



netivreg: Estimation of Peer Effects in Endogenous Social Networks

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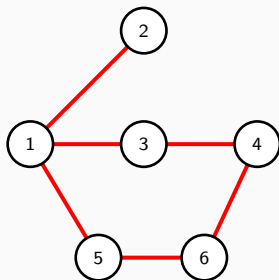
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- Estimation of network effects is becoming increasingly common
 - Interest on structural coefficients: endogenous peer effects and contextual effects
 - Estimate treatment effects and spillovers under interference
- Exogenous network formation is a commonly used assumption in empirical work
- Recent methods allowing for the presence of network endogeneity require explicit structural restrictions on the network formation process

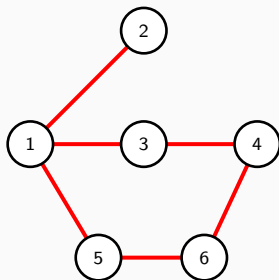
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- Exogenous network formation is a commonly used assumption in empirical work
- Recent methods allowing for the presence of network endogeneity require explicit structural restrictions on the network formation process
- **Research Question:** can the *multiplex network data structure* help with the treatment of identification issues?

- Propose novel instruments based on the topology of multiplex networks
- Provide new identification results for peer/contextual effects that generalize existing methods by accounting for potential endogenous network formation
- Computationally easy to implement estimator that is consistent and asymptotically normal
- Stata implementation: `netivreg`
- Empirical application of peer effects in coauthorship networks: positive peer effects in citations



- Contextual Effects (interference): i 's outcome depends on the characteristics of other units.
- Endogenous Peer Effects (multiplier).


$$y_i = \alpha + \beta \sum_{i \neq j} w_{i,j} y_j + \delta \sum_{i \neq j} w_{i,j} x_j + \gamma x_i + \varepsilon_i.$$



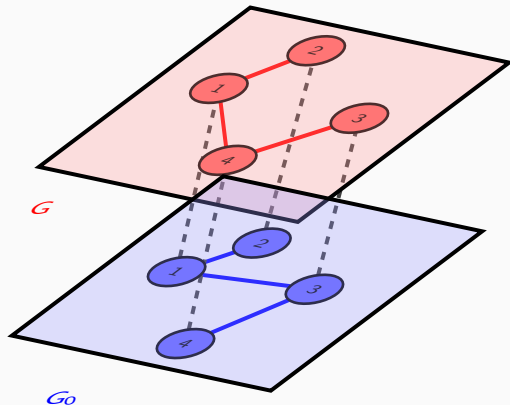
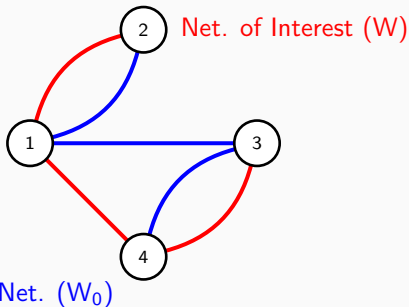
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Objective: identify and consistently estimate the parameters $(\alpha, \beta, \gamma, \delta)$.

- Simultaneity of the peer effects regressors (reflection problem).
- The decision of forming a peer connection can be correlated with unobserved characteristics or there could exist common shocks (correlated effects) 
- The network structure could induce correlation between X and ε (unobserved homophily)

$$y = \alpha^0 \iota + \beta^0 W y + \delta^0 W X \delta^0 + X \gamma^0 + \varepsilon, \text{ with } \mathbb{E}[\varepsilon \mid \mathbf{W}, X] \neq 0 \text{ and } \mathbb{E}[\varepsilon \mid \mathbf{W}_0, X] = 0.$$



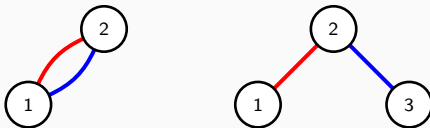
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- It is possible to observe a relevant network (for example friendship) defining W .
- This method can be used to causally estimate network friendship effects.

