AERO 417 Homework 2

```
import sympy as sy
import numpy as np
import matplotlib.pyplot as plt
import IPython
from IPython.display import display
from PIL import Image
import matplotlib
from matplotlib.lines import Line2D
import scipy
matplotlib.rcParams['mathtext.fontset'] = 'stix'
matplotlib.rcParams['font.family'] = 'STIXGeneral'
matplotlib.rcParams['font.size'] = 18


def displayH(a1,a2='', a3='', a4='', a5='', a6='', a7='',):
    latex_a1 = sy.latex(a1)
    latex_a2 = sy.latex(a2)
    latex_a3 = sy.latex(a3)
    latex_a4 = sy.latex(a4)
    latex_a5 = sy.latex(a5)
    latex_a6 = sy.latex(a6)
    latex_a7 = sy.latex(a7)
    display( IPython.core.display.Math(latex_a1 + latex_a2 + latex_a3 + latex_a4 + latex_a
```

1) A jet engine is traveling through the air with the forward velocity of $300\frac{m}{s}$. The exhaust gases leave the nozzle with an exit velocity of $800\frac{m}{s}$ with respect to the nozzle. If the massflow rate through the engine is $10\frac{kg}{s}$, determine the jet engine thrust. The exit plane static pressure is $80kPa$, inlet plane static pressure is $20kPa$, ambient pressure surrounding the engine is $20kPa$, and the exit plane area is $4.0m^2$.

$$T = \dot{m}(u_e - u_)) + A_e(P_e - P_a)$$

```
u = 300
u_exit = 800
mass_flow_rate = 10
P_exit = 80*1000
P_inlet = 20*1000
P_ambient = 20*1000
A_exit = 4

Thrust = mass_flow_rate*(u_exit-u)+A_exit*(P_exit-P_ambient)
displayH("T =",Thrust,["N"])
```

T =245000 [N]

2) Describe the differences between Brayton Cycle and a Real Gas Turbine Cycle. Make diagrams to explain the losses associated with a real engine.

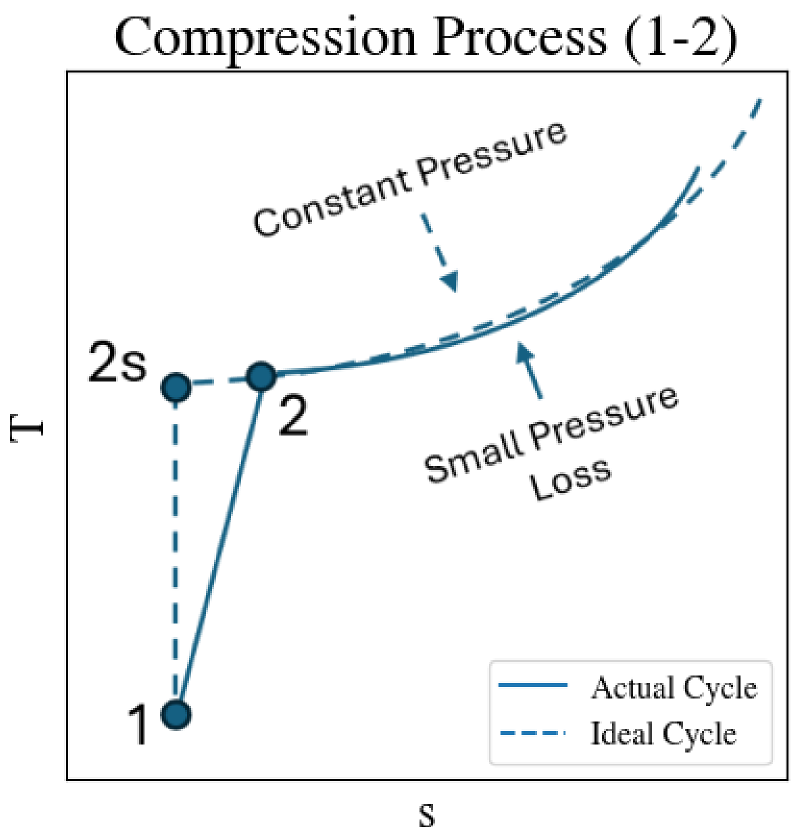
The Brayton Cycle is an idealized thermodynamic cycle that models the behavior of gas turbines. In contrast, the Real Gas Turbine Cycle accounts for the inefficiencies and losses that occur in an actual turbine system.The thermodynamic cycle consists of three processes going from states 1-4

(All figures were generated in PowerPoint, which made it hard to draw pressure lines, the Brayton Cycle pressure lines represent constant pressure, and the Real Gas Turbine Cycle pressure lines represent a small pressure loss)

Process (1-2) Compression:

In this process the pressure of the gas is increased with compression in a compressor, and the temperature also increases. The Brayton Cycle assumes that there is no entropy generation in this process, whereas in the Real Gas Turbine Cycle, entropy is produced. This entropy difference leads to a higher Temperature/ work draw in the real cycle, and is captured in the compressor efficiency term (η_c).

