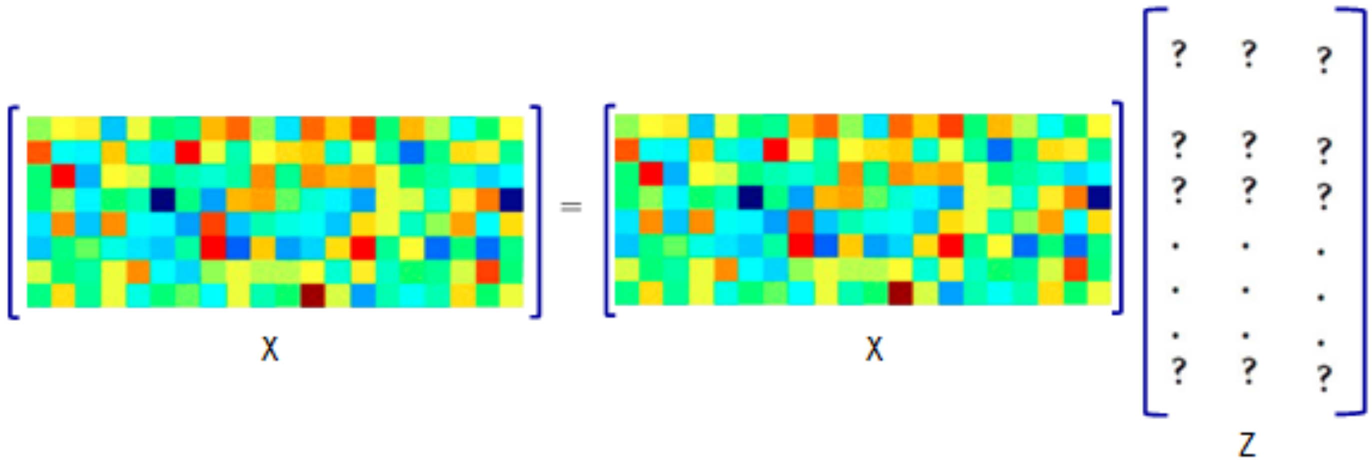


$$\min_{Z} \|Z\|_{1,1} \quad s.t. \quad X = XZ \quad diag(Z) = 0$$



B. Nasihatkon, R. Hartley, Graph Connectivity in Sparse Subspace Clustering, CVPR, 2011

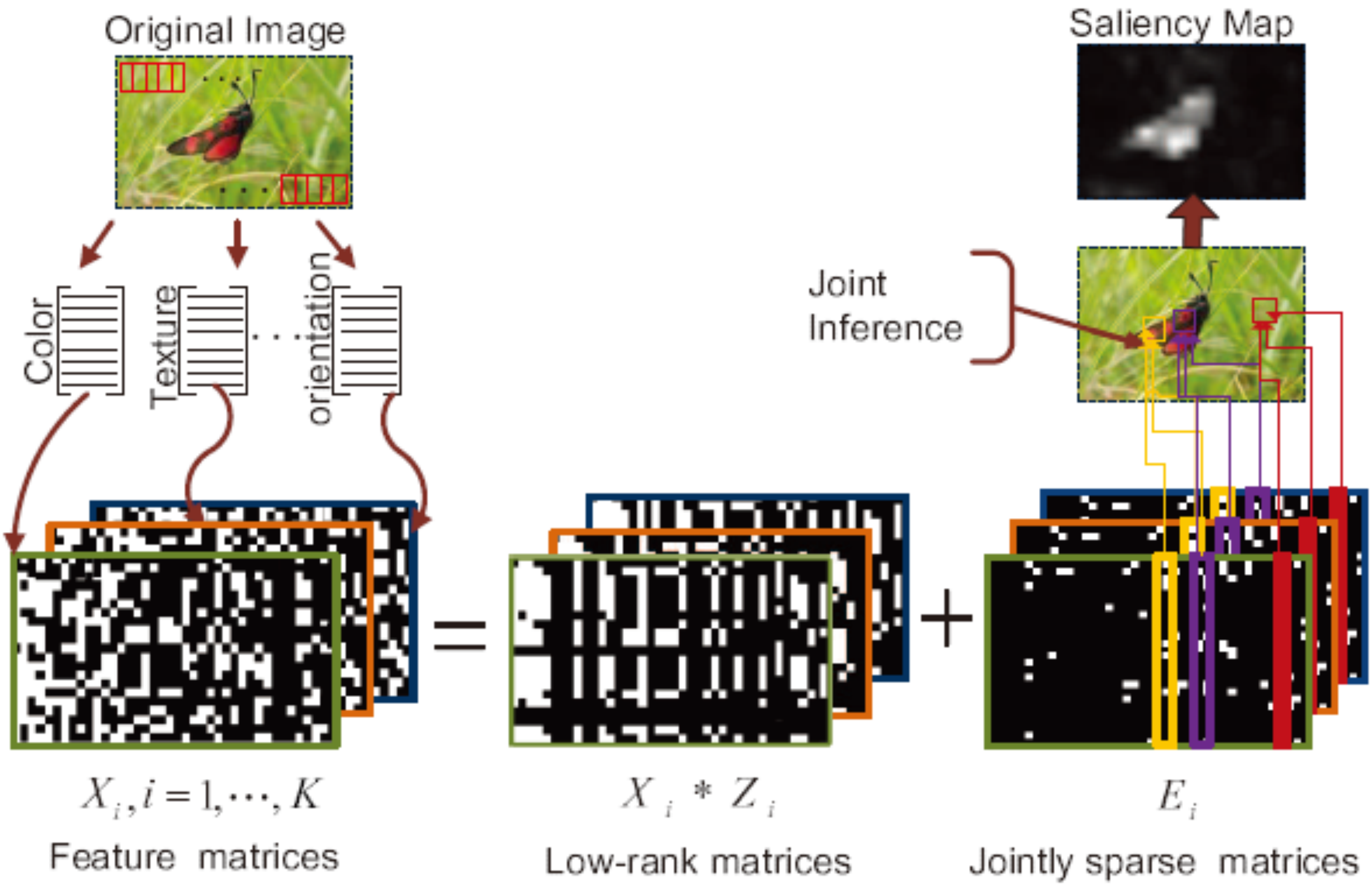
Low-Rank Representation (LRR)