1. Monolayer Linear model and Bi-layer multiplex network data $\mathcal{M} = 2$ (W and W_0).
2. Conditional distribution $\mathcal{F}(\varepsilon \mid X, \mathcal{M})$ is such that $\mathbb{E}[\varepsilon \mid W, X] \neq 0$ and $\mathbb{E}[\varepsilon \mid W_0, X] = 0$.
3. The networks generating the adjacency matrices W and W_0 are correlated in the sense that it is possible to find connections in common ($E_0 \cap E_1 \neq \emptyset$) and distance two paths that change edge type ($(i, j) \in E_0$ and $(j, k) \in E_1$).



Let Π be the projection coefficients from a regression of WS on W_0S , where $S = [y \ X]$.

Theorem:

Let Assumptions 1, 2, 3, and $\gamma^0(\pi_{11}\beta^0 + \pi_{12}\delta^0) + \pi_{21}\beta^0 + \pi_{22}\delta^0 \neq 0$ hold. If the matrices I , W_0 , W_0^2 are linearly independent, then the parameters $\alpha^0, \beta^0, \gamma^0$ and δ^0 are identified.

Remark

Note that this is a generalization of the identification result in Proposition 1 of Bramoullé et al. (2009, JoE), i.e., if $W_0 = W$, one has $\Pi = I$, and the condition reduces to $\gamma^0\beta^0 + \delta^0 \neq 0$ and the matrices I , W and W^2 being linearly independent.

► Rank Condition

$$y = \alpha^0 \iota + \text{WS}\theta^0 + X\gamma^0 + \varepsilon \quad \text{for } S = [y \ X] \quad \text{and } \theta^0 = [\beta^0 \ \delta^0]$$

$$y = \alpha^0 \iota + \text{W}_0 S\theta^* + X\gamma^0 + e, \quad \text{for } \theta^* = \Pi\theta^0.$$

Estimation Procedure

1. Estimate Π by OLS (WS on $\text{W}_0 S$).
2. 2SLS of $[\iota, X, \text{W}_0 y, \text{W}_0 X]$ with instrument $Z = [\iota, X, \text{W}_0^2 X, \text{W}_0 X]$. Calculate $\hat{\theta} = \hat{\Pi}^{-1} \hat{\theta}^*$.
3. IV of $[\iota, X, \widehat{\text{W}}_0 y, \widehat{\text{W}}_0 X]$ with instruments $\hat{Z}^* = [\iota, X, [E(\text{W}_0 y | X, \text{W}_0), \text{W}_0 X] \hat{\Pi}]$.

Estimator and Properties

$$\hat{\psi}_{G3SLS} = \left(\hat{Z}^{*\top} \hat{D} \right)^{-1} \hat{Z}^{*\top} y,$$

$$\sqrt{n}(\hat{\psi}_{G3SLS} - \psi) \xrightarrow{d} N(0, \text{V}_{\psi})$$

Stata Implementation

- Simulate data using an ideal experiment where $\beta_0 = 0.7$, $\delta_i = \gamma_i = 1/3$ for $i = \{1, 2, 3\}$, and $\delta_4 = \gamma_4 = 0$ ▶ Montecarlo
- Use the three stage procedure to calculate the efficient IV estimator in Stata

```
netivreg y-exo x1 x2 x3 x4 (edges = edges0)
```

```
. netivreg y_endo x1 x2 x3 x4 (edges = edges0)
```

Network IV Regression

Number of obs =	400
Wald chi2(10) =	822.26
Prob > chi2 =	0.0000
R-squared =	0.8176
Root MSE =	1.194

	y_endo	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
W_y	y_endo	.7059194	.0934719	7.55	0.000	.5221476 .8896911
W_x	x1	.3464024	.1277675	2.71	0.007	.0952031 .5976017
	x2	.3280795	.0870187	3.77	0.000	.1569951 .4991639
	x3	.3615469	.0926147	3.90	0.000	.1794604 .5436334
	x4	.0500988	.1476019	0.34	0.734	-.2400962 .3402939
X	x1	.3782985	.0560235	6.75	0.000	.2681526 .4884443
	x2	.3287283	.0426851	7.70	0.000	.2448066 .4126499
	x3	.3442047	.0483468	7.12	0.000	.2491518 .4392576
	x4	.0895948	.0745045	1.20	0.230	-.0568859 .2360756
	_cons	1.035534	.3189017	3.25	0.001	.4085523 1.662515

- Use of the APIs from Scopus and IDEAS/RePEc, Web scrapping and text mining.
- The for the *American Economic Review*, *Econometrica*, *Journal of Political Economy*, *Quarterly Journal of Economics*, *The Review of Economic Studies*, and *The Review of Economics and Statistics* between 2000 and 2018.
- Authors' information such as citation counts, gender, RePeC ranking, current affiliations, fields of expertise, advisor, job, and education history.

