AERO 417 Homework 1

```
import sympy as sy
import numpy as np
import matplotlib.pyplot as plt
import IPython
from IPython.display import display

def displayH(a1,a2='', a3='', a4='', a5='', a6='', a7='',):
    latex_a1 = sy.latex(a1)
    latex_a2 = sy.latex(a2)
    latex_a3 = sy.latex(a3)
    latex_a4 = sy.latex(a4)
    latex_a5 = sy.latex(a5)
    latex_a6 = sy.latex(a6)
    latex_a7 = sy.latex(a7)
    display( IPython.core.display.Math(latex_a1 + latex_a2 + latex_a3 + latex_a4 + latex_a
```

1) Create a graphic in a spreadsheet, showing the variation of the normalized specific work vs pressure ration (r) for different t values (1,2,3,4, and 5). Explain the behavior of the curves based on the technological level of metals. Use: $\gamma = 1.4$ (air).

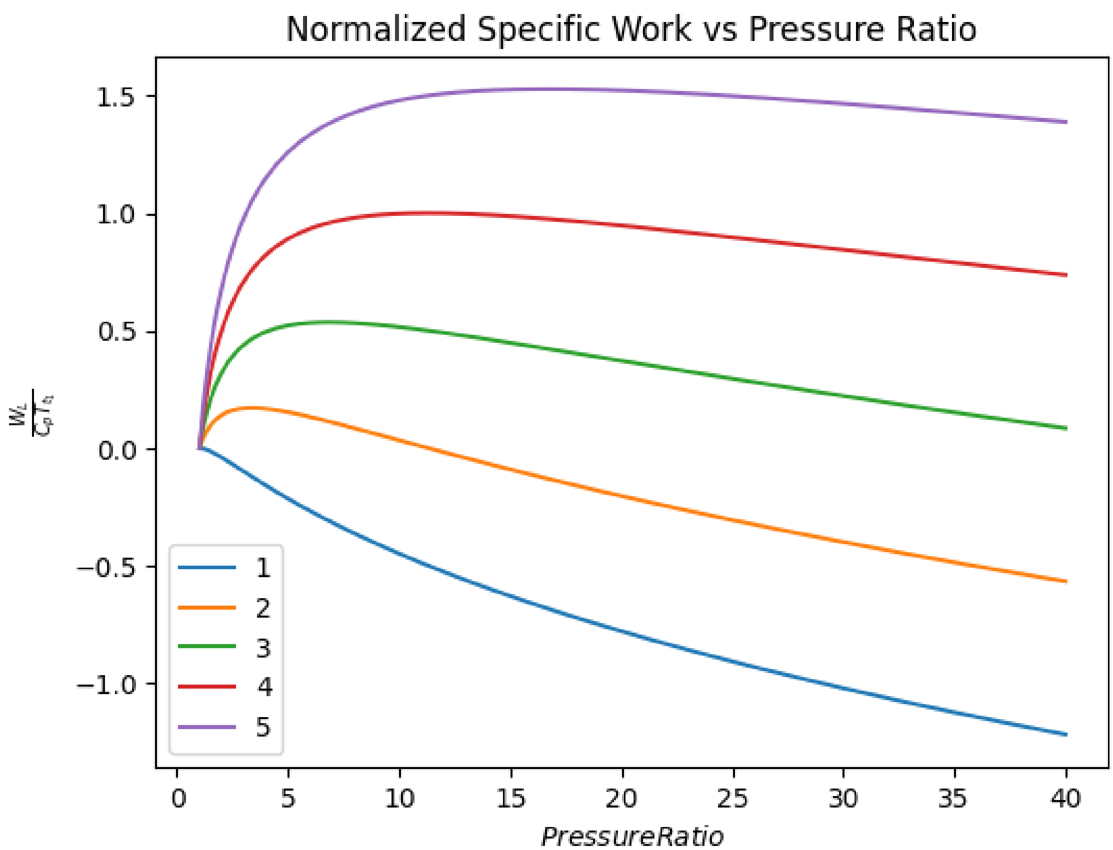
$$\frac{W_L}{C_p T_{t_1}} = t\left[1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}\right] - \left[r^{\frac{\gamma-1}{\gamma}} - 1\right]$$

```
gamma = 1.4

def normalized_specific_work(r,t):
    return t*(1-1/r**((gamma-1)/gamma))-(r**((gamma-1)/gamma)-1)

t_vals = np.arange(1,5.1,1)
r_vals = np.linspace(1,40,1000)
```

```python
plt.figure()
for t in t_vals:
    plt.plot(r_vals,[normalized_specific_work(r,t) for r in r_vals],label=int(t))
plt.title("Normalized Specific Work vs Pressure Ratio")
plt.ylabel(r"$\frac{W_{L}}{C_{p}T_{t_{1}}}$")
plt.xlabel(r"$Pressure Ratio$")
plt.legend()
plt.show()
```



What this graph shows us, is that as t (better material science/ metals), we can extract more work from the same fuels. At the same time, there is also an optimal pressure ratio that tends to increase the higher the temperature goes, meaning that our compression methods also need to be advancing with the advancements in material science.

2) About the text: for a Brayton Cycle, the maximum normalized specific work is obtained when $T_2 = T_4$ (compressor outlet temperature = turbine outlet temperature). Is this true or false?

(Use the normalized specific work equation to develop and answer this question)

Solution (True)

1) The equation for net work is given as:

$W_{net} = W_T - W_C (1)$

2) Assuming the gas to be calorically perfect:

$W_{net} = c_p \left[(T_3 - T_4) - (T_2 - T_1) \right] (2)$

2) Using isentropic relations, we can find relationships between the pressure changes and the temperature change of the turbine and the compressor:

$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} (3)$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} (4)$

3) Assuming that the pressure change across the compressor and the turbine is the same:

$\frac{T_3}{T_4} = \frac{T_2}{T_1} (5)$

4) Using the relationship defined in *Equation 5*, we can rewrite *Equation 2* as:

$W_{net} = c_p \left(T_3 - \frac{T_1 T_3}{T_2} - T_2 + T_1 \right) (6)$

5) To find the maximum of net work, we take the derivative of *Equation 6* wrt T_2 (the same result can be achieved by taking the derivative wrt T_4).

$\frac{dW_{net}}{dT_2} = 0 = c_p \left(0 + \frac{T_1 T_3}{T_2^2} - 1 + 0 \right) (7)$

6) Solving *Equation 7* for T_2:

$1 = \frac{T_1 T_3}{T_2^2} (8)$

$T_2 = \sqrt{T_1 T_3} (9)$

7) Using *Equation 5*, we can find T_2 in terms of T_4.

$T_2 = \sqrt{T_1 T_3} \cdot \frac{\sqrt{T_1 T_3}}{\sqrt{T_1 T_3}} = \frac{T_1 T_3}{\sqrt{T_1 T_3}} (10)$

$T_2 = \frac{T_1 T_3}{T_2} = T_4 (11)$