

Digital Geometry - Surface Registration

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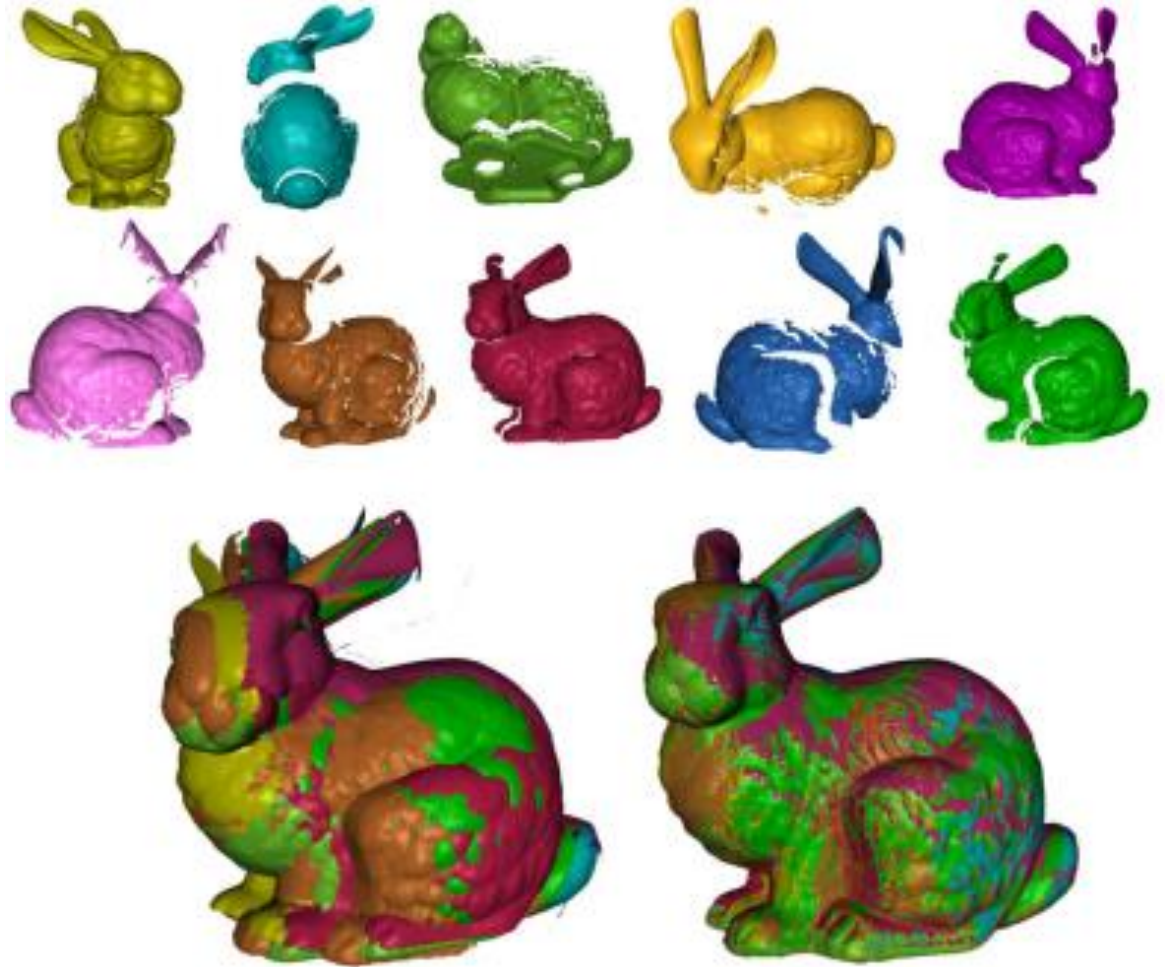
Spring 2018

<http://jjcao.github.io/DigitalGeometry/>

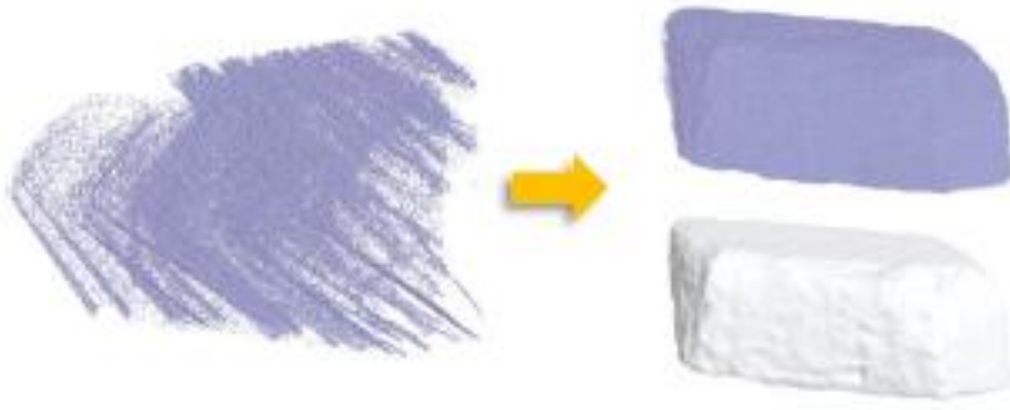
The purpose of computing is insight, not numbers

Definition

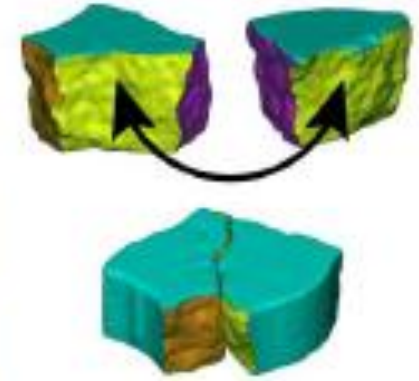
- Surface registration is the process of identifying and **matching corresponding regions**
- across **multiple scans** given in **arbitrary initial positions**,
- and estimating the corresponding **rigid transforms** that best align the scans to each other.



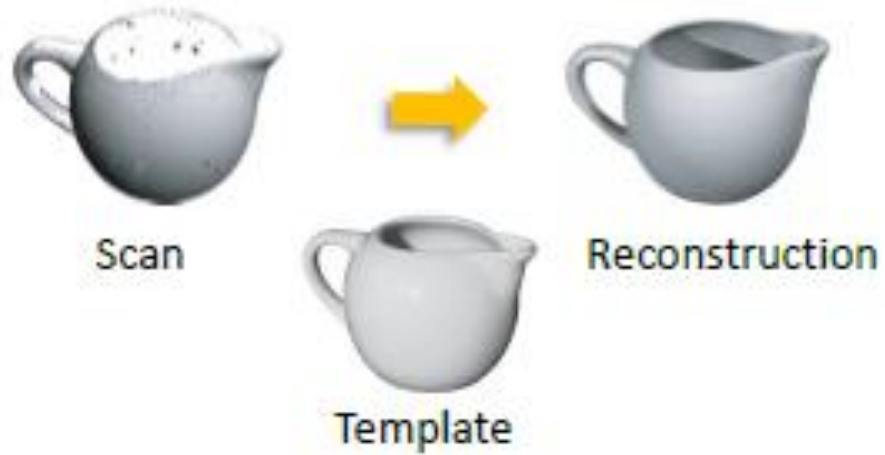
Applications



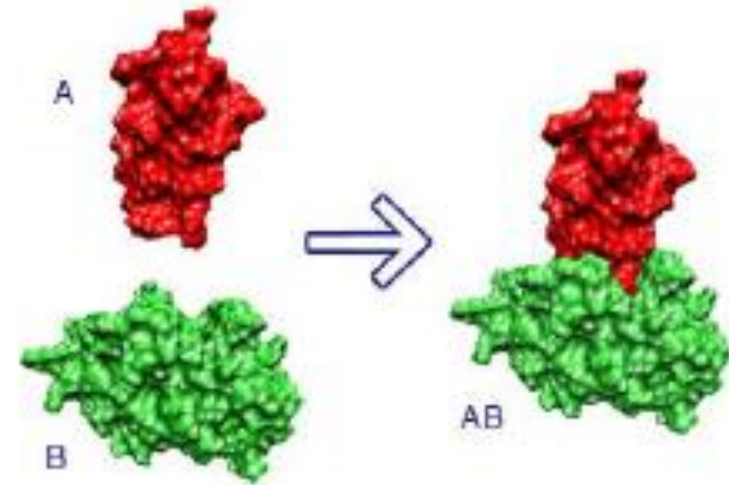
Surface reconstruction



Fragment assembly

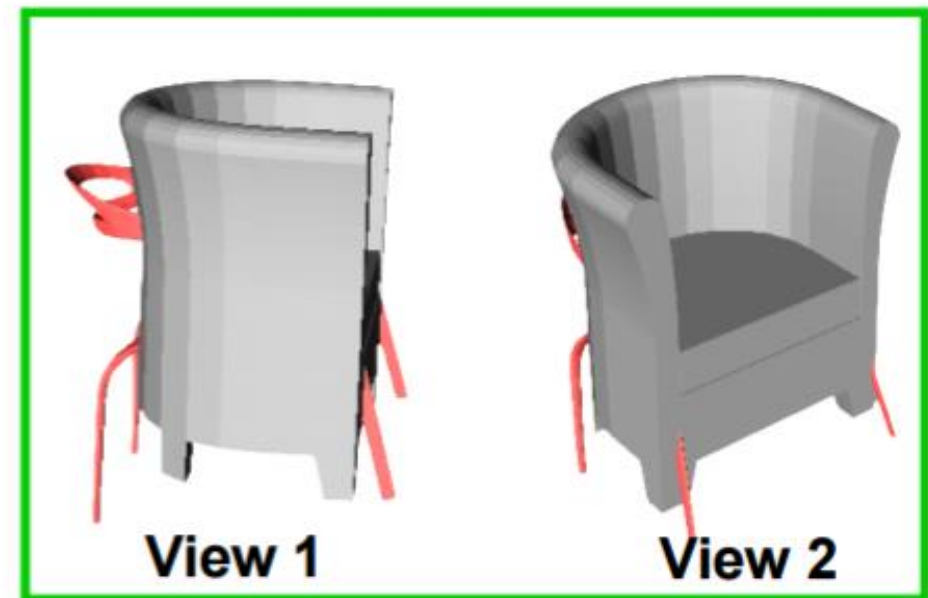
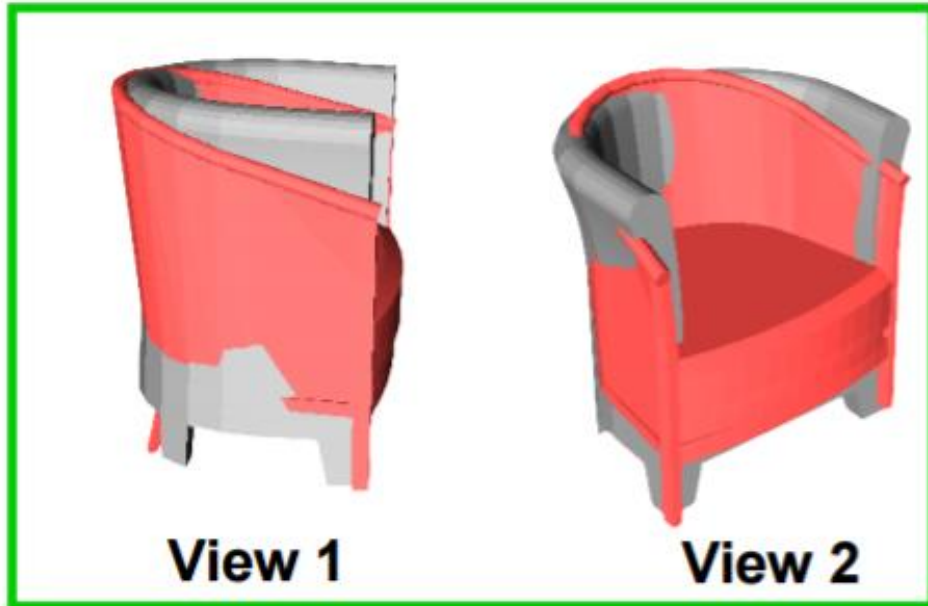
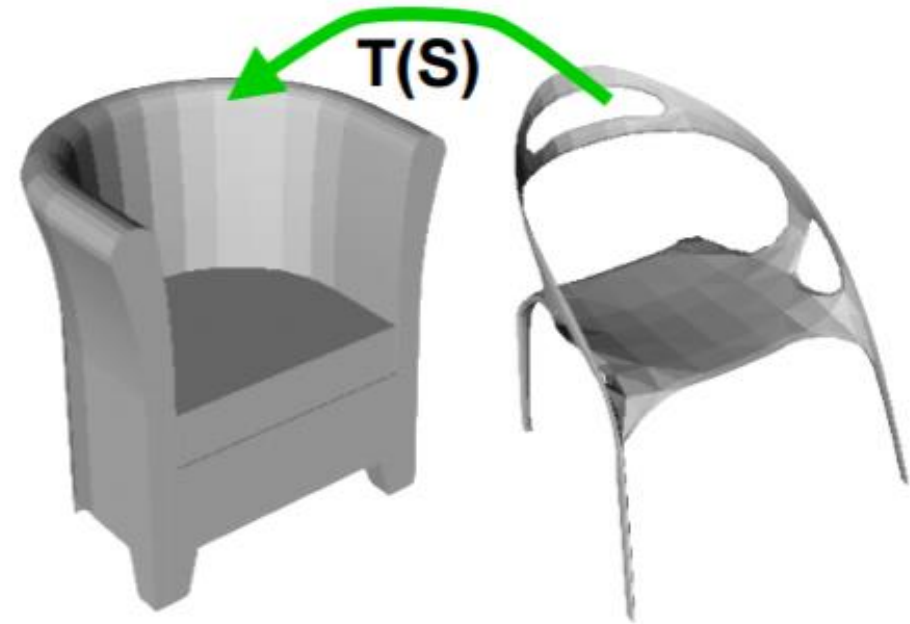
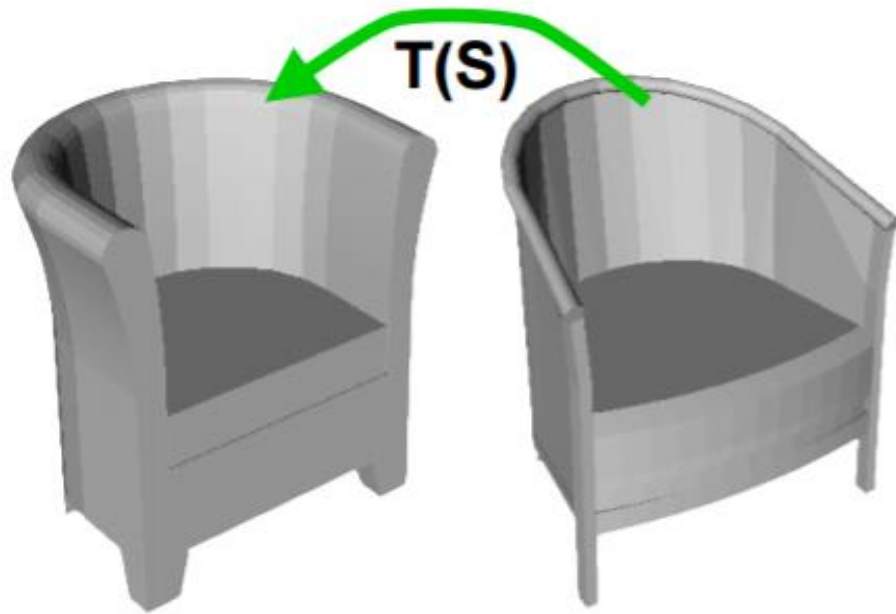