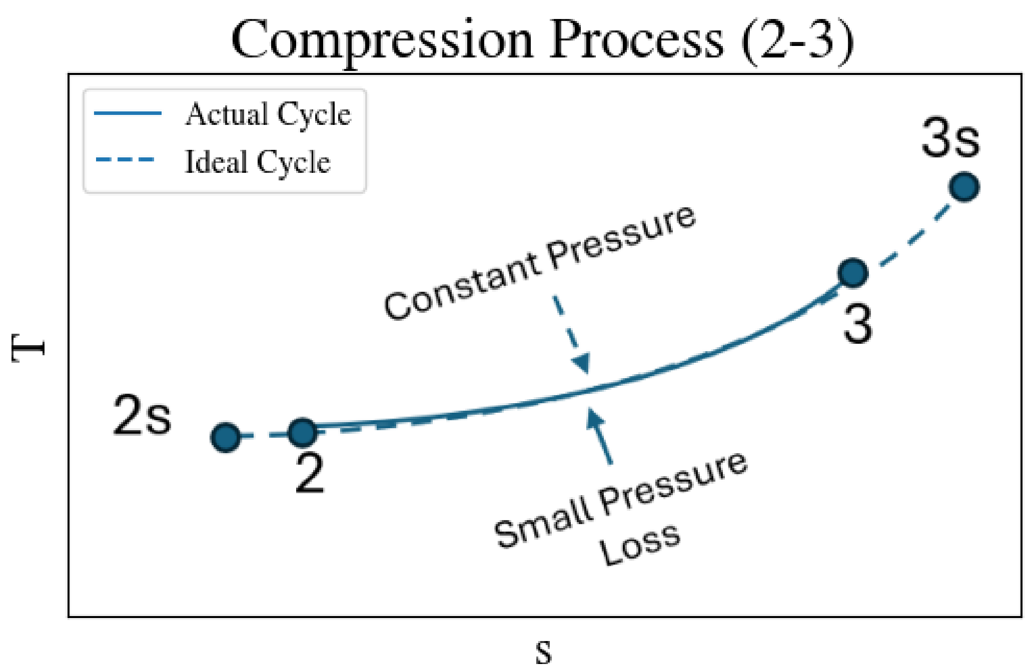
```
img = np.asarray(Image.open("Process_12.PNG"))
legend_elements = [Line2D([0], [0], color='tab:blue', label='Actual Cycle'),
                   Line2D([0], [0], color='tab:blue', linestyle="--", label='Ideal Cycle')
plt.imshow(img);plt.title("Compression Process (1-2)")
plt.tick_params(axis='x',which='both',bottom=False,top=False,labelbottom=False)
plt.tick_params(axis='y',which='both',left=False,right=False,labelleft=False)
plt.xlabel("s");plt.ylabel("T");plt.legend(handles = legend_elements,loc="lower right",pro
```



Process (2-3) Combustion:

In this process the pressure of the gas stays constant during fuel combustion, and the temperature increases by a large amount with the introduction of the fuel. The Brayton Cycle assumes that there no pressure loss, whereas in the Real Gas Turbine Cycle, there is a small pressure loss. The losses associated with this process lower the final temperature and enthalpy of state 3, and is captured in the combustion efficiency term (ξ_{comb}).

