

# **Estimating University Enrollments**

**Machine Learning Engineer Nanodegree Capstone Project**

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# Definition

## Project Overview

For this project, I am interested in producing accurate predictions of enrollment at American universities. Universities require accurate enrollment numbers in order to properly allocate resources, from financial resources to human resources. However, historically, these predictions have proven notoriously difficult to make. Many universities use rather simple prediction models. They start by looking at their historical attrition rate, usually as an average, and subtract that from the target enrollment for the year. Then they add in the number of students who have been admitted for the upcoming year.

As someone who has previously worked at institutions of higher education, I am particularly interested in using Machine Learning techniques to find efficiency gains and cost-saving opportunities like this one.

For this project, I will be using the Integrated Postsecondary Education Data System (IPEDS) Delta Cost Project Database from 2000-2012. This database includes information about higher education institutions, including finance, enrollment, staffing, completions, and student aid. It can be obtained at the following address: [http://nces.ed.gov/ipeds/deltacostproject/download/IPEDS\\_Analytics\\_DCP\\_87\\_12\\_CSV.zip](http://nces.ed.gov/ipeds/deltacostproject/download/IPEDS_Analytics_DCP_87_12_CSV.zip)

IPEDS collects this data annually via surveys distributed to all post-secondary institutions in the United States that participates in federal student financial aid programs. Every post-secondary institution with even a remotely good reputation participates in federal student financial aid programs. Many with poor reputations do as well. This data set contains 974 attributes with 215,613 observations. Among these attributes, is a straight-forward 'enrollment' value, which will be our target variable. Other attributes of interest include tuition, endowment, number of employees, faculty salaries, federal grant data, and the like.

## Problem Statement

This project is attempting to discover a model that can accurately predict student enrollment numbers at universities based on measures of institutional attributes. This is quantifiable as  $y = f(x)$ , where 'y' is the predicted student enrollment number, 'x' is a set of measures of institutional attributes, and 'f' is our model. 'y' can be measured by taking a count of enrolled students at an institution in a given year, as can the attributes associated with that institution. Furthermore, this problem recurs annually

at every institution of higher education.

A solution to this problem would accept a set of attributes about an institution ( $x$ ), run them through a model ( $f$ ), to produce a predicted enrollment number ( $y$ ). A good solution would have predictions that have a low squared error value, when compared against a withheld test set.

To find a solution, I will begin with feature selection, removing features from the data that provide no useful information (such as school name). With the remaining potential features, I will perform Independent Component Analysis (ICA) to reduce the number of dimensions to a number that is viable, given the size of our data after the cleaning from stage 1, with the curse of dimensionality in mind. I will then generate train/test splits for the data sets, probably with 80/20 proportions. With the data setup, I will then generate a series of models for comparison, using a variety of regression models (Linear Regression, Support Vector Regression, and Random Forest Regression).

## Metrics

Given that existing institutions tend to base their enrollment projections on a combination of attrition and new admissions, a benchmark model based on the retention rate and admission number should provide a good starting point. Specifically, a linear regression model that only takes the average of 'ftretention\_rate' and 'ptretention\_rate' plus the 'admitcount' to predict 'total\_enrollment', with the mean squared error for a point of comparison. This should be close to the existing simple model used at institutions. The mean squared error of each candidate model (described above) will be compared against the baseline model, and visualizations of each predicted value vs. observed value in the test set will be generated and presented.

Compared to mean absolute error, MSE is probably a better metric because MSE more heavily penalizes errors that are further away from observed values than MAE. For example, it is possible for a model to perform better on MAE by having a large number of predictions close to the observations, with the occasional prediction that is way off. By using MSE, those far-off predictions would be extra penalized (due to the squaring), and as such, the resulting models should have tighter predictions, close to actual observations.

Likewise, coefficient of determination would be as useful as MSE for the purposes of this problem. Since the ultimate goal is to get accurate predictions of enrollment for forecasting purposes, knowing the proportion of variance in the dependent variable is not as valuable as knowing the model has high accuracy (ie, low error) predictions.

Thus, MSE is the optimal metric for this problem.

# Analysis

## Data Exploration

The IPEDS Delta Cost Project Database provides survey results from American institutions of higher education that accept federal financial aid funds during the years of 1987 to 2012. This dataset provides us with 215,613 observations, across 974 attributes. Because this database is split into two separate files (1987-99, 2000-12), we will need to concatenate the two files together into a single DataFrame. Once we isolate our target variable ('total\_enrollment'), we can obtain the following summary statistics:

Table: Enrollment Summary Statistics

count	153,168
mean	2,809.486148
standard deviation	7,540.511589
minimum	0
25%	101
50%	481
75%	2344
max	380,232

From these summary statistics, it appears that there will be a rather significant issue with missing data (our count is lower than our original observations), and furthermore, it looks like there will be a significant issue with outliers (the max and minimum are very far from the 50% mark). In particular, it is worth noting that the vast majority of the observations in this data represent what is traditionally understood as small institutions. Most recognizable universities and colleges would actually be considered outliers in this dataset, as their enrollment is significantly larger than what is commonly seen in this dataset. If we use a common outlier definition of 1.5x the InterQuartile Range (IQR), it would render all universities with enrollments above 5,708 as outliers. With that in mind, it is important to remember that this analysis will apply to most institutions in the United States, but not most of the recognizable institutions.

In addition, with 974 features to work with, quite a bit of feature selection is going

to need to be done to avoid problems with the curse of dimensionality. However, most of the features present boolean, continuous, or ratio values. Some of the features are composites of other features, such as 'total\_enrollment' being a combination of 'total\_fulltime' and 'total\_parttime' (among others). A combination of feature selection and component analysis will be necessary to reduce the feature space.

