Transfer entropy for model validation

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Refresher: Information, Entropy

Discrete case:

$$H(X) = -\sum_x p(X) \log p(x) \qquad \text{(Entropy)}$$

$$H(X, Y) = -\sum_x \sum_y p(x, y) \log((p(x, y)) \qquad \text{(Joint entropy)}$$

$$I(X; Y) = H(X) + H(Y) - H(X, Y) \qquad \text{(Mutual information)}$$

$$H(X|Y) = H(X) - I(X; Y) \qquad \text{(Conditional entropy)}$$

When X and Y are time series, and X^- and Y^- are their histories,

$$\begin{aligned} T_{Y \to X} &= I(X; Y^-|X^-) \qquad \text{(Transfer entropy)} \\ &= H(X|X^-) + H(Y^-|X^-) - H(X, Y^-|X^-) \end{aligned}$$

Transfer Entropy

"How much information does X share with the history of Y, given the history of X?"

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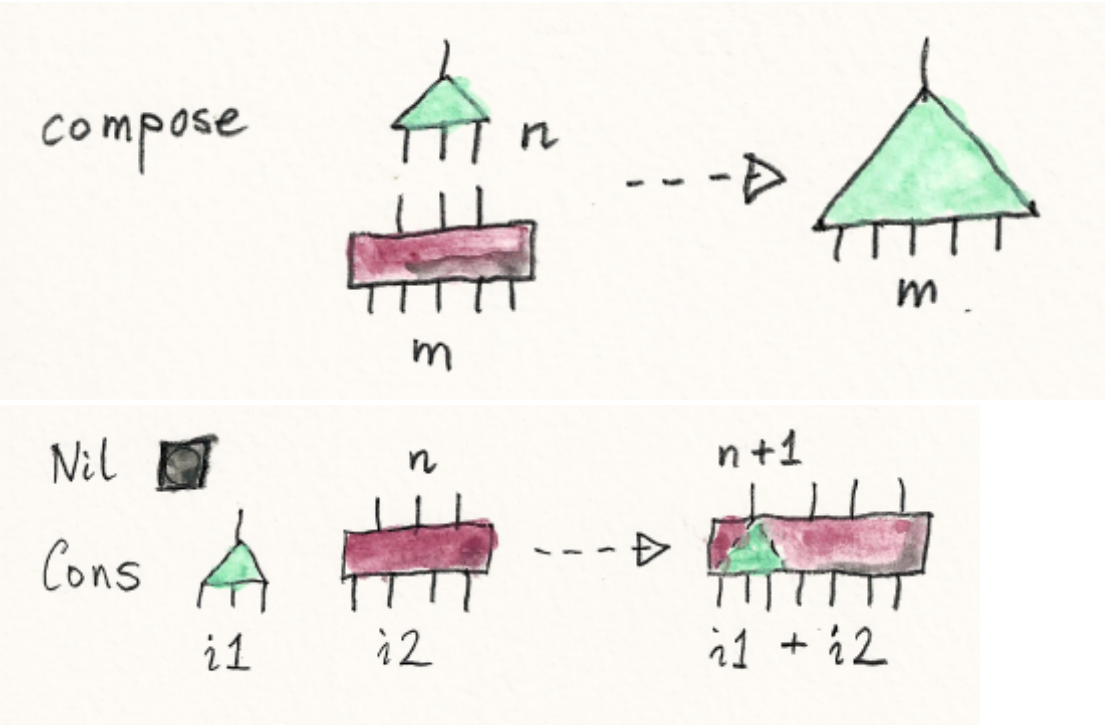
  - Should we try something with operads?



Graphics courtesy Bartosz Milewski (2015) (https://bartoszmilewski.com/)

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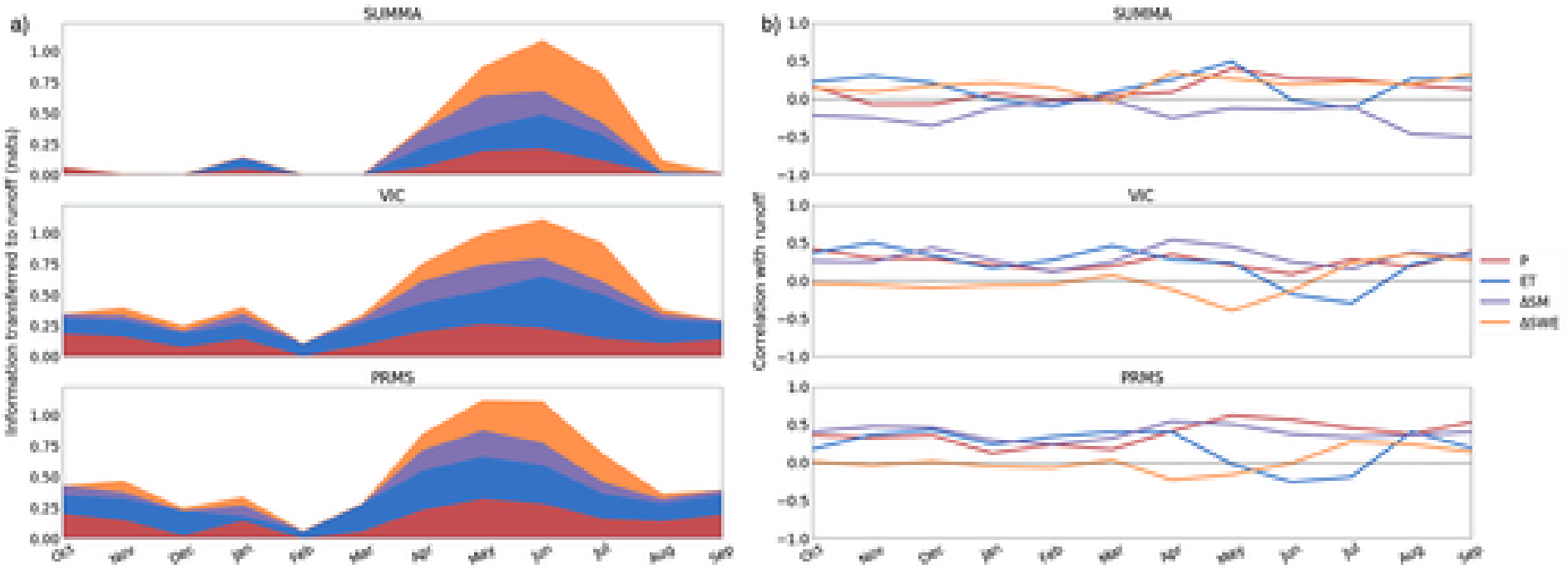
Causation?

