**Talk Title: Solving Neural Field Models on Curved Domains: An Isogeometric Approach with Geodesic Distances**

**Target Audience:** Researchers in computational neuroscience, numerical methods (FEM/IGA), geometry processing, and scientific computing.

**Duration:** (Adjust based on time slot: e.g., 15 mins for short talk, 30-45 mins for a seminar)

**I. Introduction & Motivation (5-10% of total time)**

* **A. The Brain as a Curved Surface:**
  + Brief overview of neural field models (NFMs): macroscopic models for cortical activity.
  + Traditional NFMs often solved on flat 2D domains (e.g., infinite plane, square).
  + **Problem:** The brain's cortex is a highly convoluted 3D surface. Flat approximations ignore crucial geometric effects (e.g., distances, curvature).
  + **Why does geometry matter?**
    - Local vs. long-range connections depend on true cortical distance.
    - Propagation of waves, pattern formation, and stability can be influenced by curvature.
    - Understanding neurological phenomena (e.g., epilepsy, migraines) might require realistic geometry.
* **B. Challenges of Solving PDEs on Curved Domains:**
  + Discretization difficulties (mesh generation, conforming elements).
  + Accurate representation of geometry.
  + Definition and computation of operators (Laplacian, convolution kernels) on surfaces.
* **C. Proposed Solution Overview:**
  + **Isogeometric Analysis (IGA):** A powerful numerical method for PDEs, leveraging CAD geometry representations (NURBS) for exact geometry and higher-order basis functions.
  + **Geodesic Distances:** Crucial for defining connectivity kernels in NFMs on curved surfaces.
  + **Goal:** Develop a robust framework to solve NFMs on arbitrary curved geometries, starting with a torus.

**II. Background: Neural Field Models (15-20% of total time)**

* **A. Basic NFM Equation:**
  + Introduce a simplified form of the NFM (e.g., Amari-type): ∂t∂u(x,t)​=−u(x,t)+∫Ω​w(x,x′)f(u(x′,t))dx′+I(x,t)
  + Explain components: u (neural activity), w (connectivity kernel), f (activation function), I (external input).
  + Focus on the **connectivity kernel w**: often a decaying function of distance (e.g., Gaussian, Mexican hat).
    - On flat domains: w(∣x−x′∣)
    - **On curved domains: w(dg​(x,x′)), where dg​ is the geodesic distance.** This is the key difference.
* **B. Typical NFM Behaviors:**
  + Briefly mention emergent phenomena: spatially localized bumps, traveling waves, oscillations.
  + Emphasize how these patterns are influenced by domain geometry.

**III. Isogeometric Analysis (IGA) for Curved Domains (20-25% of total time)**

* **A. Beyond Finite Elements (FEM):**
  + Briefly mention FEM's limitations for curved domains (approximation of geometry, mesh generation challenges, element quality).
  + Introduction to IGA: Seamless integration of CAD geometry (NURBS, T-splines) with numerical analysis.
* **B. NURBS Basis Functions:**
  + Review of NURBS (Non-Uniform Rational B-Splines):
    - Control points, knot vectors, weights, basis functions.
    - Key properties: exact representation of complex geometries (circles, spheres, free-form surfaces), higher-order continuity, local support.
  + **How IGA works:** Uses the same NURBS basis functions for both geometry mapping and solution approximation: x(ξ)=i=0∑n​Ni​(ξ)Pi​andu(ξ)≈j=0∑m​Rj​(ξ)Uj​ (where Ni​ are B-spline or NURBS basis functions, and Rj​ are the corresponding basis functions for the solution).
* **C. Advantages for Curved Domains:**
  + **Exact Geometry Representation:** No meshing error for geometric features (if they can be exactly represented by NURBS, like a torus).
  + **Higher-Order Continuity:** Smoother solutions, potentially higher accuracy for a given number of degrees of freedom.
  + **Simplified Mesh Refinement:** T-splines and hierarchical NURBS offer adaptive refinement capabilities.
  + **Direct Definition of Operators on the Surface:** Jacobian matrices for mapping from parametric to physical space are readily available.