Object completion



Protein docking

Rigid Shape Matching – Search a transformation

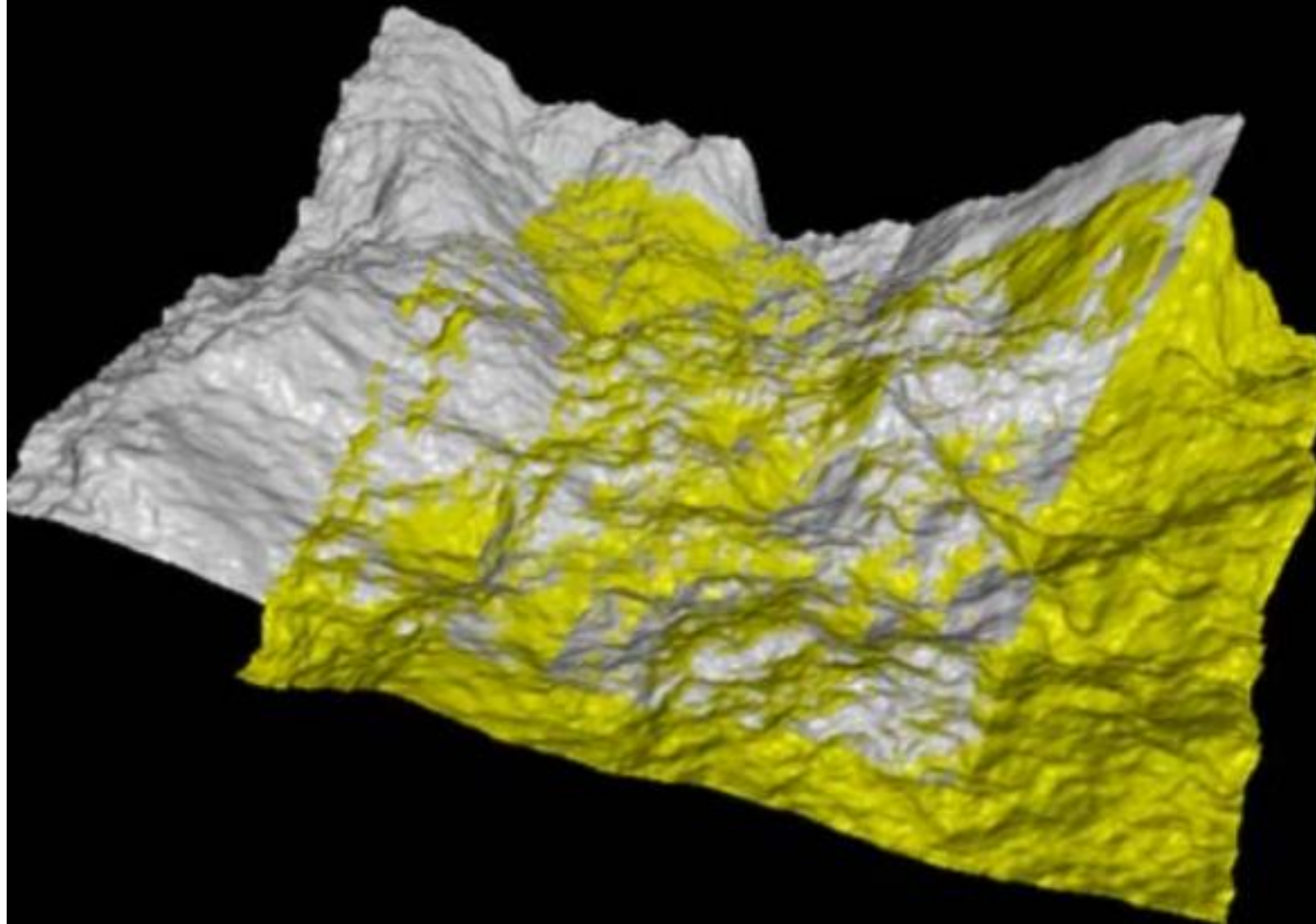


Correspondence Problem Classification

- How many meshes?
 - Two: Pairwise registration
 - More than two: multi-view registration
- Initial registration available?
 - Yes: Local optimization methods
 - No: Global methods
- Class of transformations?
 - Rotation and translation: Rigid-body
 - Non-rigid deformations

Pairwise Rigid Registration Goal

- Align two partially overlapping meshes given initial guess for relative transform



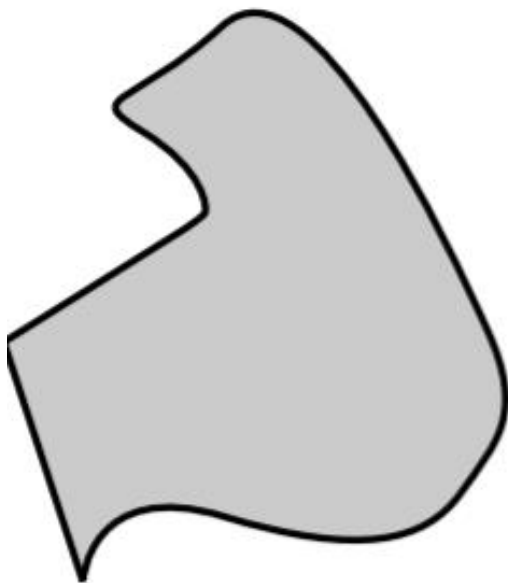
Outline

- Basic ICP: Iterative Closest Points
- Classification of ICP variants
 - Faster alignment
 - Better robustness
- Global Registration

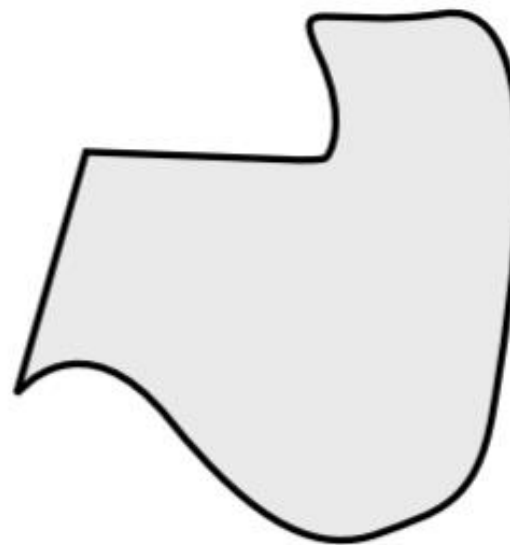
ICP: Iterative Closest Points

Objective

M_1



M_2



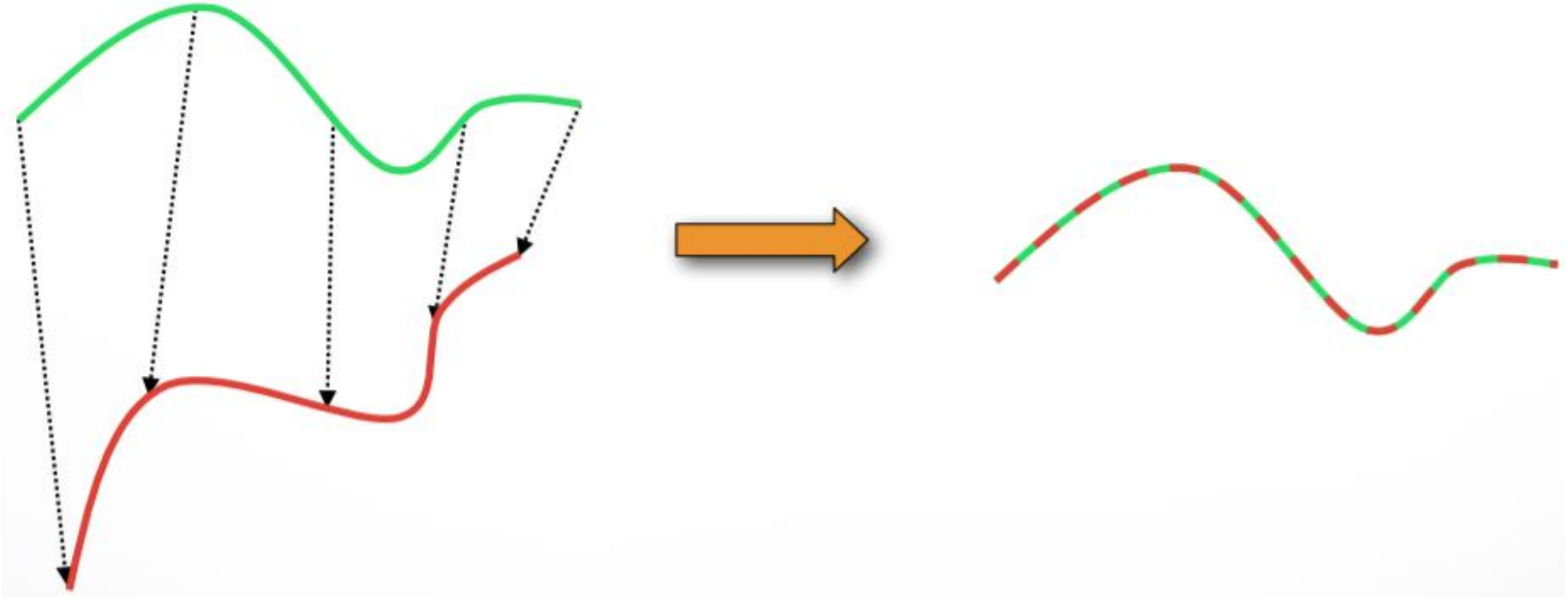
$$M_1 \approx T(M_2)$$

$T :$

translation + rotation

Aligning 3D Data

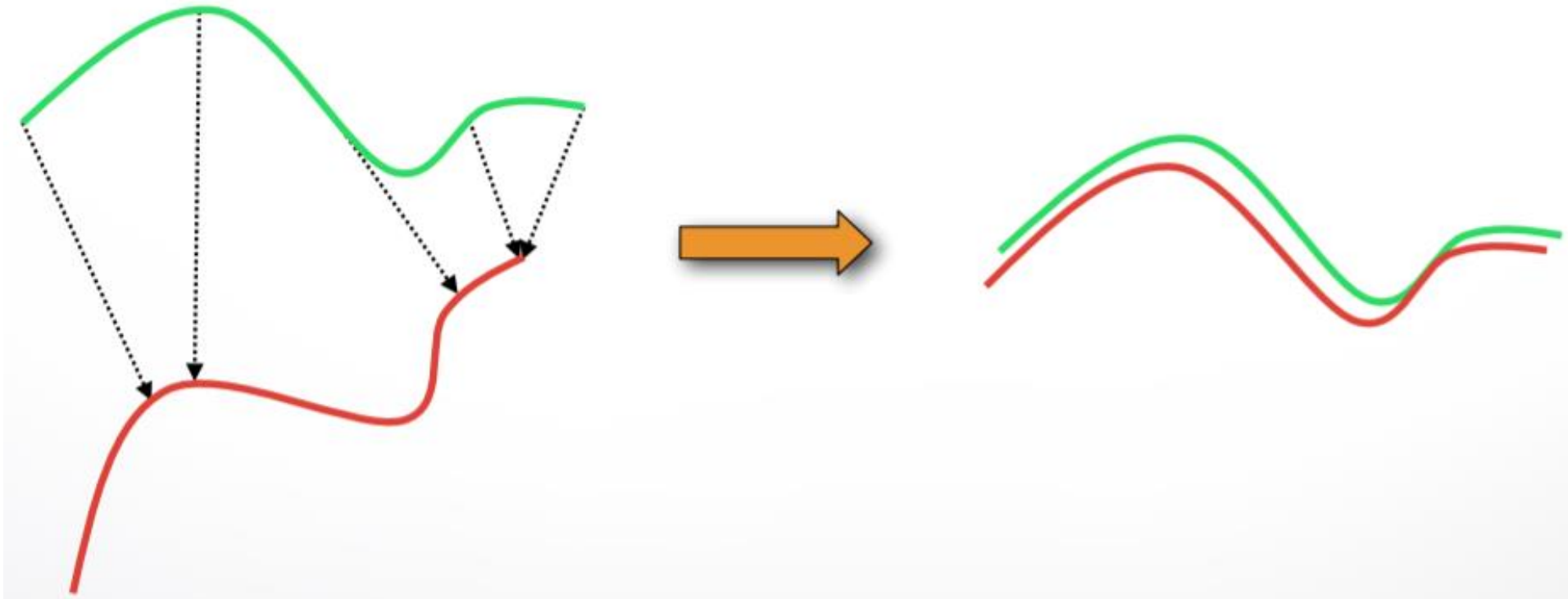
- If correct **correspondences** are **known**, can find correct relative rotation/translation



