AERO 417 Homework 3

```python
import sympy as sy
import numpy as np
import matplotlib.pyplot as plt
import IPython
from IPython.display import display
from PIL import Image
import matplotlib
from matplotlib.lines import Line2D
import scipy
matplotlib.rcParams['mathtext.fontset'] = 'stix'
matplotlib.rcParams['font.family'] = 'STIXGeneral'
matplotlib.rcParams['font.size'] = 18

def displayH(a1,a2='', a3='', a4='', a5='', a6='', a7='',):
    latex_a1 = sy.latex(a1)
    latex_a2 = sy.latex(a2)
    latex_a3 = sy.latex(a3)
    latex_a4 = sy.latex(a4)
    latex_a5 = sy.latex(a5)
    latex_a6 = sy.latex(a6)
    latex_a7 = sy.latex(a7)
    display( IPython.core.display.Math(latex_a1 + latex_a2 + latex_a3 + latex_a4 + latex_a
```

1) Why centrifugal compressors are not used in long-range aircraft as a component of the main engines?

Centrifugal compressors are not typically used in long-range aircraft engines because they generally have lower efficiency compared to axial-flow compressors. Achieving high pressure ratios is also more challenging with centrifugal compressors. While axial compressors can easily stack multiple stages to steadily increase pressure, centrifugal compressors require a much more complex design for multistage operation. Additionally, centrifugal compressors struggle to handle high Mach numbers as effectively due to the presence of shock waves. Finally, cooling is easier to implement in axial-flow compressors because of their design, further limiting the use of centrifugal compressors in aircraft engines.

2) Explain, using mathematical expressions the source of pressure increases in a centrifugal compressor and make a reflection and describe what is the source/term that indicates the pressure increase in high performance centrifugal compressors.

A centrifugal compressor works by accelerating air through an impeller, increasing its velocity and dynamic pressure $\left(\frac{1}{2}\rho v^2\right)$. As air enters the compressor, it is energized by the rotational speed of the impeller. Although the axial velocity of the air may remain similar, the radial velocity increases as the air is forced outward along the impeller's radius, due to the relationship $v = \omega r$ (where ω is angular velocity and r is radius). This increase in radial velocity raises the air's dynamic pressure. In the diffuser, this dynamic pressure is converted into static pressure, resulting in an overall increase in total pressure $\left(P_0 = P + \frac{1}{2}\rho v^2\right)$, which is the goal of the compression process.

3) Air enters the inducer blades of a centrifugal compressor at p 01 = 1.02 bar, T 01 = 335 K. The hub and tip diameters of the impeller eye are 10 and 25 cm respectively. If the compressor runs at 7200 rpm and delivers 5.0 kg/s of air, determine the air angle at the inducer blade entry and the relative Mach number.

```
P01 = 1.02*10**5
T01 = 335
dh = 10/100
dt = 25/100
RPM = 7200
RPS = RPM/60
mdot = 5
Cp = 1005
gamma = 1.4
R = 287

A = np.pi/4*(dt**2-dh**2)

rho01 = P01/T01/R
while True:
    v = mdot/rho01/A
    T1 = T01-v**2/2/Cp
    P1 = P01*(T1/T01)**(gamma/(gamma-1))
    rho1 = P1/T1/R
```

```
    if np.abs(rho01-rho1)<1e-10:
        break
    rho01 = rho1

a = (gamma*R*T1)**0.5

Uh = np.pi*RPS*dh
Ut = np.pi*RPS*dt

Wt = (v**2+Ut**2)**0.5
Wh = (v**2+Uh**2)**0.5

Mt = Wt/a
Mh = Wh/a

Betat = np.arctan2(Ut,v)
Betah = np.arctan2(Uh,v)

displayH(sy.Symbol("\\rho_{01} ="),rho01)

displayH(sy.Symbol("v_{a} ="),v)

displayH(sy.Symbol("U_{tip} ="),Ut)
displayH(sy.Symbol("U_{hub} ="),Uh)

displayH(sy.Symbol("\\Beta_{tip} ="),np.degrees(Betat))
displayH(sy.Symbol("\\Beta_{hub} ="),np.degrees(Betah))

displayH(sy.Symbol("M_{tip} ="),Mt)
displayH(sy.Symbol("M_{hub} ="),Mh)
```

$\rho_{01} = 1.00441635429464$

$v_a = 120.727732582125$

$U_{tip} = 94.2477796076938$

$U_{hub} = 37.6991118430775$

$_{tip} = 37.9778723975421$

$_{hub} = 17.3417859425384$

$M_{tip} = 0.422054446717507$

$M_{hub} = 0.348526504945584$