**IV. Computing Geodesic Distances on Point Cloud Data (15-20% of total time)**

* **A. Why Geodesic Distance?**
  + Reiterate its importance for the NFM connectivity kernel w.
  + Challenges: Computing shortest paths on complex surfaces.
* **B. The Heat Method for Geodesic Distances:**
  + Overview of the method (Crane et al.): A robust and efficient approach for computing approximate geodesic distances on arbitrary surfaces (especially from point clouds/meshes).
  + **Core idea:** Solving a heat diffusion equation on the surface, then using the gradient of the steady-state heat distribution to find distances.
  + **Steps:**
    1. Solve a heat equation with a point source.
    2. Compute the gradient of the heat distribution.
    3. Normalize the gradient field.
    4. Solve a Poisson equation with the divergence of the normalized gradient field as the right-hand side.
  + **Advantages:** Computationally efficient, handles complex topology, robust to noise, adaptable to point clouds/discrete meshes.
* **C. Integration with IGA:**
  + We use the heat method on a discrete representation (e.g., triangular mesh or point cloud generated from the NURBS surface) to pre-compute the geodesic distance map.
  + This map is then sampled and used within the IGA framework to evaluate the convolution integral.

**V. Early Results: NFM on a Torus (20-25% of total time)**

* **A. Torus Geometry:**
  + Demonstrate how a torus is exactly represented by NURBS.
  + Show the parametric domain and mapping to the 3D physical torus.
  + Highlight the periodic boundary conditions inherent in the torus geometry.
* **B. Discretization and Implementation Details:**
  + Briefly discuss basis function setup on the parametric domain.
  + Assembling the system: Mass matrix, evaluation of convolution integral (using pre-computed geodesic distances).
  + Temporal integration scheme (e.g., Euler, Runge-Kutta).
* **C. Simulation Results:**
  + **Visualizations:** Show time evolution of neural activity u(x,t) on the torus surface.
  + **Examples of patterns:**
    - Stationary bump solutions.
    - Traveling waves around the torus (if applicable with chosen parameters).
    - Impact of geodesic vs. Euclidean distance (qualitative comparison if possible).
  + **Key takeaways from torus simulations:**
    - Validation of the IGA-based approach for NFMs on curved domains.
    - Demonstration of the influence of curvature/topology on NFM dynamics.

**VI. Discussion & Future Work (5-10% of total time)**

* **A. Summary of Contributions:**
  + Successfully implemented NFM solver using IGA.
  + Integrated geodesic distance computation via the heat method.
  + Demonstrated feasibility on a non-trivial curved domain (torus).
* **B. Challenges and Limitations:**
  + Computational cost of integral term (especially for high-dimensional kernels).
  + Generalization to arbitrary cortical geometries (need real brain surface data).
  + Handling complex topologies and self-intersections.
* **C. Future Directions:**
  + Apply to realistic brain geometries (from MRI data).
  + Investigate different NFM models and their parameters.
  + Develop adaptive refinement strategies (e.g., T-splines) for localized phenomena.
  + Explore more advanced numerical integration techniques for efficiency.
  + Comparison with experimental data.

**VII. Q&A (Remaining time)**

**Visual Aids Suggestions:**

* Slides with clear equations and diagrams.
* Animations of NFM activity on the torus.
* Illustrations comparing flat vs. curved domains.
* Diagrams of the heat method steps.
* Screenshots of the IGA basis functions on the torus.

**Tips for Delivery:**

* Start strong with the motivation.
* Clearly define technical terms.
* Use analogies where helpful.
* Focus on the "why" and "what" before the "how."
* Keep the math concise; emphasize concepts.
* Practice timing!

This outline provides a solid structure. Remember to tailor the depth of each section to the allocated time and your specific findings. Good luck!