Aligning 3D Data

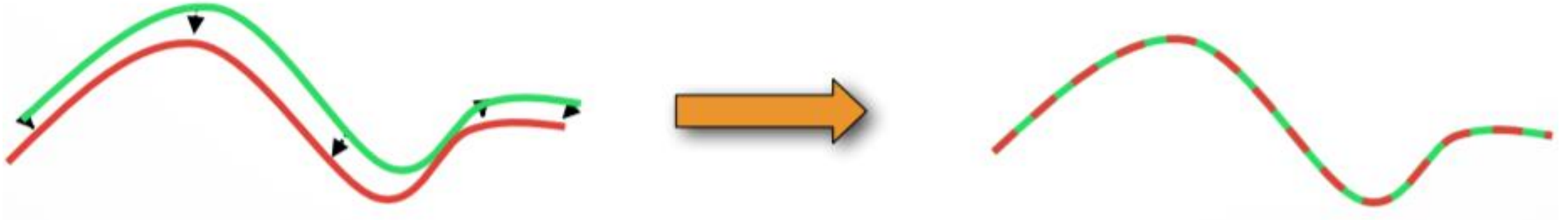
- How to find correspondences: User input? Feature detection? Signatures?
- Alternatives: assume closest points correspond

a seemingly-radical guess



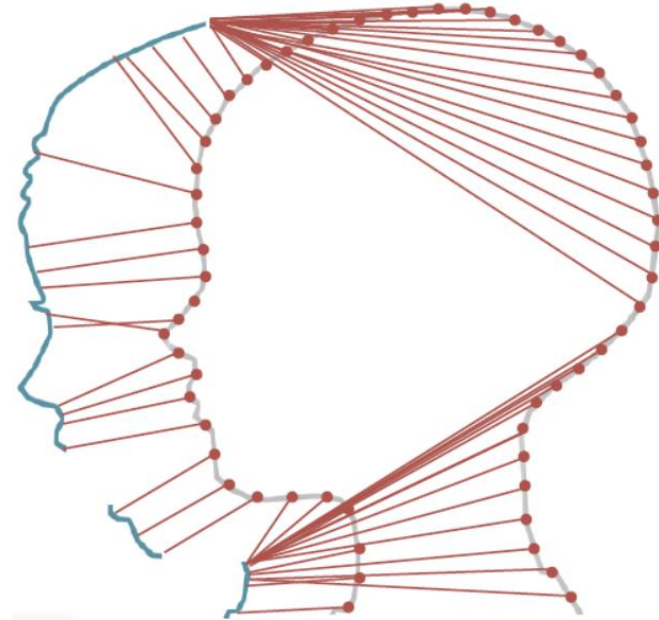
Aligning 3D Data

- ... and iterate to find alignment
 - Iterative Closest Points (ICP) [Besl & Mckay]
- Converges if starting position “close enough”

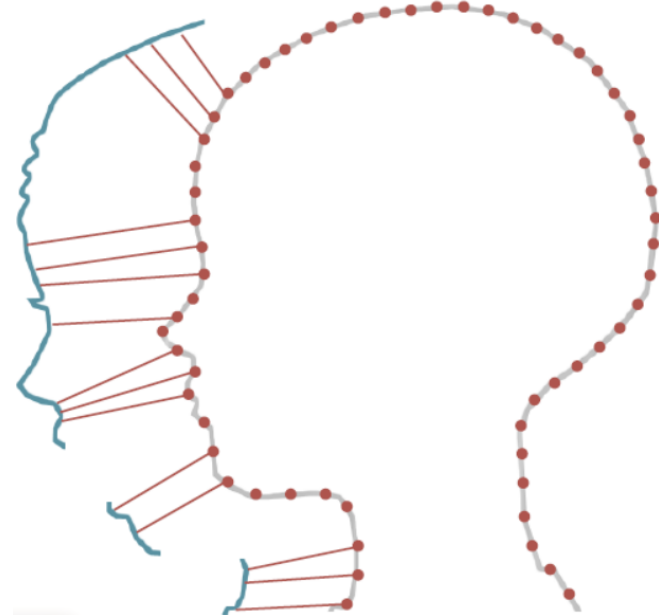
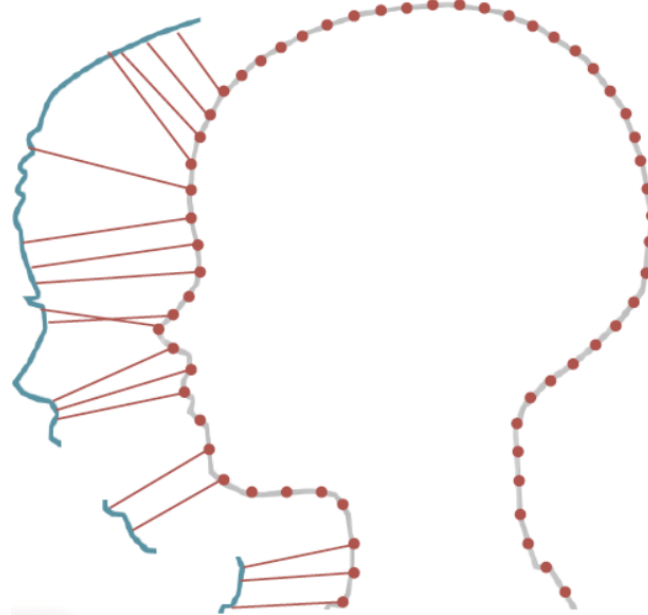
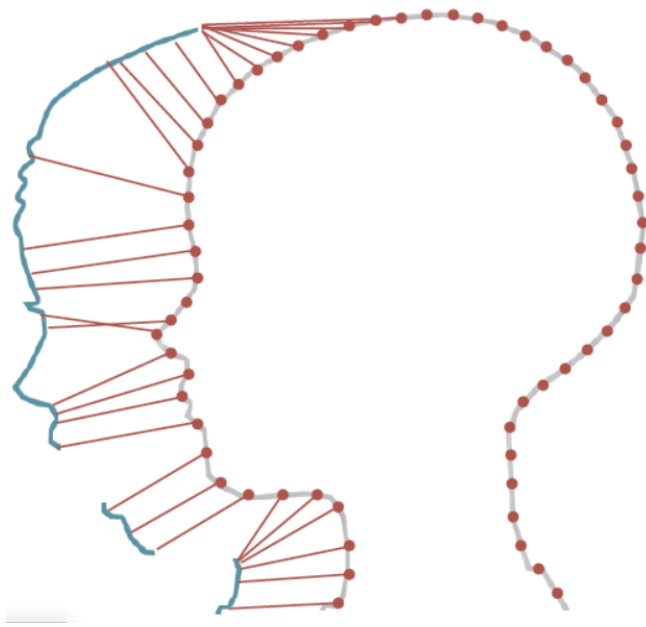
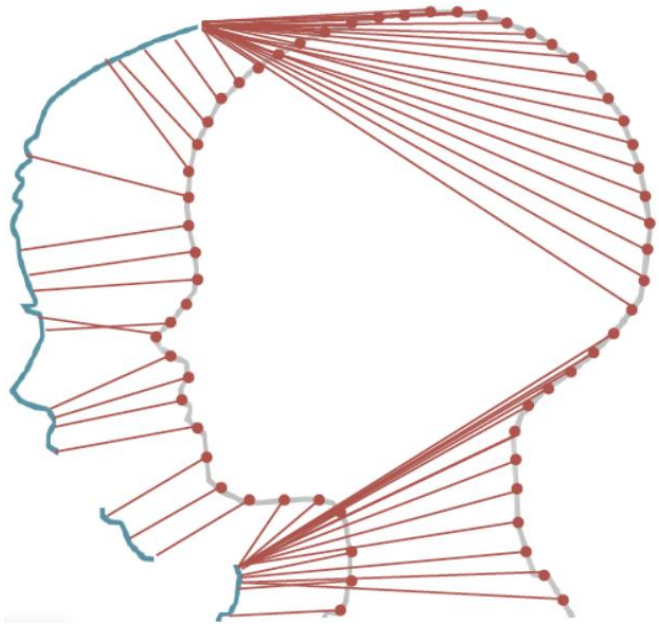


Basic ICP

- **Select** e.g., 1000 random points
- **Match** each to closest point on other scan

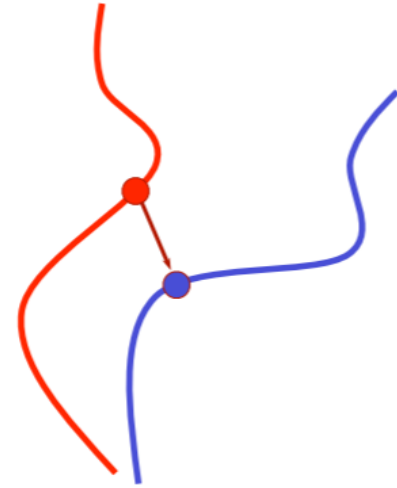


Reject some pairs



Basic ICP

- **Select** e.g., 1000 random points
- **Match** each to closest point on other scan
- **Reject** pairs with distance $> k$ times median
- Construct **error function**:



$$E = \sum ||\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i||^2$$

- Minimize (closed form solution in [Horn 87]) (Also this note: Least-Squares Rigid Motion Using SVD by Olga Sorkine)

Shape Matching: Translation first. why?

Assume R is fixed and denote $F(\mathbf{t}) = \sum_{i=1}^n w_i \|(R\mathbf{p}_i + \mathbf{t}) - \mathbf{q}_i\|^2$. We can find the optimal translation by taking the derivative of F w.r.t. \mathbf{t} and searching for its roots:

$$\begin{aligned} 0 = \frac{\partial F}{\partial \mathbf{t}} &= \sum_{i=1}^n 2w_i (R\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i) = \\ &= 2\mathbf{t} \left(\sum_{i=1}^n w_i \right) + 2R \left(\sum_{i=1}^n w_i \mathbf{p}_i \right) - 2 \sum_{i=1}^n w_i \mathbf{q}_i. \end{aligned} \quad (2)$$

Denote

$$\bar{\mathbf{p}} = \frac{\sum_{i=1}^n w_i \mathbf{p}_i}{\sum_{i=1}^n w_i}, \quad \bar{\mathbf{q}} = \frac{\sum_{i=1}^n w_i \mathbf{q}_i}{\sum_{i=1}^n w_i}. \quad (3)$$

By rearranging the terms of (2) we get

$$\mathbf{t} = \bar{\mathbf{q}} - R\bar{\mathbf{p}}. \quad (4)$$

In other words, the optimal translation \mathbf{t} maps the transformed weighted centroid of P to the weighted centroid of Q . Let us plug the optimal \mathbf{t} into our objective function:

$$\mathbf{t} = \bar{\mathbf{q}} - R\bar{\mathbf{p}}. \quad (4)$$

In other words, the optimal translation \mathbf{t} maps the transformed weighted centroid of P to the weighted centroid of Q . Let us plug the optimal \mathbf{t} into our objective function:

$$\sum_{i=1}^n w_i \|(R\mathbf{p}_i + \mathbf{t}) - \mathbf{q}_i\|^2 = \sum_{i=1}^n w_i \|R\mathbf{p}_i + \bar{\mathbf{q}} - R\bar{\mathbf{p}} - \mathbf{q}_i\|^2 = \quad (5)$$

$$= \sum_{i=1}^n w_i \|R(\mathbf{p}_i - \bar{\mathbf{p}}) - (\mathbf{q}_i - \bar{\mathbf{q}})\|^2. \quad (6)$$

We can thus concentrate on computing the rotation R by restating the problem such that the translation would be zero:

$$\mathbf{x}_i := \mathbf{p}_i - \bar{\mathbf{p}}, \quad \mathbf{y}_i := \mathbf{q}_i - \bar{\mathbf{q}}. \quad (7)$$

So we look for the optimal rotation R such that

$$R = \operatorname{argmin}_R \sum_{i=1}^n w_i \|R\mathbf{x}_i - \mathbf{y}_i\|^2. \quad (8)$$

Shape Matching: Rotation

- Approximate nonlinear rotation by general matrix

$$\min_{\mathbf{R}} \sum_i \|\hat{\mathbf{p}}_i - \mathbf{R}\hat{\mathbf{q}}_i\|^2 \quad \rightarrow \quad \min_{\mathbf{A}} \sum_i \|\hat{\mathbf{p}}_i - \mathbf{A}\hat{\mathbf{q}}_i\|^2$$

- The least squares linear transformation is

$$\mathbf{A} = \left(\sum_{i=1}^m \hat{\mathbf{p}}_i \hat{\mathbf{q}}_i^T \right) \cdot \left(\sum_{i=1}^m \hat{\mathbf{q}}_i \hat{\mathbf{q}}_i^T \right)^{-1} \in \mathbb{R}^{3 \times 3}$$

- SVD & Polar decomposition extracts rotation from \mathbf{A}

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad \rightarrow \quad \mathbf{R} = \mathbf{U}\mathbf{V}^T$$

Iterative Closest Points (ICP)-The classic version

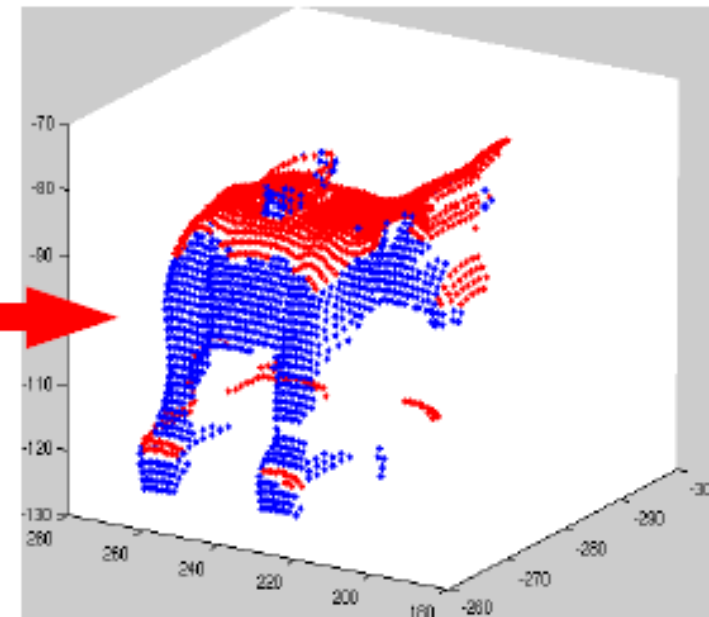
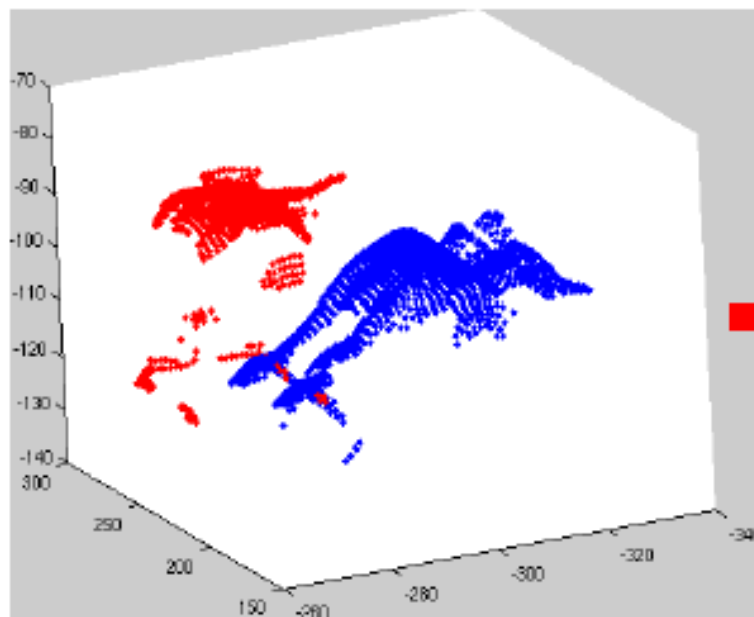
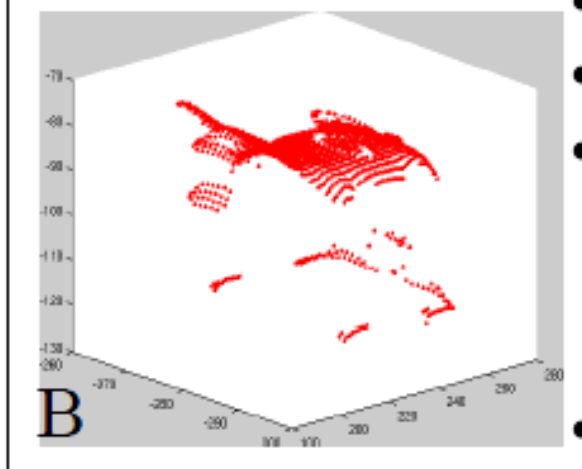
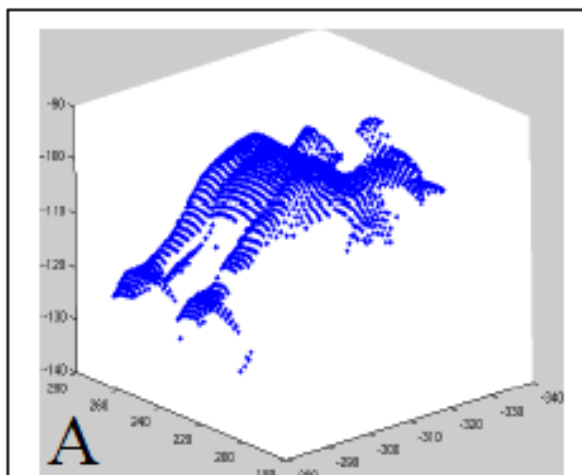
- Given two point sets P and Q , establish correspondence between each point at P and its closest point at Q .
- Iterate between two steps:
 1. Use the estimated correspondence to estimate the best rigid **transformation** and align the shapes.
 2. Derive new **correspondence** from the new alignment.
- Stop when there is no significant change.
- **The initial alignment is critical!**

