UNIVERSITY OF CALIFORNIA,SANTA BARBARA
Department of Statistics and Applied Probability
PSTAT 122, SPRING 2024
Midterm Exam
May 9, 2024, Class time: **TR** : 11:00p.m.-12:15p.m.

Name of the Student: Javier Garcia [redacted]

[redacted]

Note: Points are given in brackets at the end of each question. Partial credits will be given for using the correct methodology and showing intermediate calculations. So please show all the steps and organize your work so that it is understandable to the grader to get the full credit. Use $\alpha = 0.05$ in any problem unless otherwise specified. T-table, F-table and q-tables are provided.

Problem	1	2	3	Total
Points possible	26	30	24	80
Points earned	21	27	24	72

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1. Write or circle the correct answer.

2 (a) Which basic principle of Design of Experiments should you adopt to ensure estimation of the experimental error? [2 points]

Replication

2 (b) Using Scheffe's multiple comparison method, the confidence coefficient of a simultaneous confidence interval for two contrasts at 95% level is always equal to 95%: (i) True (ii) False. [2 points]

(ii) FALSE

2 (c) Two sample t-test for comparing two means of two Normal populations under the assumption of equality of variances is equivalent to the ANOVA F-test: (i) True (ii) False [2 points]

(i) TRUE

2 (d) In a CRD we use only replication principle of design of experiments: (i) True (ii) False [2 points]

(ii) FALSE

6 (e) Consider a one way random effects model, using 4 treatments and 5 replications for each treatment . The $MS_{Treatment}$ is 8.28 and the MS_{Error} is 1.62. What is the estimate of variance of treatments and what proportion of the total variation is due to the variability in treatments?(Show calculation)

Estimate of var → $Var(y_{ij}) = \sigma^2 + \sigma_\tau^2 = \hat{\sigma}^2 + \hat{\sigma}_\tau^2 = (MSE) + \left(\frac{MS_{trts} - MSE}{n}\right) = 1.62 + \left(\frac{8.28-1.62}{5}\right) = 0.288$

Proportion → $\frac{\sigma_\tau^2}{\sigma_\tau^2+\sigma^2} = \frac{\hat{\sigma}_\tau^2}{\hat{\sigma}^2+\hat{\sigma}_\tau^2} = \frac{1.332}{1.62+1.332} = \frac{1.332}{2.952} = 0.451$ [3+3=6 points]

0 (f) Let $X_1, \cdots X_n \overset{i.i.d}{\sim} \mathcal{N}(0,1)$ and $U = \sum_i c_i X_i$ and $V = \sum_i d_i X_i$ such that $Cov(U,V) = 0$. Then what is the distribution of

$$\boxed{\frac{U}{V}\frac{\sqrt{(\sum_i d_i^2)}}{\sqrt{(\sum_i c_i^2)}}? \sim \chi^2_{\nu = U-V}}$$

[3 points]

1 (g) To compare the means of two normal populations, when the observations from the same experimental units are correlated, we use ______t______ test. [2 points]

2 (h) For a Normal population to test $H_0 : \sigma^2 = 3$ vs. $H_0 : \sigma^2 \neq 3$, with sample size 25, the test statistics follows a χ^2 (chi-squared) distribution under H_0. [3 points]

2 (i) In a Randomized Block Design, the expectation of the difference of two observations in any block involving two specific different treatments does not depend on that block. : (i) True (ii) False [2 points]

(i) TRUE

2 (j) In a single factor fixed effects model, we cannot draw inference about the treatments not included in the experiment : (i) True (ii) False [2 points]

(i) TRUE

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2. There is an experiment involving three treatments, on $n_1 = 5, n_2 = 3$, and $n_3 = 5$, experimental units respectively. The treatment means and the sum of squares of the yields are found to be (in usual notations)

$\bar{y}_{1.} = 10.8, \quad \bar{y}_{2.} = 21.33, \quad \bar{y}_{3.} = 8.4$, and $\sum_{i=1}^{3}\sum_{j=1}^{n_i} y_{ij}^2 = 2414.$

6 (a) Estimate all the pairwise differences of treatment effects.

$|\bar{y}_{1.} - \bar{y}_{2.}| = |10.8 - 21.33| = 10.53$

$|\bar{y}_{1.} - \bar{y}_{3.}| = |10.8 - 8.4| = 2.4$

$|\bar{y}_{2.} - \bar{y}_{3.}| = |21.33 - 8.4| = 12.93$

[6 points]

16 (b) Make up the ANOVA table and test the hypothesis of the equality of the treatment effects.

One - way ANOVA Table (unbalanced case)

$n_1 = 5, \; n_2 = 3, \; n_3 = 5, \; y_{..} = (10.8 \times 5) + (21.33 \times 3) + (8.4 \times 5)$
$= 54 + 63.99 + 42 = 159.99$

$N = 13$

Source	D.F.	SS	MS	F
Trts	$3-1=2$	331.922	$SS_{trts}/2 = 165.961$	$F_0 = \frac{MS_{trts}}{MSE} = 14.68$
Error	$13-3=10$	113.093	$SSE/10 = 11.309$	
Total	$13-1=12$	445.015		

$SS_{trts} = \sum_{i=1}^{a} y_{i.}^2/n_i - y_{..}^2/N$

$= \frac{(54)^2}{5} + \frac{(63.99)^2}{3} + \frac{(42)^2}{5} - (159.99)^2/13$

$= 331.922$

$SS_{total} = \sum_i\sum_j y_{ij}^2 - y_{..}^2/N$

$= 2414 - (159.99)^2/13$

$= 445.015$

$SSE = SS_T - SS_{trts} = 113.093$

Hypothesis - $H_0: \tau_1 = \tau_2 = \tau_3 = 0$

$H_1: \tau_i \neq 0$ for at least one i,

Reject H_0 if $F_0 > F_{0.05,2,10} \Rightarrow 14.68 > 4.103$, thus we reject H_0, meaning there is differences between treatments τ_i's.