- Philosophically unsound to "give" information a direction?
  - Direct/indirect causation?
  - Coin flipping problem, overstating information
  - Differential entropy is weird.

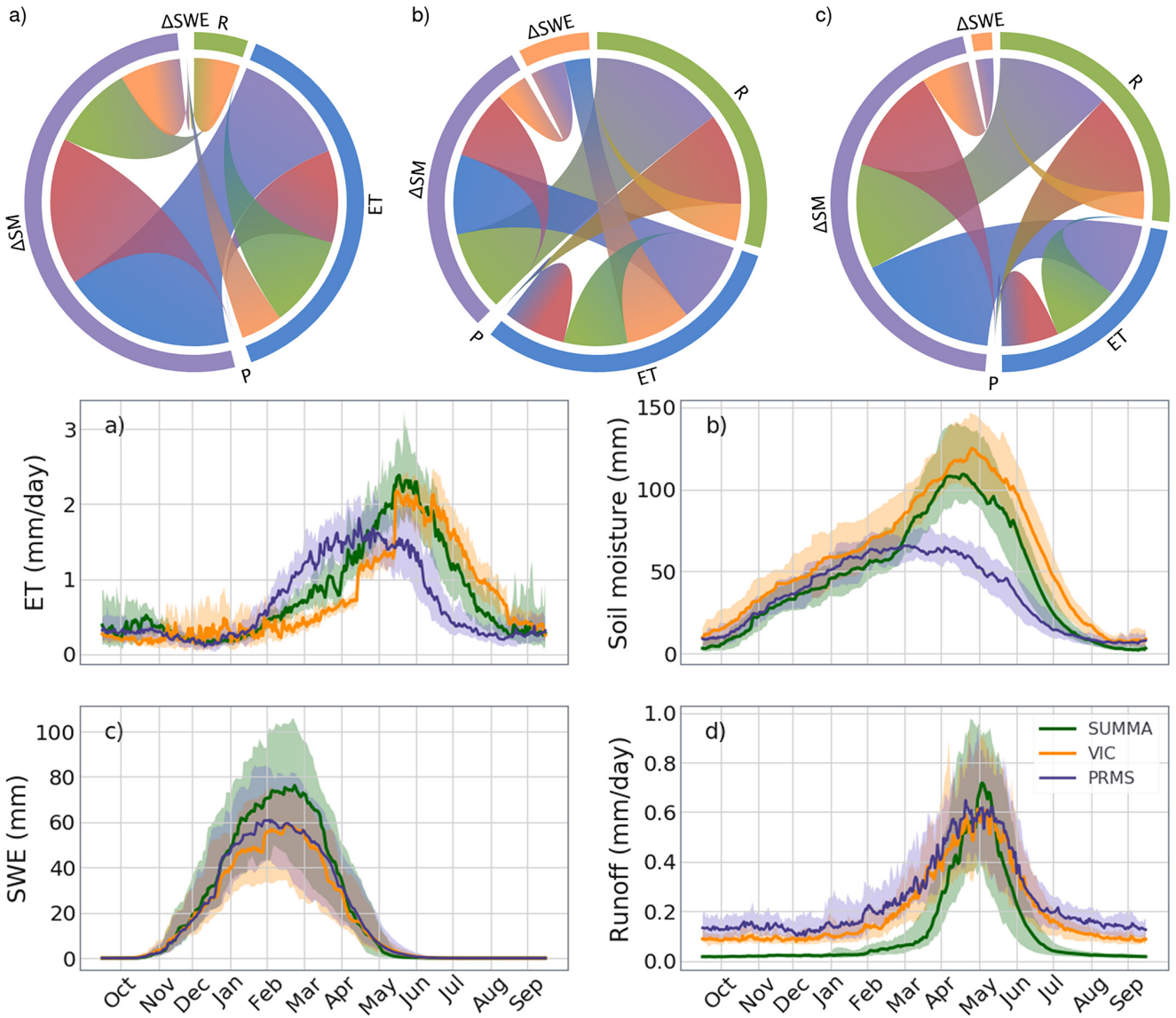
But could it still tell us something useful? Maybe?

"Runoff in SUMMA receives no information from precipitation and ET in the fall and winter. This shows that, [...] SUMMA does not respond directly to precipitation events until the SM shows a large increase due to snowmelt."



From Bennett, A, et al. (2019)

But could it still tell us something useful? Maybe not?



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