Suggestions:

1. Use a set of set of feature points/ user markers.
2. Use PCA to align the shapes.
3. Use the symmetry axes to align the shapes.

Registration – ICP algorithm

- Example : registration of 3D model parts (toy cow)




- **Input:** point clouds acquired from non-aligned viewpoints (from 3D range scanner) & initial estimation of registration
- **Output:** Transformation

ICP Variants

ICP Variants

Variants on the following stages of ICP have been proposed:

- 
1. **Selecting** source points (from one or both meshes)
 2. **Matching** to points in the other mesh
 3. **Weighting** the correspondences
 4. **Rejecting** certain (outlier) point pairs
 5. Assigning an **error metric** to the current transform
 6. **Minimizing** the error metric w.r.t. transformation

ICP Variants

- Can analyze various aspects of performance:
 - Speed
 - Stability
 - Tolerance of noise and/or outliers
 - Maximum initial misalignment
- Comparisons of many variants in
 - [Rusinkiewicz & Levoy, 3DIM 2001]: Efficient Variants of the ICP Algorithm

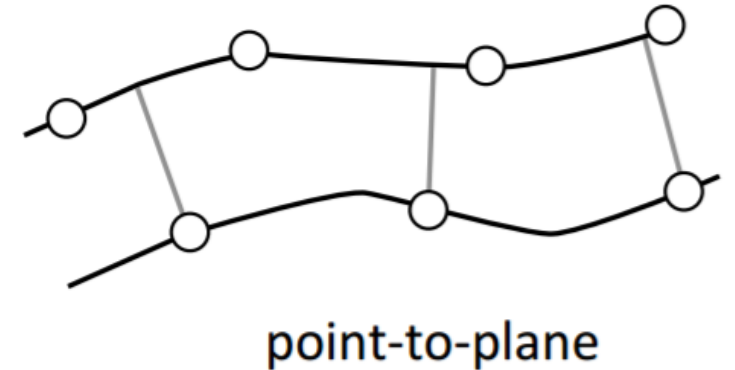
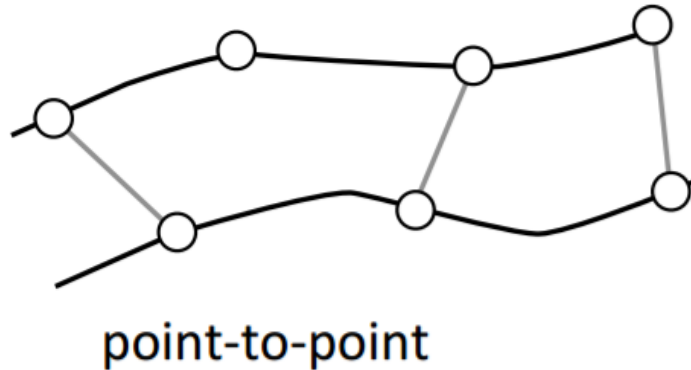
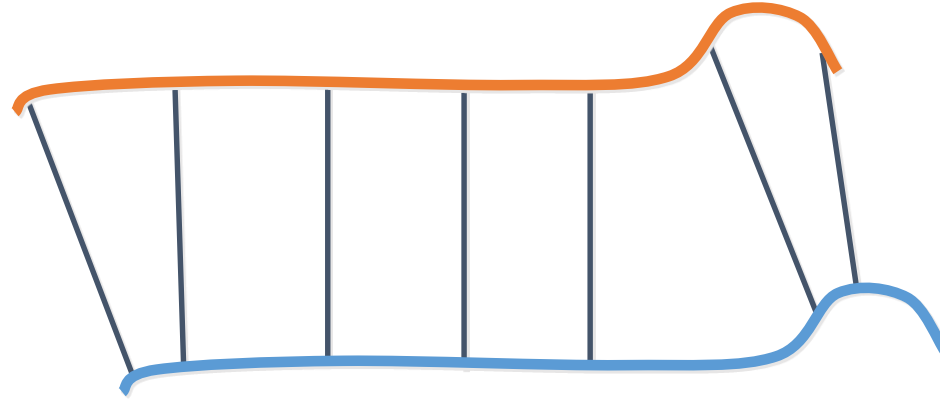
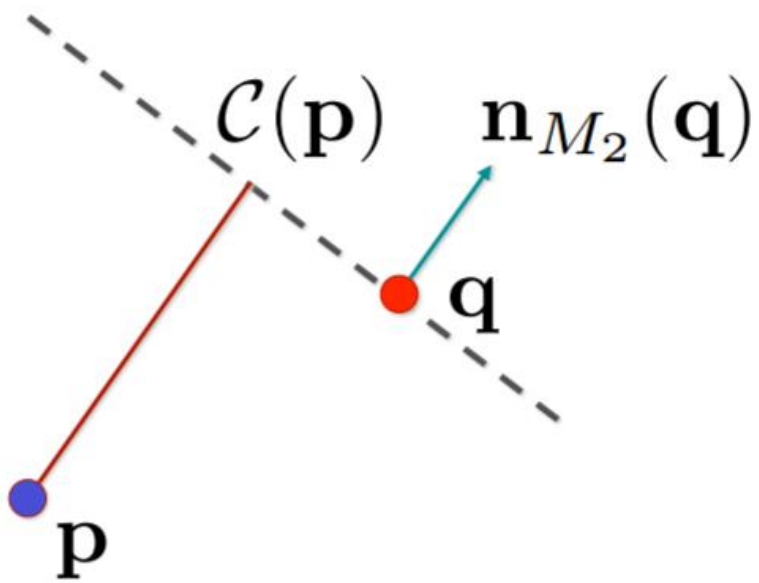
ICP Variants

1. Selecting source points (from one or both meshes)
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- 5. Assigning an error metric to the current transform**
6. Minimizing the error metric w.r.t. transformation

a huge (an order of magnitude or more)
difference in convergence

Point-to-Plane Error Metric

- Using point-to-plane distance instead of point-to-point
- allows flat regions slide along each other [Chen & Medioni 91]



Point-to-Plane Error Metric

- Error function:

$$E = \sum ((\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i)^\top \mathbf{n}_i)^2$$

where \mathbf{R} is a rotation matrix, \mathbf{t} is a translation vector

- Linearize (i.e. assume that $\sin \theta \approx \theta$, $\cos \theta \approx 1$):

$$E \approx \sum ((\mathbf{p}_i - \mathbf{q}_i)^\top \mathbf{n}_i) + \mathbf{r}^\top (\mathbf{p}_i \times \mathbf{n}_i) + \mathbf{t}^\top \mathbf{n}_i)^2 \quad \mathbf{r} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$$

- Result: overconstrained linear system

Point-to-Plane Error Metric

$$E \approx \sum ((\mathbf{p}_i - \mathbf{q}_i)^\top \mathbf{n}_i) + \mathbf{r}^\top (\mathbf{p}_i \times \mathbf{n}_i) + \mathbf{t}^\top \mathbf{n}_i)^2$$

- Overconstrained linear system

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\mathbf{A} = \begin{bmatrix} \leftarrow & \mathbf{p}_1 \times \mathbf{n}_1 & \rightarrow & \leftarrow & \mathbf{n}_1 & \rightarrow \\ \leftarrow & \mathbf{p}_2 \times \mathbf{n}_1 & \rightarrow & \leftarrow & \mathbf{n}_2 & \rightarrow \\ & \vdots & & & \vdots & \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} r_x \\ r_y \\ r_z \\ t_x \\ t_y \\ t_z \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -(\mathbf{p}_1 - \mathbf{q}_1)^\top \mathbf{n}_1 \\ -(\mathbf{p}_2 - \mathbf{q}_2)^\top \mathbf{n}_2 \\ \vdots \end{bmatrix}$$

- Solve using least squares

$$\mathbf{A}^\top \mathbf{A} \mathbf{x} = \mathbf{A}^\top \mathbf{b}$$

$$\mathbf{x} = (\mathbf{A}^\top \mathbf{A})^{-1} \mathbf{A}^\top \mathbf{b}$$

Rotation matrix from axis and angle

For some applications, it is helpful to be able to make a rotation with a given axis. Given a [unit vector](#) $\mathbf{u} = (u_x, u_y, u_z)$, where $u_x^2 + u_y^2 + u_z^2 = 1$, the matrix for a rotation by an angle of θ about an axis in the direction of \mathbf{u} is^[3]

$$R = \begin{bmatrix} \cos \theta + u_x^2 (1 - \cos \theta) & u_x u_y (1 - \cos \theta) - u_z \sin \theta & u_x u_z (1 - \cos \theta) + u_y \sin \theta \\ u_y u_x (1 - \cos \theta) + u_z \sin \theta & \cos \theta + u_y^2 (1 - \cos \theta) & u_y u_z (1 - \cos \theta) - u_x \sin \theta \\ u_z u_x (1 - \cos \theta) - u_y \sin \theta & u_z u_y (1 - \cos \theta) + u_x \sin \theta & \cos \theta + u_z^2 (1 - \cos \theta) \end{bmatrix}$$

A derivation of this matrix from first principles can be found in section 9.2. [here](#).^[4]

This can be written more concisely as

$$R = \cos \theta \mathbf{I} + \sin \theta [\mathbf{u}]_{\times} + (1 - \cos \theta) \mathbf{u} \otimes \mathbf{u}$$

where $[\mathbf{u}]_{\times}$ is the [cross product matrix](#) of \mathbf{u} , \otimes is the [tensor product](#) and \mathbf{I} is the [Identity matrix](#), or

$$\text{alternatively as } R_{jk} = \begin{cases} \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} (2u_j^2 - 1), & \text{if } j = k \\ 2u_j u_k \sin^2 \frac{\theta}{2} + \epsilon_{jkl} u_l \sin \theta, & \text{if } j \neq k \end{cases}$$

where ϵ_{jkl} is the [Levi-Civita symbol](#) with $\epsilon_{123} = 1$. This is a matrix form of [Rodrigues' rotation formula](#), (or the equivalent, differently parameterized [Euler–Rodrigues formula](#)) with^[5]

$$\mathbf{u} \otimes \mathbf{u} = \mathbf{u} \mathbf{u}^T = \begin{bmatrix} u_x^2 & u_x u_y & u_x u_z \\ u_x u_y & u_y^2 & u_y u_z \\ u_x u_z & u_y u_z & u_z^2 \end{bmatrix}, \quad [\mathbf{u}]_{\times} = \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix}.$$

$$E = \sum ((\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i)^\top \mathbf{n}_i)^2 \longrightarrow E \approx \sum ((\mathbf{p}_i - \mathbf{q}_i)^\top \mathbf{n}_i) + \mathbf{r}^\top (\mathbf{p}_i \times \mathbf{n}_i) + \mathbf{t}^\top \mathbf{n}_i)^2$$

$$R = \cos\theta I + \sin\theta [r]_\times + (1 - \cos\theta) r(r)^\top$$

- Key: $(Rp_i)^\top n_i = ((I + [r]_\times)p_i)^\top n_i$
- $([r]_\times p_i)^\top n_i \stackrel{?}{=} (r)^\top (p_i \times n_i)$

$$\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_\times \mathbf{b} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

- $(r \times p_i)^\top n_i \stackrel{?}{=} (r)^\top (p_i \times n_i)$
- $(r \times p_i)^\top n_i = n_i \cdot (r \times p_i); (r)^\top (p_i \times n_i) = r \cdot (p_i \times n_i)$
- $n_i \cdot (r \times p_i) \stackrel{?}{=} r \cdot (p_i \times n_i)$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$

Improving ICP Stability

- Closest compatible point
- Stable sampling

ICP Variants

1. Selecting source points (from one or both meshes)
2. **Matching** to points in the other mesh
3. Weighting the correspondences
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Closest Compatible Point

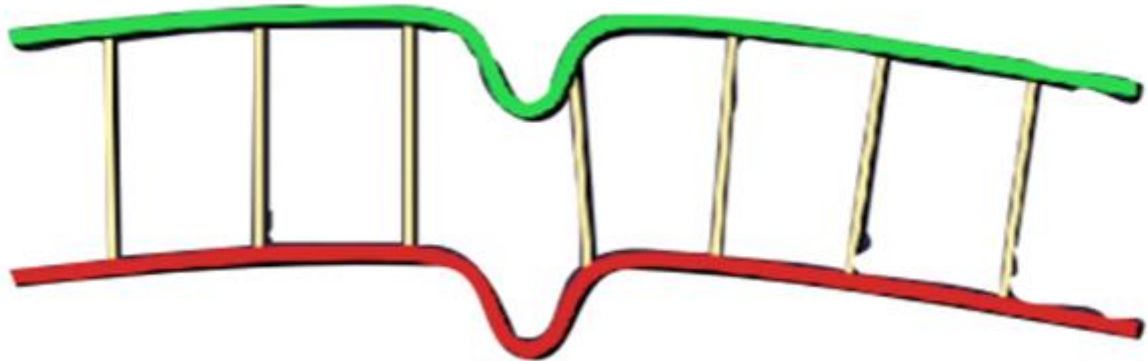
- Closest point often a bad approximation to corresponding point
- Can improve matching effectiveness by restricting match to compatible points
 - Compatibility of colors [Godin et al. '94]
 - Compatibility of normals [Pulli '99]
 - Other possibilities: curvature, higher-order derivatives, and other local features (remember: data is noisy)
- New challenge: how to assign proper weights to color, normal or ...