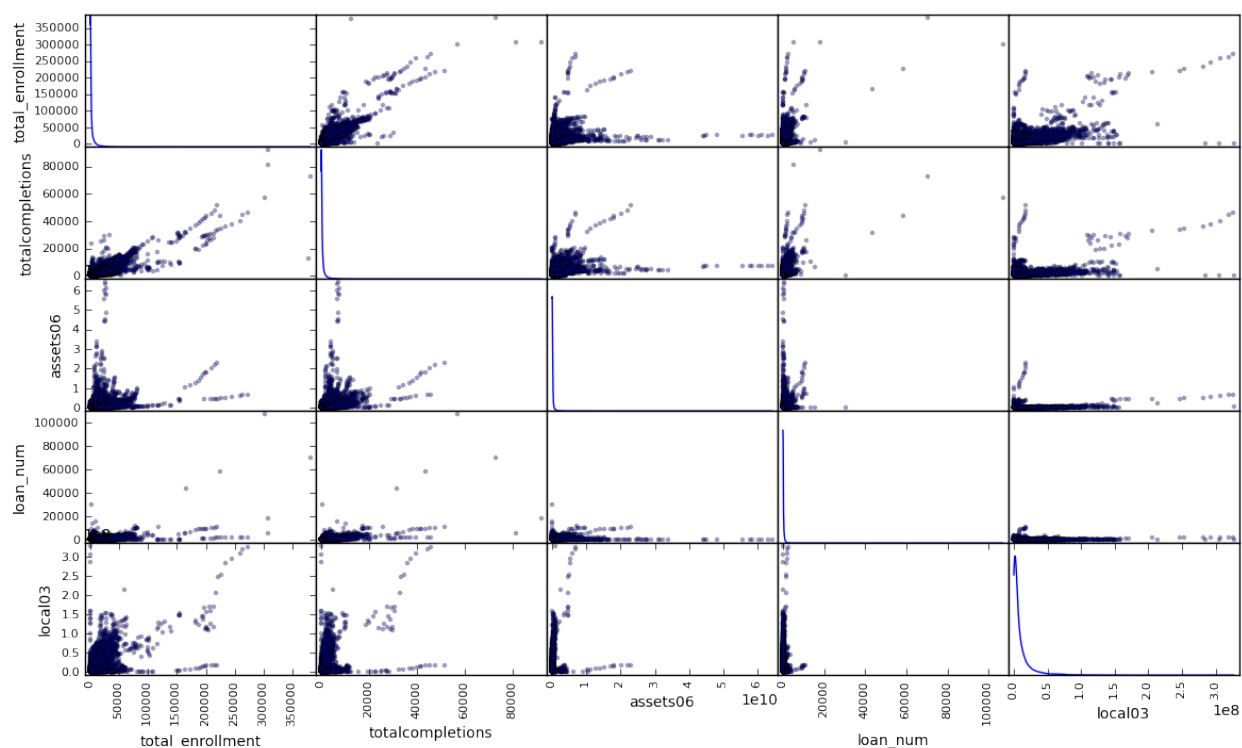
## Exploratory Visualization

Since this data set includes such a large feature space (close to a thousand features), attempts to sample some of the data and the feature behavior will be done randomly to try to get a rough sense of how the data looks. When we take a random sample of features, and plot them as pairs, we notice that the data is definitely not normally distributed, and there are quite a few outliers present. However, the scatter plotting of random samplings of pairs does seem to indicate there may be some linear relationships in the data that we may be able to capture later. Some normalization will also likely have to occur.

In addition to the target variable, the following features were randomly selected for visualization purposes, and possess the following definitions according to the IPEDS documentation:

- *total\_enrollment*: This annual component of IPEDS collects data on the number of students enrolled in the fall at postsecondary institutions. Students reported are those enrolled in courses creditable toward a degree or other formal award; students enrolled in courses that are part of a vocational or occupational program, including those enrolled in off-campus centers; and high school students taking regular college courses for credit. Institutions report annually the number of full- and part-time students, by gender, race/ethnicity, and level (undergraduate, graduate, first-professional); the total number of undergraduate entering students (including first-time, transfers-in, part-time students, and non-degree students); and retention rates.
- *totalcompletions*: This annual component of IPEDS collects number of degrees and other formal awards (certificates) conferred. These data are reported by level (associate's, bachelor's, master's, doctor's, and first-professional), as well as by length of program for some. Institutions report all degrees and other awards conferred during an entire academic year, from July 1 of one calendar year through June 30 of the following year.

- *assets06*: Total assets is the sum of cash, cash equivalents and temporary investments; receivables (net of allowance for uncollectible accounts); inventories, prepaid expenses, and deferred charges; amounts held by trustees for construction and debt service; long-term investments; plant, property, and equipment; and other assets.
- *loan\_num*: Number of full-time, first-time degree/certificate-seeking undergraduate students who received student loans. Loans to students - Any monies that must be repaid to the lending institution for which the student is the designated borrower. Includes all Title IV subsidized and unsubsidized loans and all institutionally- and privately-sponsored loans. Does not include PLUS and other loans made directly to parents.
- *local03*: Revenues from appropriations by a governmental entity below the state level. Education district taxes include all tax revenues assessed directly by an institution or on behalf of an institution when the institution will receive the exact amount collected. These revenues also include similar revenues that result from actions of local governments or citizens (such as through a referendum) that result in receipt by the institution of revenues based on collections of other taxes or resources (sales taxes, gambling taxes, etc.).