W (Coauthors)

W₀ (Alumni)

(y, X)

	source	target
4	5	478
5	5	665
6	5	705
7	8	113
8	8	133
9	8	177
10	8	189
11	8	639
12	8	658
13	10	356
14	10	527
15	11	26
16	11	639
17	13	213
18	13	428

	source	target
4	4	136
5	4	407
6	5	10
7	5	95
8	5	97
9	5	130
10	5	144
11	5	152
12	5	161
13	5	194
14	5	301
15	5	324
16	5	357
17	5	383
18	5	416

	id	lcitations	editor	diff_gender	isolated	n_pages	n_authors	n_references	journal	year
4	21	2.302585	0	0	1	15	2	27	aer	2000
5	31	3.806663	0	0	1	21	1	39	aer	2000
6	38	3.555348	0	0	1	17	2	31	aer	2000
7	51	3.583519	0	0	1	20	1	48	aer	2000
8	59	3.988984	0	0	1	17	2	50	aer	2000
9	68	2.197225	0	0	1	15	2	31	aer	2000
10	76	2.197225	0	0	0	11	2	18	aer	2000
11	86	3.218876	0	0	0	24	1	32	aer	2000
12	96	4.836282	0	0	0	24	2	57	aer	2000
13	105	4.691348	0	0	1	25	2	40	aer	2000
14	122	4.770685	0	0	1	30	1	30	aer	2000
15	139	3.850147	0	0	1	16	1	49	aer	2000
16	144	2.564949	0	0	0	21	2	16	aer	2000
17	151	4.26268	1	0	0	21	3	36	aer	2000
18	162	2.890372	0	0	1	26	1	26	aer	2000

$$y_{i,r,t} = \alpha + \beta \sum_{j \neq i} w_{\ell;i,j,t} y_{j,r,t} + \sum_{j \neq i} w_{\ell;i,j,t} \tilde{x}_{j,r,t}^{\top} \delta + x_{\ell;i,r,t}^{\top} \gamma + \lambda_r + \lambda_t + \lambda_0 + \varepsilon_{i,r,t}$$

netivreg lcitations editor diff-gender n-pages n-authors n-references isolated
(edges = edges0), wx(diff-gender editor) cluster(c-coauthor) first second

Peer Effects (β)

log(# Citations)

Contextual Effects (δ)

Editor

Different Gender

Direct Effects (γ)

Editor

Different Gender

Authors

Pages

References

Fixed Effects (λ s)

Journal

Year

Alumni Component

$$WS = W_0 S \Pi + U,$$

Projection of W on W0

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
W_lcitations						
W0_lcitations	.4956186	.0321772	15.40	0.000	.4324379	.5587993
W0_diff_gender	.0127121	.5132519	0.02	0.980	-.9950719	1.020496
W0_editor	.0085967	.7897166	0.01	0.991	-1.542033	1.559227
W_diff_gender						
W0_lcitations	.137265	.0033955	40.43	0.000	.1305979	.1439321
W0_diff_gender	.1422822	.0541602	2.63	0.009	.0359371	.2486273
W0_editor	.0325262	.0833338	0.39	0.696	-.131102	.1961544
W_editor						
W0_lcitations	.4249148	.0025367	167.51	0.000	.419934	.4298957
W0_diff_gender	.1027705	.0404624	2.54	0.011	.0233214	.1822195
W0_editor	.1367464	.0622576	2.20	0.028	.0145019	.2589909