$$\min_{Z} \|Z\|_* \quad s.t. \quad X = XZ$$

Denote the skinny SVD of X is $X = USV^T$, the closed form solution is VV^T

Saliency detection



Saliency detection

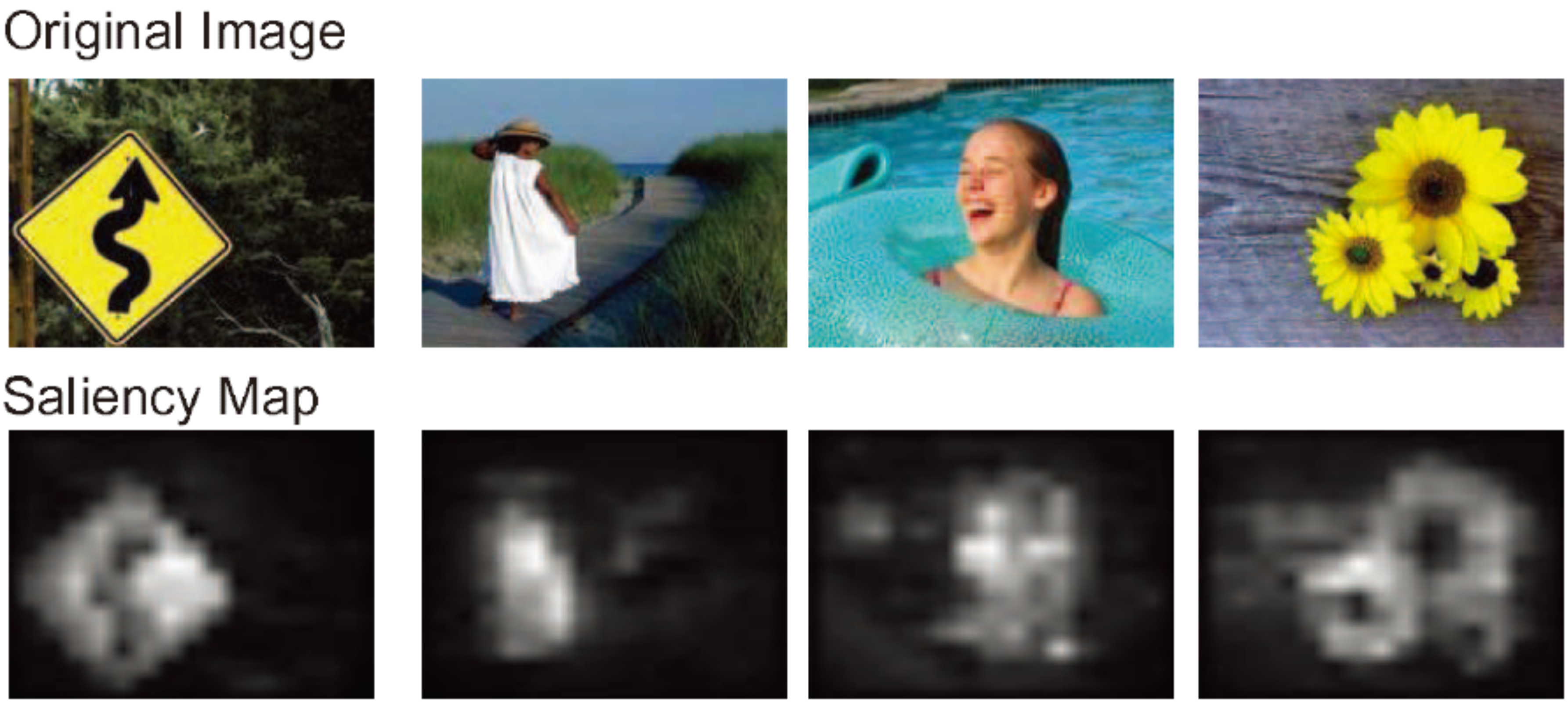
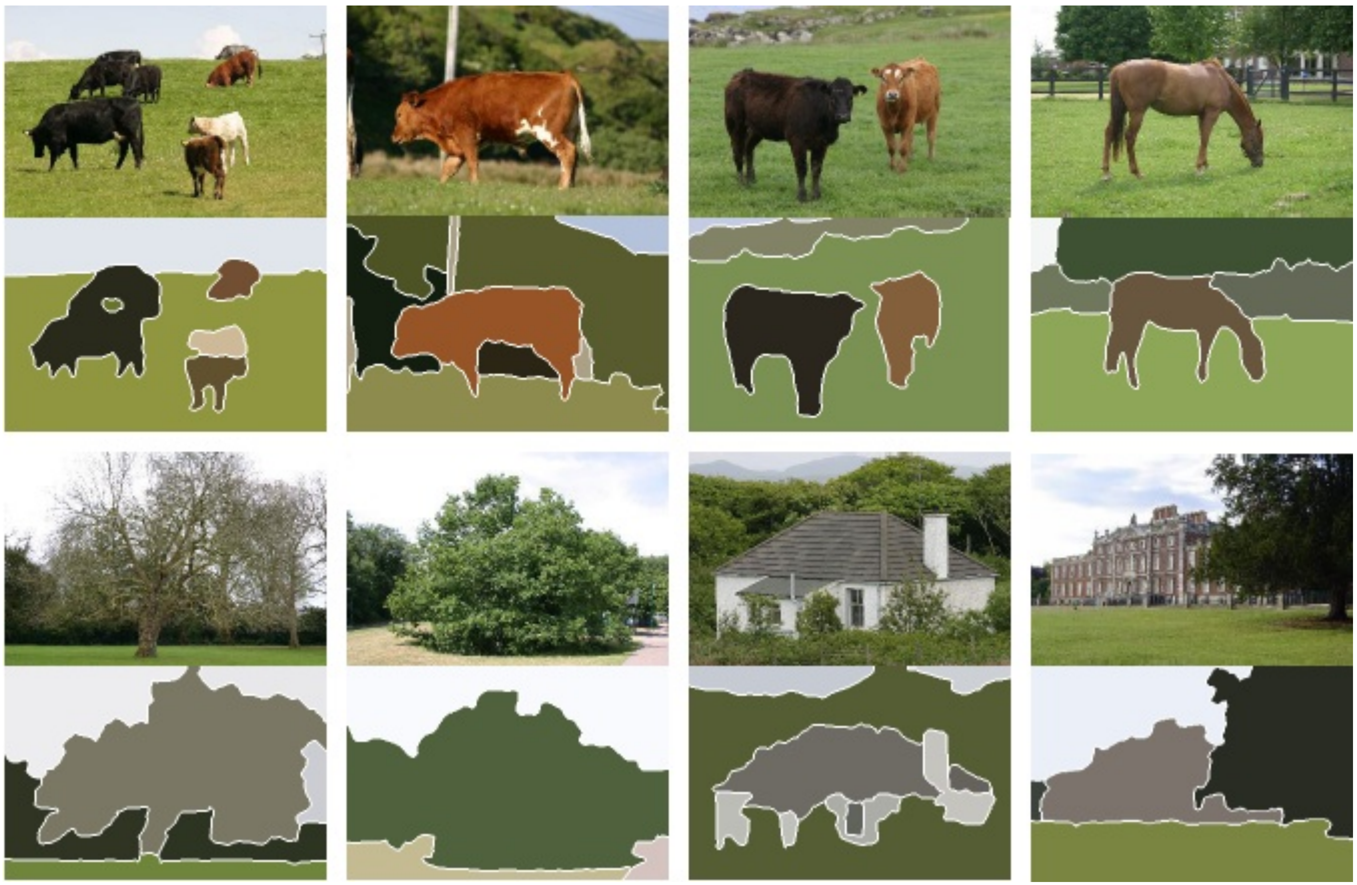