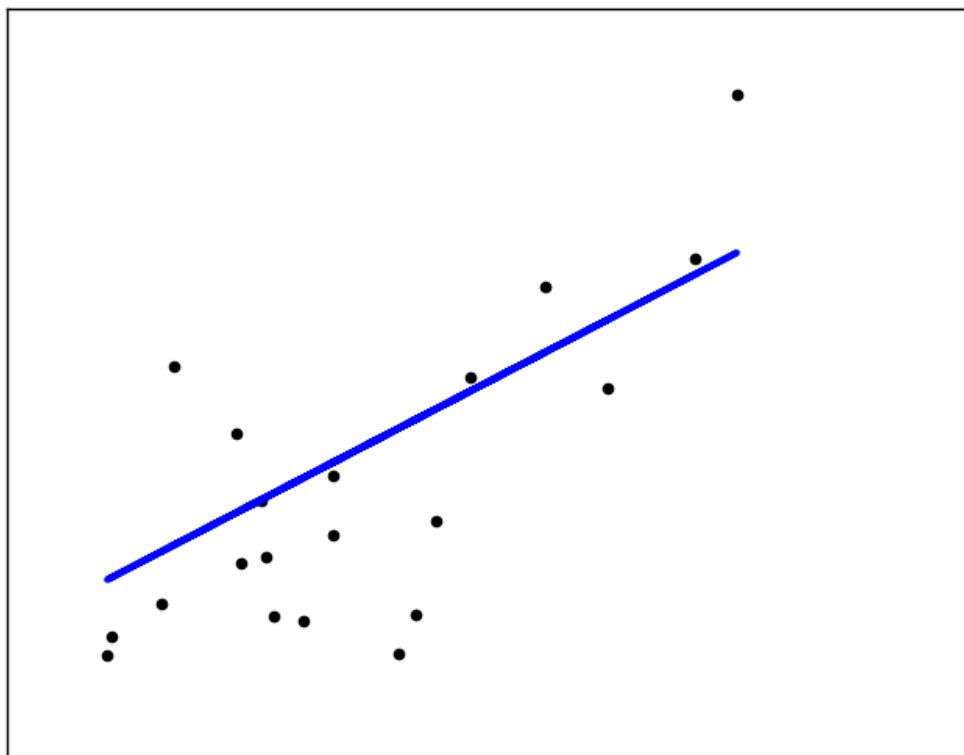
## Algorithms and Techniques

Since the data set includes the target variable of enrollment, this will be a supervised learning problem. Furthermore, since the target variable is a number and not a category, this qualifies as a regression problem. Because of that, I will be focusing on regression algorithms and techniques.

The first thing I want to note is that, with the exception of the benchmark model, I plan to use Independent Component Analysis in all my candidate models to reduce the feature space. I specifically chose ICA for this feature reduction because the data exploration suggests that many of the columns in the data are different measures of the same underlying attribute, and each of those measures seem to be capturing some cross noise from the other attributes. ICA should help to capture the independent signals that the measures seem to be indicating. Furthermore, with ICA, it will be possible to use many more independent features than the benchmark does, as the benchmark really only uses two features (admission count and retention rate). By contrast, the ICA feature reduction will allow the candidate models to use up to 17 independent component features.

Fortunately, with the population size and feature space this project is working with, ICA is not prohibitively resource intensive. As such, I am not going to do much feature selection. I think it is possible that the ICA analysis may be able to capture hidden independent measures from the repeated, but varied perspectives on similar measures. For example, there is no measure for 'diversity' directly in the data, but there are measurements for the number of asian, black, hispanic, white, multi, or unknown national origin. I'm worried that selecting features prior to the ICA may miss a chance to find hidden indicators in the data. If the dataset was significantly larger, or resources significantly more limited, proper feature selection would be considered.

In addition to the ICA done on the feature space, I will be examining three candidate regression models. Linear regression models do not tend to have great accuracy, but they train extremely fast. The linear model is developed in a way that minimizes the residual sum of squares between the observed data points and the predicted data points of the model. For example, we can see this in practice in two dimensional space with the following image<sup>1</sup>:

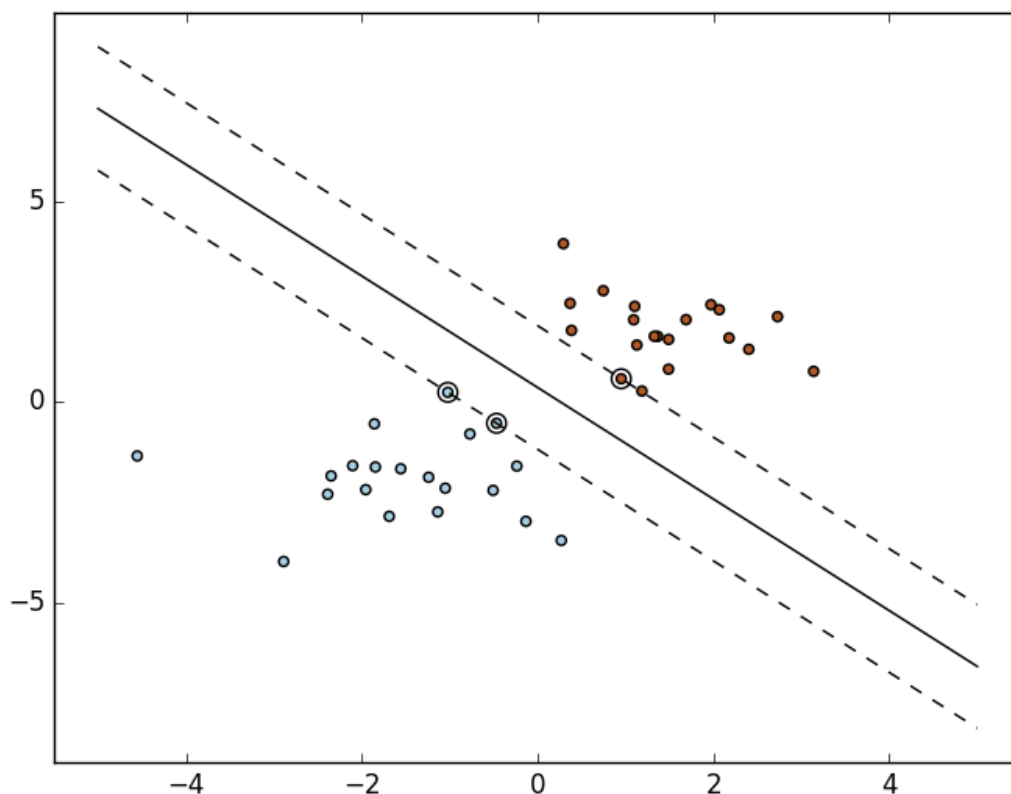


Since the current technique predominantly used to predict enrollments is a linear regression model with few features, a good candidate would be another linear regression model using the ICA features. That is, perhaps better results could be obtained by using the same model but with more (and perhaps better) features. Aside from this difference, both will be using the default parameters.

