Spring 2021

TEXAS
The University of Texas at Austin

Pydcol: Control of ODE Systems with “Minimal Effort”

May 6th, 2021

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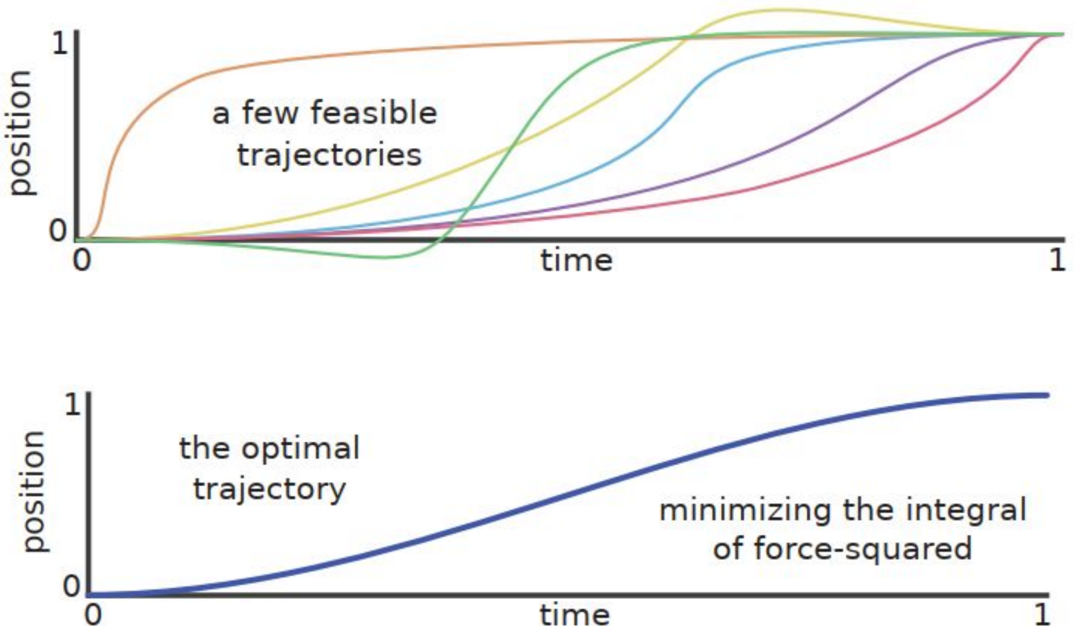
Overview

- Introduction
- Methodology
- Examples
- Demonstration & Documentation
- Conclusion



Ref. [2]

Introduction

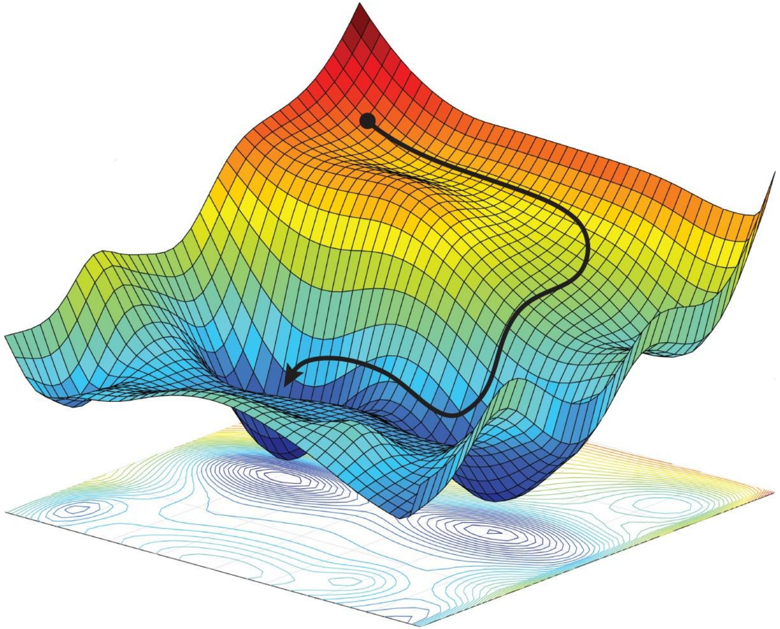
- pydcol automates direct collocation problem for
  - ODE systems: $\dot{X} = f(X, u)$
  - Fixed final time and state
  - Objective: Minimal control effort:

$$\min_{X,u} \int_{t_0}^{t_f} u^2 dt$$

$$s.t.\ X(t_0) = X_{start},\ X(t_f) = X_{goal}$$

Introduction

- Simultaneous Discretization
- Non-linear Program (NLP)
- Design a library to handle
  - Symbolic manipulation
  - Multiple integration methods
  - Choice of optimizer



Ref. [4]