2SLS of $[l, X, W_0y, W_0X]$ with instrument $Z = [l, X, W_0^2X, W_0X]$

2SLS Regression

Number of obs = 729
Wald chi2(62) = -1.1e+17
Prob > chi2 = 1.0000
R-squared = 0.1317
Root MSE = 1.846

lcitations	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
w_y lcitations	.9496092	.5481734	1.73	0.084	-.126744	2.025962
X						
diff_gender	.2224841	.1317096	1.69	0.092	-.0361313	.4810994
editor	.1691513	.1181452	1.43	0.153	-.06283	.4011327
n_pages	.0282953	.0048171	5.87	0.000	.0188369	.0377538
n_authors	.0747385	.0603238	1.24	0.216	-.043709	.1931859
n_references	.0119404	.0025597	4.66	0.000	.0069143	.0169665
isolated	-.2131575	.0942419	-2.26	0.024	-.3982041	-.0281109

IV of $[l, X, \widehat{W_y}, \widehat{W_x}]$ with instruments $\widehat{Z}^* = [l, X, [E(W_0 y | X, W_0), W_0 X] \widehat{\Pi}]$

Network IV Regression					Number of obs =	729	
Number of clusters (c_coauthor) =					575	Wald chi2(62) =	6.5e+16
						Prob > chi2 =	0.0000
						R-squared =	0.1723
						Root MSE =	1.339
(Std. Err. adjusted for 575 clusters in c_coauthor)							
lcitations		Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
W_y							
lcitations		.5200772	.3616317	1.44	0.151	-.1899963	1.230151
X							
diff_gender		.218709	.1305651	1.68	0.094	-.0376592	.4750771
editor		.1733642	.1157379	1.50	0.135	-.0538902	.4006187
n_pages		.0288947	.0044187	6.54	0.000	.0202184	.0375709
n_authors		.0719403	.0597035	1.20	0.229	-.0452891	.1891696
n_references		.0119892	.0025599	4.68	0.000	.0069628	.0170156
isolated		-.2230689	.0897056	-2.49	0.013	-.3992083	-.0469295

- Identification of a linear-in-means model with endogenous network.
- Computationally simple estimation that uses two-layered multiplex network structure with Stata implementation.
- Robust to different types of network endogeneity. It does not require to model unobserved heterogeneity and network formation.

Appendix

Distortions Induced by Social Effects: Consumption Examples

- If individuals care about **status** (conspicuous consumption models), the proportion of conspicuous consumption may increase with respect to other goods.
- If conspicuous consumption is considered wasteful, peer effects might have noticeable welfare consequences.
- Savings may differ from the optimal in an attempt to keeping up with the peers.

► Empirical Work

Aggregate Effects: Consumption Example

- Unanticipated tax changes to the rich might have aggregate consequences.
- If individuals who are not affected by the shock change their consumption after observing changes in consumption of the rich, the shock can spread through the network.
- Social multipliers depend on the size of the endogenous peer effects and the connectedness of the affected groups.

► Empirical Work

Angrist's (2014) Critique: Group Regressions

- **Reflection Problem:** a regression of individual outcomes on group mean outcomes is tautological.
- **Correlated Effects:** even the leave-one-out estimator does not provide information of human behavior. “Like students in the same school, households from the same village are similar in many ways”.
- **Mechanical Relationship:** the coefficient on group averages in a multivariate model of endogenous peer effects does not reveal the action of social forces. He interprets the value $1/(1-\beta)$ as approximately the ratio of the 2SLS to OLS estimands for the effect of individual covariates on outcomes (using dummy groups as instruments).

Angrist's (2014) Critique: Network Regressions

- Start by a saturated model $E[y_i | x_i] = \gamma_0 + \gamma_1 x_i$ satisfying $E[u_i | x_i] = 0$, for $u_i \equiv y_i - \gamma_0 - \gamma_1 x_i$.
- Individuals are ordered from left to right. Each person i is connected only with the individual to her left $i - 1$. Friends are only similar on unobservables: $u_i = \beta u_{i-1} + \varepsilon_i$.