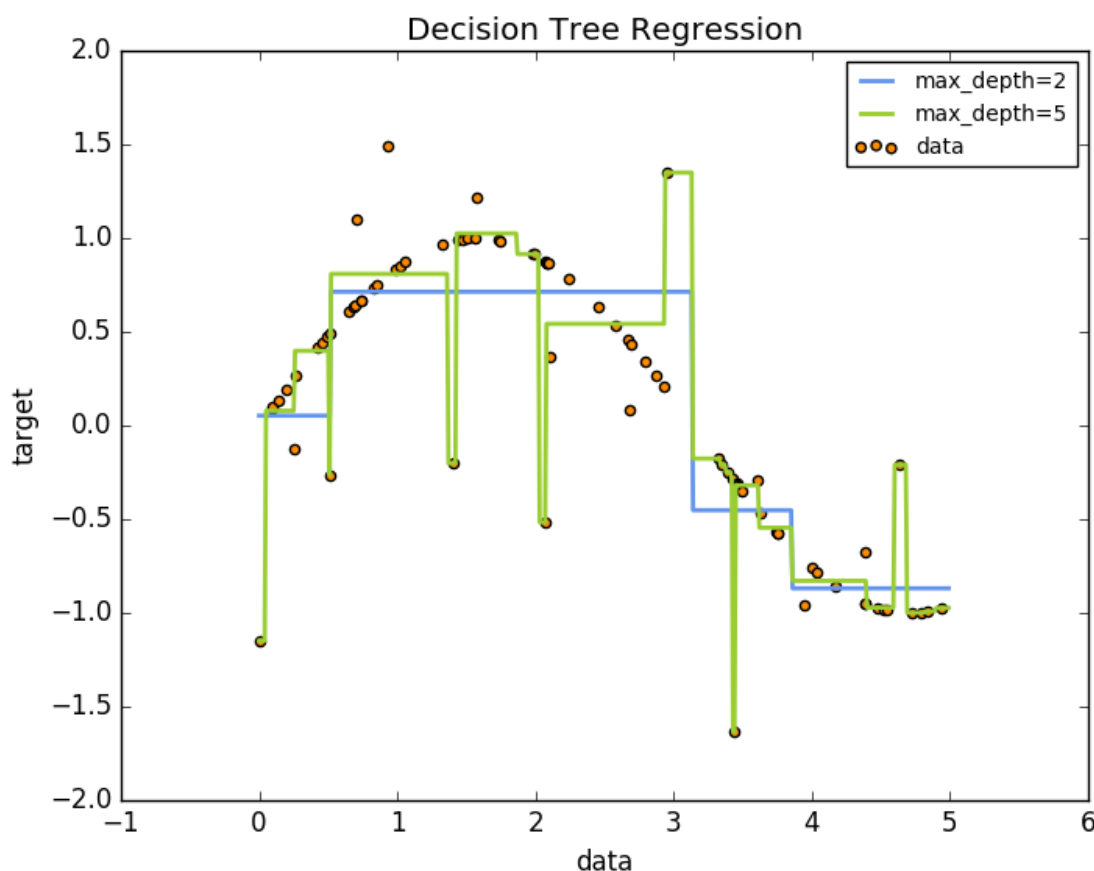
As a separate condition, perhaps the underlying data is linearly separable. Considering the high degree of dimensionality, it is hard to tell this in advance. However, if it turns out that the data is linearly separable, a support vector approach to the problem could be promising. As such, Candidate #2 will be a Support Vector Regressor (SVR), using the default settings. This model would be expected to perform well if it is possible to create a hyper-plane that has the largest distance to the nearest training data-points, even in high dimensional space. If such a large margin could be found, an SVR model could theoretically perform much better on measures of overfitting, compared to other models. Furthermore, SVMs are generally effective in high-dimensional space, while also being memory efficient. scikit-learn visualizes this idea of maximizing the margin with a two-dimensional example in the following



image:<sup>2</sup>



Finally, ensemble methods have had a high degree of success with a number of machine learning problems. Thus, my third candidate will be a Random Forest Regressor. That is, it will be a regressor that will take multiple Decision Tree Regressors on the data, and average the result, bagging technique. A decision tree attempts to approximate the data by a series of if-then-else decision rules. An example of this in two dimensional space can be seen in this scikit-learn example image<sup>3</sup>:



For a Random Forest Regressor, each of these decision trees is generated from a sample of the training data, and as such, each decision tree ends up being slightly different. The averaging of the results of all these decision trees tends to smooth out any decision trees that randomly produce an outlier result, by picking a non-representative subsection of the data. This smoothing effect improves with more trees in the forest. Random Forests tend to have high accuracy, and good training times, though they do not take advantage of linear separability. Again, this will be done using the default parameters.

After all four models have been tested, I will take the best performing model and run a Grid Search to attempt to find the optimal parameters, and better tune the model.

## Benchmark

Existing institutions tend to base their enrollment projections on a combination of attrition and new admissions,<sup>4</sup> a benchmark model based on the retention rate and admission number should provide a good starting point. Specifically, a linear regression model of 'ftretention\_rate' and 'ptretention\_rate' plus the 'admitcount' to predict

'total\_enrollment', with the mean squared error for a point of comparison. This should be close to the existing simple model used at institutions.

It should be noted, I cannot get the exact same model as is usually used, as I do not have that full data available. Most institutions have a precise count of how many are re-enrolling, and they also have an institutional target goal for enrollment. With these two pieces, institutions actively aim for an admit count that will produce their target enrollment. As a result, I am using the retention rate as a proxy for a re-enrollment measure, and the admit count to give an estimate of the target enrollment for the institution.

Mean squared error (MSE) is a good metric for comparison here, as it captures how close the model predictions are to the actual observed enrollments, with a penalty for being further away. The benchmark model has a MSE of approximately 2.913 (normalized). This is not a great model, but it does provide a starting point to evaluate from.

## Methodology

### Data Preprocessing

A significant amount of data preprocessing will be required, based on what was seen in the data exploration section. I proceed with the following procedures: minimal feature selection, drop missing target values, handling outliers, imputing missing values, scaling and normalizing the data, splitting the data for cross validation, and finally, Independent Component Analysis (ICA) to reduce the feature space.