ICP Variants

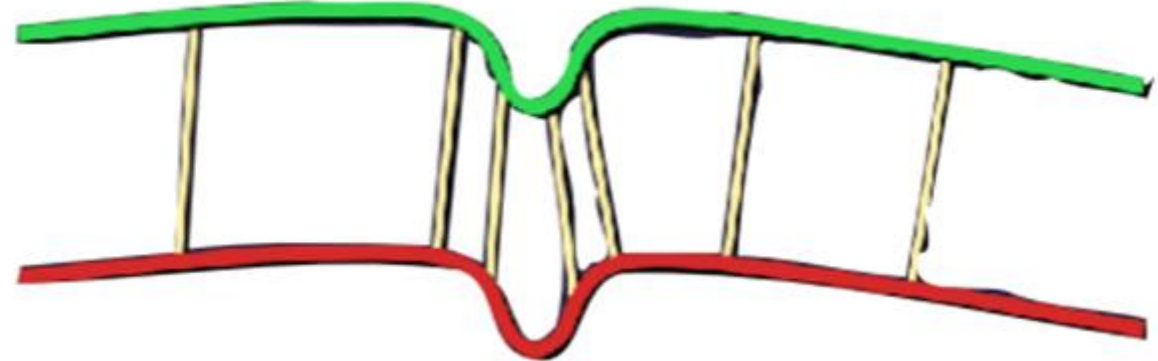
- 1. Selecting source points (from one or both meshes)**
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Selecting Source Points

- Use all points
- Uniform subsampling
- Random sampling
- Stable sampling [Gelfand et al. 2003]
 - Select samples that constrain all degrees of freedom of the rigid-body transformation



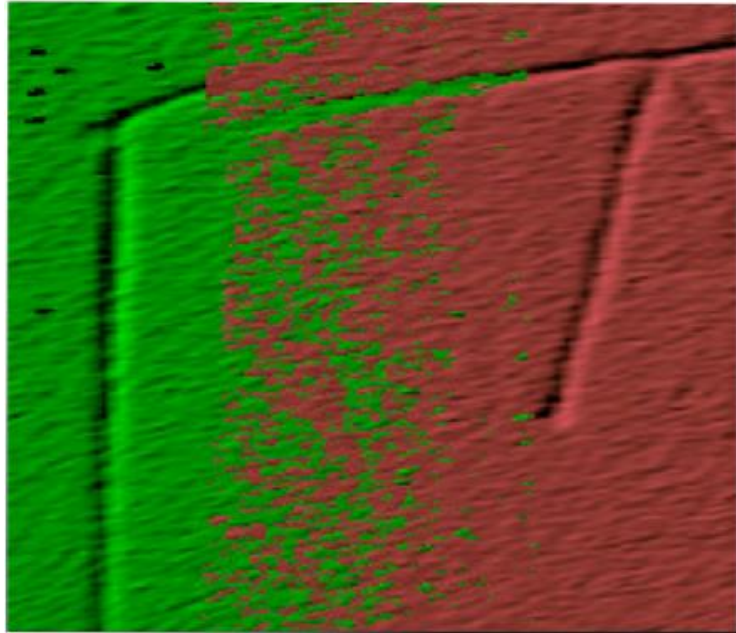
Uniform Sampling



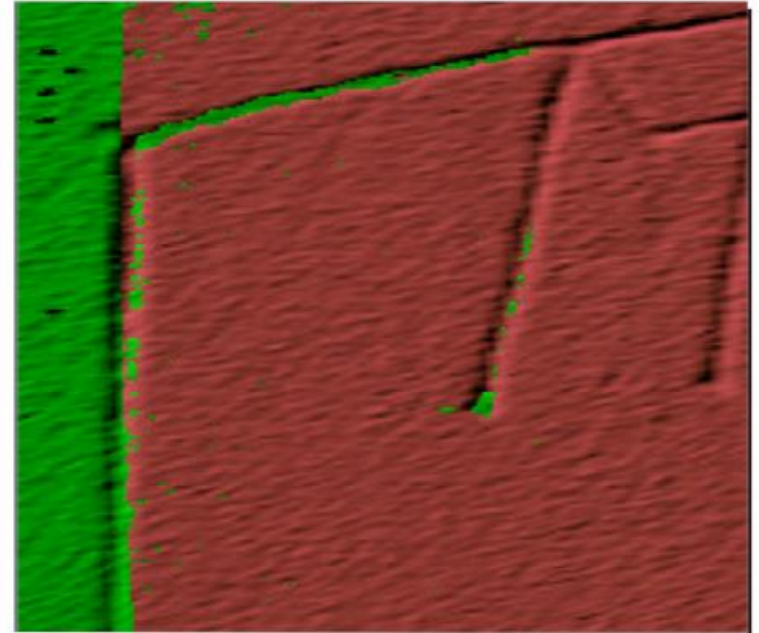
Stable Sampling

Sample Selection

- Simpler variant: normal-space sampling
 - select points with uniform distribution of normals
 - Pro: faster, does not require eigenanalysis
 - Con: only constrains translation
- Stability-based or normal-space sampling important for smooth areas with small features



Random Sampling



Normal-space Sampling

Selection vs. Weighting

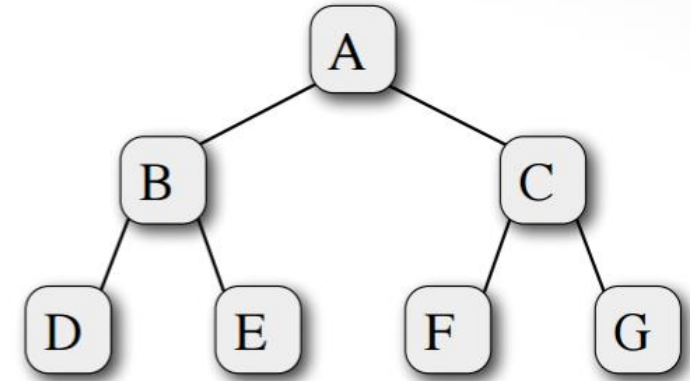
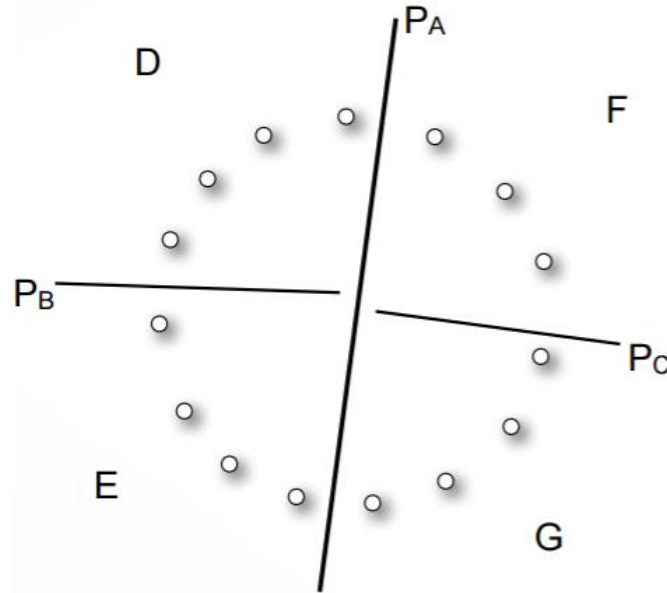
- Could achieve same effect with weighting
- Hard to ensure enough samples in features except at high sampling rates
- However, have to build special data structure
- Preprocessing / run-time cost tradeoff

ICP Variants

- Can analyze various aspects of performance:
 - **Speed**
 - Stability
 - Tolerance of noise and/or outliers
 - Maximum initial misalignment
- Comparisons of many variants in
 - [Rusinkiewicz & Levoy, 3DIM 2001]

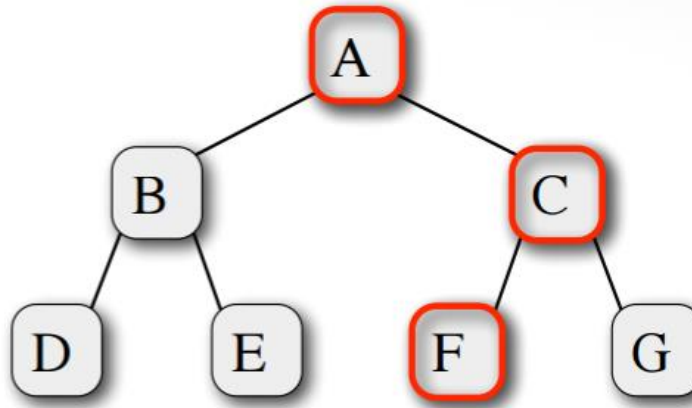
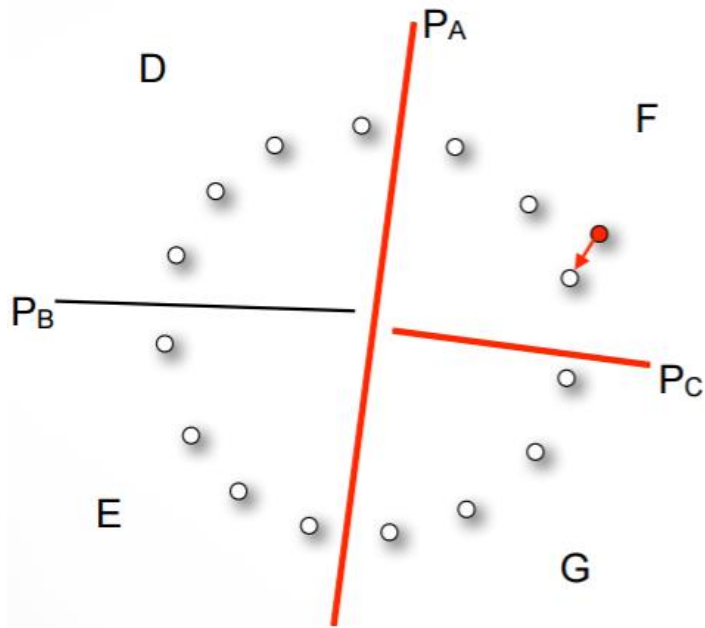
Closest Point Search

- most expensive stage of the ICP algorithm
 - Brute force search – $O(n)$



- Use Hierarchical BSP tree
 - Binary space partitioning tree (general kD-tree)
 - Recursively partition 3D space by planes
 - Tree should be balanced, put plane at median
 - $\log(n)$ tree levels, complexity $O(n \log n)$

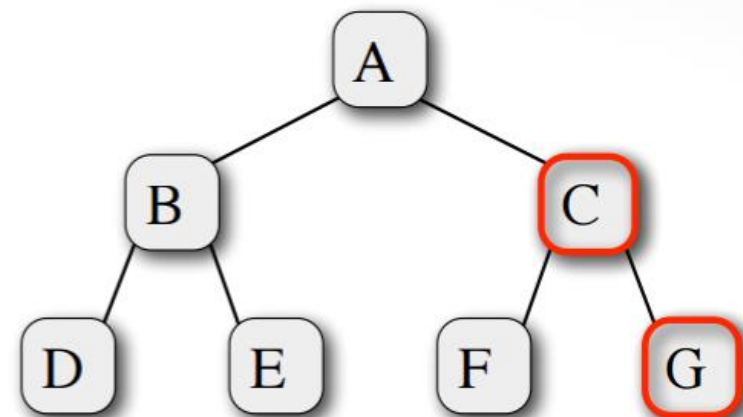
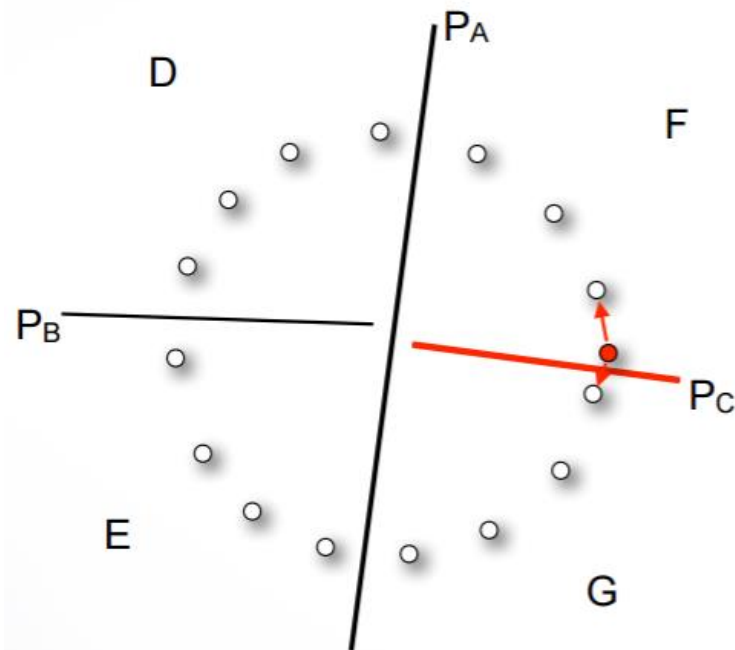
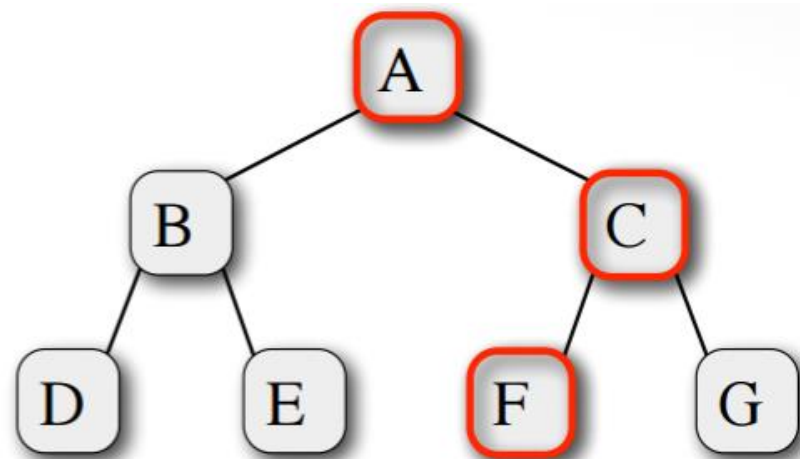
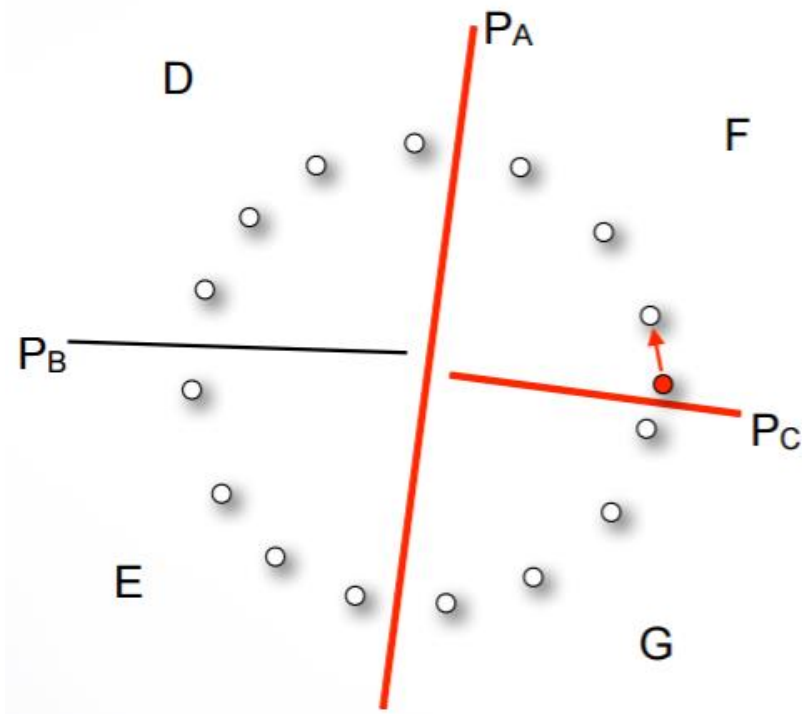
BSP Closest Point Search