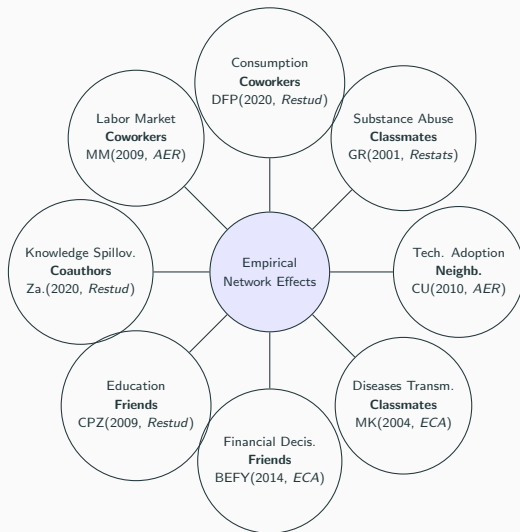
- The outcome can be written in a linear-in-means (lmm) model form:

$$y_i = \gamma_0(1 - \beta) + \beta y_{i-1} + \gamma x_i - \beta \gamma x_{i-1} + \varepsilon_i$$

- Flaw in Angrist's example:** let $\delta = -\beta\gamma$ to write this model exactly as a lmm. Note that $\delta + \gamma\beta = 0$ so that the outcome equation can be written as (for $\alpha = \gamma_0(1 - \beta)$)

$$y_i = \frac{\alpha}{1 - \beta} + \gamma x_i + v_i$$

Different Network Effects



Critique to Randomly Assigned Groups

- In principle, randomization of peers would guarantee identification in a monolayer linear in means model where endogenous network formation is ruled out.
- It can completely eliminate the problem of unobserved common variables.
- However, if individuals endogenously form groups (homophily), there can be a subsequent resorting. If resorting happens faster than the effects of social interactions, identification is not possible.
- Even with random peers, researchers face a classical problem of **omitted variables** when trying to estimate contextual effects ($\mathbb{E}[x_i \varepsilon_j \mid w_{i,j} = 1] \neq 0$).

Multilayers Networks in Economics

Labor Supply

- Sisters, Cousins and Neighbors networks (NST (2018, *AEJ*))

Education

- Friendship network in t and $t - 1$ (GI (2013, *JBES*))
- Roommates, classmates, Study-mate, Friendship networks (CL (2015))
- Siblings and Classmates networks (NR (2017, *JAE*))

Consumption

- Coworker and Spouses networks (DFP (2020, *Restud*))

Publication Outcomes

- Coauthors, Alumni and Same Advisor networks (EHJS (2020))

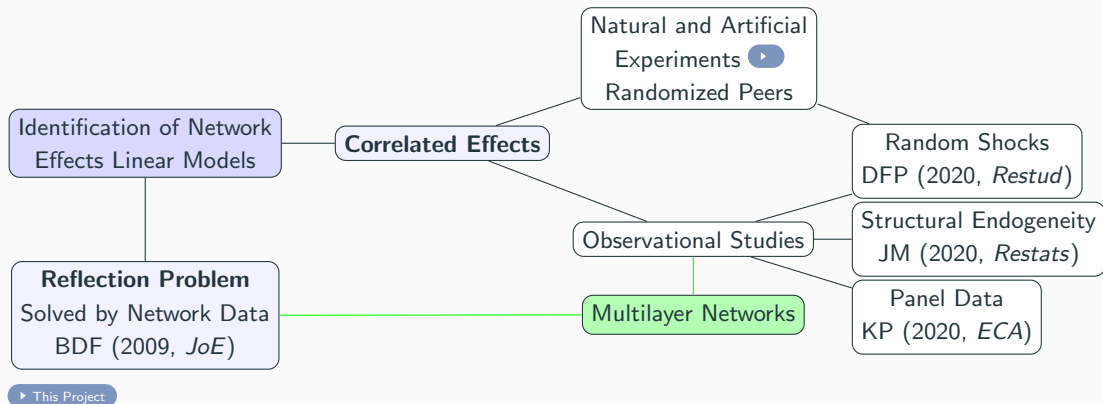
Microfoundations

- The monolayer linear model of interest corresponds with the best response of a Bayesian Game of Social Interactions as proposed by Blume, Brock, Durlauf and Jayaraman (2015, JPE).
- Quadratic utility with social pressure or strategic complementarities

$$U_i(\omega_i, \omega_{-i}) = \left(\gamma x_i + z_i + \delta \sum_j c_{ij} x_j \right) \omega_i - \frac{1}{2} \omega_i^2 - \frac{\phi}{2} \left(\omega_i - \sum_j a_{ij} \omega_j \right)^2$$