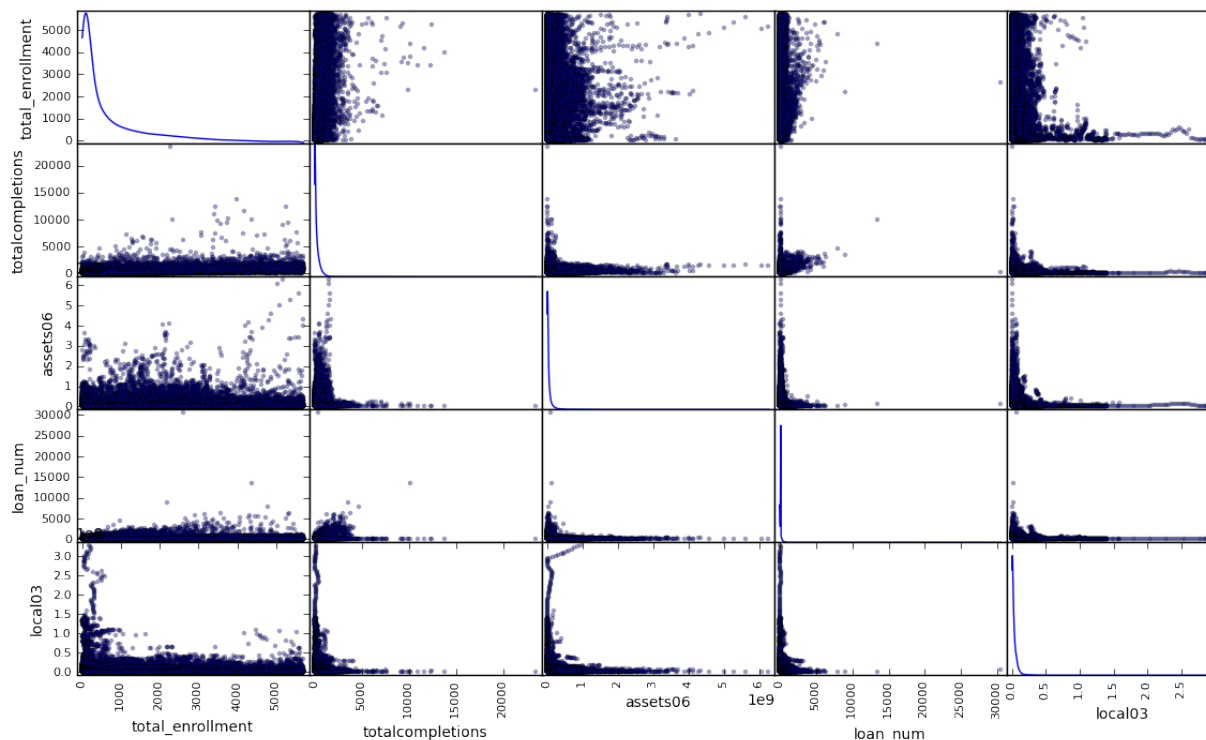
Feature selection was fairly minimal. All columns that contained strings were dropped, as these columns either contained no relevant information (such as institution name), or they contained data that was implied in other columns. For example, the string column of state names can more or less be deduced by the zip code column. Since the strings are harder to work with, and provide no clear advantage, those columns are dropped from the data. I also made sure to convert the remaining values to numerics, as there were some cells that imported invisible characters, making the values appear as non-numbers.

Next, I dropped all rows that were missing the target value of 'enrollment'. While it is worth attempting to impute missing values for the other columns, my worry is that imputing values for the target variable may end up distorting the results of the model. This reduces the number of observations we're working with from 215,613 down to

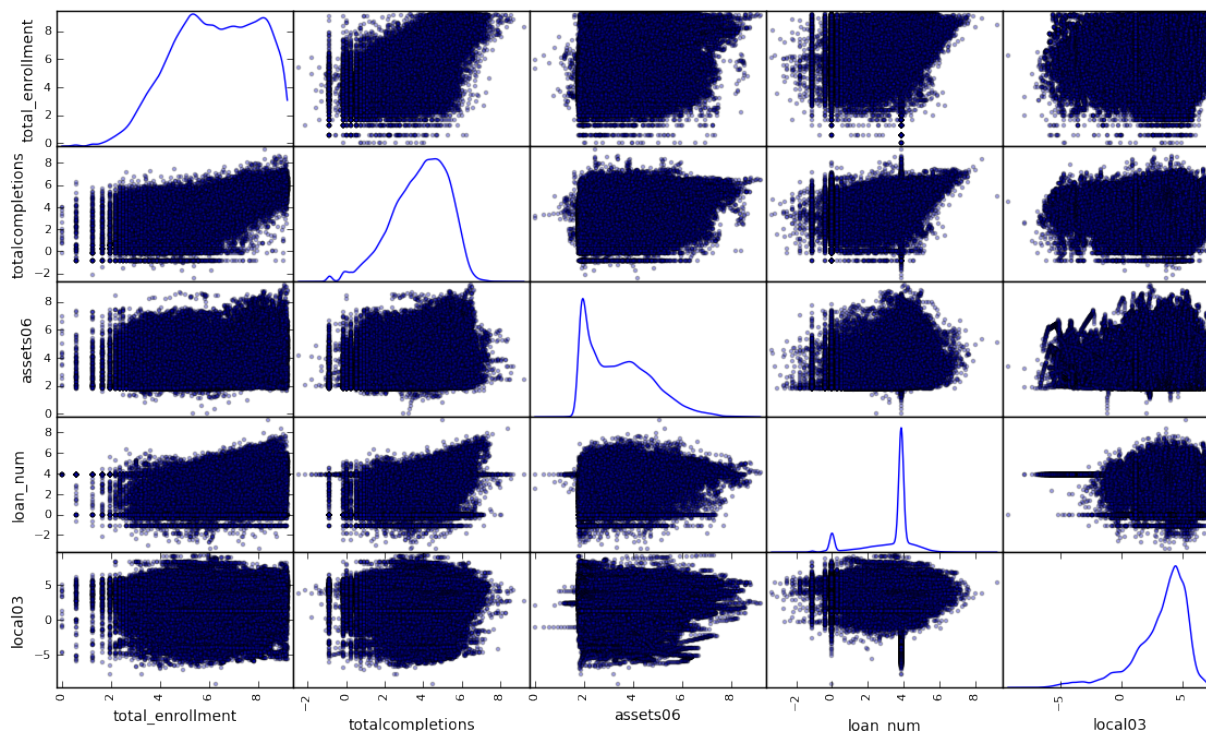
153,168. This is still quite a few observations, so there is no worry of a small  $n$  problem.

In the data exploration section, the visualizations revealed some significant outliers in the data that may produce odd results. Furthermore, since the plan is to impute the missing data, large outliers can have a significant impact on the results. So, at this point, I dropped any rows of the target variable which were greater or less than 1.5 times the interquartile range. I limited this outlier removal to only the target variable, on the assumption that the largest and smallest institutions would be the least normal along multiple dimensions.