Implement `BSPNode::dist()` with the following info:

```
BSPNode{
    BSPNode left_child, right_child;
    vector<Point> p; // p[i] is the ith point
    ...
    bool leaf_node();
    void dist(Point x, Scalar& dmin): x: the query point, dmin: min distance
    between x and its closest point in the Tree.
};
float dist = dist_to_plane(x); // a signed distance for assigning x to left
or right of the plane & ...
```

How to handle this?



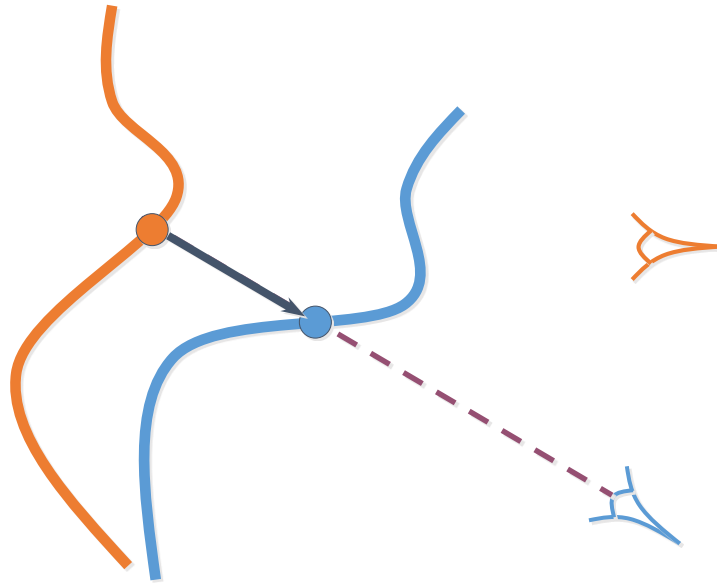
BSP Closest Point Search

```
BSPNode::dist(Point x, Scalar& dmin)
{
    if (leaf_node())
        for each sample point p[i]
            dmin = min(dmin, dist(x, p[i]));

    else
    {
        d = dist_to_plane(x);
        if (d < 0)
        {
            left_child->dist(x, dmin);
            if (|d| < dmin) right_child->dist(x, dmin);
        }
        else
        {
            right_child->dist(x, dmin);
            if (|d| < dmin) left_child->dist(x, dmin);
        }
    }
}
```

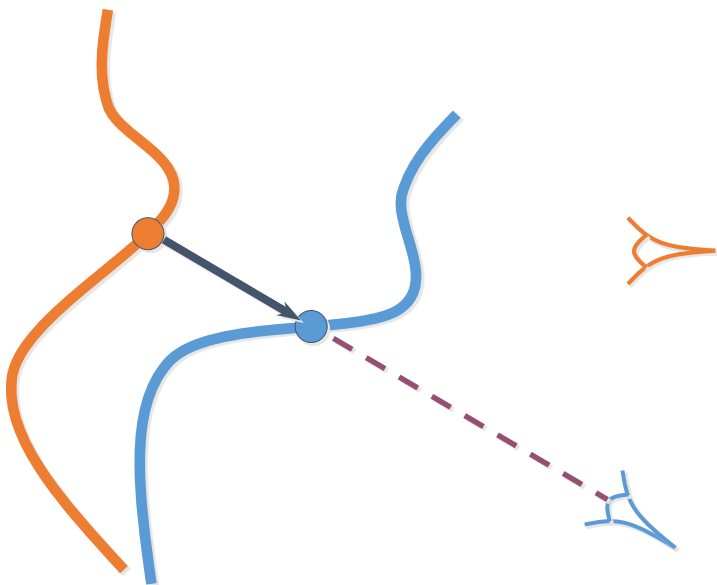
Closest Point Search

- most expensive stage of the ICP algorithm
 - Brute force search – $O(n)$
 - Binary space partitioning tree (general kD-tree) - $O(n \log n)$



Projection to Find Correspondence

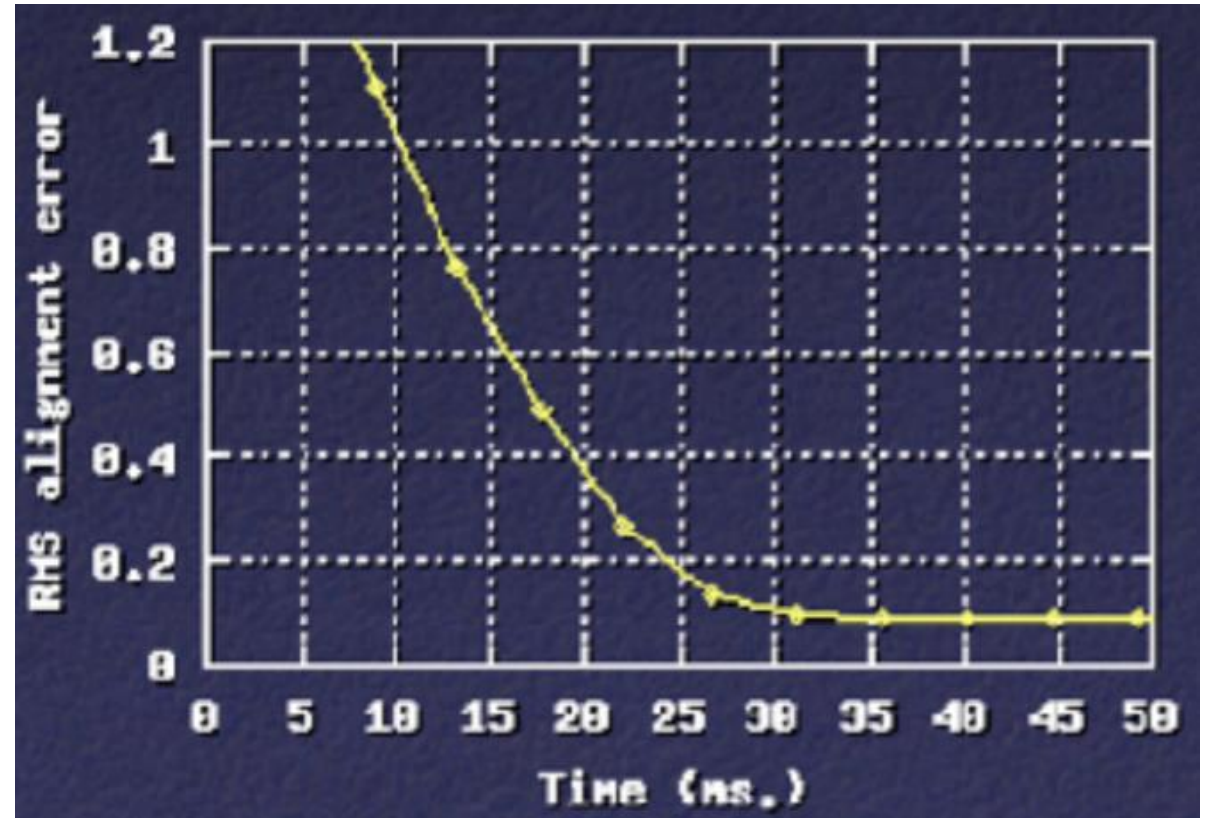
- Idea: use a simpler algorithm to find correspondences
- For range images, can simply project point by “reverse calibration” [Blais 95]
 - Constant-time
 - Does not require precomputing a spatial data structure



$$\tilde{\mathbf{x}}_s = \underset{\substack{\uparrow \\ \text{Calibration Matrix}}}{\mathbf{K}} \left[\mathbf{R} \mid \mathbf{t} \right] \mathbf{p}_w = \underset{\substack{\uparrow \\ \text{Camera Matrix}}}{\mathbf{P}} \mathbf{p}_w$$

Projection-Based Matching

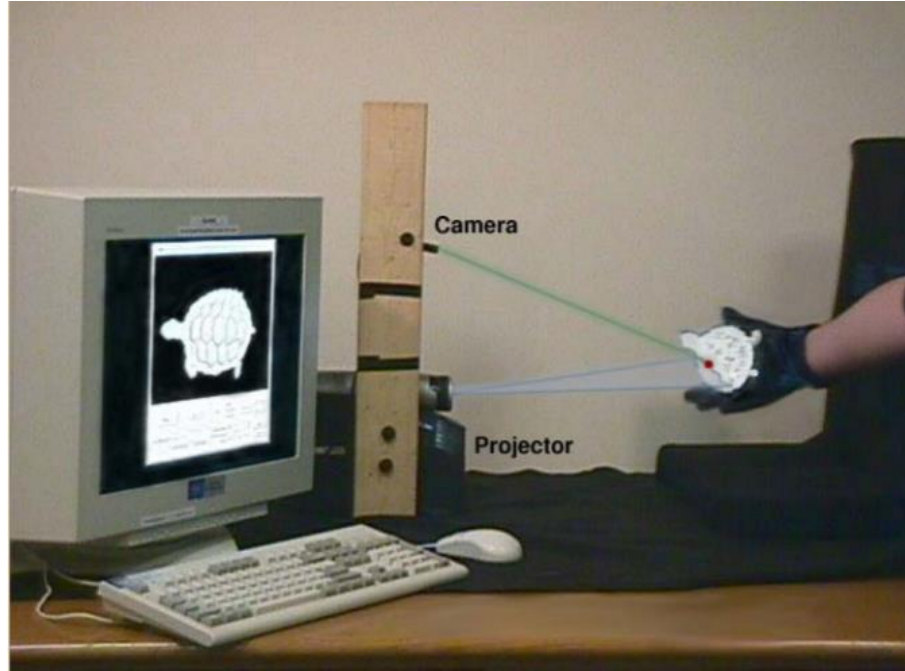
- Slightly worse performance per iteration
- Each iteration is one to two orders of magnitude faster than closest point
- Result: can align two range images in a few milliseconds (**0.001 seconds**), vs. a few seconds



Applications

- Given:
 - A scanner that returns range images in real time
 - Fast ICP
 - Real-time merging and rendering
- Result: 3D model acquisition
 - Tight feedback loop with user
 - Can see and fill holes while scanning

Scanner Layout



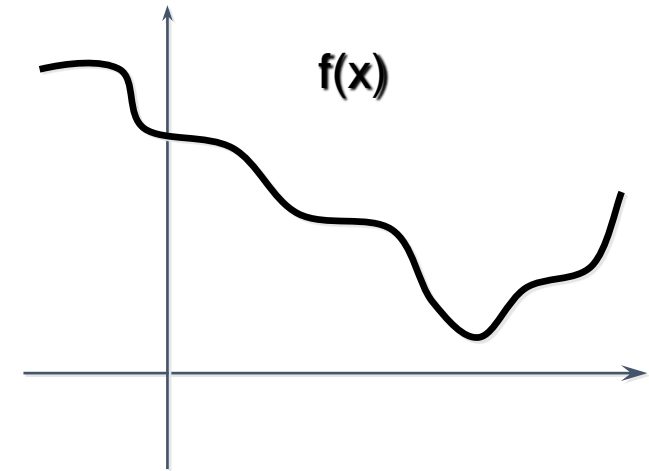
Artec Group



[Newcombe et al. '11]
KinectFusion

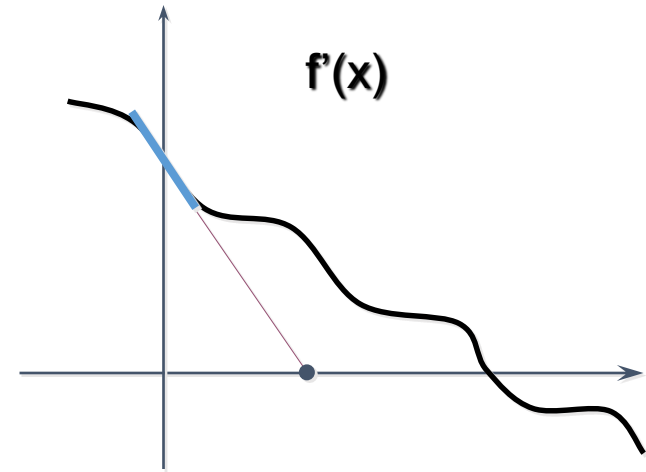
What Does ICP Do?