[16 points].

5 (c) Test if the effect of treatment 1 is same as the average of the effect of the effects of treatment 2 and treatment 3? [8 points]

$H_0: \frac{\tau_2 + \tau_3}{2} = \tau_1 \qquad \equiv \qquad H_0: \tau_2 + \tau_3 - 2\tau_1 = 0$

$H_1: \frac{\tau_2 + \tau_3}{2} \neq \tau_1 \qquad\qquad H_1: \tau_2 + \tau_3 - 2\tau_1 \neq 0$

Since $t_{0.020,10} = 2.76$

$\Downarrow$

$t_{0.025,10} \approx 2.75$

$\hat{\Gamma} = C = (\bar{y}_{2.} + \bar{y}_{3.} - 2\bar{y}_{1.}) = (21.33 + 8.4 - 2(10.8)) = 8.13$

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Reject H_0 if $|t_0| > t_{0.025,10} \Rightarrow |t_0| > 2.75$

$|t| = \left|\frac{C}{\sqrt{MSE(\sum_{i=1}^{3} c_i^2/n_i)}}\right| = \left|\frac{8.13}{\sqrt{11.309(\frac{1}{3} + \frac{1}{5} + \frac{4}{5})}}\right| = |2.09| \Rightarrow 2.09 > 2.75$, reject H_0

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then we can conclude effect of trt 1 and average effect of trt 2 and trt 3 are different.

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3. In a Randomized Complete Block design with 4 treatments and 5 blocks the summary statistics in usual notations are as follows:
$y_{1.} = 5.7,\quad y_{2.} = 8.8,\quad y_{3.} = 6.9,\quad y_{4.} = 17.8$ $= 39.2 = y_{..}$
$y_{.1} = 9.2,\quad y_{.2} = 10.1,\quad y_{.3} = 3.5, y_{.4} = 8.8,\quad y_{.5} = 7.6,$ $= 39.2 = y_{..}$
$\sum_{i=1}^{4}\sum_{j=1}^{5} y_{ij}^2 = 102.52$

(a) Make up the ANOVA table for this problem and test if the treatments affect the response. [18 points]

ANOVA for RBD

$N = ab = (4)(5) = 20$

Source	D.F.	SS	MS	F
Blocks	$5-1=4$	6.693	$6.693/4 = 1.673$	$F_{01} = \frac{MS_{trts}}{MSE} = 76.14$
Trts	$4-1=3$	18.044	$18.044/3 = 6.015$	$F_{02} = \frac{MS_{block}}{MSE} = 21.18$
Error	$(4-1)(5-1)=12$	0.951	$0.951/12 = 0.079$	
Total	$(4)(5)-1=19$	25.688		

$SS_{Blocks} = \sum_{j=1}^{5} y_{.j}^2/4 - y_{..}^2/20 = \frac{(9.2)^2+(10.1)^2+(3.5)^2+(8.8)^2+(7.6)^2}{4} - \frac{(39.2)^2}{20}$

$= 83.525 - 76.832 = 6.693$

$SS_{trts} = \sum_{i=1}^{4} y_{i.}^2/5 - y_{..}^2/20 = \frac{(5.7)^2+(8.8)^2+(6.9)^2+(17.8)^2}{5} - 76.832$

$= 94.876 - 76.832 = 18.044$

$SS_{total} = \sum\sum y_{ij}^2 - y_{..}^2/20 = 102.52 - 76.832 = 25.688$

$SSE = SS_{total} - SS_{blocks} - SS_{trts} = 0.951$

Hypothesis $- H_{10}: \tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$
$H_{11}: \tau_i \neq 0$ for at least one i

reject H_{10} if $F_{01} > F_{0.05,3,12} \Rightarrow 76.14 > 3.490,$

thus we reject H_{10} therefore we can conclude the treatments do affect the response.

(b) Using Tukey's Method test if there is any significant difference in the effects of Treatment 1 and Treatment 4 as well as Treatment 2 and Treatment 3? [6 points]

$\bar{y}_{i.} = y_{i.}/b$

$\bar{y}_{1.} = \frac{5.7}{5} = 1.14$

$\bar{y}_{2.} = \frac{8.8}{5} = 1.76$

$\bar{y}_{3.} = \frac{6.9}{5} = 1.38$

$\bar{y}_{4.} = \frac{17.8}{5} = 3.56$

$q_\alpha(a,(a-1)(b-1)) \cdot \sqrt{\frac{MSE}{b}} \Rightarrow q_{0.05}(4,12) \cdot \sqrt{\frac{0.079}{5}} = (4.20) \cdot (\sqrt{0.0158})$
$= 0.528$

$|\bar{y}_{1.} - \bar{y}_{4.}| = |1.14 - 3.56| = 2.42 > 0.528$ ← significantly different

$|\bar{y}_{2.} - \bar{y}_{3.}| = |1.76 - 1.38| = 0.38 < 0.528$ ← Not significantly Different

By Tukey's Method test we can conclude that the effects of Trt 1 and Trt 4 are significantly different but the effects of Trt 2 and Trt 3 are NOT significantly different.