After outliers were removed, I then imputed the missing data from the non-target variables. I should mention, my first attempt was not to impute the missing data, but simply drop any row that had any missing data. This resulted in zero observations. More selective dropping might be possible, but with over 150,000 observations and close to 1,000 feature dimensions, manually figuring out what could and could not be dropped was impractical. Imputing the missing values provided an alternative. I did this first by using the pandas's DataFrame interpolate method, which would attempt to sort the values in a column, and would estimate that the missing values between actual observations followed a linear pattern. For values that could not be interpolated, the column mean was used, by using pandas's DataFrame fillna and mean methods. These are not perfect estimates, but since they are not done to the target values, they should not artificially inflate the predictive value of the resulting models.



As we see above, at this state the pre-processed data appeared to have some issues with skew. So I wanted to normalize the data so that it would have a roughly gaussian distribution, since many models assume that. To do this, I scaled the data using a min-max scale. This removed any negative values. I then multiplied that by 10,000 to avoid potential problems with really tiny numbers, and finally I took the natural logarithm of the result. Furthermore, any values that resulted in negative infinity, as the result of taking the logarithm of zero, I converted those values back to zero.



As we see in this image, the resulting data looks more normally distributed, but we can also spot a couple of irregularities. In many of the images, there are observations that appear extremely regular, almost like perfectly horizontal or vertical lines. Those points are artifacts of the imputing process. That is, the estimated values end up looking a little too perfect in the data. Again, since the imputing process was not done to the target variable, only the inputs, this artifact should not artificially improve the result of our model. Worst case scenario, the imputing artifacts make our model have a worse MSE score. However, if the estimates are good enough, they may make the model more accurate in reality.

After all this was done, the data was then split into 80% training set, 20% testing set, for cross validation purposes.

Finally, since the data set contained such a large number of features, I needed to dramatically reduce the feature space to deal with the issue of the curse of dimensionality. To do this, I used Independent Component Analysis (ICA) to transform the inputs down to 17 independent dimensions. I used ICA because the feature space often had several columns that seemed to be taking measurements of similar values (such as full time and part time admissions, or in-state and out-of-state admission, etc), but often these measures would have noise from some other hidden feature as well. ICA should help to isolate out the actual independent features behind the measures. I

chose 17, because we have sufficient observations to handle up to 17 features without running into problems with the curse of dimensionality.

Once all this has been done, I have accounted for the missing values, the skew in the distribution of the data, and excess of feature space in light of the curse of dimensionality. The data is now ready for analysis.

## Implementation

By contrast to the data preprocessing, the model implementation was very straight forward. Four regression models from the scikit-learn libraries were tested: the benchmark model (linear regression with 3 non-ICA features), linear regression with ICA, Support Vector Regressor (SVR) with ICA, and Random Forest Regressor with ICA.

For each candidate, the model was trained with the training inputs and the training enrollment targets. The model was then used to make predictions for the test inputs. Finally, I generated two sets of mean squared error results, for both the training data and the testing data, by using the predicted values from the inputs compared to the actual observed values of enrollment. The MSE for the training data was compared to the test to get a sense model overfit, while the MSE test value is our primary metric for comparing usefulness of each candidate model. No expected complications or changes arose in the implementation.

<b>Model</b>	<b>Training Normalized MSE</b>	<b>Testing Normalized MSE</b>
Benchmark (Linear no ICA)	2.92670403437	2.9127528257
Candidate 1: Linear w/ ICA	0.572164100866	0.568313314229
Candidate 2: SVR w/ ICA	2.73706191612	2.71240878953
Candidate 3: Random Forest w/ ICA	0.0683715696409	0.357933570385

As can be seen in the summary results table, the Benchmark model did not perform particularly well, but neither did the Support Vector Regressor. Furthermore, the SVR model ran extraordinarily slow, while producing results almost as bad as the benchmark. This makes it easy to dismiss the SVR as not providing much value over

the benchmark model.

Choosing between candidate 1 and candidate 3 is a bit more tricky. Both perform significantly better than the benchmark model. The Random Forest model ultimately has the lower mean squared error on the testing data, but the MSE on the training data is worrying. There does appear to be a real danger of overfitting with candidate 3, where candidate 1 does not appear to have the same problem. Having said that, even with the overfitting on the training data, candidate 3 still performs much better than any other candidate on the test data.

Thus, the Random Forest Regressor with ICA is our initial solution to the problem of enrollment prediction.

## Refinement

With the Random Forest selected as the final model for the solution, I performed a Grid Search to find optimal parameters. Specifically, I had the Grid Search test various numbers of trees developed for the forest (5, 10, 20 or 40; default is 10). I also had the grid search experiment with using fewer features (auto, sqrt, or log2; default auto=17).

After the grid search was performed, the best and final parameters (`max_features='auto', 'n_estimators=40'`) resulted in a testing normalized MSE of 0.317, which is better than the untuned model. Unfortunately, the normalized MSE of the training data was 0.048, suggesting there is still a real danger of overfit with this model.

# Results

## Model Evaluation and Validation

The final model appears to be a reasonable solution to the problem of estimating enrollment at institutions of higher education. By using a wide variety of institutional attributes related to the student body makeup, the faculty and staff makeup, funding sources and the like, give us a reasonable way to estimate current enrollment. This is true even if some of the institutional attributes need to be crudely estimated. By running Independent Component Analysis, it becomes possible to isolate distinct signals from the attributes, which can then be run through a Regressor. A Random Forest Regressor – running multiple Decision Tree Regressors and bagging the results – produces the best final estimate.

To get the true estimate of enrollment, a little bit more work needs to be done. Remember, the model predicts a transformed estimate of enrollment that has been



normalized. To get the actual estimate, the value produced by the model must be used as an exponent for  $e$  to get the reverse of the natural logarithm, and then divided by the enlarging factor that was used to avoid small numbers (10,000), and then de-normalized by multiplying by the original non-outlier maximum enrollment, minus the original non-outlier minimum enrollment, plus the non-outlier minimum enrollment.

Because the model was tested using cross validation, there is reasonable confidence that the model can produce reasonably accurate results, with most institutions. The estimates are, on average, only off by about 1 student on the unseen testing data. If this sounds unreasonably accurate, it is worth remembering that the median institution — once outliers are removed — only has 314 students enrolled. This is true because the huge outlier institutions have been removed from the data set. Less accuracy would likely be found if the huge outlier institutions were re-introduced, or if attempts were made to predict on the outlier data.