- Two ways of thinking about ICP:
 - Solving the correspondence problem
 - Minimizing point-to-surface squared distance
- ICP is like Newton's method on an approximation of the distance function



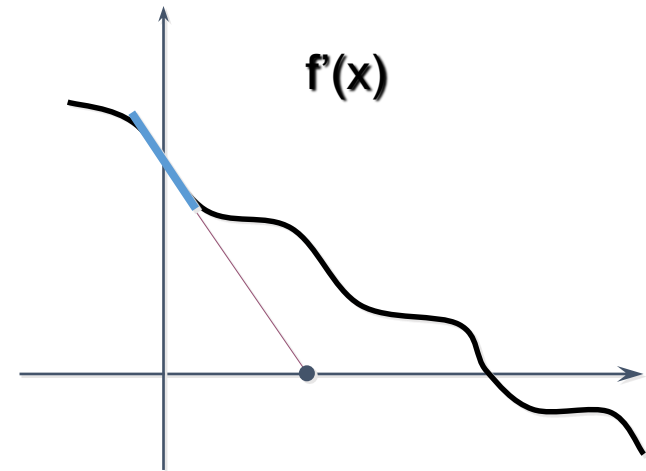
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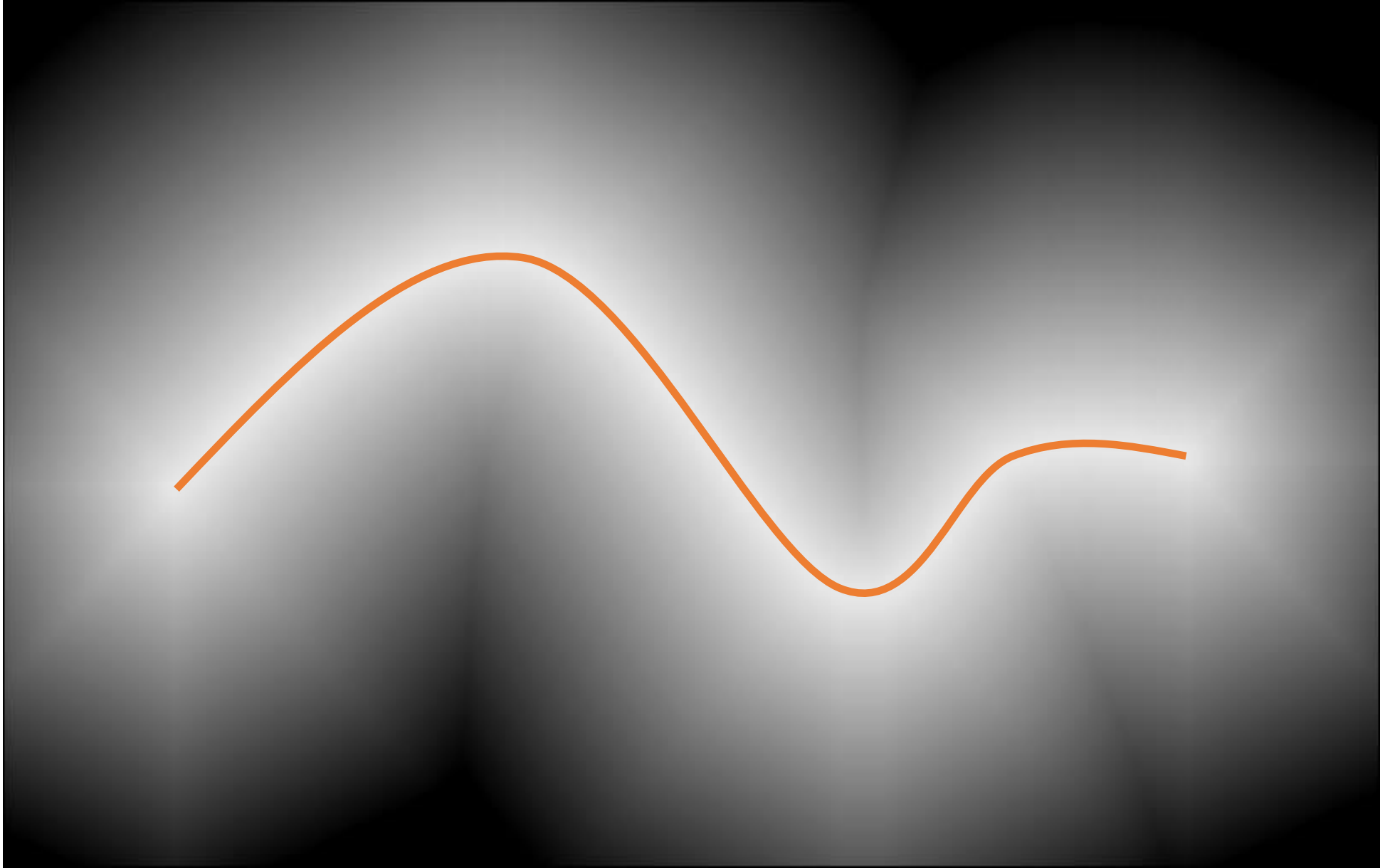


What Does ICP Do?

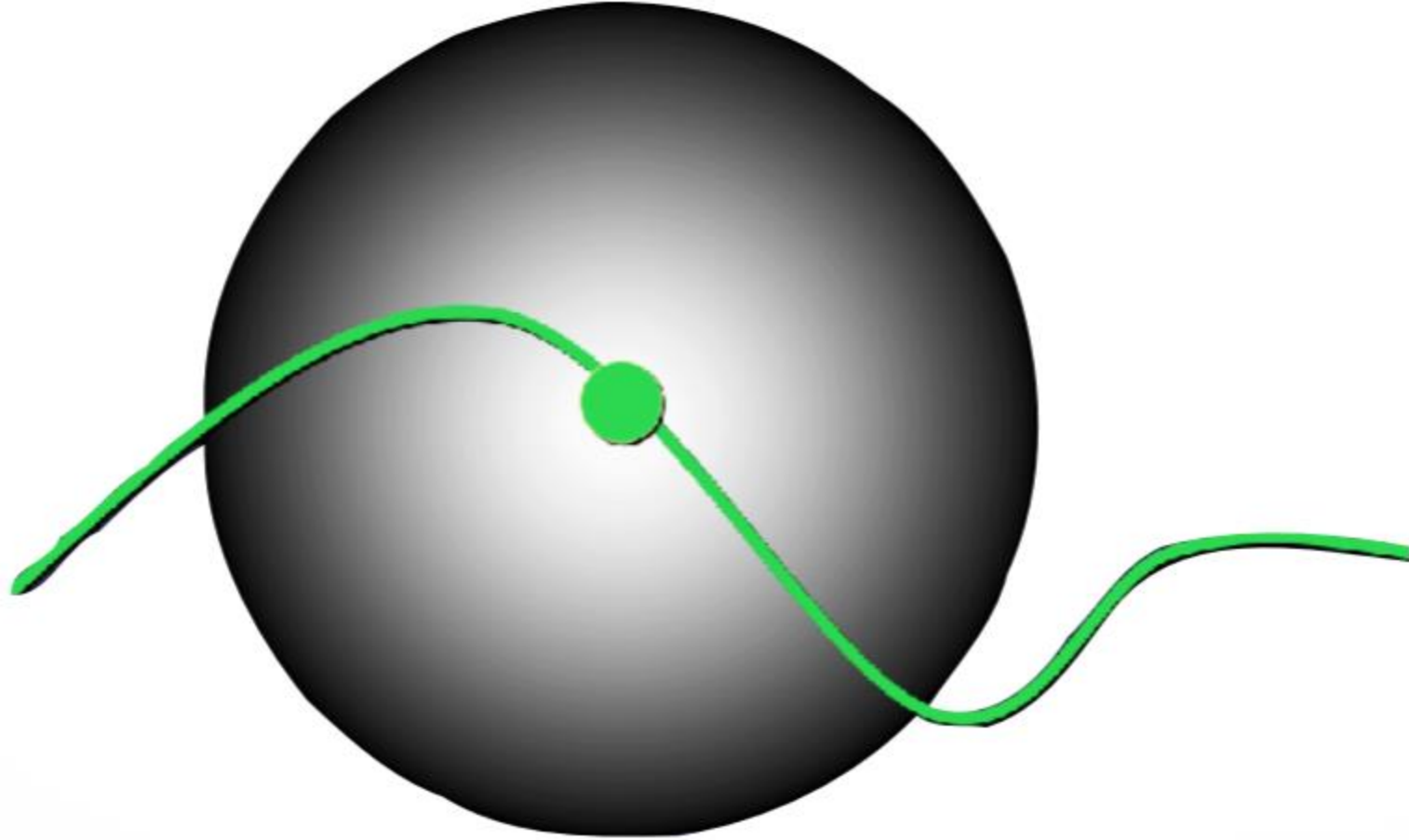
- Two ways of thinking about ICP:
 - Solving the correspondence problem
 - Minimizing point-to-surface squared distance
- ICP is like Newton's method on an approximation of the distance function
 - ICP variants affect shape of global error function or local approximation



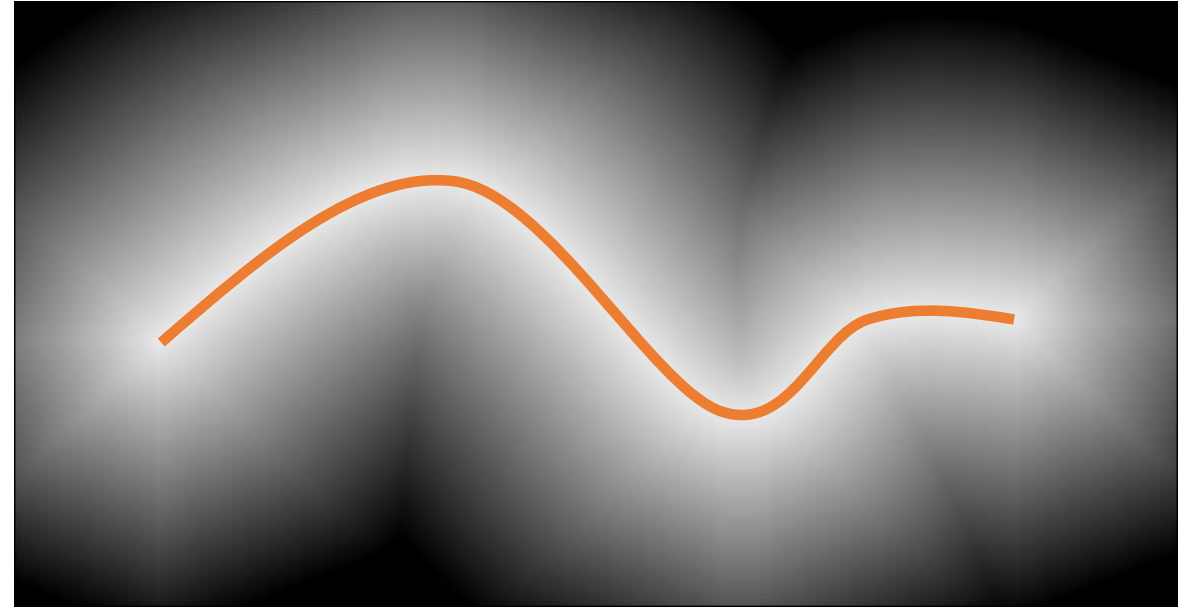
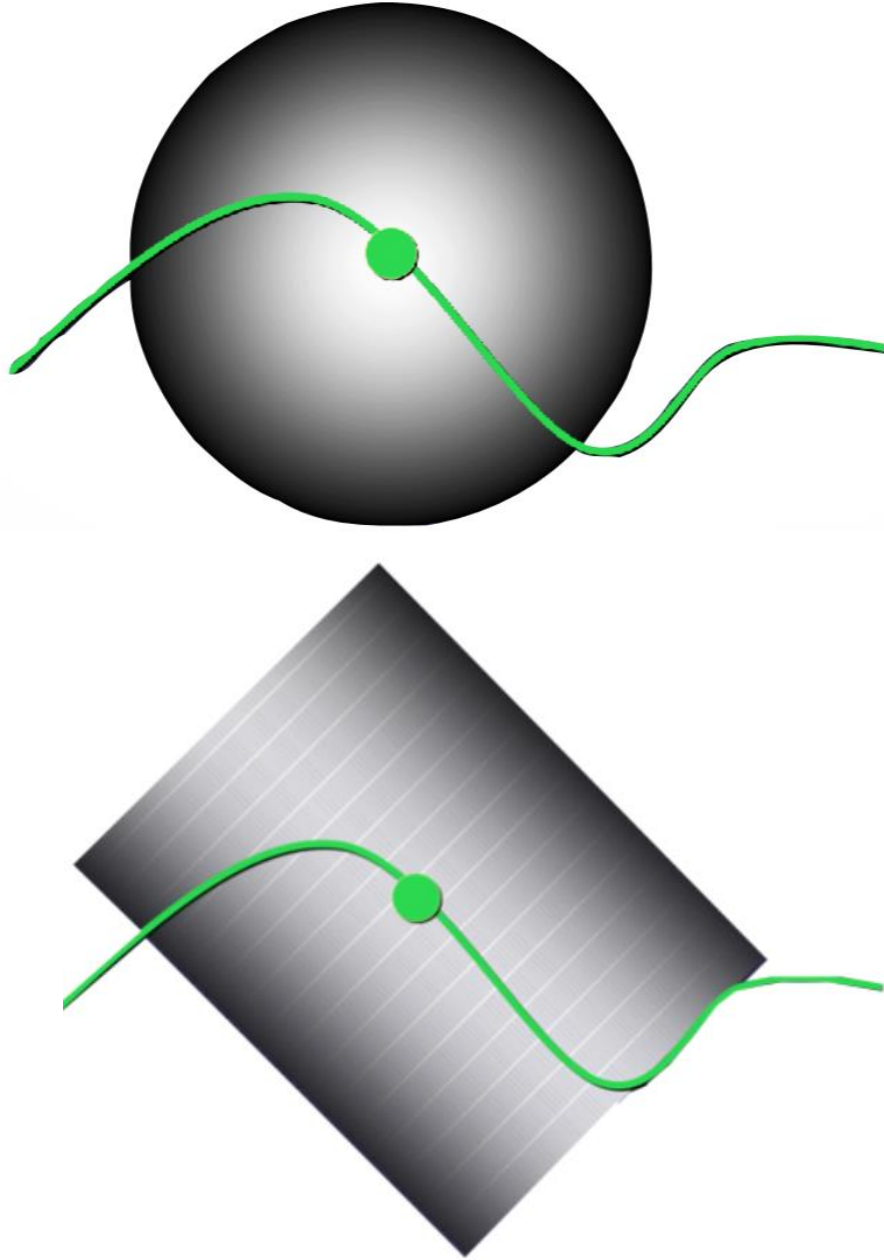
Point-to-Surface Distance