Methodology

$$\dot{X} = f(X, U)$$

$$X(t_0), X(t_f)$$

$$N$$

User Input:
Problem Definition

→ SymPy

Sym Objective
Sym Constraints
Sym Derivatives

→ **Numpy**

Num Objective
Num Constraints
Num Derivatives

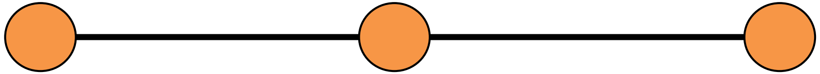
→ NLP Solver

↓

Output: Locally-Optimal X,U Trajectory

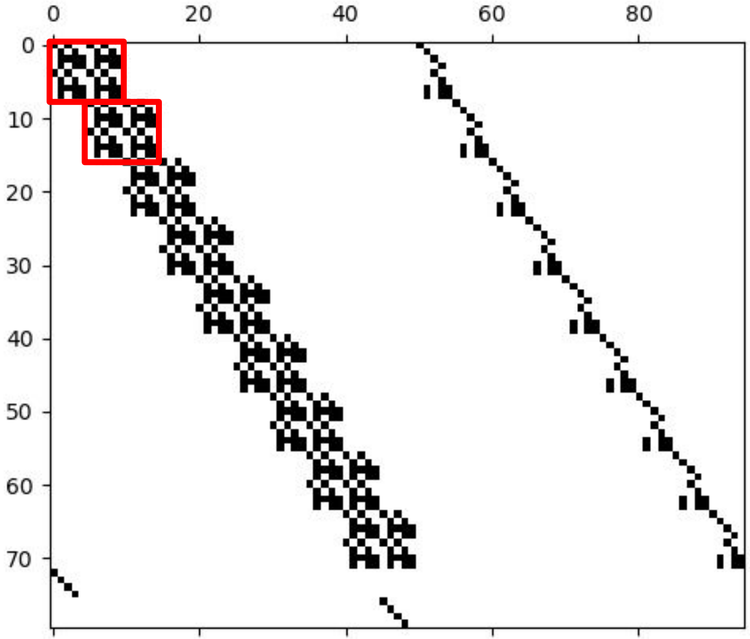
Methodology

- General equation for a node used instead of writing N equations
- Reduces computational cost of Jacobian and Hessian



$X_i \qquad X_{i+0.5} \qquad X_{i+1}$

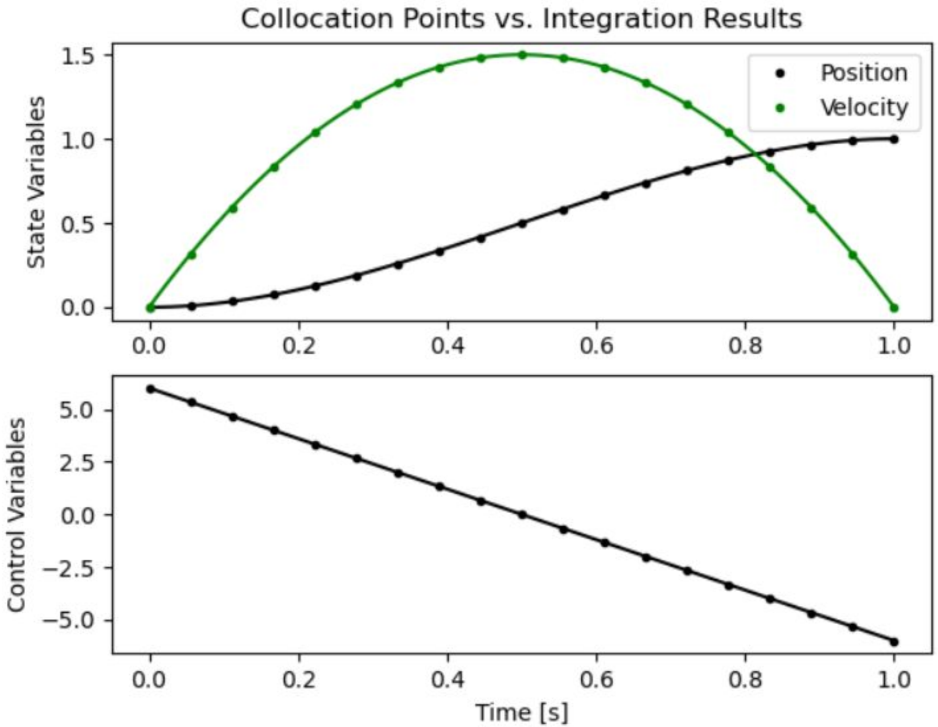
Sparsity Pattern for Jacobian of Equality Constraints

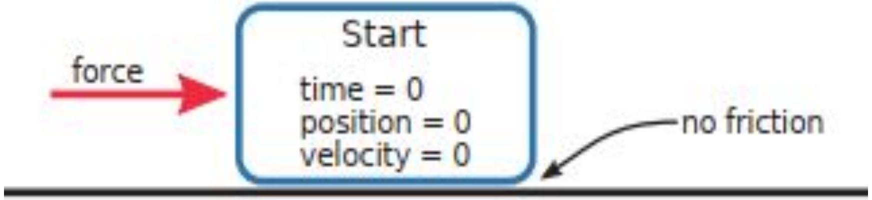


Examples

- A set of test problems was selected (sorted by difficulty to solve):
  - Box move (linear)
  - Cartpole swing-up (slightly nonlinear)
  - Double pendulum swing-up (very nonlinear)
  - Lunar lander (stiff, multiple inputs)

Box Move



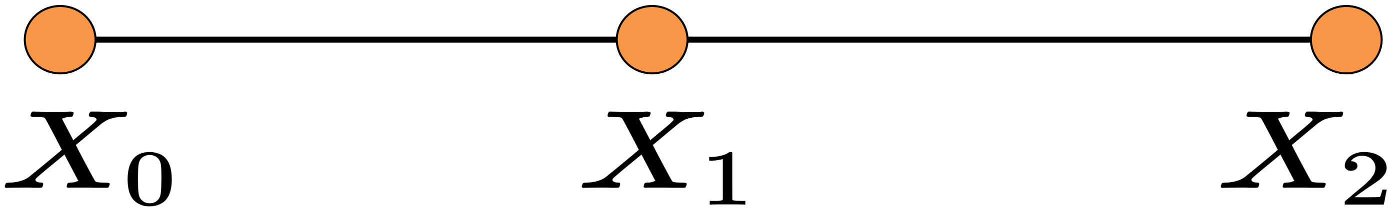


Ref. [2]