- In their model endogeneity arises because an individual i , observing that he is connected to j , make an inference about the value of z_j that is dependent on x_j . Then, x_j will be correlated with ε_i in my equation of interest.
- Their critique of instrumental variable is that if individual i observe the instruments v_j , he can use it to predict z_j which will induce correlation between ε_i and the instrument.
- Our instrument is based on x_r of individuals r connected to i in a network that is independent of the individuals' utilities. Therefore x_r is not useful to predict z_j . [► Model](#)

Positioning the Research Agenda in the Literature



Assumptions

Assumption 1

There exists a $n \times n$ adjacency matrix W_0 such that: $\mathbb{E}[v|x, W_0] = 0$

Assumption 2

Let Π be the full-rank matrix of coefficients from the system regression

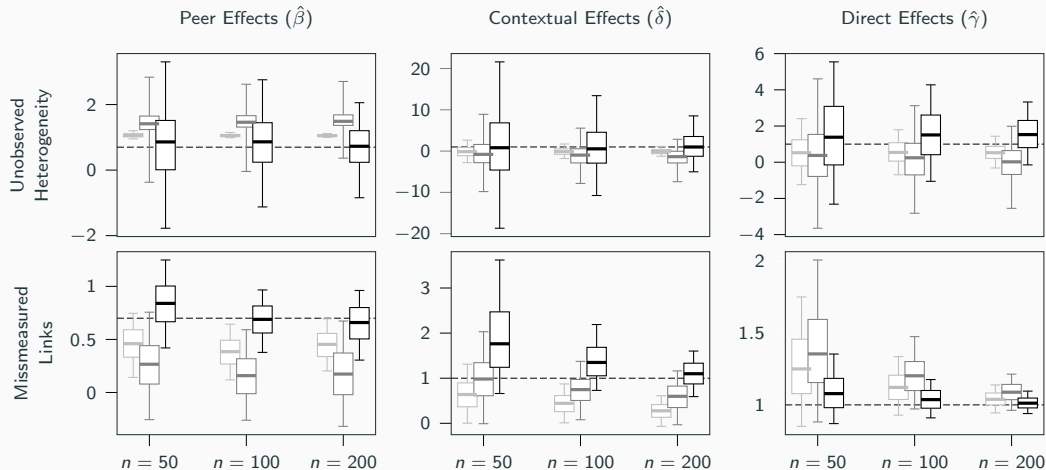
$$WS = W_0 S \Pi + U,$$
$$E[U|W_0 y, W_0, X] = 0.$$

where $\mathbb{E}[S^\top w_{0;i} w_{0;i}^\top S] > 0$. Furthermore, the first row of Π is such that $\pi_{11}\beta + \pi_{12}\delta < 1/\lambda_{\max}$, where λ_{\max} is the largest eigenvalue of W_0 .

Rank Condition

- Given that $\text{rank}(\Pi) \leq \min\{\text{rank}(E[S^\top w_{0;i} w_{0;i}^\top S]^{-1}), \text{rank}(E[S^\top w_{0;i} w_i^\top S])\}$, a necessary condition for $\text{rank}(\Pi) = k + 1$ is that $\text{rank}(E[S^\top w_{0;i} w_i^\top S]) = k + 1$ which would be equivalent to the **relevance** condition in the classical Instrumental Variable literature.
- For large enough sample, this condition imposes some restriction on the matrix $W_0 W$. This matrix contains the connections in common across the two networks in the main diagonal, and length two paths that change color in the off- diagonal.
- It cannot be zero so there have to be enough connections in common and indirect triads that change colors. This is a way to think about the **correlation** between the two matrices.

Monte Carlo Experiments



Empirical Application: Data

- 1,628 articles published in the *American Economic Review*, *Econometrica*, the *Journal of Political Economy*, and the *Quarterly Journal of Economics* between 2000 and 2006. Source: RePEc, Scopus, and Journal Websites.
- Employment, education, and research interest information for 1,985 unique authors and 42 unique editors (37 of which also published papers in these journals in this time period). Source: Web scrapping/text mining and Colussi (2018, ReStat).
- *Co-authorship* ($\ell = 1$) and *Alumni* ($\ell = 0$) networks are constructed for all 2,027 scholars.

Articles i and j are connected in network W_ℓ if at least one of the authors of article i shares a professional connection of type ℓ with at least one of article j 's authors.