Image segmentation



homework

- 分别使用kmeans, normalized cut(用高斯核构造权图), lrr, ssc对所给两组数据集进行聚类，并用7个指标评价聚类结果。

$$aff(x,y)=\exp\left\{-\left(\frac{1}{2\sigma_d^2}\right)\left(\|x-y\|^2\right)\right\}$$

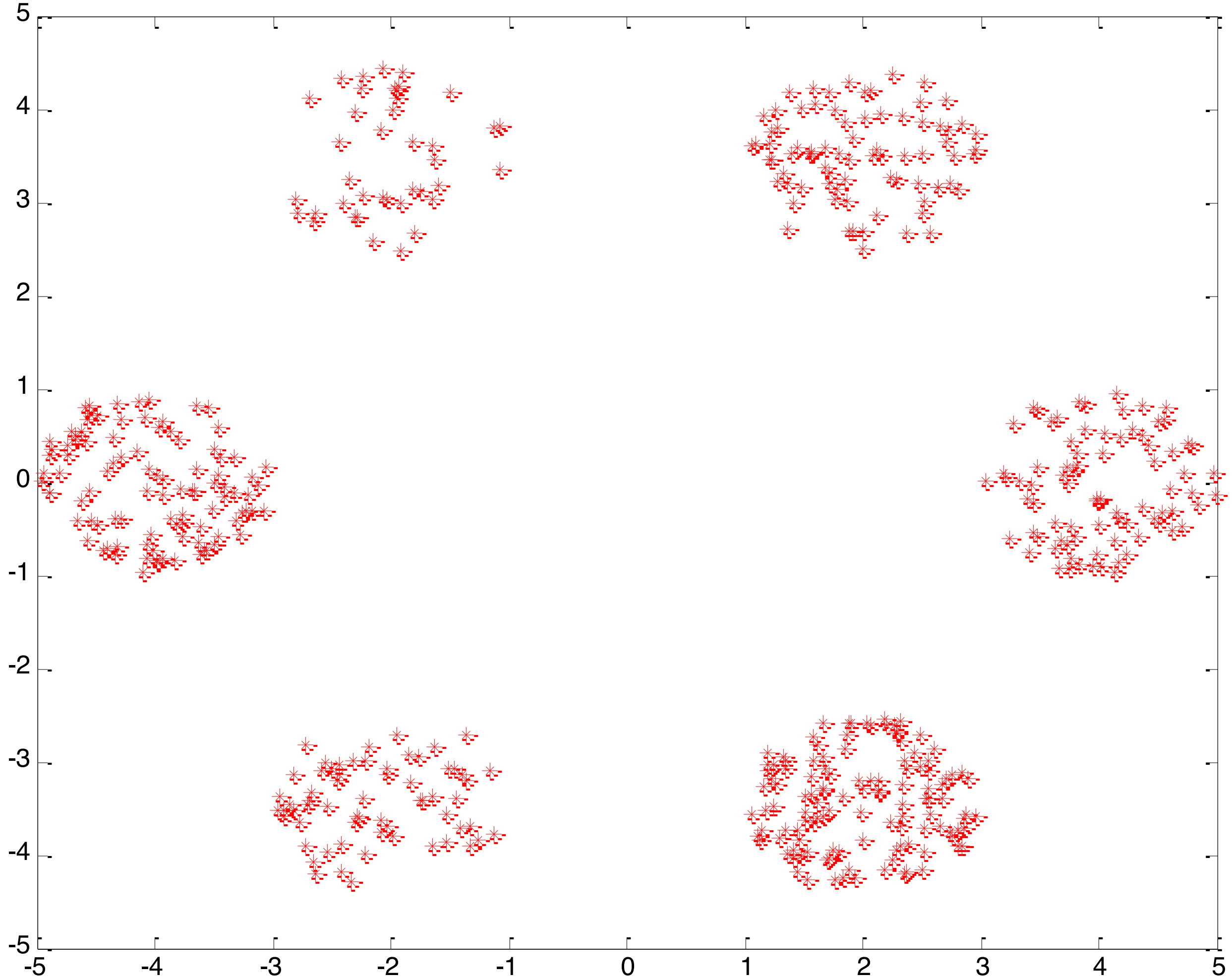
data1.mat (MAT File)