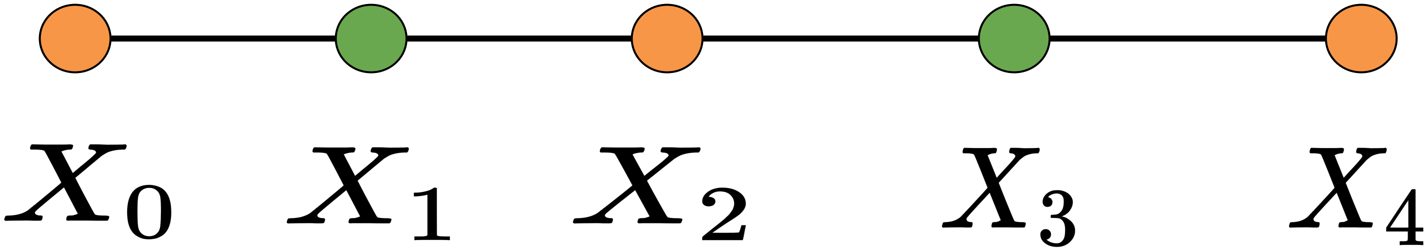
Dots = Collocation, Lines = IVP

Error Analysis

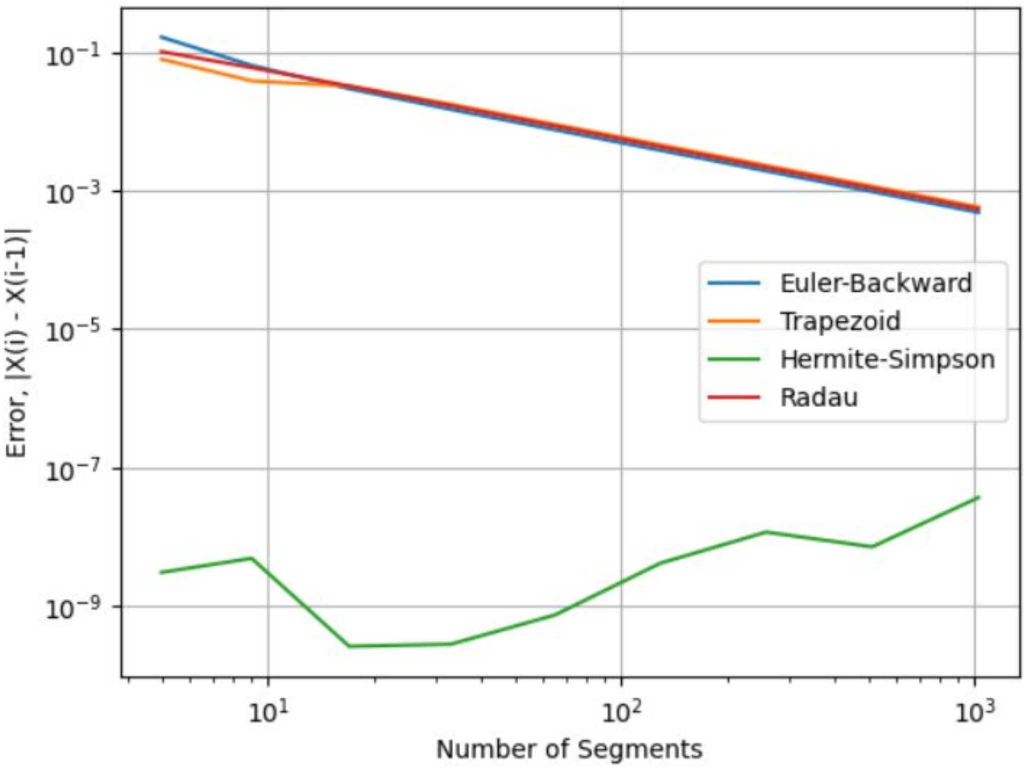
Iteration 1:



Iteration 2:



Box Move - Error Analysis

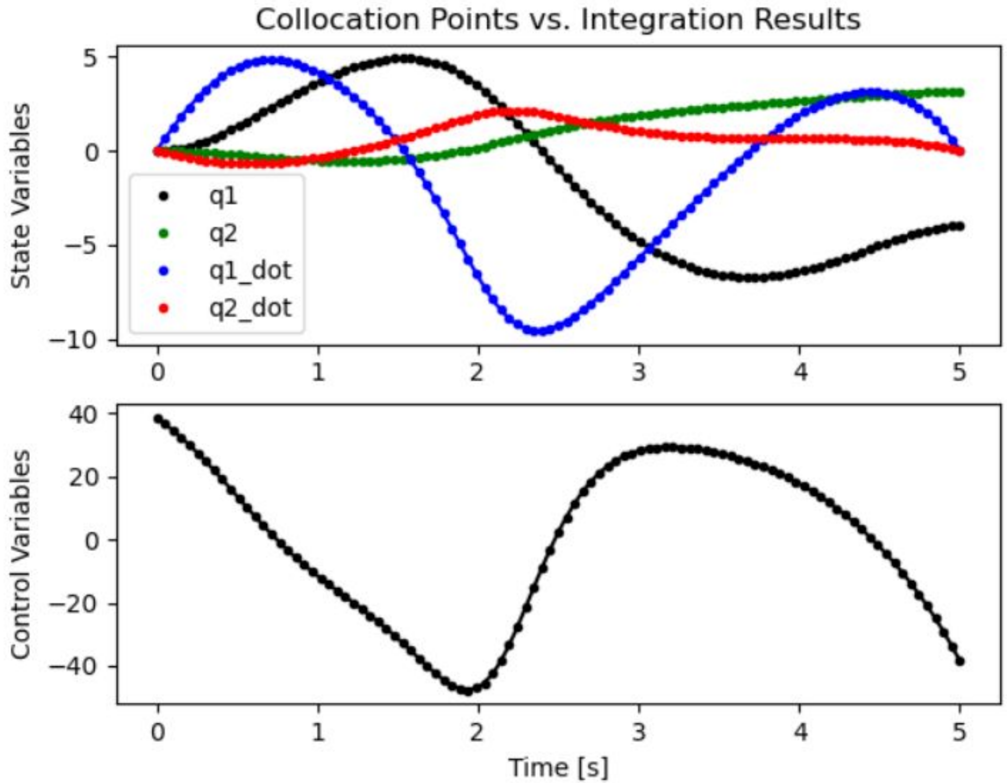


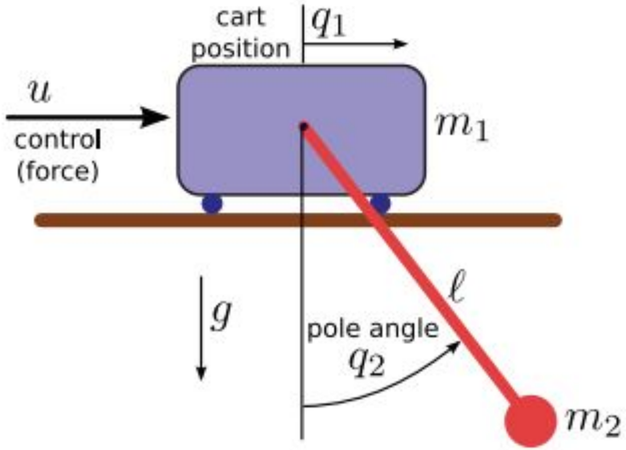
$$e_i = \frac{1}{N} \sum_{j=1}^{N} ||X_{i,2*j} - X_{i-1,j}||_2^2$$

$$O(e) \approx log(\frac{e_{i-1}}{e_i})/log(2)$$

Integration Method	Estimated Order of Accuracy
Euler Backward	1
Trapezoid	1
Hermite-Simpson	**N/A**
Radau IIA	**1**

Cartpole Swing-up





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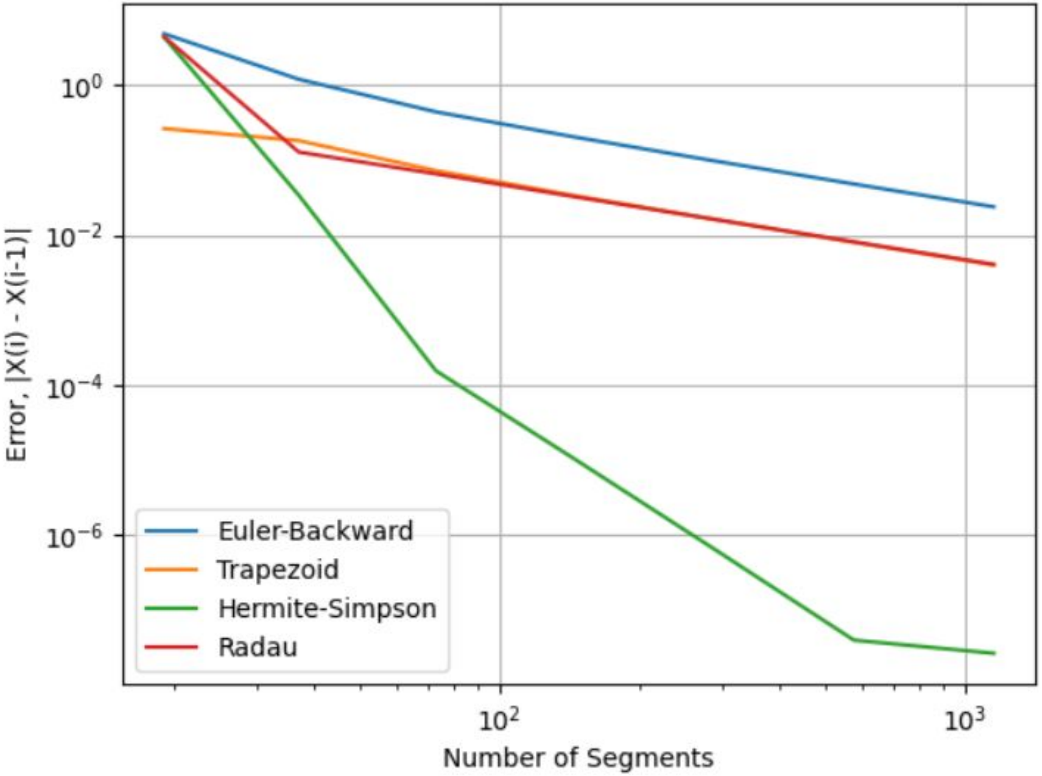
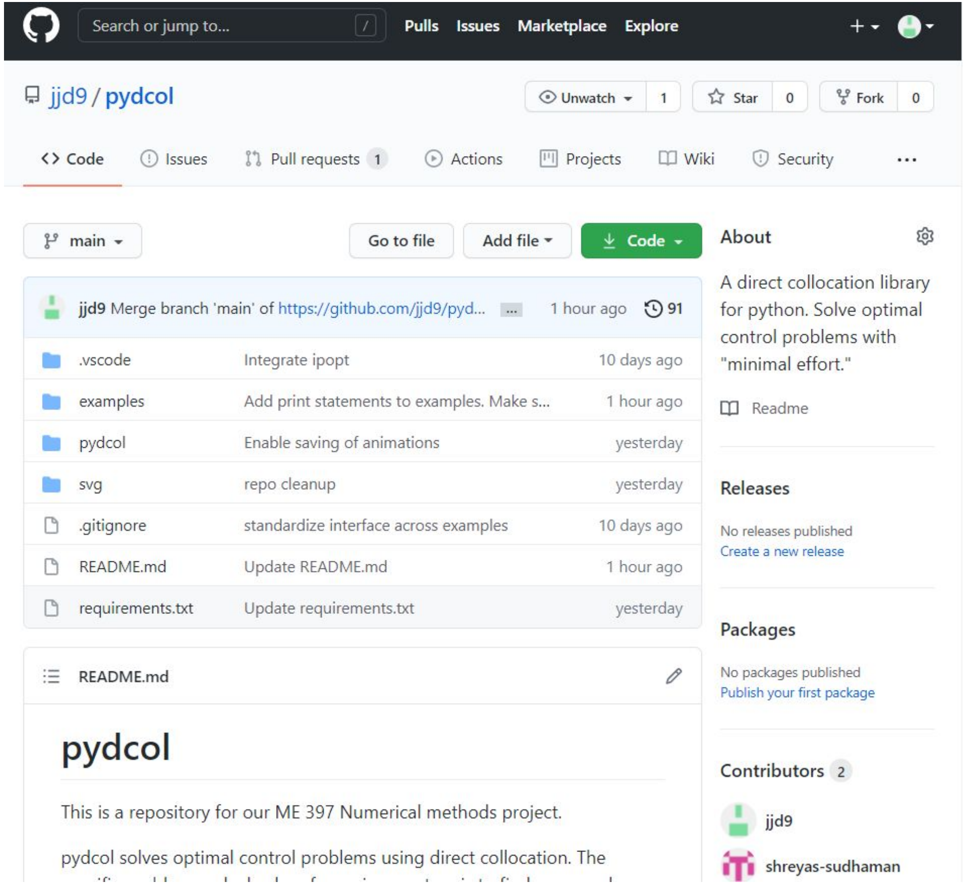
Cartpole - Error Analysis



