method described [1]

$$xy' + y = \ln x + 1$$

first we solve the homogeneous equation $xy' + y = 0$:

$$xy' + y = 0$$
$$\frac{dy}{y} = -\frac{dx}{x}$$
$$\ln y = A - \ln x$$
$$y = \frac{C}{x}$$

now assume that $C = C(x)$ and substitute $y = \frac{C(x)}{x}$ into the original equation (for more complex situations see [1]):

$$x(\frac{C'(x)}{x} - \frac{C(x)}{x^2}) + \frac{C(x)}{x} = \ln x + 1$$
$$C'(x) = \ln x + 1$$

now, integrating (the $\ln x$ term is integrated by parts):

$$C(x) = x\ln x - \int \frac{x}{x}dx + x + \hat{C}$$
$$C(x) = x\ln x + \hat{C}$$

so getting back to our y

$$y = \ln x + \frac{\hat{C}}{x}$$

References

[1] https://en.wikipedia.org/wiki/Variation_of_parameters