Name	Value
data	<2x479 double>
label	<1x479 double>

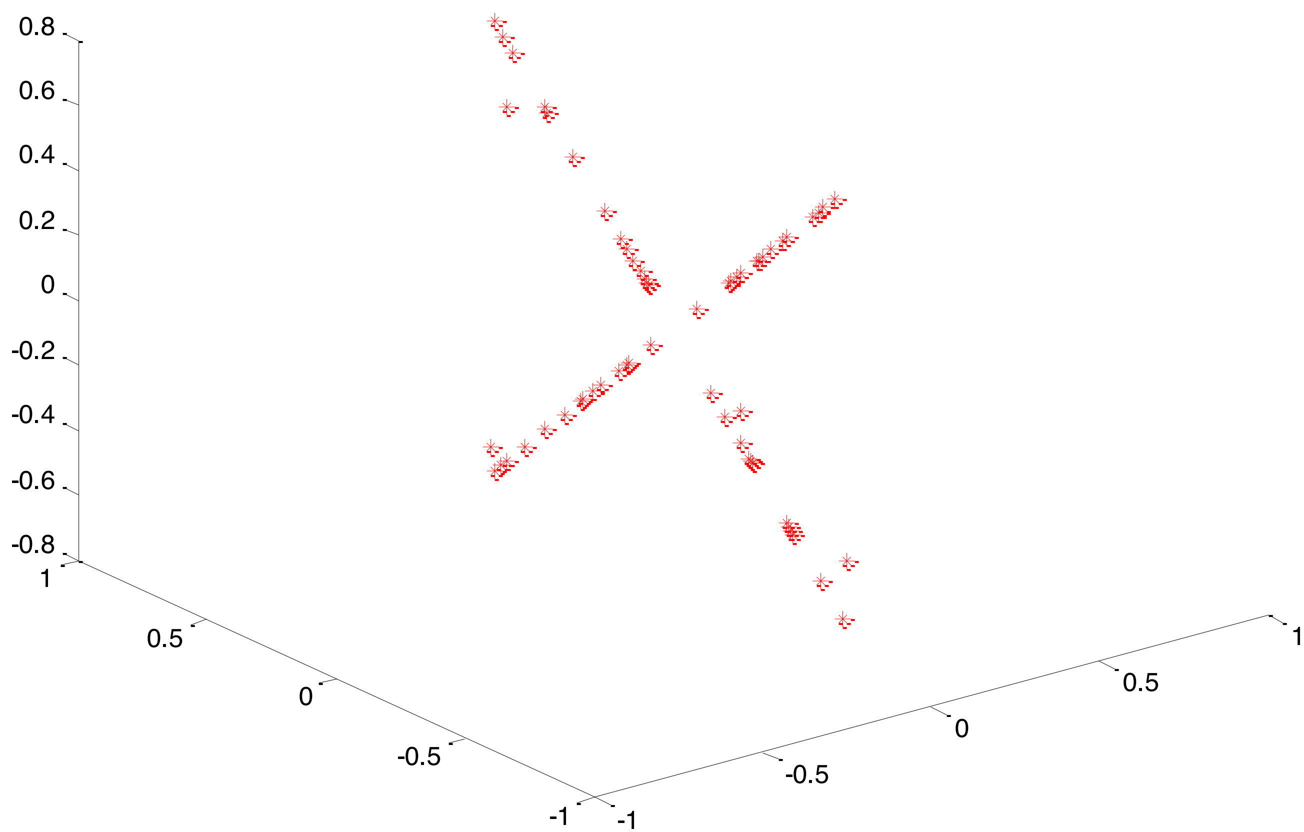
data2.mat (MAT File)