Integration Method	Estimated Order of Accuracy
Euler Backward	1
Trapezoid	1
Hermite-Simpson	4
Radau IIA	1

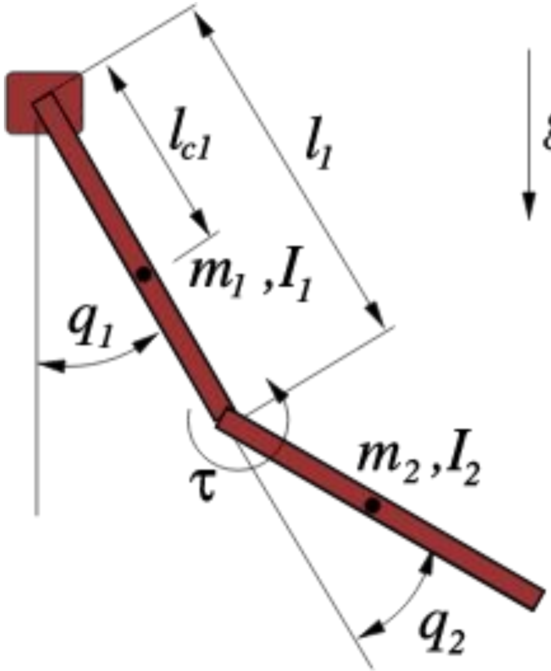
Documentation

- The documentation and source code for pydcol are available on Github
- We welcome feedback through issues or pull requests



Double Pendulum Swing-up

Go to the code demo!



Ref. [1]

Conclusions

- pydcol is a tool for solving a common variation of the optimal control problem
- Our testing showed that pydcol's hermite-simpson method is well suited for optimal control of mechanical systems
- Developing this tool yielded some practical insights into direct collocation (how you integrate the objective matters, sparsity is great but gets complex quickly, etc.)

References

[1] Russ Tedrake. Underactuated Robotics: Algorithms for Walking, Running, Swimming, Flying, and Manipulation (Course Notes for MIT 6.832). Downloaded on [date] from http://underactuated.mit.edu/

[2] Kelly, M. An Introduction to Trajectory Optimization: How to Do Your Own Direct Collocation. SIAM Rev. 2017, 59 (4), 849–904. https://doi.org/10.1137/16M1062569

[3] Assignment 4: Lunar Lander Solution https://web.aeromech.usyd.edu.au/AMME3500/Course_documents/material

[4] A. Amini et al. “Spatial Uncertainty Sampling for End-to-End Control”. NeurIPS Bayesian Deep Learning 2018.