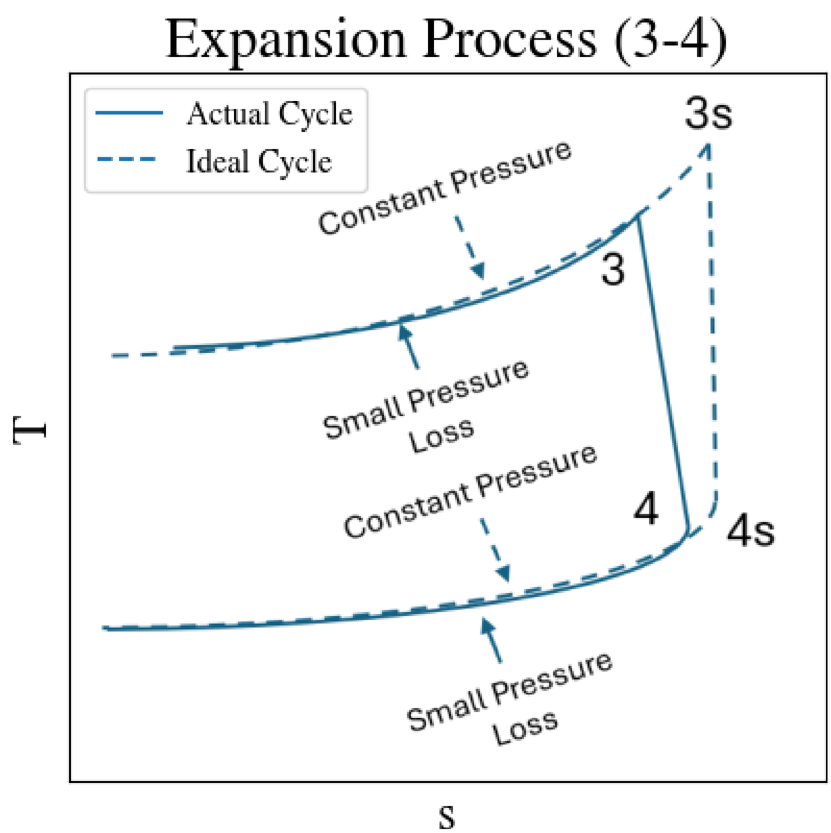
```
img = np.asarray(Image.open("Process_23.PNG"))
legend_elements = [Line2D([0], [0], color='tab:blue', label='Actual Cycle'),
                   Line2D([0], [0], color='tab:blue', linestyle="--", label='Ideal Cycle')
plt.imshow(img);plt.title("Compression Process (2-3)")
plt.tick_params(axis='x',which='both',bottom=False,top=False,labelbottom=False)
plt.tick_params(axis='y',which='both',left=False,right=False,labelleft=False)
plt.xlabel("s");plt.ylabel("T");plt.legend(handles = legend_elements,loc=2,prop={'size': 1
plt.show()
```



Process (3-4) Expansion:

In this process the pressure of the gas is lowered with expansion, and the temperature decreases as the flow is cooled. The Brayton Cycle assumes that there is no entropy generation in this process, whereas in the Real Gas Turbine Cycle, entropy is produced. This entropy difference leads to a lower Temperature/ work production in the real cycle, and is captured in the turbine efficiency term (η_t).

```
img = np.asarray(Image.open("Process_34.PNG"))
legend_elements = [Line2D([0], [0], color='tab:blue', label='Actual Cycle'),
                   Line2D([0], [0], color='tab:blue', linestyle="--", label='Ideal Cycle')
plt.imshow(img);plt.title("Expansion Process (3-4)")
plt.tick_params(axis='x',which='both',bottom=False,top=False,labelbottom=False)
plt.tick_params(axis='y',which='both',left=False,right=False,labelleft=False)
plt.xlabel("s");plt.ylabel("T");plt.legend(handles = legend_elements,loc=2,prop={'size': 1
```