There is one final concern about trusting the model: there remain signs of overfit, mentioned previously. However, given that the Random Forest does not perform absurdly better than the Linear Regression Model — which does not show signs of overfit — there is reason to think that the results of the final model are a plausible fit for the data.

## Justification

The final parameters for the model only differ from the default parameters on the 'n\_estimators' value. This parameter refers to the number of trees that are generated in the random forest that are ultimately bagged together in the ensemble method. The tuned model now uses 40 trees, which is four times the default value of 10. This is not too surprising, as adding more bagged decision trees should smooth out any decision trees that deviate too far from the optimal tree. Increasing this value further may provide even higher accuracy, but at the cost of much higher processing time.

'max\_features', the number of features the decision trees consider, remained the same at 'auto', which simply uses all 17 available ICA features. I was hoping that a smaller number, either by taking the square root, or the log2 of the number of features would help in avoiding overfitting, but neither of these results performed as well as using all features.

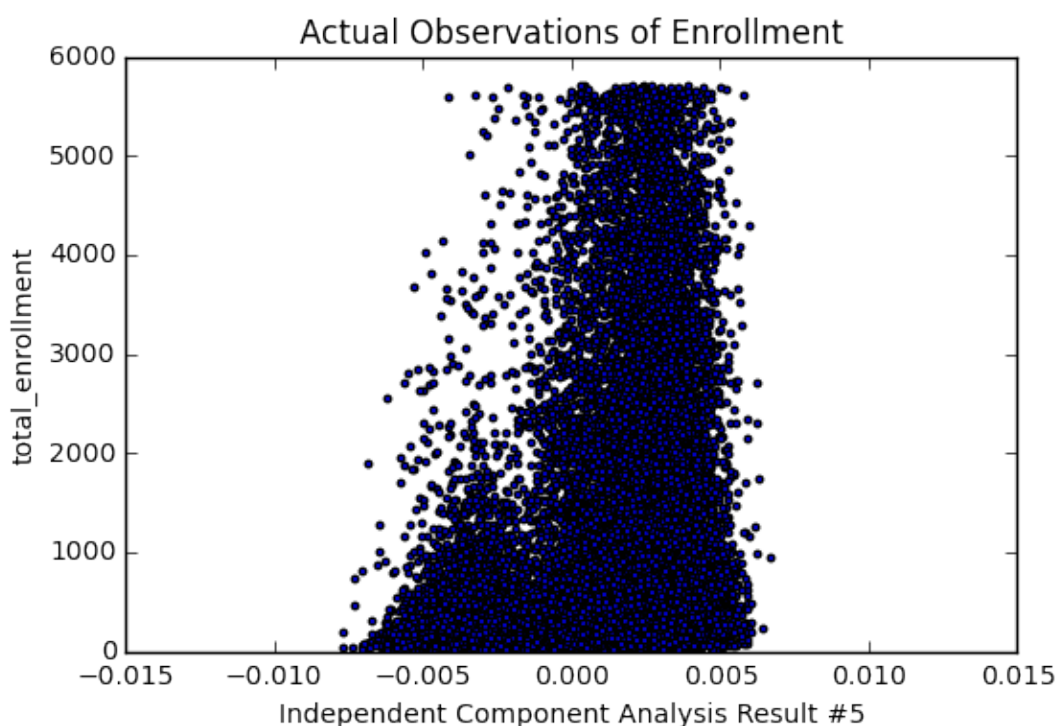
To summarize, the tuned Random Forest Regressor with ICA, using all features and producing 40 bagged decision trees does outperform the benchmark established earlier, when comparing Mean Squared Errors of predictions on the test data. The difference is

significant enough to suggest that the model presented here is enough of an improvement to consider it the new benchmark. I would not suggest the problem is solved, despite these improvements. In my conclusion, I will discuss possible improvements I think can be made to the model.

## Conclusion

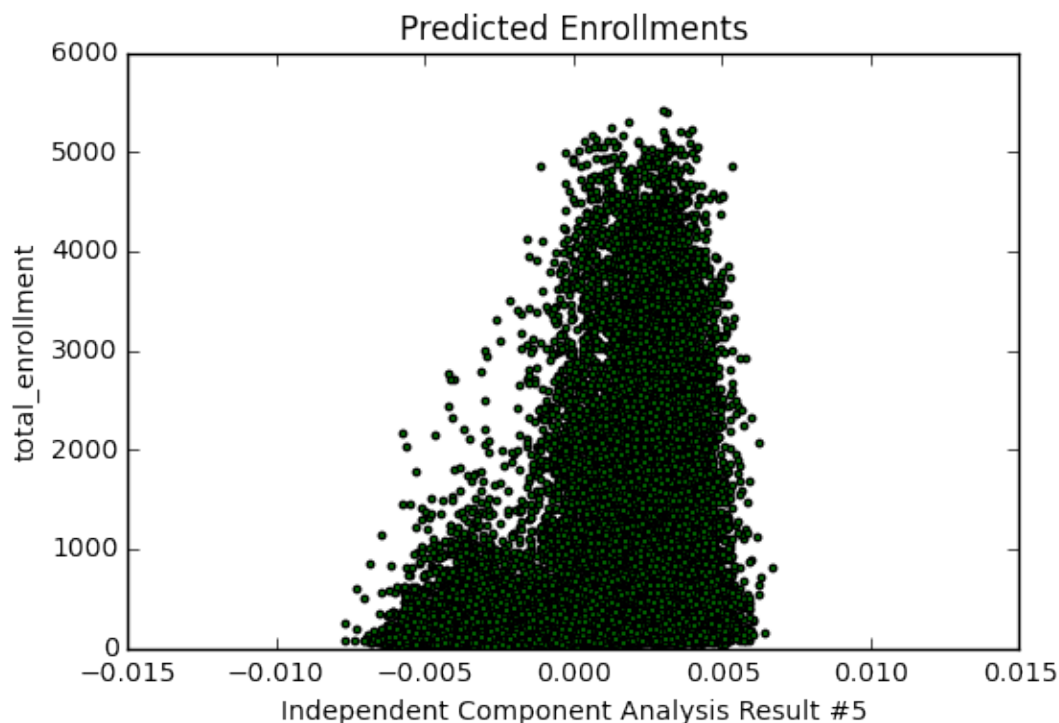
### Free-Form Visualization

Working with a data set that provides such a large number of observations and such a large number of features makes it a bit difficult to visualize. However, I want to show at least one visual comparison that gives a sense of how well the final model tends to predict the target variable.