Name	Value
data	<3x60 double>
label	<60x1 double>

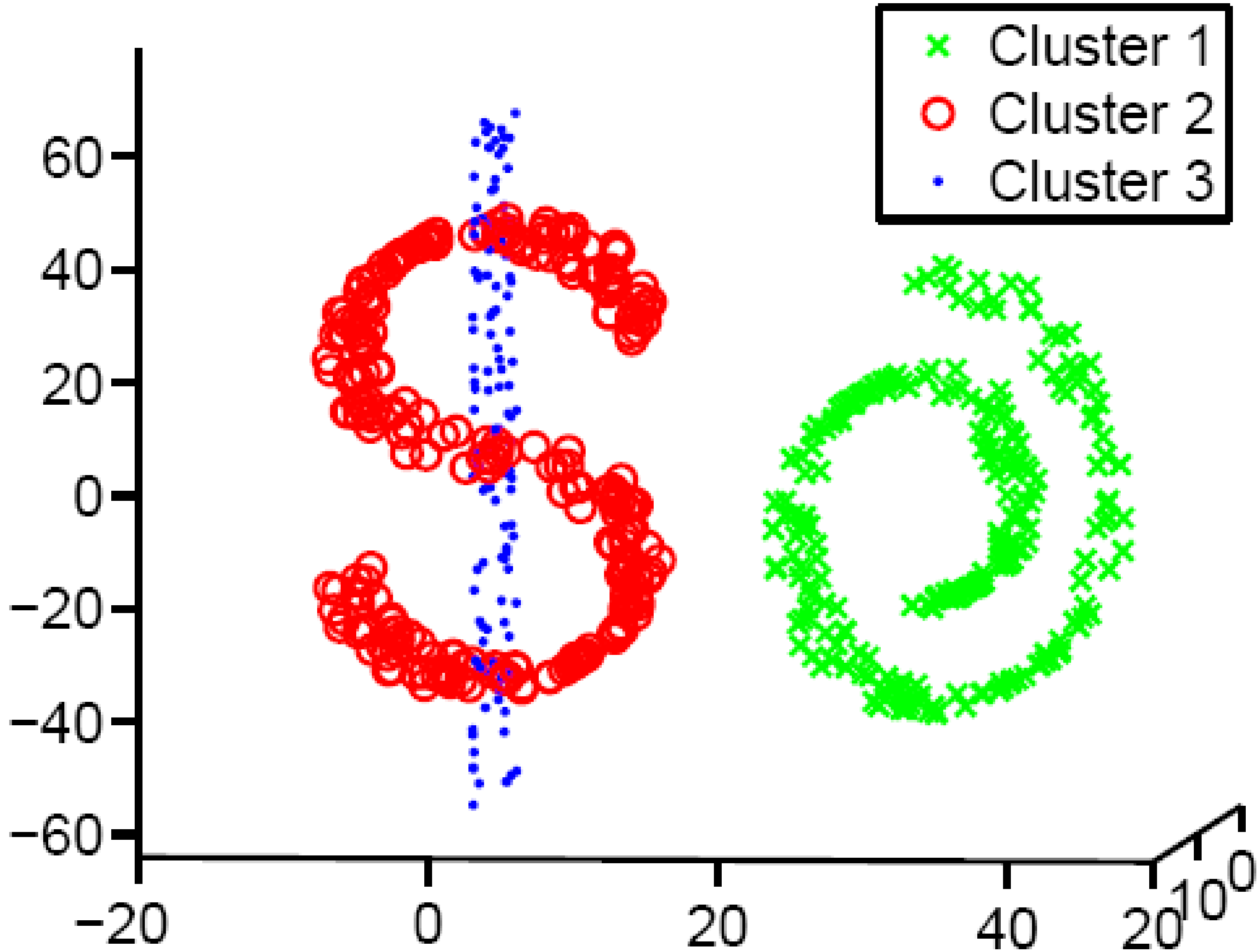
Data1



Data2



Manifold clustering



Idea for manifold clustering

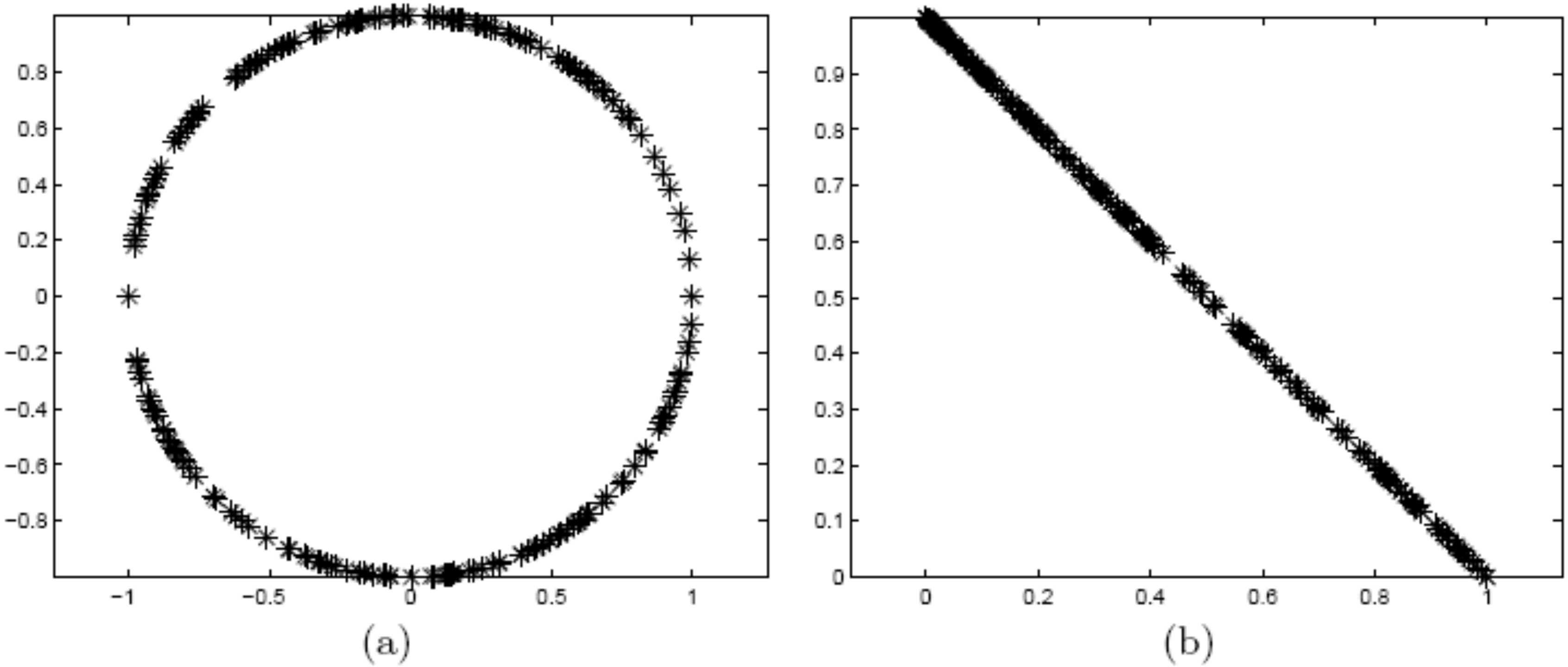


Fig. 1. The structure of the data in (a) is one circle. After the transformation $\phi(\mathbf{x}) = \|\mathbf{x}\|_2^2$, it becomes a line shown in (b).

E.g. Kernel SSC, Kernel LRR

Idea for manifold clustering

Kernel SSC

Vishal M. Patel, Rene Vidal, Kernel Sparse Subspace Clustering, ICIP, 2014

Ming Yin, Yi Guo, Junbin Gao, Zhaoshui He, Shengli Xie, Kernel Sparse Subspace Clustering on Symmetric Positive Definite Manifold, CVPR, 2016

Kernel LRR

Shijie Xiao, Mingkui Tan, Dong Xu, Zhaoyang Dong, Robust Kernel Low-Rank Representation, IEEE Transactions on Neural Networks and Learning Systems, 2016