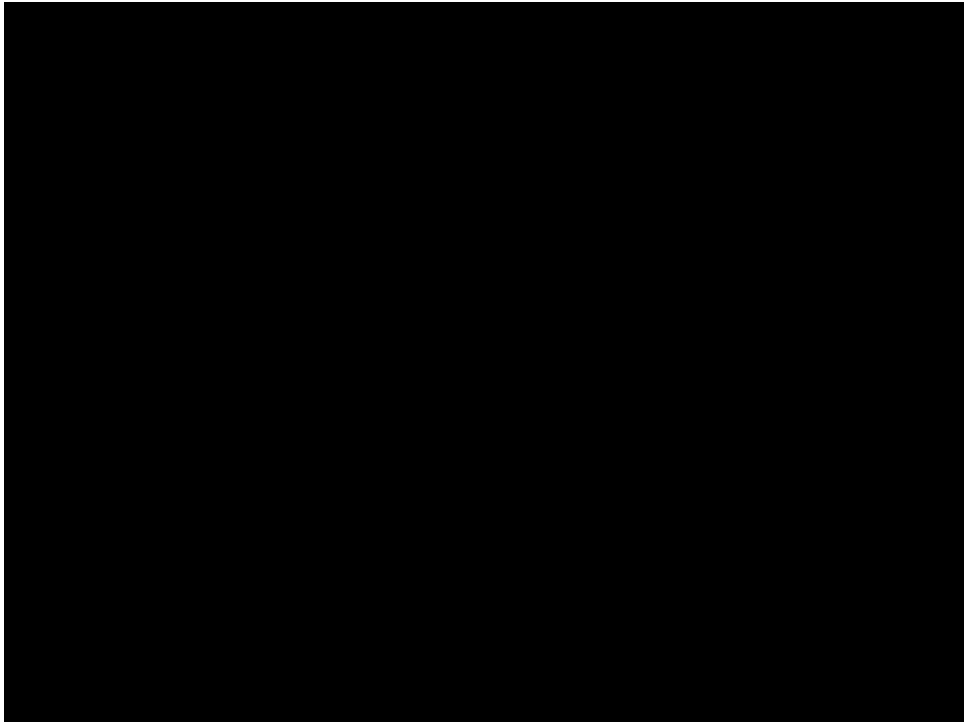
Acknowledgement

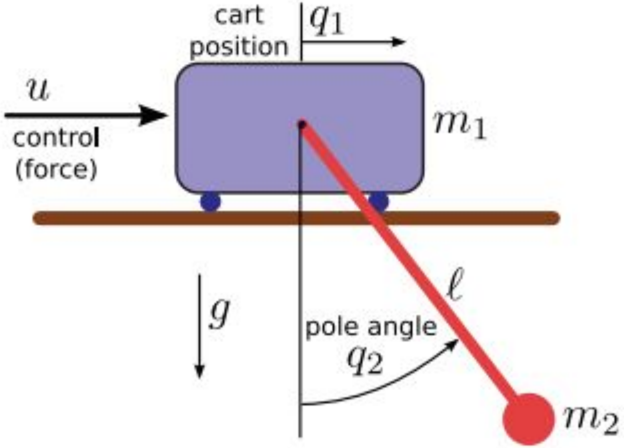
Thank you to Professor Subramanian for his instruction in this course and his assistance with this project.

Thank you!

Questions?

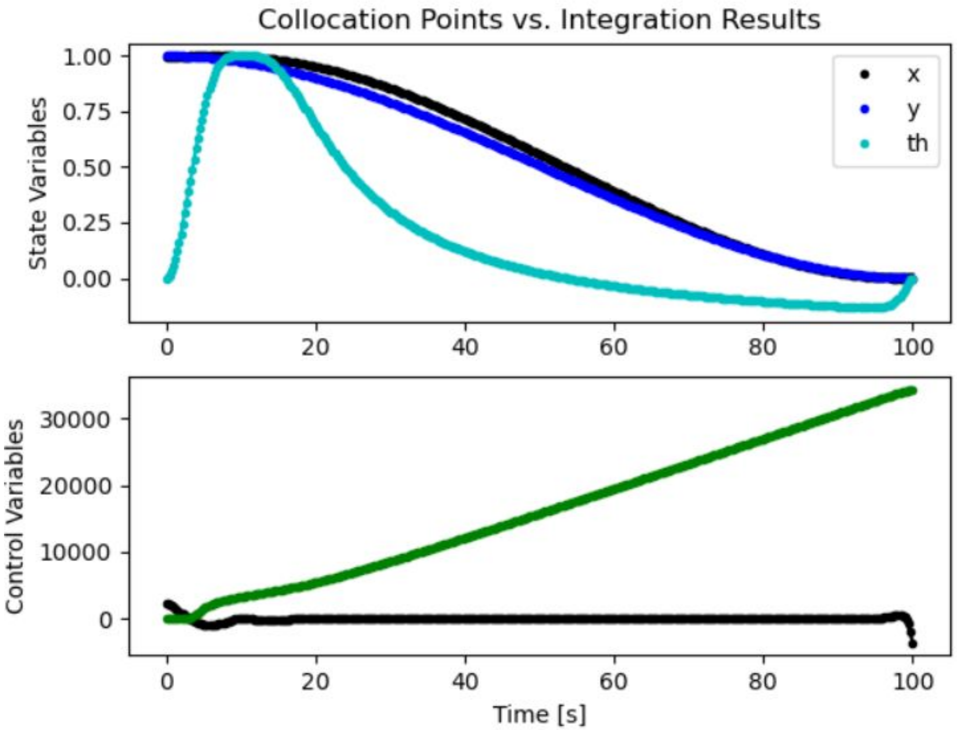
Cartpole Swing-up

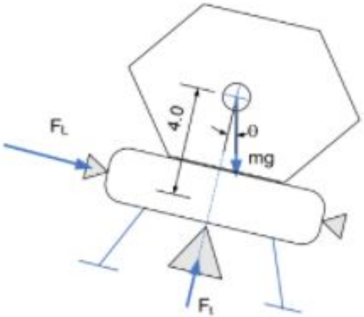


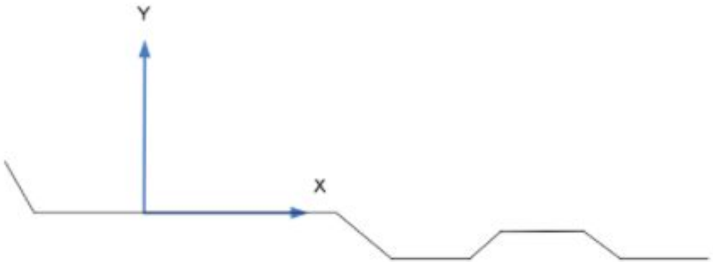


Ref. [2]