Point-to-Point Distance



Point-to-Plane Distance

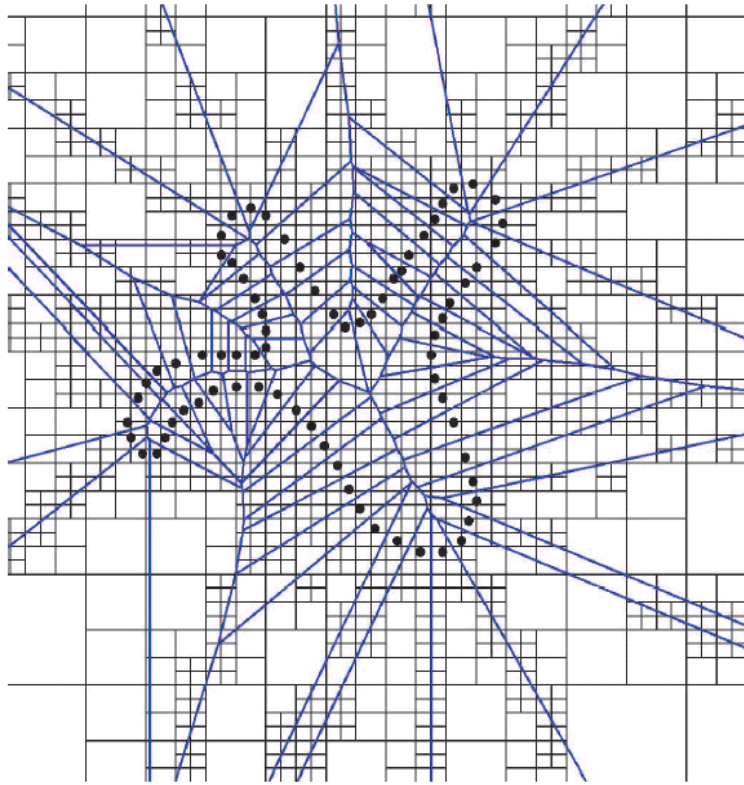


Soft Matching and Distance Functions

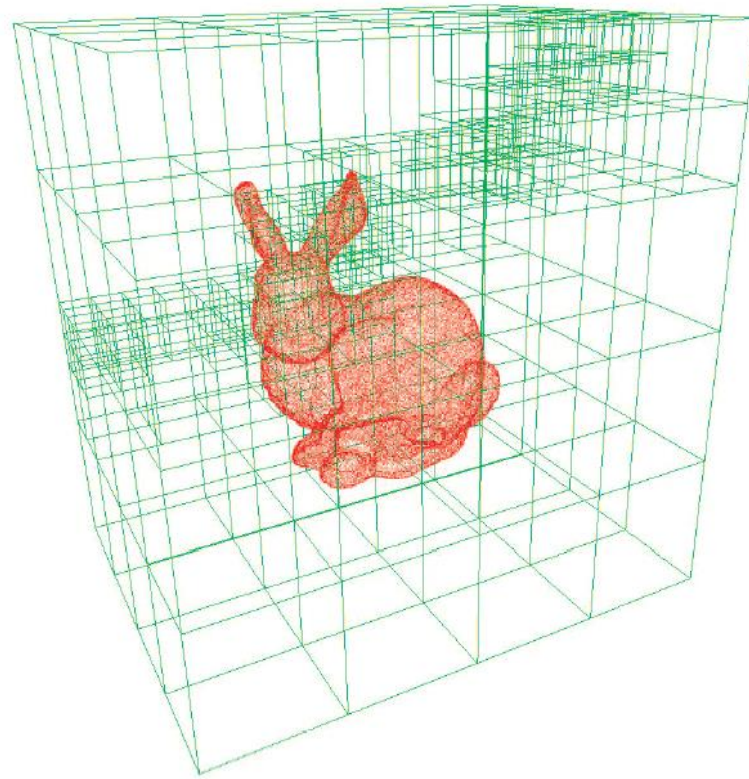
- Soft matching equivalent to standard ICP on (some) filtered surface
- Produces filtered version of distance function
⇒ fewer local minima
- Multiresolution minimization [Turk & Levoy 94]
or soft assign with simulated annealing (good description in [Chui 03])

Mitra et al.'s Optimization

- Precompute piecewise-quadratic approximation to distance field throughout space
- Store in “d2tree” data structure



2D



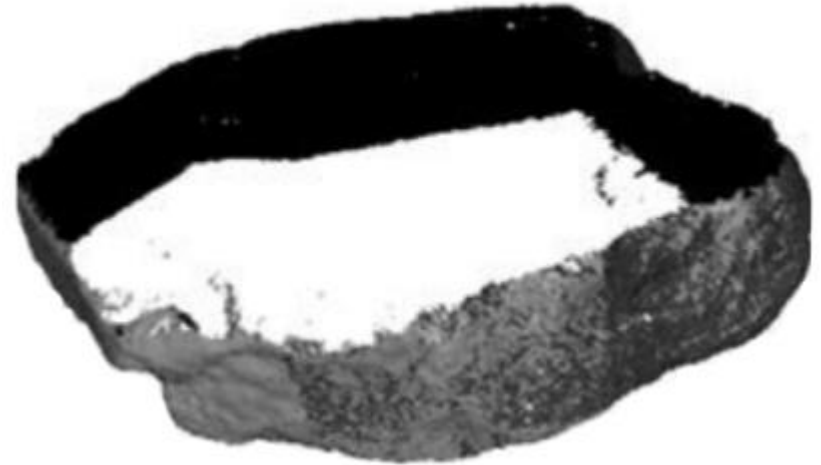
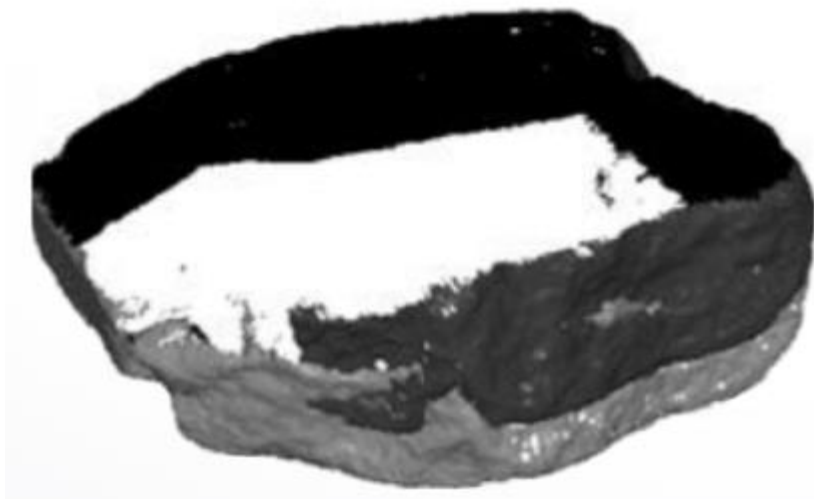
3D

Mitra et al.'s Optimization

- Precompute piecewise-quadratic approximation to distance field throughout space
- Store in “d2tree” data structure
- At run time, look up quadratic approximants and optimize using Newton's method
 - More robust, wider basin of convergence
 - Often fewer iterations, but more precomputation

Global Registration Goal

- Given: n scans around an object
- Goal: align them all
- First attempt: apply ICP to each scan to one other
- Want method for distributing accumulated error among all scans



Approach #1: Avoid the Problem

- In some cases, have 1 (possibly low-resolution) scan that covers most surface
- Align all other scans to this “anchor” [Turk 94]
- Disadvantage: not always practical to obtain anchor scan

Approach #2: The Greedy Solution

- Align each new scan to the union of all previous scans [Masuda '96]
- Disadvantages:
 - Order dependent
 - Doesn't spread out error
 - This sometimes avoids catastrophic accumulation of error, but really isn't guaranteed to do anything.

Approach #3: The Brute-Force Solution

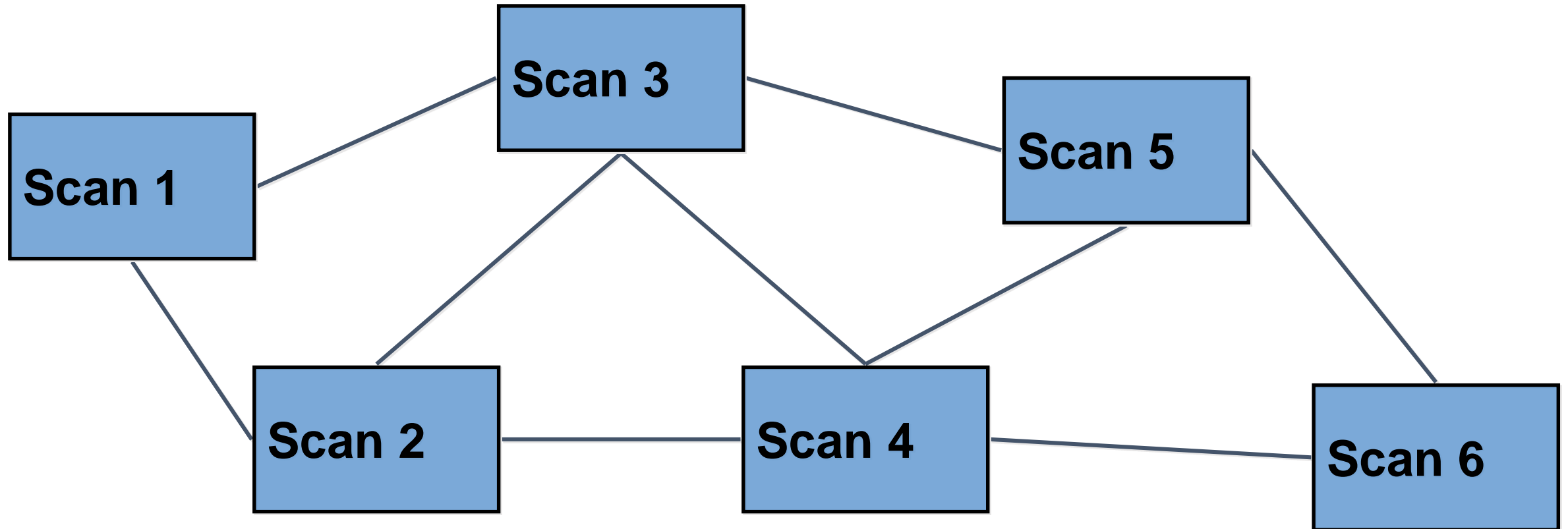
- While not converged:
 - For each scan:
 - For each point:
 - For every other scan
 - Find closest point
 - Minimize error w.r.t. transforms of all scans
- Disadvantage:
 - Solve $(6n) \times (6n)$ matrix equation, where n is number of scans

Approach #3a: Slightly Less Brute-Force Solution

- While not converged:
 - For each scan:
 - For each point:
 - For every other scan
 - Find closest point
 - Minimize error w.r.t. transforms of this scans
- Faster than previous method (matrices are 6x6) [Bergevin '96, Benjemaa '97]
- Previous: Solve $(6n) \times (6n)$ matrix equation, where n is number of scans

Graph Methods

- Many globalreg algorithms create a graph of pairwise alignments between scans



Pulli's Algorithm

- **Perform pairwise ICPs, record sample (e.g. 200) of corresponding points**
- For each scan, starting w. most connected
 - Align scan to existing set
 - While (change in error) > threshold
 - Align each scan to others
- **All alignments during globalreg phase use precomputed corresponding points**

Sharp et al. Algorithm

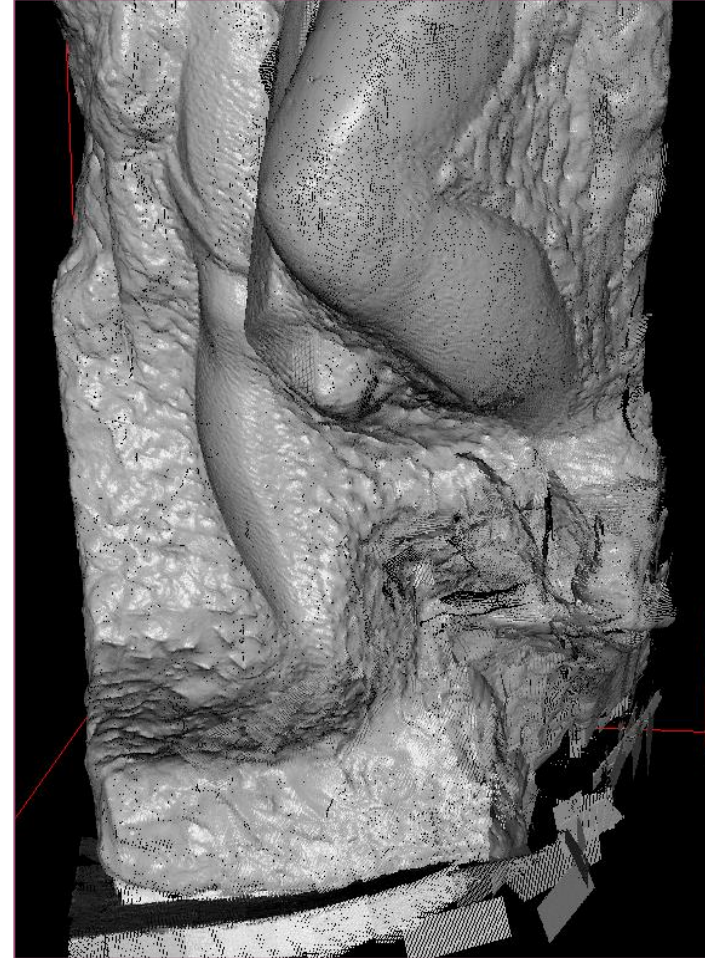
- Perform pairwise ICPs, record only optimal rotation/translation for each
- Decompose alignment graph into cycles
- While (change in error) > tolerance
 - For each cycle:
 - Spread out error equally among all scans in the cycle
 - For each scan belonging to more than 1 cycle:
 - Assign average transform to scan

Bad ICP in Globalreg

One bad ICP can throw off the entire model!



Correct Globalreg



Globalreg Including Bad ICP

Acknowledgement

- Data-Driven Shape Analysis - CS468 @Stanford.edu, Spring 2014
CSCI 621: Digital Geometry Processing SS 2017 @ USC
- 3D Scan Matching and Registration, Part II, ICCV 2005 Short Course

Literature

- Rusinkiewicz & Levoy, Efficient Variants of the ICP Algorithm, 3DIM 2001
- Chen & Medioni, “Object modeling by registration of multiple range images”, ICRA1991
- Besl & McKay: A method for registration of 3D shapes, PAMI 1992
- Horn: Closed-form solution of absolute orientation using unit quaternions, Journal Opt. Soc. Amer. 4(4), 1987
- Gelfand et al: Geometrically Stable Sampling for the ICP Algorithm, 3DIM, 2001.
- Pulli, Multiview Registration for Large Data Sets, 3DIM 1999

Thanks