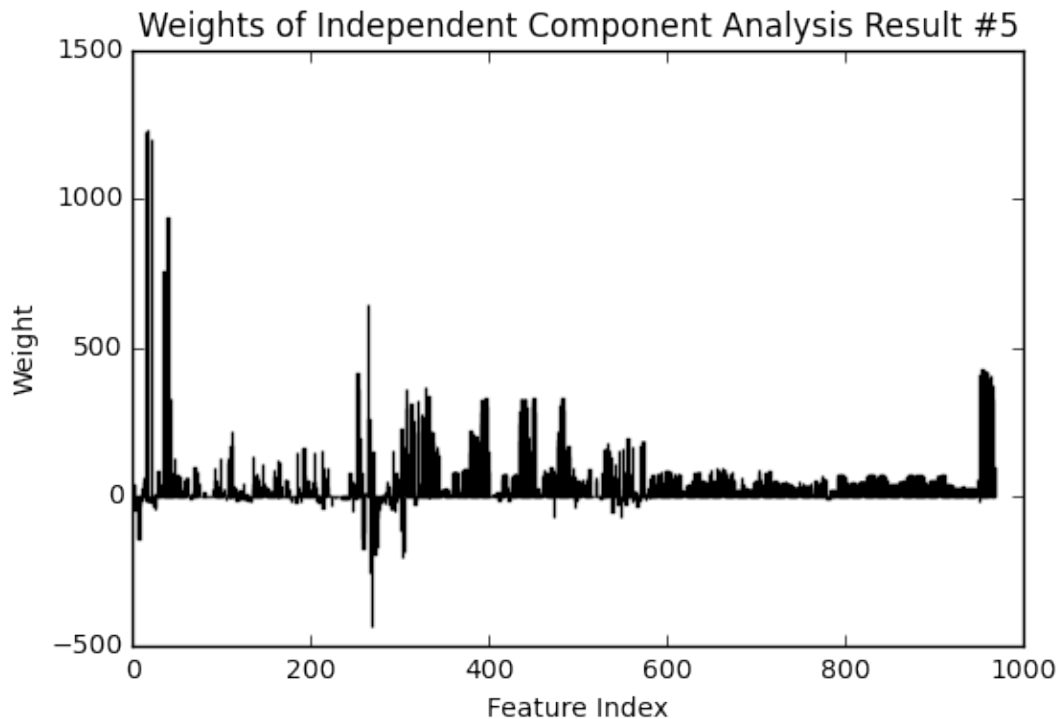
In this scatterplot, we see all of the actual observed enrollment values plotted against one of the results of the Independent Component Analysis, #5. #5 was selected because the final model registers it as having the highest feature importance when making predictions. The general shape of the plot is a bit odd. In general, as Feature #5 increases, so does enrollment, but there is a huge amount of variance. There is also a

peculiar dip as feature #5 approaches zero from the negative side.



This graph, with the green plot, is the same measures (total\_enrollment plotted against Feature #5), but these are the predicted values, not the observed ones. Not only is the general shape the same, the predictor actually has the dip that the actual observations display as Feature #5 approaches zero from the negative side. This gives a pretty good visual sense in two dimensional space as to how the shape of the predictions appears pretty close to the shape of the actual observations.

If the reader is curious as to what Feature #5 actually is, that is a lot harder to explain. As mentioned above in the Data Preprocessing, ICA was used to reduce the 900+ features down to 17. The specific makeup of #5 can be seen in the following visualization of the feature weights:



As can be seen above, #5 combines signals from a large number of the original features, and especially weights some of the earlier features. Individually picking these out would take an extraordinary amount of time.

## Reflection

This project attempted to solve the problem of estimating enrollments at institutions of higher education. Using the PEDS Delta Cost Project Database, it is possible to evaluate a large number of institutional attributes that may help in predicting total enrollment. To do this, extensive preprocessing needed to be done to deal with outliers, missing data, and reduce the feature space. Once the data was prepared, four regression models were tested, comparing their Mean Squared Error on their predictions. Finally, the best performing model, the Random Forest Regressor, was run through a Grid Search to optimize its parameters.

The data preprocessing was by far the hardest part of this process. By having so many observations and so many feature columns, it was basically impossible to eyeball anything. It required taking a much more systematic approach to figure out how to make the data useful. The missing values on the inputs, in particular, was the hardest to work around. I am quite happy that the imputed estimates did not seem to hurt the analysis at all.

I think the final model works quite well as a solution to the problem for average institutions of higher education. The error rate ends up being quite low, and the predictions look to take a very similar shape to what is actually observed.

Perhaps unfortunately, most of the time what we ordinarily think of as average institutions are much larger than what this analysis actually ends up looking at. That is, the big well known schools are actually outliers, so this solution probably would not work well for institutions that are best known, or cater to the most students.

## Improvement

More careful – and painstakingly slow – feature selection would probably help the analysis here. Right now, the model takes almost all of the feature space into account, to some extent or another. Manually going through and removing features that appear highly correlated with one another – and appear to be measures of the same thing – might reduce the dangers of overfitting.

On consideration, perhaps the more important improvement for this analysis would be to be a *less* general model rather than a more general one. In particular, given that most students tend to be in larger institutions that were treated as outliers here, it may be more interesting to only do the analysis on the top quartile of institutions. Since most institutions are quite small, the solution here works for most schools, but not for the schools that contain the most students.

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<sup>1</sup> [http://scikit-learn.org/stable/modules/linear\\_model.html#logistic-regression](http://scikit-learn.org/stable/modules/linear_model.html#logistic-regression)

<sup>2</sup> <http://scikit-learn.org/stable/modules/svm.html#svm-regression>

<sup>3</sup> <http://scikit-learn.org/stable/modules/tree.html#regression>

<sup>4</sup> See: • <http://sites.williams.edu/wpehe/files/2011/06/DP-26.pdf>

• <http://www.uwsp.edu/enrollmanage/Documents/Predicting%20Enrollments.pdf>

• [http://spu.edu/depts/idm/docs/publications/JW\\_Publication07.pdf](http://spu.edu/depts/idm/docs/publications/JW_Publication07.pdf)