Lunar Lander







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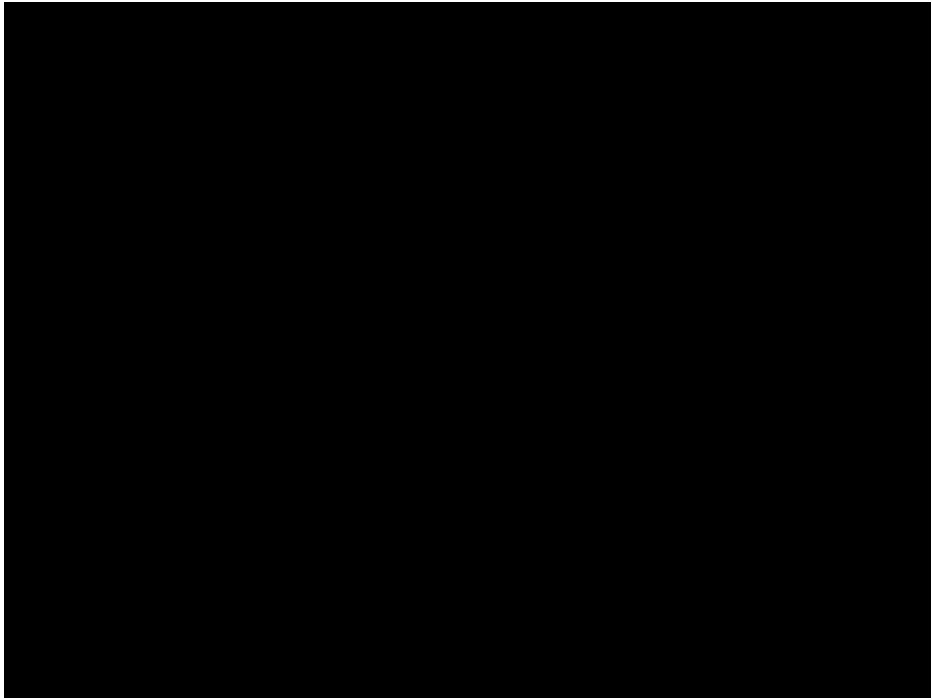
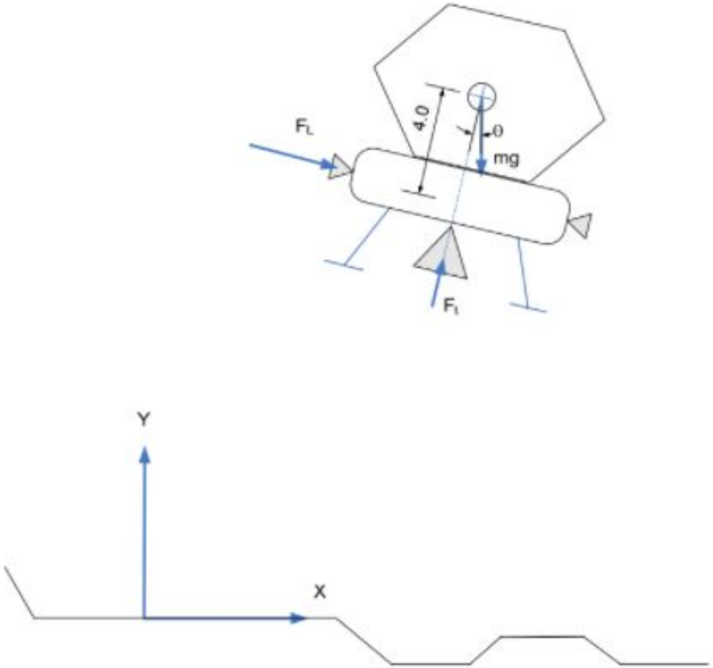
Lunar Lander