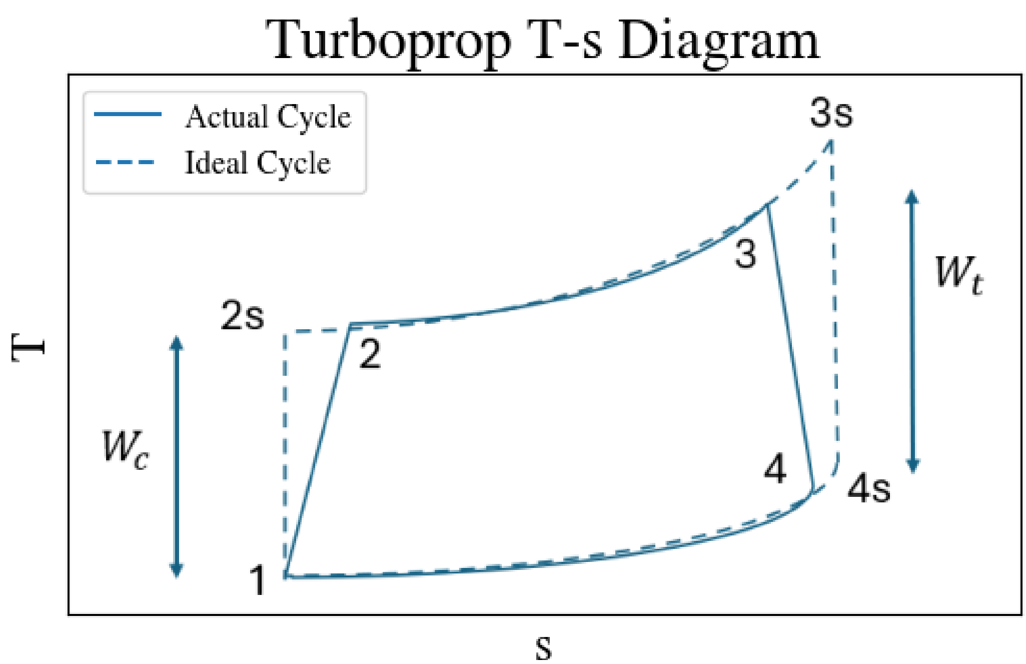


3) A turboprop engine has the following data:

- $T_1 = 288K$
- $P_1 = 101325Pa$
- $\frac{P_{02}}{P_{01}} = 10.3$
- $\eta_c = 0.87$
- $\eta_{mech} = 0.99$
- $\frac{P_{03}}{P_{02}} = 0.95$
- $LHV = 41.00\frac{MJ}{kg}$
- $\xi_{comb} = 0.99$
- $T_3 = 1190K$
- $\eta_t = 0.88$
- $T_4 = 873K$
- $\dot{m} = 108\frac{kg}{s}$
- $C_{p_{air}} = 1005\frac{J}{kgK}$
- $C_{p_{gas}} = 1150\frac{J}{kgK}$
- $R = 287\frac{J}{kgK}$

Draw the T-s diagram:

```
img = np.asarray(Image.open("Ts_Diagram.PNG"))
legend_elements = [Line2D([0], [0], color='tab:blue', label='Actual Cycle'),
                   Line2D([0], [0], color='tab:blue', linestyle="--", label='Ideal Cycle')
plt.imshow(img)
plt.title("Turboprop T-s Diagram")
plt.tick_params(axis='x',which='both',bottom=False,top=False,labelbottom=False)
plt.tick_params(axis='y',which='both',left=False,right=False,labelleft=False)
plt.xlabel("s")
plt.ylabel("T")
plt.legend(handles = legend_elements,loc=2,prop={'size': 12})
plt.show()
```



Determine the turbine shaft power:

```
T_1 = 288
P_1 = 101325
P_2_P_1 = 10.3
eta_c = 0.87
eta_mech = 0.99
P_3_P_2 = 0.95
LHV = 41.000*10**6
xi_comb = 0.99
T_3 = 1190
eta_t = 0.88
T_4 = 873
mass_flow_rate = 108
cp_air = 1005
cp_gas = 1150
R = 287
gamma_air = 1/(1-R/cp_air)
gamma_gas = 1/(1-R/cp_gas)
```

$$W_{t_s} = C_{p_{gas}}(T_3 - T_4)$$

$$W_t = \eta_t W_{t_s}$$

$$P_t = \dot{m} W_t$$

```
work_turbine_ideal = cp_gas*(T_3-T_4)
work_turbine = work_turbine_ideal*eta_t

turbine_power = work_turbine*mass_flow_rate
displayH(sy.Symbol("P_{t} ="),turbine_power)
```

$P_t = 34646832.0$

Determine the air-fuel ratio:

$$T'_{02} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{02} = T_{01} + \frac{T'_{02} - T_{01}}{\eta_c}$$

$$f = \frac{1}{\lambda minL}$$

$$h_{02} + \frac{\xi_{comb}LHV}{\lambda minL} = \left(1 + \frac{1}{\lambda minL}\right)h_{03}$$

$$h_{02} + \xi_{comb}LHVf = (1+f)h_{03}$$

$$f = \frac{h_{02} - h_{03}}{h_{03} - \xi_{comb}LHV} = \frac{(C_{p_{air}}T_{02}) - (C_{p_{gas}}T_{03})}{(C_{p_{gas}}T_{03}) - \xi_{comb}LHV}$$

```
T_2prime = T_1*P_2_P_1**((gamma_air-1)/gamma_air)
T_2 = T_1+(T_2prime-T_1)/eta_c

f = (cp_air*T_2-cp_gas*T_3)/(cp_gas*T_3-xi_comb*LHV)
displayH(sy.Symbol("f ="),f)
```

$f = 0.0194840474843885$