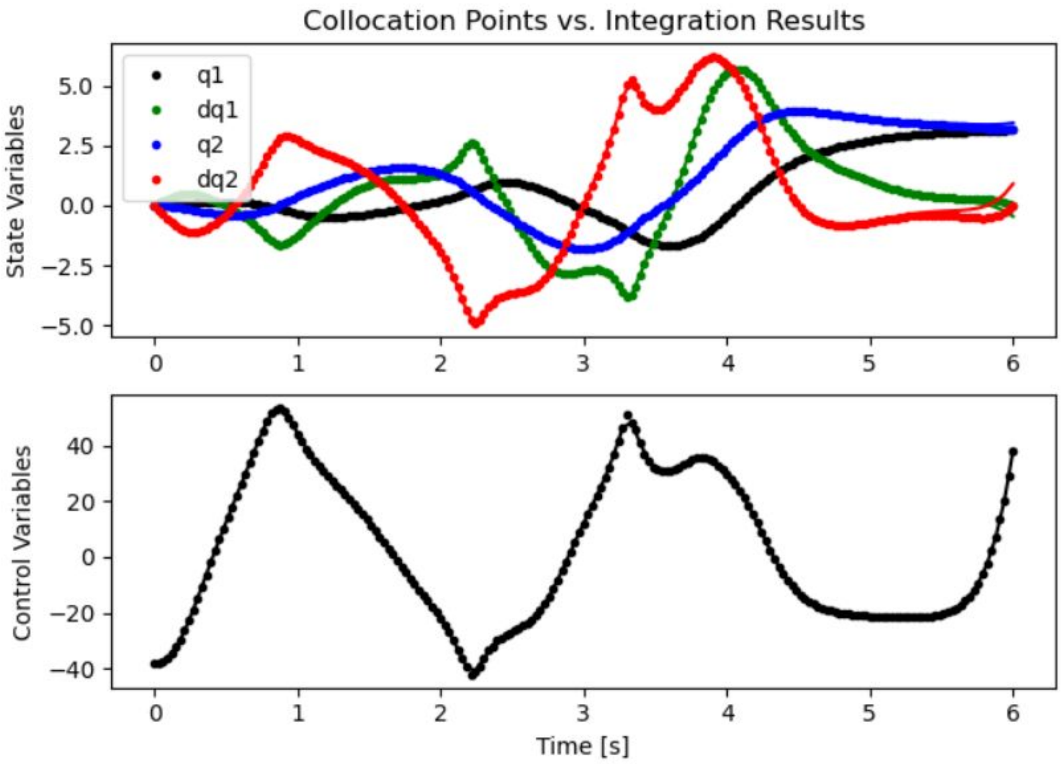


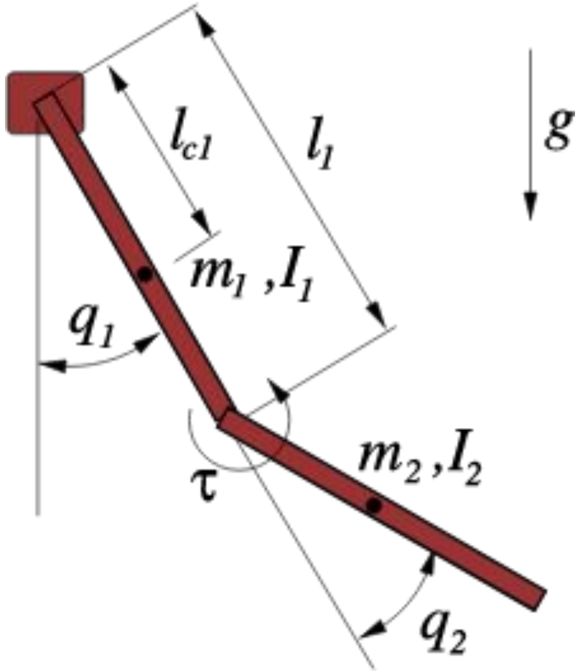
Ref. [3]

Room for Improvement

- Using final time as an optimization variable
- Interfacing with a Sequential Quadratic Programming solver
- Handling arbitrary objectives and integration schemes
- Handling DAE’s

Double Pendulum Swing-up





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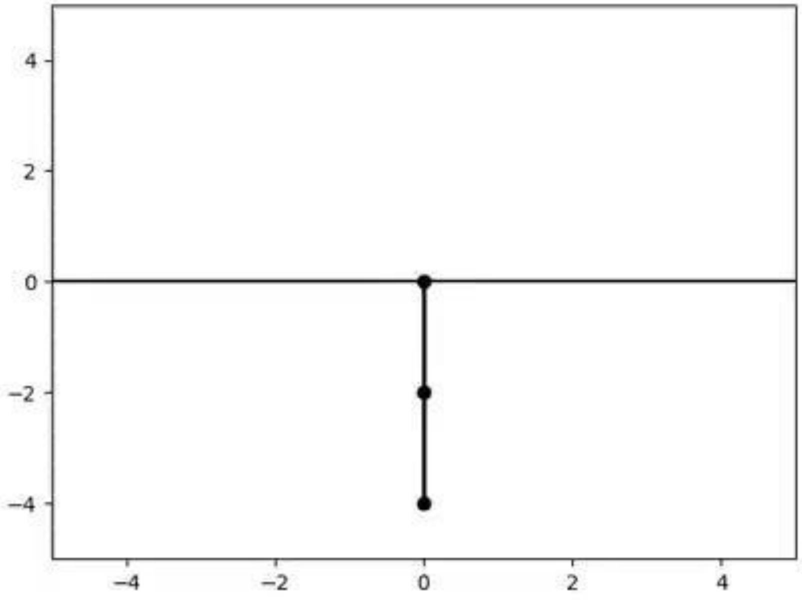
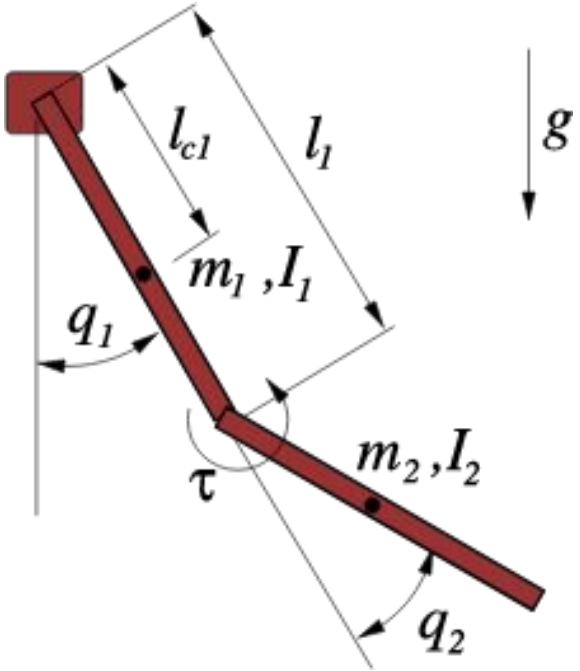
Double Pendulum Swing-up





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